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THE ION BEAM MOMENTUM SPREAD IN STORAGE RINGS WITH ELECTRON COOLING AND INTERNAL TARGETS

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The ion beam momentum spread in storage rings with electron cooling and internal targets

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Abstract

In this paper we study an increase in the antiproton beam momentum spread due to finite betatron emittance of the beam. This effect is specific for the spectromentic experiments with internal targets in ion storage rings with electron cooling.

1 Introduction

Without additional perturbations, the electron cooling of ion beams stops when the temperature of the ion beam reaches the temperature of the cooling electron beam. If m is the mass of the electron and M is the mass of the ion, the corresponding momentum spreads in the ion beam become $\sqrt{m/M}$ times smaller than that in the electron beam. Generally, this enables the spectrometric experiments with extremely high energy resolutions (e.g. in Ref.[1]). In such cases, even small perturbations increasing the beam energy spread may limit the experiment performance.

In this paper we discuss one of such effects which is specific for spectrometric experiments with internal targets in ion storage rings with electron cooling. Ionizing the target atoms, the ions gradually lose their energies thus, leaving the beam. For the particle with the energy $E = \gamma Mc^2$, $\gamma = 1/\sqrt{1-(v/c)^2}$, the average rate of this energy loss reads

$$\frac{dE}{dt} = -mc^2 \frac{4\pi Z_A t_A r_e^2 c^2}{v\Pi} \ln \frac{E_{\max}}{I}. \tag{1}$$

Here, t_A is the thickness of the target, Z_A is the atomic number of the target material, $r_e = e^2/mc^2$ is the electron classical radius, Π is the closed orbit perimeter, $E_{\max} \simeq 2m\gamma^2 v^2$, I is the ionization potential of the target atoms. If the beam is cooled, this deceleration will not result in the particle losses provided that the power of the cooling force (vF) exceeds the power of the ionization energy losses given in Eq.(1). Since the force due to electron cooling vanishes when the ion velocity is equal to the average velocity of the electron beam (v_0), the equation

$$vF(\Delta E) = mc^2 \frac{4\pi Z_A t_A r_e^2 c^2}{v\Pi} \ln \frac{E_{\max}}{I} \tag{2}$$

defines the equilibrium energy of the ion due to the balance between the cooling and ionization energy losses. Here, $\Delta E = E - E_0$, where $E_0 = \gamma_0 Mc^2$ and $\gamma_0 = 1/\sqrt{1-(v_0/c)^2}$. Generally, due to a decrease in the value of the cooling force with an increase in the value of ΔE Eq.(2) has two roots (two fixed points in ΔE). The vicinity near the fixed point at the decreasing slope of the curve $F(\Delta E)$ corresponds to stable solutions. During the cooling, the ions are collected near energy corresponding to this fixed point. The root of Eq.(2) on the increasing slope of the curve $F(\Delta E)$ corresponds to unstable solutions and define the momentum aperture of the ring. As is known, for a given particle in the beam the power of the electron cooling force depends both on ΔE and on the amplitudes of its betatron oscillations. For this reason, the equilibrium energies defined by Eq.(2) are different for the particles with different amplitudes of betatron oscillations. So that in the ring with electron cooling the balance between the cooling power and the power of the ionization energy losses increases the energy spread of the beam thus, limiting the achievable monochromaticity of the beam. In general, such a possibility was mentioned in Ref.[1].

In this paper we study this increase in the beam energy spread for a simplest case, when a coasting antiproton beam interacts with a target consisting of H_2 molecules ($Z_A = 2$).

2 The equilibrium energy

For the sake of simplicity we calculate the roots of Eq.(2) assuming that cooling electron are spiralling along the guiding magnetic field of the cooling device with the Larmour velocity spread v_L and that the cooling force can be calculated using a simple expression, which was suggested e.g. in Ref.[2]:

$$\mathbf{F} = \frac{4n_e e^4}{m} \frac{\mathbf{u}}{\left(u^2 + v_{eff}^2\right)^{3/2}} \ln\left(1 + \frac{\rho_{\max}}{\rho_{\min} + \rho_L}\right). \tag{3}$$

Here, all values are calculated in the beam rest frame system, $\mathbf{u}$ is the ion velocity, v_{eff} is the effective velocity spread of the electron Larmour circles, $\rho_{\max}$ is the maximum impact parameter for collisions of an ion with the electrons:

$$\rho_{\max} = \min\left(\frac{lu}{\gamma v}, \frac{u}{\omega_e}\right),$$

l is the length of the cooling region, $\omega_e = \sqrt{4\pi n_e e^2/m}$ is the plasma frequency of the electron beam, $\rho_{\min} = e^2/mu^2$ and $\rho_L = v_L/\omega_L$ is the (rms) Larmour radius in the electron beam, m is the mass of the electron.

Systematic variations of the particle energy due to the cooling force are obtained averaging the power $vF_{\|}$ over the periods of the particle betatron oscillations. Taking that the dispersion function in the cooling section is equal to zero and neglecting the variations of the betatron functions of the ring along the cooling section we write

$$x = \sqrt{J_x\beta_x}\cos\phi_x,\ \theta_x = \frac{p_x}{p_0} = -\sqrt{\frac{J_x}{\beta_x}}\sin\phi_x,\ \theta_{\|} = \frac{1}{(v/c)^2}\frac{\Delta E}{E_0}, \quad (4)$$

$$z = \sqrt{J_z\beta_z}\cos\phi_z,\ \theta_z = \frac{p_z}{p_0} = -\sqrt{\frac{J_z}{\beta_z}}\sin\phi_z.$$

Here, x and z stand for the horizontal and for the vertical planes respectively. Then, the average variation of the particle energy due to the cooling force in Eq.(3) reads

$$\frac{d\Delta E}{dt} = -\frac{4n_e e^4}{m}\frac{l}{\Pi}vu_{\|}\int_0^{2\pi}\frac{d\phi_x d\phi_z}{(2\pi)^2}\frac{\ln\left(1+\dfrac{\rho_{\max}}{\rho_{\min}+\rho_L}\right)}{\left(u^2+v_{eff}^2\right)^{3/2}}. \quad (5)$$

In terms of the values in the laboratory system we substitute in this equation $n_e \to n_e/\gamma$ and

$$u^2 + v_{eff}^2 = v^2\left(a^2 + \gamma^2\left[\frac{J_x\cos^2\phi_x}{\beta_x} + \frac{J_z\cos^2\phi_z}{\beta_z}\right] + \theta_{\|}^2\right). \quad (6)$$

Then, we rewrite Eq.(2) in the following form

$$\frac{d\Delta E}{dt} = -mc^2 \frac{8\pi t_A r_e^2 c^2}{v\Pi} \ln \frac{E_{\max}}{I}(1+Q) = 0, \tag{7}$$

where

$$Q = -\frac{n_e l}{2\pi t_A \ln\left(\dfrac{E_{\max}}{I}\right)}\theta_{\parallel} \tag{8}$$

$$\times \int_0^{2\pi} \frac{d\phi_x d\phi_z}{(2\pi)^2} \frac{\ln\left(1+\dfrac{\rho_{\max}}{\rho_{\min}+\rho_L}\right)}{\left(a^2+\gamma^2\left[\dfrac{J_x\cos^2\phi_x}{\beta_x}+\dfrac{J_z\cos^2\phi_z}{\beta_z}\right]+\theta_{\parallel}^2\right)^{3/2}}.$$

Solutions to Eq.(7) define in the space $\theta_{\parallel}$, J_x, J_z a surface $\left(\theta_{\parallel}\right)_{st} = \theta_{\parallel}(J_x, J_z)$. Since the amplitudes of betatron oscillations in the beam ($\propto \sqrt{J_{x,z}}$) are distributed within some range, this will increase the beam momentum spread by the amount which depends on the betatron emittances of the beam. Generally, the value of the power of the cooling force decrease with increases in the amplitudes of betatron oscillations of particles. Hence, this additional momentum spread is lower the lower are betatron beam emittances.

3 Numerical examples

Usually Eq.(7) cannot be solved analytically. In order to evaluate the described additional momentum spread of the ion beam we solved this equation numerically assuming the beam and the target parameters which are specific for the internal target experiments in the antiproton ring HESR (see. e.g. in Ref.[3]). The desired collision monochromaticity in these experiments is of about 100 keV. For simplicity, in our numerical calculations we assumed $J_x = J_z = J$ and in the cooling section $\beta_x = \beta_z = \beta$. The longitudinal temperature of the electrons for these calculations was taken as $mv_{eff}^2 = e^2 n_e^{1/3}$. Remaining necessary parameters were taken from the Table 1. Inspecting dependencies to

Table 1: The beam and the target parameters used for numerical calculations

Kinetic energy range	0.8–14.5	GeV
Perimeter	424.7	m
Betatron tunes	$\simeq 10$	
Length of the cooling region	30	m
β-function in the cooling region (β_c)	60	m
Number of antiprotons	$10^{10} - 10^{11}$	
Betatron emittances (experiment)	0.001–0.1	mmmrad
Momentum spread	0.002–0.02	%
Target (H_2-jet)	up to 10^{16}	$1/[\mathrm{cm}^2]$
Maximum luminosity	2×10^{32}	$1/[\mathrm{cm}^2\mathrm{s}]$

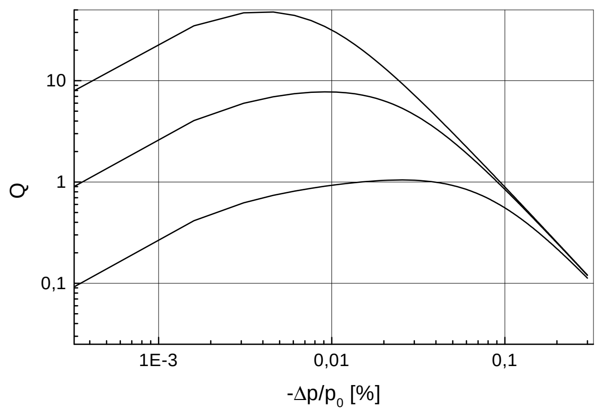

Figure 1: Dependence of the factor Q in Eq.(8) on $-\Delta p/p$. From top to bottom $J_x = J_z = J$: 0.001 mmmrad, 0.01 mmmrad and 0.1 mmmrad, kinetic energy 14.5 GeV, $n_e = 10^9$ 1/cm^3.

$Q(J, \theta_{\|})$ (Fig.1) we find out that for a taken electron beam density of $n_e = 10^9$ 1/cm^3 the roots of the Eq.(7) ($Q(J, -\theta_{\|}) = 1$) corresponding the stable fixed point occur in the region below $\left|\theta_{\|}\right| = 0.001\%$ for the beam with emittance only slightly exceeding 0.01 mmmrad. On the contrary, for particles with the amplitudes of betatron oscillations in the range J =0.01–0.1 mmmrad the equilibrium values of energy varies in the range corresponding to

$$3 \times 10^{-6} < \theta_{\|} < 1 \times 10^{-4}.$$

Moreover, at the upper end of this range the momentum aperture of the ring shrinks dramatically. Inspecting dependencies $\Delta E_{st}(J)$ (Fig.2), we see that for taken target thickness, the antiproton beam parameters

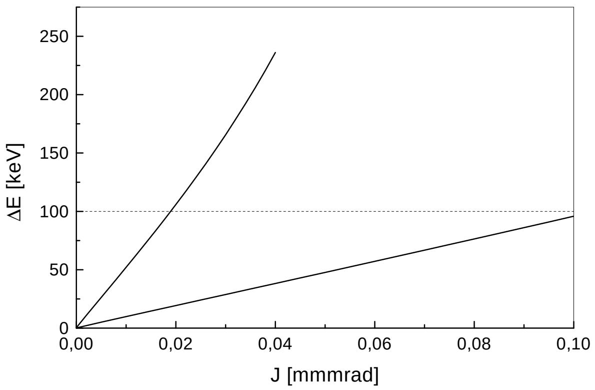

Figure 2: Dependence of the equilibrium energy of an antiproton on the amplitude of its betatron oscillations (J). Kinetic energy is 14.5 GeV, upper curve $n_e = 10^9$ 1/cm^3, lower curve $n_e = 10^{10}$ 1/cm^3.

and $n_e = 10^9$ 1/cm^3 the requirement $\Delta E_{st} \leq 100$ keV holds only for betatron amplitudes corresponding to $J = 0.02$ mmmrad. Taking this value as the beam border in its phase space of the betatron oscillations and corresponding to 2σ, we find that in such a case the required collision monochromaticity will be achieved for the beam with the betatron emittances of 0.005 mmmrad. The lower curve in Fig.2 shows that in the same conditions the collision monochromaticity $\Delta E_{st} \leq 100$ keV is achieved for the 10 times dense electron beam ($n_e = 10^{10}$ 1/cm^3).

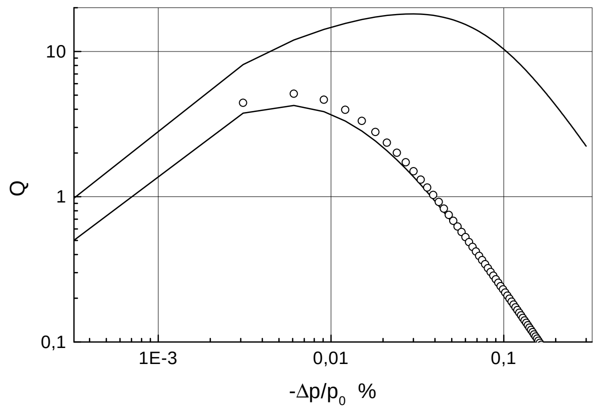

Figure 3: Dependence of the factor Q in Eq.(8) on $-\Delta p/p$. Kinetic energy of antiprotons is 14.5 GeV, $J = 0.1$ mmmrad, $n_e = 2.66 \times 10^{10}$ 1/cm^3. Upper solid curve: electron density is uniform within $\sqrt{J\beta_c}$ (electron beam current is about 24 A), lower solid curve: the current of the electron beam is I_e =1 A, its radius is 0.05 cm, $n_e(r)$ is a Gaussian function, open circles: $I_e = 1$ A, but n_e is uniformly distributed within $r \leq 0.05$ cm.

4 Electron beam compression

For a given betatron emittance (ϵ_0) of the antiproton beam the generation of so dense electron beam with a uniform density inside the radius e.g. $\sqrt{\epsilon_0\beta_c}$ can be limited at high electron energies by effects of the total electron beam current. If, for example, for $n_e = 10^{10}$ 1/cm^3, $\epsilon_0 = 0.1$ mmmrad and $\beta_c = 60$ m the current of the electron beam is evaluated as 9 A. The required current of the electron beam can be reduced compressing the electron before it will arrive at the cooling region. In this case, the radius of the electron beam is reduced to achieve an acceptable beam current for a given value of its density. Generally, such a reduction in the beam radius decreases the achievable values of the average power of the cooling force thus, increasing the antiproton beam momentum spread (see e.g. in Fig. 3). Comparing the lower solid line and open circles in Fig. 3, we find that for a given electron beam radius such a reduction in the power of the cooling force does not strongly depends on the shape of $n_e(r)$.

As is seen from Fig.4, for the beam emittances up to 0.1 mmmrad the collision energy spread will be below 100 keV, if the electron beam with the current of 1 A is compressed till the radius of 0.05 cm (lower curve in Fig.4). A twice wider electron beam with the sane current will enable same collision monochromaticity for a twice lower antiproton beam emittance.

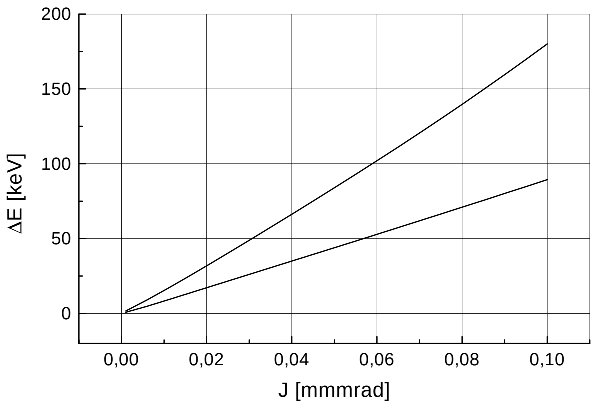

Figure 4: Dependence of the equilibrium energy of an antiproton on the amplitude of its betatron oscillations (J). Kinetic energy is 14.5 GeV, $I_e = 1$ A, upper curve: electron beam radius is 0.1 cm, lower curve: 0.05 cm.

5 Conclusion

The discussed dependence of the equilibrium energy of the antiprotons on the amplitudes of their betatron oscillations in the storage ring with electron cooling may substantially limit the collision energy monochromaticity which can be achieved in such a ring during experiments with internal targets. The limitation occurs, if the desired operation mode demands the beam with a rather large transverse emittances (e.g. 0.1 mmmrad in our examples). With taken beam and target parameters, an extension of the operation region for such experiments on the beam emittances of 0.1 mmmrad demands a 10 times lower target thickness (e.g. $t_A = 10^{15}$ 1/cm^2), or a 10 times dense cooling electron beam. In the first case, the collision luminosity will drop 10 times. In the second case, the desired electron beam density can be achieved using e.g. the electron beam compression. In the last case, the required compression ratio should provide sufficient both the power of the cooling force and the lifetime of the beam in the desired operation mode.

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