Siberian Branch of Russian Academy of Science
BUDKER INSTITUTE OF NUCLEAR PHYSICS

Valery S. Cherkassky, Boris A. Knyazev,
Igor A. Kotelnikov

ELECTROMAGNETIC SCREENING BY A THIN CONDUCTING CAN

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Electromagnetic screening by a thin conducting can

Valery S. Cherkassky[1], *Boris A. Knyazev*[2], *Igor A. Kotelnikov*[3]

Budker Institute for Nuclear Physics
630090 Novosibirsk, Russia

Abstract

Detailed analysis of the penetration of external AC magnetic field into an arbitrary-shaped cylindrical conducting can is performed for the case when magnetic field is parallel to the axis of the can. The effect of narrow slits cut in the can walls on the field screening is also analyzed. Analytical solution is found for the can of elliptical cross section immersed into transverse magnetic field.

[1]E-mal: cherk@phys.nsu.ru
[2]E-mal: B.A.Knyazev@inp.nsk.su
[3]I.A.Kotelnikov@inp.nsk.su

In the most simple case, the problem of the penetration of alternating magnetic fields into normal metals can be discussed in terms of a single length, the skin depth δ, defined in Gaussian units as

$$\delta = c/\sqrt{2\pi\sigma\omega}\,, \tag{1}$$

where σ is the electrical conductivity. For simplicity, we have taken the permeability as unity and we have also assumed that the carrier mean free path $\ell \ll \delta$, in order to exclude the regime of anomalous skin effect.

Historically, skin depth calculations leading to eq. (1) refers to a cylindrical wire that carries AC electric current along a wire, with wavelength exceeding the wire diameter, $\lambda \gg d$. In the limit $\delta \ll d$ the problem reduces to the penetration of the AC field into a semi-infinite solid bounded by a plane. In this case the current density and electromagnetic field fall off exponentially from the outside of the solid with a characteristic length given by (1). This problem is analyzed in detail in many textbooks (see e. g. Tamm [1], Smythe [2], and Jackson [3]).

Such approach might create illusion that the problem involves the only the parameter, the skin depth, and that the AC magnetic field should penetrate into a metal can if its wall thickness h does not exceed the skin depth: $h < \delta$. This conclusion, however, ignores the importance of the specific geometry of eddy currents. In fact, even for $h \ll \delta$, a conducting cylinder of radius $a \gg h$ can effectively screen external AC magnetic field provided that $h \gg \delta^2/a$. This effect has purely geometrical nature and escape attention of most textbook writers. A rare exception is the monograph by Meshkov and Chirikov [4], where one can find an elegant "physical" solution of the following problem: "Calculate the screening factor for a cylindrical screen of radius a, wall thickness h of which is much smaller than the skin depth δ. Magnetic field is parallel to the axis of the cylinder." A solution, similar to Meshkov's one, was published later by Fathy, Kittel, and Louie [5]; they did not know the monograph [4] but mentioned exact "mathematical" solution given in Ref. [6] in terms of Bessel's functions.

Strong screening effect of a conducting thin-walled cylinder has geometrical origin. We reproduce below a simple explanation which one of us (I.K.), being a student, heard from Dmitry Ryutov in the end of 1970s.

Let us consider a cylindrical conducting screen of radius a immersed into magnetic field $H_{\rm out}$, parallel to the axis of the screen and alternating with the period T. As follows from the equation

$$\frac{\partial \mathbf{H}}{\partial t} = \frac{c^2}{4\pi\,\sigma}\,\Delta \mathbf{H}\,, \tag{2}$$

derived in the theory of skin effect, the magnetic field penetrates into the conductor to a depth equal to the wall thickness for the time $\tau \sim 2\pi\sigma h^2/c^2$. We assume that $\tau \ll T$; otherwise we come to a case of strong skin-effect where the skin depth is less then the wall thickness. When associated magnetic flux $\Delta\Phi \sim \pi a h H_{\rm out}$ penetrates into the screen interior, it changes internal magnetic field $H_{\rm in}$ by the magnitude $\Delta H_{\rm in} \sim \Delta\Phi/\pi a^2 \sim (h/a)\,H_{\rm out}$. It is small compared to $H_{\rm out}$ since $h \ll a$ but such flux portions penetrate T/τ times for the total period T of the magnetic field alternation. Consequently, the internal magnetic field rises up to $H_{\rm in} \sim (T/\tau)\,\Delta H_{\rm in} \sim (T/\tau)\,(h/a)\,H_{\rm out}$. Evaluating T as $2\pi/\omega$, we conclude that $H_{\rm in}/H_{\rm out} \sim \delta^2/ah$, and the internal field is small compared to the external one if $h \gg \delta^2/a$.

In this paper, we extend the above mentioned "physical" solution to a case of a cylindrical screen of arbitrary cross section. We discuss the effect of slits cut in the screen walls, which can be important for practical use of metallic screens in the experimental environment. We also consider screening of magnetic field perpendicular to the screen axis. In this case exact solution can be found for definite screen cross sections, such as circular or elliptic. In particular, we show that the circular cylindrical screen has numerically equal screening factors for both longitudinal and transverse magnetic field. In final section we calculate electric field inside thin screen since it is the electric, rather than magnetic AC field is of primary concern in experimental environment.

1 Longitudinal magnetic field

Let a thin conducting cylinder is immersed into external magnetic field, parallel to its axis directed along z-coordinate. Cross section of the cylinder can be of an arbitrary shape, not necessarily circular one, but both the area of the cross section S and the wall thickness $h \ll \sqrt{S}$ do not depend on z, while h may vary along the circumference of the cylinder. We assume the cylinder to be long enough to consider it as infinite. This reduces the problem to two dimensions. For simplicity we assume that time dependance of magnetic field is $\exp(-i\omega t)$.

The screen does not disturb the external longitudinal magnetic field $H_{\rm out}$. In the quasi-static limit, when the wavelength $\lambda = 2\pi c/\omega$ exceeds the characteristic cylinder dimension $\sqrt{S}$, both external $H_{\rm out}$ and internal fields $H_{\rm in}$ are uniform. In the case of weak skin-effect, $\delta \gg h$, the current density within the wall $j = \sigma E_\tau$, as well as the electric field E_τ, is uniform across the wall but can vary along the circumference of the cross section. One can mention that electric field is tangential to the wall surface at each point. The total current $I = jh$ per unit length of the can is a constant value, and E_τ can be found from the equation

$$E_\tau = I/\sigma h \,, \tag{3}$$

being the magnitude I is given. The latter can be found from the Faraday law:

$$\mathcal{E} = -\frac{1}{c}\frac{d\Phi}{dt} \,, \tag{4}$$

which states that the emf

$$\mathcal{E} = \oint E_\tau \, dl \tag{5}$$

along the circumference of the cross section of the screen is proportional to the rate of change of the magnetic flux $\Phi = H_{\rm in} S$. The thinner are can walls the more accurate are the expressions for E_τ and Φ. The magnetic fields inside and outside the screen obey the Ampere law:

$$H_{\rm out} - H_{\rm in} = -\frac{4\pi}{c} I \,. \tag{6}$$

Combining all together, we obtain the ratio of the internal magnetic field to the external one:

$$H_{\rm in}/H_{\rm out} = \left[1 - \frac{2i}{\delta^2} S \Big/ \oint \frac{dl}{h}\right]^{-1} \,. \tag{7}$$

Since the ratio is a complex value the internal magnetic field has phase lag in comparison with the external one. Absolute value of the ratio yields the screening factor

$$|H_{\rm in}/H_{\rm out}| = \left[1 + \left(2S/\delta^2 \oint dl/h\right)^2\right]^{-1/2} \,.$$

The can effectively screens the field provided that $\delta^2 \ll (h/l)\, S$, where l is perimeter of the can.

Figure 1: Rectangular screen with a cut slit.

Figure 2: Rectangular screen with a tunnel slit.

Eq. (7) is the main result of this section. It effectively solves the problem of electromagnetic screening for arbitrary shape of conducting can in the case of weak skin effect. To illustrate the flexibility of our method we apply the Eq. (7) to three specific geometries.

In particular case of a circular cylinder with the walls of a constant thickness h, Eq. (7) reveals well known result [4, 5]:

$$H_{\rm in}/H_{\rm out} = \left[1 - i\, a\, h/\delta^2\right]^{-1} . \tag{8}$$

For a rectangular screen with the cross section $2a \times 2b$ and wall thicknesses h_a and h_b, shown in Fig. 1 (but without any slit), we have

$$H_{\rm in}/H_{\rm out} = 1/[1 - 2i\, a\, b/(a/h_a + b/h_b)\, \delta^2] . \tag{9}$$

For the elliptical cross section with the half-axes $b < a$ and the walls of a constant thickness h we find:

$$H_{\rm in}/H_{\rm out} = 1/[1 - \pi i\, b\, h/2\, E(\sqrt{1 - b^2/a^2})\, \delta^2] , \tag{10}$$

where $E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{1/2}\, d\phi$ is a complete elliptic integral of second kind.

Taking the limit $a \to \infty$ in Eqs. (9) or (10), we obtain formal solution of 1D problem where two parallel plates of constant thickness h are separated by the gap of width $2b$:

$$H_{\rm in} = H_{\rm out}\, \left[1 - i\, \mathcal{A}\, b\, h/\delta^2\right]^{-1} .$$

Though the final geometry are the same for each initial geometry, the numerical coefficient $\mathcal{A}$ acquires different values. This means that the one-dimensional problem is badly established because its solution depends on

Figure 3: Screening factor $\kappa(\omega) = |H_{\rm in}/H_{\rm out}|$ for a screen with cut slit (solid line) and without slit (dashed line) vs AC field frequency. The frequency ω is normalized to $\omega_{RL} = c^2 R/L$ where $L = 4\pi S$ is the inductance of the screen per unit length. For very thin walls (less than a micron) magnetic field inside the screen may exceed the outer field near the resonance frequency $\omega_{LC} = c/\sqrt{LC}$ as shown in the right figure.

how the two plates are connected at the "infinity". Otherwise it is not possible to determine uniquely the currents flowing along the plates. One can imagine also that the remote parts of the wall have slits that are able to drastically change screening factor because AC field freely penetrates into the interior of the screen throughout the slits. The effect of a slit can readily be accomplished with minor amendments in the method which led us to Eq. (7).

A narrow slit of the width $d \ll h$ is equivalent to a plane capacitor. Its capacity per unit length is equal to $C = \varepsilon\, h_{\rm cut}/4\pi\, d$, if the slit is filled with the material with permittivity ε, and the wall thickness near the slit is $h_{\rm cut}$. Electric current I is to be found from the equation

$$RI + Q/C = \mathcal{E} \tag{11}$$

instead of Eq. (5), where $R = \oint dl/\sigma h$ is the resistance of the screen walls per unit length, and $Q = I/(-i\omega)$ is the charge of the capacitor. Simple algebra yields:

$$H_{\rm in}/H_{\rm out} = \frac{1 + i/\omega\tau}{1 + i/\omega\tau - (2iS/\delta^2)/\oint dl/h}\,, \tag{12}$$

where $\tau = RC$. Typical plots of the screening factor (12) versus frequency are drawn in Fig. 3.

Because of low capacitance of the slit, the magnitude of $\omega\tau$ is typically very small even at very high frequencies, $\omega\tau \ll 1$. As a result, screening of

AC field now occurs at much greater frequency when $\delta^2 \ll (h/l)\, S\, \omega\tau$. Since even very narrow slit of the kind shown in Fig. 1 significantly reduces the ability of a conducting can to screen AC field, it is rarely used in practice. Instead, the slit is usually has the form of a maze or a tunnel shown in Fig. 2. In particular, similar screen usually shields Rogovsky's belt which is widely used in fusion facilities to measure plasma pressure and currents.

Accurate treatment of the tunnel slit is somewhat cumbersome but the result is rather simple in the limit $\omega h/\sigma d \ll 1$, which covers practically important range of frequencies. In this case, one can still use Eq. (12) with the capacitance and resistivity given by $C = \varepsilon d/4\pi\Delta l$ and $R = (l - \frac{1}{3}\Delta l)/\sigma h$, where l is the circumference length of the screen, Δl is the length and the maze (so that total length of the conducting wall is $l + \Delta l$), d is thickness of the maze; for simplicity the latter is assumed to be constant as well as the thickness of the conducting wall h. This result can be obtained "by fingers" if one notes that the voltage between interfacing points of the tunnel slit is almost constant. It means that capacitance C of the slit is equal to that of the plane capacitor of the width Δl and the gap d between the capacitor's plates though the plates are not equipotential. To calculate the effective resistance R we note that total current flowing along the circumference of the wall from one end of the cut to the opposite end, linearly rises from zero to maximal value I, which enters the equation (11), on the length of the tunnel Δl, then remains constant outside the gap on the length $l - \Delta l$, and finally drops linearly to zero on the opposite side of the gap, again on the length Δl. Calculation of the Joule-Lenz heating then yields the above mentioned result, $R = (l - \frac{1}{3}\Delta l)/\sigma h$.

Another way to diminish penetration of the AC field through open parts of the screen is to use a cut, limited both in width and length. For example, a small circle hole of radius a perturbs magnetic field only in the close proximity of the whole–at large distance from the hole, $r \gg a$, the perturbation falls down as m/r^3, where $m = 2(H_{\text{out}} - H_{\text{in}})a^3/3$ (see Ref. [3]).

2 Transverse magnetic field

A conducting can immersed into the transverse magnetic field perturbs external magnetic field, and magnetic field inside the can becomes non-uniform. This complicates the problem as compared to the case of the longitudinal magnetic field. In this section we show that simple solution exists for a number of cross sections of special shapes. We consider a cylindrical screen with the elliptic cross section and with thin walls. Magnetic field inside such screen can be made uniform by special profiling of the wall thickness along circumference of the cylinder. We consider the latter case in more details.

It is natural to solve this problem in elliptic coordinates ξ, η, attached to the focuses of the ellipse: $x = p\,\cosh\xi\,\cos\eta$, $y = p\,\sinh\xi\,\sin\eta$. The distance p between the focuses is to be found from the pair of equations: $a = p\,\cosh\xi_*$, $b = p\,\sinh\xi_*$, with ξ_* being the elliptic coordinate ξ of the screen. Having written the square of the arc length $(dx)^2 + (dy)^2 = (h_\xi\,d\xi)^2 + (h_\eta\,d\eta)^2$, we find Lamé coefficients: $h_\xi = h_\eta = p\,(\cosh^2\xi - \cos^2\eta)^{1/2}$.

The problem is fully described by z-component A_z of the vector potential $\mathbf{A}$. Note that $\mathbf{H} = \nabla A_z \times \hat{\mathbf{z}}$, and hence

$$H_\xi = \frac{1}{h_\eta}\frac{\partial A_z}{\partial\eta}, \qquad\qquad H_\eta = -\frac{1}{h_\xi}\frac{\partial A_z}{\partial\xi}.$$

Outside the wall, where there is no current, the function A_z satisfies the scalar Laplace equation $\Delta A_z = 0$. In elliptic coordinates it takes the form

$$\frac{\partial^2 A_z}{\partial\xi^2} + \frac{\partial^2 A_z}{\partial\eta^2} = 0\,. \tag{13}$$

Since physical solution of this equation is single-valued, A_z is a periodic function of the angle coordinate η. Hence, A_z can be expand into a series, each term of which has the form of $A_n\exp(in\eta)\exp(\pm n\xi)$ with integer n (positive or negative). Every term of the series satisfies the equation (13), and the coefficients A_n are to be determined from the boundary conditions at $\xi = 0$, $\xi = \xi_*$, and $\xi \to \infty$.

At large distance from the screen ($\xi \to \infty$) external field tends to the unperturbed vector with components $H_{x,\mathrm{out}}$, $H_{y,\mathrm{out}}$. It means that outside the screen, in the region $\xi > \xi_*$, one needs to discard fast growing terms:

$$A_{z,\mathrm{out}} = H_{x,\mathrm{out}}\,p\,\sinh\xi\,\sin\eta - H_{y,\mathrm{out}}\,p\,\cosh\xi\,\cos\eta \\ + \sum_{n=0}^{\infty}[a_n\,\mathrm{e}^{-n\xi}\,\sin(n\eta) + b_n\,\mathrm{e}^{-n\xi}\,\cos(n\eta)] = 0\,. \tag{14}$$

Boundary conditions at $\xi \to 0$ can be derived from the fact that magnetic field is continuous at the line connecting the focuses of the elliptic system of coordinates. It means that the derivatives $\frac{\partial}{\partial\xi}A_z$ и $\frac{\partial}{\partial\eta}A_z$ are odd functions of η and leads to the following series

$$A_{z,\mathrm{in}} = \sum_{n=0}^{\infty}[c_n\,\sinh(n\xi)\,\sin(n\eta) + d_n\,\cosh(n\xi)\,\cos(n\eta)] = 0 \tag{15}$$

for $\xi < \xi_*$.

We match the series (14) and (15) using the boundary conditions at the screen boundary $\xi = \xi_*$. The continuity of $E_z = -\frac{1}{c}\frac{\partial}{\partial t}A_z$ field here yields the first equation $A_{z,\rm out} = A_{z,\rm in}$. Second equation follows from the Ampere law:

$$\frac{1}{h_\xi}\left[\frac{\partial}{\partial\xi}A_{z,\rm out} - \frac{\partial}{\partial\xi}A_{z,\rm in}\right] = -\frac{4\pi i\,\omega\,\sigma\,h}{c^2}\,A_z\,.$$

Since the Lamé coefficient h_ξ depends upon the angle variable η, various harmonics in the series (14) and (15) are entangled in general case, so that all the coefficients a_n, b_n, c_n, d_n, generally speaking, are not zeros. In other words, magnetic field both inside and outside the screen is not uniform.

A notable exception supplies the case when the wall thickness h is inversely proportional to h_ξ so that the parameter $Z = 2h\,h_\xi/\delta^2$ is constant. In this case only the coefficients a_1, b_1, c_1, d_1 are not equal to zero. If external magnetic field is not parallel to the ellipse axes, magnetic field inside the screen is uniform but not collinear to the external field:

$$\begin{aligned} H_{x,\rm in}/H_{x,\rm out} &= 1/[1 - iZ\ {\rm e}^{-\xi_*}\ \sinh\xi_*]\,, \\ H_{y,\rm in}/H_{y,\rm out} &= 1/[1 - iZ\ {\rm e}^{-\xi_*}\ \cosh\xi_*]\,. \end{aligned} \tag{16}$$

In the limit where $\xi_* \to \infty$ but h_ξ is finite, $h_\xi \to a$, we come to the circular screen of radius a, and Eq. (16) yields the same result as for the circular screen in longitudinal magnetic field (8). This means that electromagnetic screening is effective for arbitrary orientation of external AC field. Note however that for elongated shape of the screen cross section, $a/b = \tanh\xi_* \ll 1$, screening factor is significantly larger when external magnetic field is directed along short axis of the ellipse.

3 Electric field inside the screen

Practical goal of electromagnetic screening in experimental environment is to diminish electric AC field rather than magnetic one since the instrumentation indications are more sensitive to electric field. If the magnetic field is transverse to the axis of the screen, the electric field can readily be found from the results of previous section since in this case $\mathbf{E}$ is directed along the axis z of the screen, and $E_z = -(1/c)\partial A_z/\partial t$. Below we focus on the longitudinal magnetic field.

In this case, electric field is a two-dimensional vector perpendicular to the axis z. It can be cast into the form

$$\mathbf{E} = \nabla\psi \times \hat{\mathbf{z}} \tag{17}$$

where ψ is a function of x, y, and $\hat{\mathbf{z}}$ is the unit vector directed along z. The form (17) satisfies the equation $\operatorname{div}\mathbf{E} = 0$. In the interior of the screen the Faraday's law yields

$$\Delta\psi = -\frac{i\omega}{c}\, H_{\rm in} \tag{18}$$

where $H_{\rm in}$ is given by Eq. (7). The equation (18) should be supplemented with the boundary condition (3). It can be solved for various shape of the screen. For rectangular screen shown in Fig. 1 we find

$$\begin{aligned} E_x &= \frac{\partial\psi}{\partial y} = -\frac{i\omega}{c}\,\frac{h_b\, a\, y}{h_b\, a + h_a\, b}\, H_{\rm in}\,, \\ E_y &= -\frac{\partial\psi}{\partial x} = \frac{i\omega}{c}\,\frac{h_a\, b\, x}{h_b\, a + h_a\, b}\, H_{\rm in}\,, \end{aligned}$$

where the point $x = y = 0$ is assumed to be placed at the geometrical center of the cross section of the screen. Note that at given magnitude of $H_{\rm in}$ electric field inside the screen can be additionally diminished along a desired direction by a proper choice of the wall thickness.

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Б.К. Князев, И.А. Котельников, В.С. Черкасский

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