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LONGITUDINAL COOLING FORCE
DUE TO MAGNETIZED ELECTRONS

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Abstract

We study effects of the tails of the longitudinal velocity distribution function of cooling electrons on dependencies of the longitudinal magnetized electron cooling force on ion velocities. For the case, when ions move parallel to the guiding magnetic field of the cooling device we calculate the longitudinal cooling force beyond the logarithmic approximation.

1 Introduction

The most important characteristics of the electron cooling device are the attainable values of the cooling forces as well as the dependencies of the cooling force on the ion positions in its phase space. Due to magnetization of electrons, a strong guiding magnetic field \mathbf{H}_0 of the electron cooling device eliminates the contributions of their Larmour rotations to the cooling force [1]. This results in a strong enhancement of the cooling forces experienced by slow ions in the cooling device. According to measurements at NAP-M [2] and at other storage rings which use the electron cooling (see, e.g. in Ref.[3]) the designs of new electron cooling devices, or of the ion storage rings with electron cooling definitely should take into account this effect.

Unfortunately, the possibilities of comprehensive calculations of the cooling force, or of its measurements are embarrassed by the complexity of the kinetics of the electron cooling process. For this reason, the ion motion in such rings is usually simulated using various model, or interpolation expressions for the cooling force. A simple expression of this kind was suggested in Ref.[4]:

$$\mathbf{F} = \frac{4n_e e^4}{m} \frac{\mathbf{V}}{(V^2 + v_{eff}^2)^{3/2}} \ln \left(1 + \frac{\rho_{\max}}{\rho_{\min} + \rho_L} \right). \quad (1)$$

Here, \mathbf{V} is the ion velocity, v_{eff} is the effective velocity spread of the electron Larmour circles, $\rho_{\max} = V/\omega_e$, $\omega_e = 4\pi e^2 n_e/m$ is the plasma frequency of the electron beam, $\rho_{\min} = e^2/mV^2$ and $\rho_L = v_L/\omega_L$ is the (rms) Larmour radius in the electron beam, m is the mass of the electron. The interpolation of the cooling force using Eq.(1) does not

assume the logarithmic approximation ($\rho_{\max} \gg \rho_{\min} + \rho_L$). Arguing for Eq.(1), the authors in Refs.[3] and [4] compare e.g. the projection of the cooling force from Eq.(1) on the direction of \mathbf{H}_0 (the longitudinal cooling force) for an ion moving along \mathbf{H}_0 , the longitudinal cooling force obtained using numerical simulations and the longitudinal cooling force obtained using the following simple expression [1]:

$$F_{\parallel}(v, u) = -\frac{2\pi e^4 n_e L_C}{m} \frac{3uv^2}{(v^2 + u^2)^{5/2}}, \quad L_C = \ln \frac{k_{\max}}{k_{\min}}. \quad (2)$$

Here, $V^2 = v^2 + u^2$, u is the deviation of the longitudinal ion velocity from the average longitudinal velocity of the electron beam, $k_{\max} = 1/\rho_{\min}$ and $k_{\min} = 1/\rho_{\max}$. Contrary to Eq.(1) and to similar predictions of the simulations, the value of F_{\parallel} in Eq.(2) vanishes, if $v = 0$ and provided that $u \neq 0$. However, the reason for such a behavior of F_{\parallel} in Eq.(2) is that this equation holds only for the monochromatic electron beam ($v_{eff} = 0$), or for the case, where $V \gg v_{eff}$, where mv_{eff}^2 is an effective temperature of the electron larmour circles. As was mentioned in Refs.[1] and [5] in the region where $v = 0$ and $|u| \ll v_{eff}$, Eq.(2) should be replaced by another one yielding $F_{\parallel} \propto -u/v_{eff}^3$. So that a comparison of Eqs.(1) and (2) is irrelevant for that purpose. Indeed, the value of the cooling force in Eq.(2) tends to infinity, if v and u tend to zero simultaneously ($F \propto -1/u^2$, if $u = v \rightarrow 0$). This infinite growth of the cooling force is limited, when the ion velocities become smaller than the velocity spread of the electron Larmour circles. Closer inspection of general expressions for the cooling force shows that for magnetized electrons it is very sensitive to the shape of the longitudinal velocity distribution function of the cooling electrons (see. e.g. in Ref.[5]).

In this paper, within the framework of the perturbation theory we study two subjects. First, within the framework of the pair collision approximation we inspect the dependence of the longitudinal cooling force on the shape of the electron distribution function in their longitudinal velocities. The second, within the framework of the plasma theory approach (see. e.g. in Ref.[6]) we calculate the longitudinal cooling force for ions moving parallel to the guiding magnetic field

of the cooling device beyond the so-called logarithmic approximation. We do not take into account the non-perturbative contributions to the cooling force (see. e.g. in Ref.[5]).

2 Effect of tail electrons on magnetized cooling

If $f_e(u_e)$ is the distribution function of the cooling electrons in the longitudinal velocities, the longitudinal cooling force calculated in the pair collision approximation and assuming $H_0 \rightarrow \infty$ reads

$$F_{\parallel}(v, u) = -\frac{2\pi e^4 n_e L_C}{m} \int_{-\infty}^{\infty} \frac{3v^2(u - u_e)f(u_e)}{(v^2 + (u - u_e)^2)^{5/2}} du_e \quad (3)$$

$$= \frac{2\pi e^4 n_e L_C}{m} v^2 \int_{-\infty}^{\infty} \frac{du_e}{(v^2 + (u - u_e)^2)^{3/2}} \frac{df(u_e)}{du_e}. \quad (4)$$

First, let us calculate the longitudinal cooling force assuming the following distribution function in the longitudinal electron velocities

$$f = \frac{1}{2\sigma} \begin{cases} 1, & |u_e| \leq \sigma, \\ 0, & |u_e| > \sigma. \end{cases} \quad (5)$$

Substituting this expression in Eq.(4), we obtain

$$F_{\parallel} = -\frac{2\pi e^4 n_e L_C}{m} \frac{v^2}{2\sigma} \left[\frac{1}{(v^2 + (\sigma - u)^2)^{3/2}} - \frac{1}{(v^2 + (\sigma + u)^2)^{3/2}} \right]. \quad (6)$$

The value of F in this equation also vanishes when $v \rightarrow 0$ unless $|u| = \sigma$. For example, for small longitudinal velocity of the ion ($|u| \ll \sigma$) Eq.(6) yields an expression

$$F_{\parallel} \simeq -\frac{2\pi e^4 n_e L_C}{m} \frac{3v^2}{(v^2 + \sigma^2)^{5/2}} u, \quad (7)$$

which is very similar to that in Eq.(2). Again, if the values v and $|u| - \sigma$ in Eq.(6) tend to zero simultaneously, the value of the cooling

force tends to infinity. According to Eq.(4), we may expect that it occurs due to sharp edges of the distribution function. To figure out the role of the tails of the distribution function on the behavior of the cooling force we calculate F_{\parallel} taking as $f(u_e)$ the following expression

$$f(u_e) = \frac{1}{2\pi\sigma} \left[\arctan \left(\frac{u_e + \sigma}{\delta} \right) - \arctan \left(\frac{u_e - \sigma}{\delta} \right) \right]. \quad (8)$$

Here, the value of δ determines the widths of the tails of f . In particular, the distribution function in Eq.(5) is obtained as a limit of the right-hand side in Eq.(8), if δ tends to zero. Using Eq.(8), we rewrite Eq.(4) like follows

$$F_{\parallel} = \frac{2\pi e^4 n_e L_C}{m} \frac{z^2}{\sigma^2} \frac{b}{2\pi} \int_{-\infty}^{\infty} \frac{1}{(z^2 + (y-x)^2)^{3/2} (b^2 + (x+1)^2)} dx \\ - \frac{2\pi e^4 n_e L_C}{m} \frac{z^2}{\sigma^2} \frac{b}{2\pi} \int_{-\infty}^{\infty} \frac{1}{(z^2 + (y-x)^2)^{3/2} (b^2 + (x-1)^2)} dx. \quad (9)$$

Here, all components of the ion velocity are measured in units of σ (e.g. $y = u/\sigma$) and $b = \delta/\sigma$. Substituting in Eq.(9) $x = y + z \tan \alpha$, we obtain

$$F_{\parallel} = \frac{2\pi e^4 n_e L_C}{m\sigma^2} \frac{b}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{\cos \alpha d\alpha}{(b^2 + (y+1+z \tan \alpha)^2)} \\ - \frac{2\pi e^4 n_e L_C}{m\sigma^2} \frac{b}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{\cos \alpha d\alpha}{(b^2 + (y-1+z \tan \alpha)^2)}. \quad (10)$$

The integrands in the right-hand side of Eq.(10) are well converging functions for all values of z . For this reason, we can put in this equation $z = 0$ to find

$$F_{\parallel}(v=0, u) = \frac{2\pi e^4 n_e L_C b}{m\sigma^2 \pi} \left[\frac{1}{(b^2 + (y-1)^2)} - \frac{1}{(b^2 + (y+1)^2)} \right]. \quad (11)$$

Although $v=0$, this function does not vanish unless $u=0$. Due to smooth tails of the distribution function in Eq.(8) the cooling force in Eq.(11) is also a non-singular function of u .

Similar calculations for a Gaussian function

$$f_e = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{u_e^2}{2\sigma^2}\right),$$

result in

$$F_{\parallel}(v, u) = -\frac{2\pi e^4 n_e L_C}{m\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \frac{3v^2(u-u_e)}{(v^2 + (u-u_e)^2)^{5/2}} \exp\left(-\frac{u_e^2}{2\sigma^2}\right) du_e,$$

or, measuring all velocities in units of σ ,

$$F_{\parallel} = -\frac{2\pi e^4 n_e L_C}{m\sigma^2} \frac{z^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{x}{(z^2 + (y-x)^2)^{3/2}} \exp\left(-\frac{x^2}{2}\right) dx. \quad (12)$$

Here, $z = v/\sigma$ and $y = u/\sigma$. Without other limitations the value of k_{\min} in the Coulomb logarithm L_C in Eq.(12) is determined by the Debye screening in the electron beam, so that

$$L_C = \ln \frac{k_{\max}}{k_D}, \quad k_D^2 = \frac{\omega_e^2}{\sigma^2} = \frac{4\pi n_e e^2}{m\sigma^2}. \quad (13)$$

Substituting in Eq.(12) $x = y + |z| \tan \alpha$, we obtain

$$F_{\parallel} = -\frac{\sqrt{2\pi} e^4 n_e L_C}{m\sigma^2} \int_{-\pi/2}^{\pi/2} (y \cos \alpha + |z| \sin \alpha) \times \exp\left(-\frac{[y + |z| \tan \alpha]^2}{2}\right) d\alpha \quad (14)$$

Since the integrand in Eq.(14) is a nonsingular function of its arguments we can put in this equation $z = 0$ to find

$$F_{\parallel}(v = 0, u) = -\frac{4\pi e^4 L_C}{m\sigma^2} \frac{ye^{-y^2/2}}{\sqrt{2\pi}}. \quad (15)$$

We note that in the region where longitudinal velocities of ions do not exceed substantially the longitudinal velocity spread of electrons, all obtained expressions of the longitudinal cooling force are proportional to the first derivative of the electron longitudinal velocity distribution function over the longitudinal ion velocity.

Let us also calculate the power of the cooling force

$$Q = \mathbf{F}_{\perp} \mathbf{v} + F_{\parallel} u. \quad (16)$$

Using (see, e.g. in Ref.[1])

$$\mathbf{F}_{\perp} = -\frac{2\pi e^4 n_e L_C}{m} \mathbf{v} \int_{-\infty}^{\infty} du_e \frac{(u - u_e)}{(v^2 + (u - u_e)^2)^{3/2}} \frac{df(u_e)}{du_e} \quad (17)$$

and measuring the velocities in units of σ , after simple transformations we obtain

$$-Q = \frac{Q_0 \sin^2 \theta}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{x^2 e^{-r^2 x^2/2}}{(1 - 2x \cos \theta + x^2)^{3/2}} dx. \quad (18)$$

Here, $r^2 = (v^2 + u^2)/\sigma^2$, $\tan \theta = v/u$ and

$$Q_0 = \frac{2\pi e^4 n_e L_C}{m\sigma} r^2. \quad (19)$$

According to Eq.(18) the dependence of Q on the angle (θ) between the ion velocity and the direction of the guiding magnetic field of the cooling device varies with an increase in the value of r (Fig.1 and Fig.2).

For the finite longitudinal velocity spread, the dependence of $-Q$ on θ reminds that calculated for the monochromatic magnetized electron beam only, when $r > 4$ (Fig.2). This behavior agrees with that

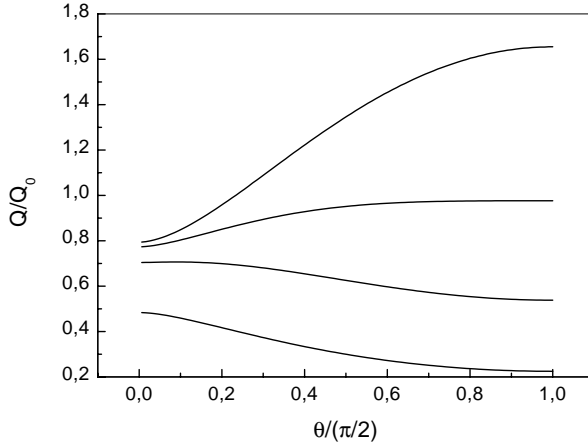


Figure 1: Dependence of the power of the cooling force (Q) on α . From top to bottom $r = 0.1, 0.25, 0.5$ and 1 .

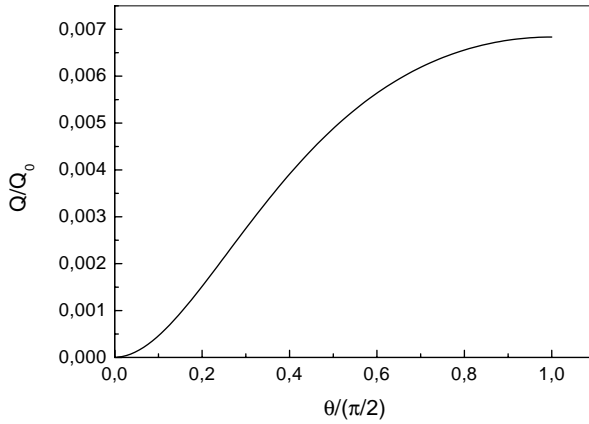


Figure 2: Same as in Fig.1, but $r = 5$.

predicted in Ref.[1] For small ion velocities the graphs shown in Fig.1 seem to be at least in a qualitative agreement with the simulation results given in Fig.7 of Ref.[3].

3 The magnetized cooling force of ions with zero transverse velocities

More comprehensive calculations of the cooling force is achieved treating the electron cooling process as a dynamical Debye screening of the ion in the electron beam. Within this approach and neglecting the transient effects, we write (see, e.g. in Ref.[6])

$$F_{\parallel} = \frac{-ie^2}{2\pi^2} \int \frac{d^2qdkk}{(q^2 + k^2) \varepsilon(qv, ku)}. \quad (20)$$

Here, $q^2 = q_x^2 + q_z^2$ is the square of the transverse to \mathbf{H}_0 wave vector of the Coulomb electric field of the ion in the electron beam and k is its longitudinal component, $\varepsilon(qv, ku)$ is the dielectric permeability of the electron beam, calculated for the frequency $\omega = q_x v_x + q_z v_z + ku$. For the strongly magnetized electron beam ($|\mathbf{H}_0| = \infty$), the combination $(q^2 + k^2) \varepsilon(qv, ku)$ reads

$$(q^2 + k^2) \varepsilon(qv, ku) = q^2 + k^2 + \omega_e^2 k \int \frac{du_e (df_e/du_e)}{q_x v_x + q_z v_z + k(u - u_e) + i0}, \quad (21)$$

where $\omega_e^2 = 4\pi n_e e^2/m$ is the plasma frequency of the electron beam and f_e is the longitudinal velocity distribution function of the electron beam. For an electron beam in the magnetic field of a finite strength, Eq.(21) holds, if $q\rho_L \ll 1$ and $k|u| \ll \omega_L$. In this Section we calculate the longitudinal cooling force for a particular case, where $v_x = v_z = 0$ while f_e is a Gaussian function

$$f_e = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{u_e^2}{2\sigma^2}\right). \quad (22)$$

Using these assumptions, we rewrite Eq.(21) in the following form

$$(q^2 + k^2) \varepsilon(qv, ku) = q^2 + k^2 + k_D^2 P(u) + ik_D^2 \frac{k}{|k|} J(u), \quad (23)$$

where

$$P = - \lim_{\lambda \rightarrow 0} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} \frac{(y-x)xe^{-x^2/2}}{(y-x)^2 + \lambda^2}, \quad y = \frac{u}{\sigma}, \quad (24)$$

$$J = \sqrt{\frac{\pi}{2}} ye^{-y^2/2}. \quad (25)$$

Substituting Eq.(23) in Eq.(20), we find

$$F_{\parallel} = - \frac{e^2 k_D^2 J(u)}{2\pi^2} \int d^2 q \int_{-\infty}^{\infty} \frac{dk |k|}{(q^2 + k^2 + k_D^2 P(u))^2 + k_D^4 J^2(u)}.$$

Changing here the integration variables according to $k^2 = k_D^2 x$ and $q^2 = k_D^2 t$, we transform F_{\parallel} to the following form

$$F_{\parallel} = - \frac{e^2 k_D^2 J(u)}{2\pi} \frac{1}{|J(u)|} \int_0^{\infty} dx \left\{ \frac{\pi}{2} - \arctan \frac{x + P(u)}{|J(u)|} \right\}. \quad (26)$$

The integral over x in the last expression logarithmically diverges at the upper limit of the integration. This divergency can be regularized integrating in Eq.(26) over x from zero to some finite value $x = x_{\max}$. The value k_{\max} in $x_{\max} = k_{\max}^2/k_D^2$ is chosen as a smallest value between $e^2 k_{\max} \ll m\sigma^2$ and $k_{\max} |u| \ll \omega_L$, where $\omega_L = eH_0/mc$. The first inequality enables the application of the perturbation theory for the calculation of the cooling force. The second condition enables an application of Eq.(21) for such calculations. Now, the integration in Eq.(26) yields

$$F = F_L + F_w, \quad (27)$$

where

$$F_L = -F_0 y e^{-y^2/2} \frac{1}{2} \ln \left[\frac{\left(\frac{k_{\max}^2}{k_D^2} + P \right)^2 + J^2}{P^2 + J^2} \right], \quad (28)$$

and

$$F_w = -F_0 \sqrt{\frac{2}{\pi}} \frac{y}{|y|} \left\{ \frac{\pi k_{\max}^2}{2 k_D^2} - \left(\frac{k_{\max}^2}{k_D^2} + P \right) \arctan \frac{k_{\max}^2 + k_D^2 P}{k_D^2 |J|} + P \arctan \frac{P}{|J|} \right\} \quad (29)$$

Here,

$$F_0 = \frac{e^2 k_D^2}{2\sqrt{2}\pi}. \quad (30)$$

In the region where the logarithmic approximation holds well ($k_{\max} \gg k_D$) the logarithmic part of the cooling force (F_L) coincides with the right-hand side in Eq.(15). For a given value of k_{\max} an expression in Eq.(27) yields the value of the cooling force in both logarithmic and in non-logarithmic approximations. According to Eq.(26), the non-logarithmic contributions are important in the regions where both $q^2 + k^2 + k_D^2 P(y)$ and $J(y)$ approach zero values. This region of parameters corresponds to conditions where the ion radiates in the electron beam real (or, almost real) plasma waves. As is seen from (Fig.3), in the region where the logarithmic approximation holds more or less well (e.g. $L_C = 4.6$), the logarithmic part of the cooling force describes its dependence on the ion velocity only in the region where u is relatively small (e.g. in Fig.3 it holds, if $u < \sigma$). For larger ion velocities the non-logarithmic contribution to F_{\parallel} due to plasma waves radiation is important. It dominates, if $|u| > 4\sigma$.

Beyond the logarithmic approximation (e.g. $L_C = 0.04$ in Fig.4) the logarithmic part of the cooling force is negligible small while the value of F is determined mainly by the plasma wave radiation processes (Fig.4).

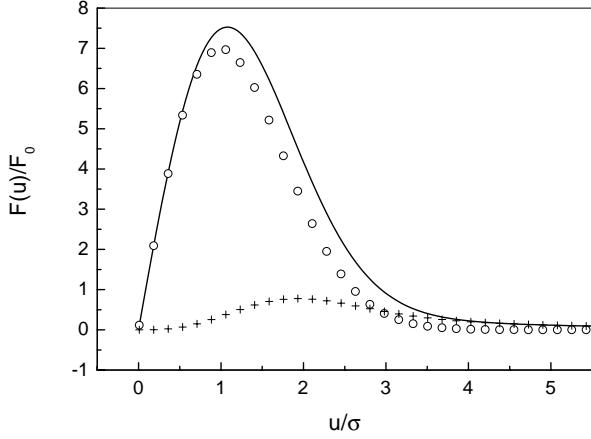


Figure 3: Dependence of the longitudinal cooling force on the longitudinal ion velocity (solid line). Open circles show the contribution from the logarithmic region, crosses – the contribution of the plasma wave radiation, $L_C = 4.6$.

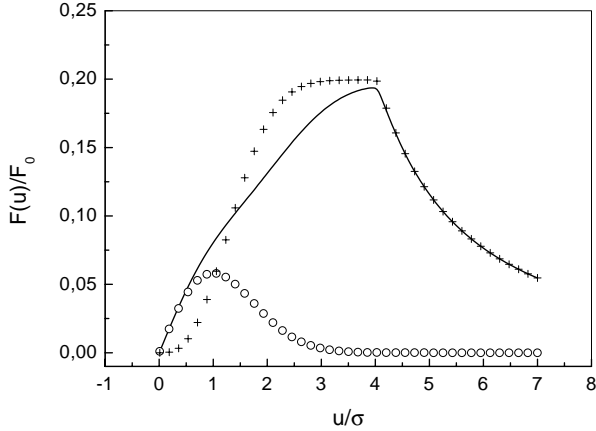


Figure 4: Same as in Fig.3, but $L_C = 0.04$.

4 Conclusion

Presented calculations indicate the following important features of the electron cooling theory. First, in the cases where the magnetized cooling dominates the calculations of the cooling force are very sensitive to the smoothness of the tails of the distribution function in the electron longitudinal velocities. For example, in the case of a Gaussian distribution the predicted behavior of the cooling force and of its power do not indicate significant deviations from the results of the simulations discussed in Refs.[3] and [4]. For the cooling in a strong guiding magnetic fields the comparison of the results of the described analytic calculations and of simulations presented in Refs.[4] and [3] indicates better qualitative agreement than between these simulations and the interpolation formula in Eq.(1).

Second, the calculations of the cooling force due to strongly magnetized electrons for ions with zero transverse velocities indicate that the radiation by an ion of the almost real plasma waves may give significant contributions to the cooling force. In particular, in the cases when the value of the Coulomb logarithm is not very large, the values of the cooling forces affecting the fast ions (e.g. $u > 4\sigma$) occurs due to these radiation processes. The cooling force for such fast ions should be calculated beyond the logarithmic approximation.

In both cases the value of the cooling force is very sensitive to the smoothness of the distribution function over longitudinal velocities of the cooling electrons. In particular, the longitudinal cooling force for ions with zero transverse velocities is proportional to the first derivative of this distribution function over the ion longitudinal velocity. Provided that transverse components of the guiding magnetic field of the cooling device are suppressed, this fact can be used for direct measurements of the longitudinal velocity distribution function in the electron beam.

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*Продольная сила трения
в замагниченном электронном пучке*

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