RUSSIAN ACADEMY OF SCIENCE
Lenin Order of Siberian Branch
G.I. Budker INSTITUTE OF NUCLEAR PHYSICS

T.D. Akhmetov, V.I. Davydenko, A.A. Ivanov

MODEL OF A NEUTRAL BEAM WITH GEOMETRIC FOCUSING AND APERTURES

Budker INP 2007-25

Novosibirsk
2007

T.D. Akhmetov, V.I. Davydenko, A.A. Ivanov

Model of a neutral beam
with geometric focusing and apertures

Budker Institute of Nuclear Physics,
630090, Novosibirsk, RF

Abstract

A model of a neutral beam with geometric focusing and angular divergence is described. An algorithm is presented for calculation of a two-dimensional current density profile at an arbitrary distance from a flat circular emitter with account of multiple plane apertures in a beamline limiting cross-section of the beam. Numerical code is applied to calculation of current density profiles and power load on circular apertures due to neutral particles.

Т.Д. Ахметов, В.И. Давыденко, А.А. Иванов

Модель пучка нейтральных атомов с геометрической
фокусировкой и апертурами

Институт ядерной физики им. Г.И.Будкера
630090, Новосибирск, РФ

Аннотация

Рассматривается модель пучка нейтральных атомов с геометрической фокусировкой и угловой расходимостью. Представлен алгоритм расчета двумерного профиля плотности тока на произвольном расстоянии от плоского кругового эмиттера атомов с учетом плоских апертур, ограничивающих поперечное сечение пучка. Написанный численный код использован для расчета профилей плотности тока пучка в инжекционном тракте и определения тепловой нагрузки круглых лимитеров за счет попадания быстрых атомов.

1. Introduction

Low-divergent long-pulse neutral beams are often used in modern magnetic fusion devices as a diagnostic tool providing unique information about plasma parameters [1]. The most important requirements to these beams are sufficiently large current and energy of the particles, so that the beam can penetrate to the plasma core. Also the duration of the beams must be long enough, i.e. close to that of a plasma discharge, amounting to at least a few seconds for large fusion devices. In particular, this implies limitations on power flux onto the beam limiters and determines the necessity of their water cooling.

Here we describe a model of a circular neutral beam with geometric focusing passing through a beamline with plane apertures. The beam profile at desired positions is calculated as a function of beamlet (elementary cell of an ion-optical system) divergence and accelerator focal length. It is assumed that the grid curvature, determining the focal length, is uniform and beamlet divergence distribution is of Gaussian type. Considering the number of beamlets large enough, the local current density of the beam at a distance z can be evaluated by using a simple analytical approximation. Namely, we assume that the ion current density is constant over the circular plasma emitter and at each point of the emitter the divergence is equal to that in a single beamlet.

The current density distribution of a focused neutral beam at a given distance from an ion-optical system with uniform current density and without limiting apertures in the beamline was obtained in [1]. In the presence of apertures absorbing the outer part of the beam the task of finding the current density distribution becomes more complicated. The current density distribution can be reliably calculated using the already existing numerical code PADET [3]. In this code the current density in a certain point is determined by summing up contributions of many elementary beams formed in the beamlets of the ion-optical system. This procedure includes possible absorption of the elementary beams by the apertures. The current density profile and absorption of the beam by the apertures can be also calculated by Monte Carlo method taking sufficiently large number of random trajectories starting from the surface of the plasma emitter.

In this work the analytical expressions are obtained for calculation of the current density profile of a focused beam of fast neutral atoms with the account of the limiting apertures. Specific current density profiles needed for practical applications can be calculated from these analytical formulas using computing software like Mathcad or Mathematica.

2. Model of a neutral beam

Geometry of the beam is shown in Fig.1. A circular emitter of radius a at $z = 0$ produces a neutral beam with uniform equivalent current density, and every elementary beam is aimed at the point $z = R$ on the z axis, where R is the curvature radius of grids or the focal length of the accelerator. Atom current density profile is of primary practical importance for neutral beam application in plasma devices and other purposes. On the way to the target the elementary beam is subject to geometrical focusing and angular spreading which is assumed of the Gaussian form, $\exp(-\theta^2/\theta_0^2)$, a typical approximation used for experimental data. Let us use cylindrical coordinates and find current density at radius r of the observation plane at distance z produced by the elementary current from the emitter infinitesimal area $\rho d\rho d\phi$ with the polar coordinates (ρ, φ). Since the beam emitter is axisymmetric we can without loss in generality place the point (r, z) at the observation plane at the x axis. The axis of the elementary beam due to focusing is directed to the point $z = R$ and crosses the observation plane at the point with polar coordinates (r', φ) where $r' = \rho(R - z)/R$ from geometry.

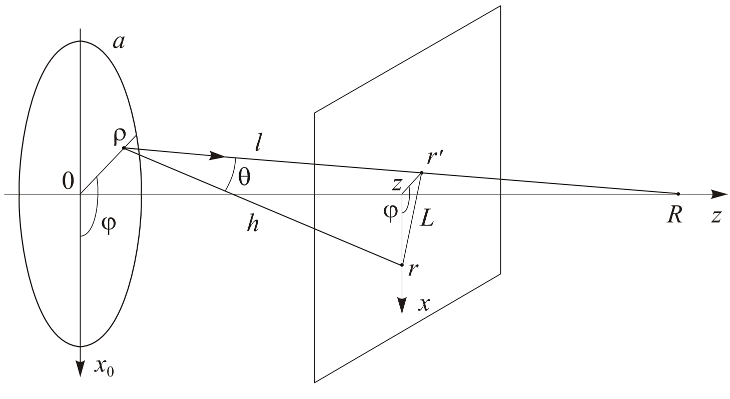

Fig. 1. Geometry of a beam and observation plane.

We need further to calculate the angle θ between the elementary beam axis and direction to the point (r, z) which determines the current density. Let us find it from a triangle with the sides L, l, h (see Fig.1)

$$L^2 = l^2 + h^2 - 2lh\cos\theta . \tag{1}$$

From Fig.1 we also find

$$l^2 = z^2 + (\rho - r')^2 = z^2\left(1 + \rho^2/R^2\right),$$

$$h^2 = z^2 + (r - \rho\cos\varphi)^2 + (\rho\sin\varphi)^2 = z^2 + r^2 + \rho^2 - 2r\rho\cos\varphi ,$$

$$L^2 = r^2 + r'^2 - 2rr'\cos\varphi .$$

Substitution of r', l, h, L into (1) yields the exact expression for $\theta(r,z,\rho,\varphi)$

$$\cos\theta = \frac{1+\rho(\rho - r\cos\varphi)/zR}{\sqrt{\left(1+\rho^2/R^2\right)\left(1+(\rho^2+r^2-2\rho r\cos\varphi)/z^2\right)}}. \tag{2}$$

In practice the focal length and distances from the atom emitter to apertures in the beamline and to a plasma target are much greater than the emitter radius and characteristic aperture sizes b_n, $z >> \rho, b_n$ and $R >> \rho, b_n$. Moreover, since the angular divergence of the beam is usually small, of the order $\theta_0 \sim 1^\circ \approx 1.7\cdot 10^{-2}$ rad $<< 1$, and the current density depends on the angle exponentially, we may consider only small θ angles. For $\theta << 1$ $\cos\theta \approx 1-\theta^2/2$, so we expand (2) into Taylor series and keep only the terms of the order $O(1/z^2)$ to obtain

$$\theta^2 \approx \frac{r^2}{z^2} + \rho^2\left(\frac{1}{z}-\frac{1}{R}\right)^2 - \frac{2\rho r}{z}\left(\frac{1}{z}-\frac{1}{R}\right)\cos\varphi \tag{3}$$

with neglected terms of the order of $O(1/z^4)$. The same approximate result can be obtained directly from Fig.1 under condition $\theta << 1$, when it is sufficient to write

$$\theta^2 \approx \left(\frac{L}{z}\right)^2 = \frac{r^2 + r'^2 - 2rr'\cos\varphi}{z^2} = \frac{r^2}{z^2} + \rho^2\left(\frac{1}{z}-\frac{1}{R}\right)^2 - \frac{2\rho r}{z}\left(\frac{1}{z}-\frac{1}{R}\right)\cos\varphi.$$

It is easy to show by straightforward expansion into series that account of the neglected terms of the order of $O(1/z^4)$ (or $O(\theta^4)$) in simplification from (2) to (3) becomes important only for relatively large angles $\theta/\theta_0 >> 1$. But for such θ values the exponent $\exp(-\theta^2/\theta_0^2)$ is negligibly small. For example, for typical divergence of $\theta_0 \sim 1^\circ \sim 1.7\cdot 10^{-2}$ rad a 1% relative error in the current density arising from omitting the $O(1/z^4)$ terms appears only at θ as large as $\theta/\theta_0 \sim 4.5$, when $\exp(-\theta^2/\theta_0^2) \sim 2\cdot 10^{-9}$ and for smaller θ angles the error is even less. Therefore, the expression (3) can be assumed accurate.

Let us also consider the influence of the angle χ between the elementary beam axis and normal to the observation plane on the current density. In practical situation this angle is small, $\max(\tan\chi) = (a+b)/z << 1$, where b is the aperture radius. The axial component of the elementary current at the observation plane is $dj_z = dj\cdot\cos\chi \approx dj\cdot(1-\chi^2/2)$, and correction to the current density is of the order of $(a+b)^2/2z^2$. In practice, typical values of the emitter radius and aperture cross-size are comparable $a \sim b \sim 10$ cm and $z \geq 100$ cm, therefore, $(a+b)^2/2z^2 \sim 0.02$, so the maximum relative error is about 2% at $z = 100$ cm at large radii $r \sim b$. Further we neglect this angle for the sake of simplicity and keep in mind that our formulas are correct to within about 2% at all distances of interest, and that this systematic error rapidly decreases with distance from the emitter. Of the same order is the error from replacing the spherical form of the accelerator grid by a flat one. It leads to $O(1/z^2)$ errors which are maximal at the beam periphery. However, in numerical calculation there is no problem to include both χ-dependence and exact expression for $\theta(r,z,\rho,\varphi)$ into the expression for the current density.

The beam current density at the observation plane is calculated by integration of elementary currents over the whole emitter surface

$$j_{\text{beam}}(r,z) = B\int_0^a \rho d\rho \int_0^{2\pi} \exp\left(-\frac{\theta^2(r,z,\phi,\rho)}{\theta_0^2}\right) d\varphi\,, \tag{4}$$

where B is a normalization constant to be found later. Integration over φ using the relation $\int_0^\pi e^{z\cos\theta} d\theta = \pi I_0(z)$, where $I_0(z)$ is the modified Bessel function, gives

$$j_{\text{beam}}(r,z) = 2\pi B \exp\left(-\frac{r^2}{z^2\theta_0^2}\right)\int_0^a \exp\left(-\frac{\rho^2}{\theta_0^2}\left(\frac{1}{z}-\frac{1}{R}\right)^2\right) I_0\left(\frac{2r\rho}{z\theta_0^2}\left(\frac{1}{z}-\frac{1}{R}\right)\right)\rho d\rho\,. \tag{5}$$

This expression for the current density distribution of a focused neutral beam at a given distance from the ion-optical system and without limiting apertures in the beamline was obtained in [1].

The constant B is determined from total current conservation, i.e. from condition that the total beam current at any z position must be equal to the total emitter current

$$\int_0^\infty j_{\text{beam}}(r,z) 2\pi r dr = \int_0^a j_0(r) 2\pi\rho d\rho\,,$$

which in the case of the uniform emitter current density J_0 yields

$$\int_0^\infty j_{\text{beam}}(r,z) r dr = J_0\, a^2/2\,.$$

Performing integration in the left hand side using [2] we find $B = J_0/\pi z^2\theta_0^2$. Thus, the elementary current density at the point (r,z) from the infinitesimal area with coordinates (ρ,φ) on emitter surface is

$$dj_{\text{beam}}(r,z,\rho,\varphi) = (J_0/\pi z^2\theta_0^2)\exp\left(-\theta^2(r,z,\phi,\rho)/\theta_0^2\right)\rho d\rho d\varphi\,. \tag{6}$$

At the focus ($z = R$) the current density distribution follows from (5) and becomes a simple Gaussian type

$$j_{\text{focus}}(r) = J_0(a^2/R^2\theta_0^2)\exp\left(-r^2/R^2\theta_0^2\right)\,. \tag{7}$$

with the beam half-width at $1/e$ equal to $R\theta_0$.

Equation (5) can be also obtained in straightforward manner by using a model distribution function of the beam at the plane at which it starts:

$$f = (J_0/\pi V_0^3\theta_0^2)\,\delta(v_z - V_0)\,H\left(a - \sqrt{x_0^2 + y_0^2}\right)\exp\left(-(v_x^2 + v_y^2)/\theta_0^2\,v_z^2\right), \quad (8)$$

where V_0 is the atom velocity, x_0 and y_0 are the Cartesian coordinates in the emitter plane, and H is the Heaviside step function indicating that the beam particles are emitted within a circular aperture with radius a. For $z > 0$ the distribution function can be obtained by solving a collisionless kinetic equation. The particle trajectories are straight lines, so there are constants of motion, that are essentially initial coordinates at the plane $z = 0$. These constants of motion can be written as $x_0 = x - v_x z/v_z$, $y_0 = y - v_y z/v_z$. The components of the velocity vector are also constants of motion. So, since the distribution function is a function of the constants of motion it can be immediately written at a plane at distance z from the source as

$$\begin{aligned} f = (J_0/\pi V_0^3\theta_0^2)\,\delta(v_z - V_0)\,H\left(a - \sqrt{(x - v_x z/v_z)^2 + (y - v_y z/v_z)^2}\right)\times \\ \times\exp\left(-(v_x^2 + v_y^2)/\theta_0^2\,v_z^2\right). \end{aligned} \quad (9)$$

The current density profile is given by $j_{beam}(x, y, z) = \iiint f\,v_z dv_x dv_y dv_z$ which after simple calculations gives the same result as (5), but for $R = \infty$, since we did not take into account inclination of the particle trajectories due to bending of the grids. In order to include the focusing effect, equation (8) should be modified as follows

$$\begin{aligned} f = (J_0/\pi V_0^3\theta_0^2)\,\delta(v_z - V_0)\,H\left(a - \sqrt{x_0^2 + y_0^2}\right)\times \\ \times\exp\left(-((v_x - v_z x/R)^2 + (v_y - v_z y/R)^2)/\theta_0^2\,v_z^2\right) \end{aligned}. \quad (10)$$

This expression includes the inclination of the particle trajectories, which assuming the facts that $\theta_0^2 << 1$ and that the angle of focusing is small, is equal to $v_z x/R$ or $v_z y/R$ for the displacements along x and y-axes respectively. Calculation of the flux density profile using (10) gives the same relationship as (5) for the finite R.

This approach can be applied to obtain the profile of a beam passing through a set of circular apertures of different radii. For the case of one aperture of radius b placed at $z = z_0$, the distribution function of the beam without focusing immediately after the aperture has the form

$$\begin{aligned} f = (J_0/\pi V_0^3\theta_0^2)\,\delta(v_z - V_0)\,H\left(a - \sqrt{(x - v_x z_0/v_z)^2 + (y - v_y z_0/v_z)^2}\right)\times \\ \times\exp\left(-(v_x^2 + v_y^2)/\theta_0^2\,v_z^2\right)H\left(b - \sqrt{x^2 + y^2}\right), \end{aligned}$$

where the last step function describes transmission of the aperture. Using the same constants of motion, beyond the aperture, at $z > z_0$ the distribution function can be written as

$$f = (J_0/\pi V_0^3\theta_0^2)\,\delta(v_z - V_0)\,H\left(a - \sqrt{(x - v_x z/v_z)^2 + (y - v_y z/v_z)^2}\right)\times$$
$$\times\exp\left(-(v_x^2 + v_y^2)/\theta_0^2 v_z^2\right)H\left(b - \sqrt{(x - v_x(z - z_0)/v_z)^2 + (y - v_y(z - z_0)/v_z)^2}\right).$$

The current density profile can be again found by multiplying the distribution function by v_z and integration over the velocity space. The final result has the following form

$$j_{\text{beam}}(r,z) = (J_0/\pi z^2\theta_0^2)\exp\left(-\frac{r^2}{z^2\theta_0^2}\right)\int_0^{2\pi} d\varphi\int_0^a \exp\left(-\frac{\rho^2}{\theta_0^2 z^2} + \frac{2r\rho}{z^2\theta_0^2}\cos\varphi\right)\times$$
$$\times H\left(b - \sqrt{\rho^2(1 - z_0/z)^2 + z_0^2 r^2/z^2 + 2z_0/z(1 - z_0/z)\rho r\cos\varphi}\right)\rho d\rho \quad .$$

Addition of other apertures leads to multiplying the distribution function by corresponding step functions. Generalization of this procedure for non-circular apertures and for a focused beam is straightforward. Another, probably more vivid approach to calculation of the current density profile of a beam passing through apertures is considered in the next section.

3. Account of apertures in the beamline

Consider the system shown in Fig.1 with addition of a singly-connected plane aperture of a general form. The aperture plane perpendicular to the z-axis is located at the distance z_0 from the beam emitter, and the shape of the aperture edge is described by the plane curve C given in parametric form with $x(t)$ and $y(t)$ being functions of parameter t measured along C (Fig.2). We are again interested in the current density produced by the beam in an arbitrary point (r,z) of the observation plane at the distance z from the emitter. The elementary current at the point (ρ,φ) at the emitter produces the current density at the observation plane only if these two points can be connected by a straight line which passes through the aperture. Let us formulate this condition mathematically. The assumed straight line passes through two points with the Cartesian coordinates $(\rho\cos\varphi, \rho\sin\varphi, 0)$ at the emitter and (x,y,z) at the observation plane (Fig.2).

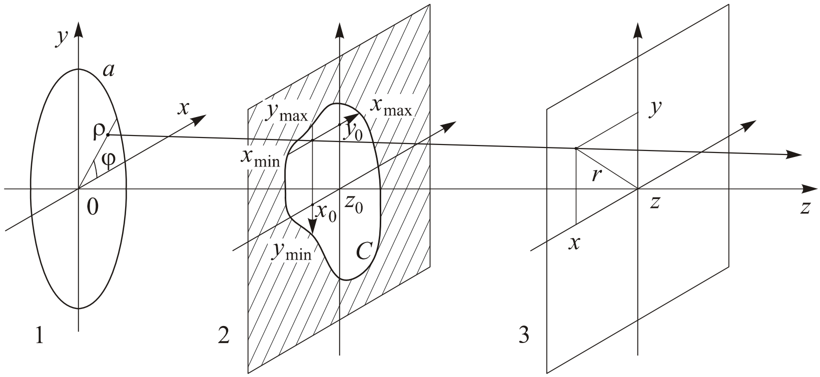

Fig. 2. Beam emitter 1, aperture plane 2, and target plane 3.

In projection to the plane $y = 0$ the equation of this line is $x(Z) = \rho\cos\varphi + Z\,(x - \rho\cos\varphi)/z$ and in the plane $x = 0$ the equation is $y(Z) = \rho\sin\varphi + Z\,(y - \rho\sin\varphi)/z$. Thus, this line crosses the aperture plane $z = z_0$ in the point

$$x_0 = \rho\cos\varphi + (x - \rho\cos\varphi)z_0/z\ ,\ \ y_0 = \rho\sin\varphi + (y - \rho\sin\varphi)z_0/z\ , \qquad (11)$$

where coordinates x_0, y_0 in the aperture plane should not be confused with x_0, y_0 temporarily used in (8)–(10) as coordinates in the emitter plane.

For beam passing through the aperture this point (x_0, y_0) must lie inside the closed curve *C*, being the edge of the aperture i.e. inequalities $x_{\min,C}(y_0) < x_0 < x_{\max,C}(y_0)$ and $y_{\min,C}(x_0) < y_0 < y_{\max,C}(x_0)$ must be satisfied (for simplicity we assume only convex shapes of the curve *C* which is suitable for almost all practical cases). Let us define an "aperture function" *A* which is equal to unity if both of the latter inequalities hold, i.e. if the elementary beam under consideration passes through the aperture and adds to the current density at the observation plane, and equal to zero otherwise. Using the Heaviside step function $H(x)$ which is equal to unity for $x > 0$ and zero for $x < 0$, we may write the aperture function in the explicit form

$$A(z_0, x, y, z, \rho, \varphi) = \qquad (12)$$
$$= H(x_0 - x_{\min,C}(y_0))H(x_{\max,C}(y_0) - x_0)H(y_0 - y_{\min,C}(x_0))H(y_{\max,C}(x_0) - y_0),$$

where $x_0(z_0, x, z, \rho, \varphi)$ and $y_0(z_0, y, z, \rho, \varphi)$ are taken from (11). The current density after the aperture is written explicitly using the expression (6) for the current density of the freely propagating beam

$$dj_{\text{after}}(r, z, \rho, \varphi) = dj_{\text{beam}}(r, z, \rho, \varphi) \cdot A(z_0, x, y, z, \rho, \varphi)\,.$$

In practice circular and rectangular apertures are used most often, and we will specify conditions of elementary beam passing for these cases in more detail.

1. Rectangular aperture. Consider a rectangular aperture with the sides of length $2b_1$ along the x-axis and $2b_2$ along the y-axis, and with the center of the rectangle lying at the point with Cartesian coordinates (x_c, y_c). Conditions for elementary beam passing through the aperture are: $x_c - b_1 < x_0 < x_c + b_1$ and $y_c - b_2 < y_0 < y_c + b_2$, therefore the aperture function takes the form

$$A(x, y, z, \rho, \varphi) = H(x_0 - x_c + b_1)H(x_c + b_1 - x_0)H(y_0 - y_c + b_2)H(y_c + b_2 - y_0).$$

In the limiting case when, for example, $b_2 \to \infty$, the rectangle degenerates into an infinite slit of constant width $2b_1$, and only one condition remains $x_c - b_1 < x_0 < x_c + b_1$. The aperture function in this particular case becomes

$$A(z_0, x, y, z, \rho, \varphi) = H(x_0 - x_c + b_1)H(x_c + b_1 - x_0).$$

2. Circular aperture. It is quite natural that in practice circular neutral beams are used in combination with circular apertures.

a) *Off-axis circular aperture.* This is the most general case. Let the center of the circle have coordinates (x_c, y_c), then the edge of the circular aperture of radius b is given by the equation $(x - x_c)^2 + (y - y_c)^2 = b^2$ and conditions of the elementary beam passing through the aperture are $x_c - \sqrt{b^2 - y_0^2} < x_0 < x_c + \sqrt{b^2 - y_0^2}$ and $y_c - \sqrt{b^2 - x_0^2} < y_0 < y_c + \sqrt{b^2 - x_0^2}$, or combining them we obtain one condition $(x_0 - x_c)^2 + (y_0 - y_c)^2 < b^2$. The aperture function is

$$A(z_0, x, y, z, \rho, \varphi) = H(b^2 - (x_0 - x_c)^2 - (y_0 - y_c)^2).$$

b) *Concentric circular aperture.* In this special case the center of the circular aperture lies on the beam axis $x_c = y_c = 0$, the aperture edge equation becomes $x^2 + y^2 = b^2$, and conditions of elementary beam passing are $-\sqrt{b^2 - y_0^2} < x_0 < \sqrt{b^2 - y_0^2}$ and $-\sqrt{b^2 - x_0^2} < y_0 < \sqrt{b^2 - x_0^2}$ or combining both conditions, $x_0^2 + y_0^2 < b^2$. If the system of the beam and apertures is fully axisymmetric, it is convenient to use polar coordinates in the aperture plane, and this condition using (11) can be rewritten in the form $\frac{r^2 z_0^2}{z^2} + \frac{2rz_0}{z}\left(1 - \frac{z_0}{z}\right)\rho\cos\varphi + \left(1 - \frac{z_0}{z}\right)^2 \rho^2 < b^2$.

The corresponding aperture function is

$$A(z_0, x, y, z, \rho, \varphi) = \theta(b^2 - x_0^2 - y_0^2) = \tag{13}$$

$$= H\left(b^2 - \frac{r^2 z_0^2}{z^2} - \frac{2rz_0}{z}\left(1 - \frac{z_0}{z}\right)\rho\cos\varphi - \left(1 - \frac{z_0}{z}\right)^2 \rho^2\right).$$

3. Multiple apertures. It is easy to generalize these single-aperture functions to the case of several consecutive apertures. The elementary beam would reach a certain aperture at the distance z_n only if it already passed through all previous apertures located at the distances z_i between the beam emitter and the given aperture. In other words, the straight line of sight from (ρ, φ) to (r, z) must not cross any aperture. Thus, the condition of passing through the n-th aperture requires that $A(z_i, x, y, z, \rho, \varphi) = 1$ for all i from the interval $1 \le i \le n$, and the net aperture function for $z_n < z < z_{n+1}$ may be written in a compact form as a product

$$A_{\text{total}}(x, y, z, \rho, \varphi) = \prod_{i=1}^{n} A(z_i, x, y, z, \rho, \varphi).$$

The elementary current density at an arbitrary distance from the emitter with the account of all apertures between the emitter and the observation plane is

$$dj(x, y, z, \rho, \varphi) = dj_{\text{beam}}(\sqrt{x^2 + y^2}, z, \rho, \varphi) \cdot A_{\text{total}}(x, y, z, \rho, \varphi),$$

and finally the two-dimensional current density distribution in x-y plane at the distance z from the emitter of fast neutrals is given by integration of the elementary currents over the emitter surface

$$j(x, y, z) = \int_0^a \rho d\rho \int_0^{2\pi} (dj(x, y, z, \rho, \varphi)/\rho d\rho) d\varphi. \tag{14}$$

For practical use it is important to know the portion of the initial beam that passes through each aperture. Let us call this value a "transmission coefficient". It is calculated as a ratio of the total current that passes through the n-th aperture to the total beam current at the emitter I_0

$$\eta_n \equiv I(z_n + 0)/I_0. \tag{15}$$

The total current after the aperture is calculated using the current density (14)

$$I(z_n + 0) = \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} j(x, y, z_n + 0)\, dx.$$

Since immediately after the aperture the current density is nonzero only in over the aperture area, the $I(z_n + 0)$ can be also calculated as an integral over this area

$$I(z_n + 0) = \int_S j(x, y, z_n)\, dS. \tag{16}$$

Hitting of the aperture surface by an energetic beam may lead to melting and destruction of the aperture in case of long-pulse powerful beams. It may become a serious concern and would require special measures to cool the apertures in order to keep them under acceptable temperature. Power density can be calculated by multiplying the current density immediately before the aperture plane by the accelerating voltage in the ion-optical system

$$P(x, y, z_n)\,[\mathrm{Watt}/\mathrm{cm}^2] = j(x, y, z_n - 0)\,[\mathrm{A}/\mathrm{cm}^2]\cdot U\,[\mathrm{Volt}].$$

4. Numerical calculations for realistic neutral beam system

The model calculation presented below is made for the beam and beamline with the following parameters: the beam emitter radius is $a = 10$ cm, the focal length $R = 340$ cm, and the angular divergence $\theta_0 = 1.2^\circ = 2.1\cdot 10^{-2}$ rad. The total beam current at the emitter is $I_0 = 40$ A, and the accelerating voltage is $U_0 = 40$ kV, therefore the power density at the emitter is $P_0 = I_0 U_0 / \pi a^2 = 5.1$ $[\mathrm{kW}/\mathrm{cm}^2]$. There are four circular apertures in the beamline, all of them concentric with the beam (Table 1).

Table 1. Axial positions and radii of circular concentric apertures in the beamline.

Aperture number	Distance from emitter z, cm	Aperture radius b, cm
1	79	9.85
2	210	9.75
3	263	12.5
4	312	12.5

Results of numerical calculations are presented in Fig.3 in terms of power density profiles immediately before all four apertures and at the focal plane instead of current density using the conversion formula $P = jU_0$, so the current density can be easily found by dividing the power density by the accelerating voltage $U_0 = 40$ kV.

In the focal plane (curve “5”) the beam half-width at the $1/e$ level found using (7) is $R\theta_0 = 7.1$ cm. The power load profiles at the apertures are shown in Fig.4.

The power load at the aperture #3 at $z_3 = 263$ cm is relatively small, because its radius is larger than the radius of the aperture #2, therefore the beam does not manage to spread radially due to angular divergence since the distance from the aperture #2 at $z_2 = 210$ cm is not enough for that, $(z_3 - z_2)\cdot\theta_0 < b_3 - b_2$, and moreover the beam focusing attempts to decrease the beam diameter with *z*, the maximal focusing angle is $a/R = 3\cdot 10^{-2}$ rad.

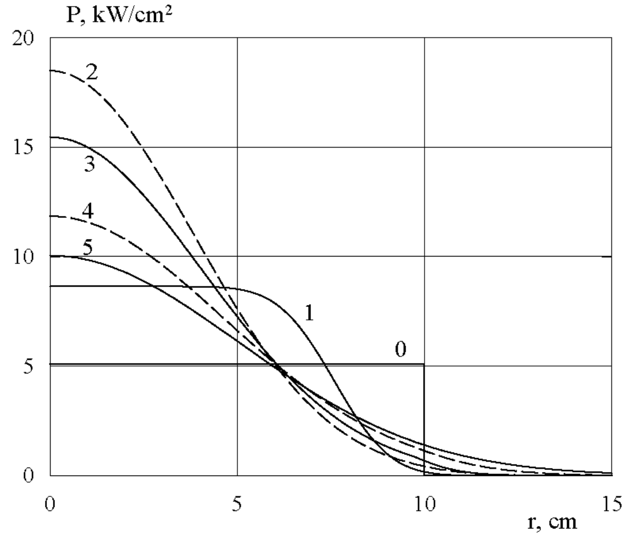

Fig. 3. Power density radial profiles at various positions: 0 – emitter, 1 – 79 cm, 2 – 210 cm, 3 – 263 cm, 4 – 312 cm, 5 – at the focal plane $z = R = 340$ cm.

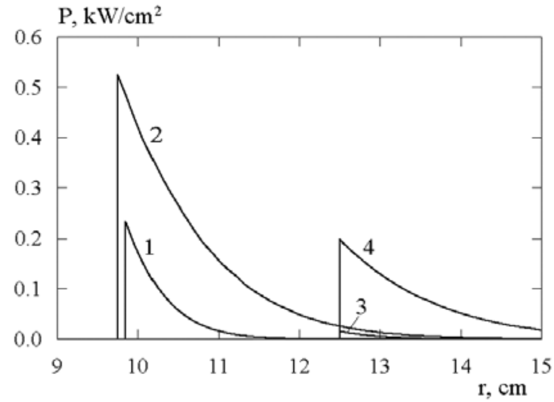

Fig. 4. Power load distribution at the apertures:
1 – 79 cm, 2 – 210 cm, 3 – 263 cm, 4 – 312 cm.

For the case of concentric circular apertures calculation of the aperture power loads and transmission coefficients is particularly simple. Expression (16) for the total beam current that passes through the n-th aperture becomes $I(z_n+0) = 2\pi\int_0^{b_n} j(r, z_n-0)\, rdr$, thus the transmission coefficient (15) is $\eta_n = 2\pi\int_0^{b_n} j(r, z_n-0) rdr \Big/ I_0$ and the total power load on the n-th aperture is

$Q_n = U_0 \int_{b_n}^{\infty} j(r, z_n) r dr = W_0 (\eta_{n-1} - \eta_n)$, where $W_0 = I_0 U_0$ is the beam power at the emitter.

Integral characteristics of the beamline with this specific neutral beam, namely the aperture transmission coefficients and the integral power loads are presented in Table 2.

Table 2. Transmission coefficients and power loads of the apertures.

Aperture number	Aperture position z, cm	Transmission coefficient, η, %	Total aperture power load, Q, kW
1	79	99.6	6.8
2	210	97.4	34.7
3	263	97.4	0.5
4	312	96.3	16.7

Analyzing the results presented in Table 2, one should be aware of the model limitations discussed in Section 2, which cause some systematic errors due to simplifications in geometrical expressions.

5. Conclusion

The physical model of the circular neutral beam with geometric focusing and Gaussian angular divergence in the beamline with multiple plane apertures of arbitrary shape is described. Formulas for two-dimensional current density profile of the beam at any distance from the emitter are derived. Rectangular and circular apertures limiting the cross-size of the beam are considered in detail. Numerical code based on this model is applied to the case of four circular apertures concentric with the beam. Current density radial profiles, aperture transmission coefficients and aperture power loads have been calculated.

The authors are grateful to Dr. G.Fiksel (University of Wisconsin, Madison, USA) and Dr. R.Uhlemann (IEF-Plasma Physics, FZ Jülich, Germany) for valuable discussions.

References

[1] *V.I. Davydenko, P.P. Deichuli, A.A. Ivanov et al.* Radio frequency ion source for plasma diagnostics in magnetic fusion experiments, Rev. Sci. Instrum., 2000, v.71(10), pp.3728-3735.

[2] *I.S. Gradshtein, I.M. Ryzhik.* Tables of Integrals, Series and Products, Academic Press, NewYork, 1965.

[3] *R. Uhlemann and J. Ongena.* Variation of injected neutral beam power at constant particle energy by changing the beam target aperture of the TEXTOR neutral beam injectors, Fusion Technol., 1999, v.35, p.42-53.

T.D. Akhmetov, V.I. Davydenko, A.A. Ivanov

Model of a neutral beam
with geometric focusing and apertures

Т.Д. Ахметов, В.И. Давыденко, А.А. Иванов

Модель пучка нейтральных атомов с геометрической фокусировкой и апертурами

Budker INP 2007-25

Ответственный за выпуск А.М. Кудрявцев
Работа поступила 6.09. 2007 г.

Сдано в набор 7.09. 2007 г.
Подписано в печать 7.09. 2007 г.
Формат 60x90 1/16 Объем 1.0 печ.л.,0.8 уч.-изд.л.
Тираж 105 экз. Бесплатно. Заказ № 25

Обработано на PC и отпечатано
на ротапринте «ИЯФ им. Г.И. Будкера» СО РАН,
Новосибирск, 630090, пр. Академика Лаврентьева, 11