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COHERENT SCATTERING OF HIGH-ENERGY PHOTON IN A MEDIUM

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Abstract

The coherent scattering of photon in the Coulomb field (the Delbrück scattering) is considered for the momentum transfer $\Delta \ll m$ in the frame of the quasiclassical operator method. In high-energy region this process occurs over rather long distance. The process amplitude is calculated taking into account the multiple scattering of particles of the intermediate electron-positron pair in a medium. The result is the suppression of the process. Limiting cases of weak and strong effects of the multiple scattering are analyzed. The approach used is the generalization of the method developed by authors for consideration of the Landau-Pomeranchuk-Migdal effect.

1 Introduction

The nonlinear effects of QED are due to the interaction of a photon with electron-positron field. These processes are the photon-photon scattering, the coherent photon scattering (the elastic scattering of a photon by the static Coulomb field called often the Delbrück scattering), the photon splitting into two photons, and the coalescence of two photons into photon in the Coulomb field. Among these processes the coherent scattering of photon has been observed and investigated experimentally (see reviews [1] and [2]) during last decades, the most accurate measurement of the coherent scattering has been performed not long ago in the Budker Institute of Nuclear Physics [3]. The photon splitting into two photons in the Coulomb field was observed for the first time at Budker INP only recently [4]. Observation of photon-photon scattering is still a challenge.

History of the coherent photon scattering study can be found in the mentioned reviews. There is a special interest to the process for heavy elements because contributions of higher orders of $Z\alpha$ ($Z|e|$ is the charge of nucleus, $e^2 = \alpha = 1/137,\ \hbar = c = 1$) into the amplitude of photon scattering are very important. This means that one needs the theory which is exact with respect to the parameter $Z\alpha$. The amplitudes of the coherent photon scattering valid for any $Z\alpha$ for high energy photon ($\omega \gg m$) and small scattering angle (or small momentum transfer Δ) were calculated in [5], [6]. The approximate method of summing of the set of Feynman diagrams with an arbitrary number of photons exchanged with the Coulomb source was used. Another representation of these amplitudes (in the Coulomb field) was found in [7], using the quasiclassical Green function of the Dirac equation in the Coulomb field. This Green function in a spherically symmetrical external field was obtained in [8] where the coherent photon scattering in the screened Coulomb potential was investigated as well. Lately this Green function was calculated for "localized potential", and the coherent photon scattering was analyzed using it [9]. Recently the process of the coherent photon scattering was considered [11] in frame of the quasiclassical operator method (see e.g. [10]) which appears to be very adequate for consideration of this problem.

The coherent photon scattering belong to the class of electromagnetic processes which in high-energy region occurs over rather long distance, known as the formation length. Among other processes there are the bremsstrahlung and the pair creation by a photon. If anything happens to an electron while traveling this distance, the emission can be disrupted. Landau and Pomeranchuk [12] showed that if the formation length of bremsstrahlung becomes comparable to the distance over which a mean angle of the multiple scattering becomes comparable with a characteristic angle of radiation, the bremsstrahlung will be suppressed. Migdal [13] developed a quantitative theory of this phenomenon which is known as the Landau-Pomeranchuk-Migdal (LPM) effect.

A very successful series of experiments (see [14], [15]) was performed at SLAC during last years. In these experiments the cross section of bremsstrahlung of soft photons with energy from 200 KeV to 500 MeV from electrons with energy 8 GeV and 25 GeV is measured with an accuracy of the order of a few percent. The LPM was observed and investigated. These experiments were the challenge for the theory since in all the previous papers calculations are performed to logarithmic accuracy which is not enough for description of the new experiment. The contribution of the Coulomb corrections (at least for heavy elements) is larger then experimental errors and these corrections should be taken into account.

Recently authors [16] developed the new approach to the theory of the LPM effect in frame of the quasiclassical operator method. In it the cross section of bremsstrahlung process in the photon energies region where the influence of the LPM effect is very strong was calculated with term $\propto 1/L$, where L is characteristic logarithm of the problem, and with the Coulomb corrections taken into account. In the photon energy region, where the LPM effect is "turned off", the obtained cross section gives the exact Bethe-Heitler cross section (within power accuracy) with the Coulomb corrections. This important feature was absent in the previous calculations. The contribution of an inelastic scattering of a projectile on atomic electrons is also included. We have analyzed (see [16], [18]) the soft part of spectrum, including all the accompanying effects: the boundary photon emission, the multiphoton radiation and the influence of the polarization of a medium. Perfect agreement of the theory and SLAC data was achieved in the whole interval of measured photon energies. Very recently we apply this approach to the process of pair creation by a photon in [20].

In the quasiclassical approximation the amplitude M of the coherent photon scattering is described by diagram where the electron-positron pair is created by the initial photon with 4-momentum k_1 $(\omega, \mathbf{k}_1)$ and then annihilate into the final photon with 4-momentum k_2. For high energy photon $\omega \gg m$ this process occurs over a rather long distance, known as the time of life of the virtual state

$$l_f = \frac{\omega}{2q_c^2}, \qquad (1.1)$$

where $q_c \geq m$ is the characteristic transverse momentum of the process, the system $\hbar = c = 1$ is used. When the virtual electron (or positron) is moving in a medium it scatters on atoms. The mean square of momentum transfer to the electron from a medium on the distance l_f is

$$q_s^2 = 4\pi Z^2\alpha^2 n_a L l_f, \quad L(q_c) = \ln\frac{q_c^2}{q_{min}^2}, \quad q_{min}^2 = a^{-2} + \Delta^2 + \frac{m^4}{\omega^2}, \qquad (1.2)$$

where $\alpha = e^2 = 1/137$, Z is the charge of nucleus, n_a is the number density of atoms in the medium, Δ is the photon momentum transfer ($\Delta = |\mathbf{k}_2 - \mathbf{k}_1| \ll q_c$), a is the screening radius of atom.

The coherent photon scattering amplitude M can be obtained from general formulas for probabilities of electromagnetic processes in the frame of the quasiclassical operator method (see e.g. [10]). It can be estimated as

$$M \sim \frac{\alpha\omega}{2\pi l_f n_a}\frac{q_s^2}{q_c^2} = \frac{\alpha}{\pi n_a}q_s^2. \tag{1.3}$$

We use the normalization condition Im$M = \omega\sigma_p$ for the case $\Delta = 0$, where σ_p is the total cross section of pair creation by a photon.

In the case of small momentum transfer $q_s \equiv \sqrt{q_s^2} \ll m$ we have in the region of small $\Delta \ll m$

$$q_c^2 = m^2, \quad M \sim \frac{2Z^2\alpha^3\omega}{m^2}\ln\frac{m^2}{a^{-2}+\Delta^2+\frac{m^4}{\omega^2}}. \tag{1.4}$$

At an ultrahigh energy it is possible that $q_s \gg m$. In this case the characteristic momentum transfer q_c is defined by the momentum transfer q_s. The self-consistency condition is

$$q_c^2 = q_s^2 = \frac{2\pi\omega Z^2\alpha^2 n_a L(q_c)}{q_c^2}, \tag{1.5}$$

where $L(q_c)$ is defined in (1.2). So using (1.3) one gets for the estimate of the coherent photon scattering amplitude M (the influence of the multiple scattering manifests itself at the high photon energies such that $m^2a/\omega \ll 1$)

$$M \sim \frac{2Z^2\alpha^3\omega}{\Delta_s^2}\sqrt{\ln\frac{\Delta_s^2}{a^{-2}+\Delta^2}}, \quad \Delta_s^2 = \sqrt{2\pi\omega Z^2\alpha^2 n_a} \gg \Delta^2. \tag{1.6}$$

In the present paper we use consider the influence of the multiple scattering on the process of the coherent photon scattering for $\Delta \ll q_c$. The theory of the coherent photon scattering in the Coulomb field in frame of the quasiclassical operator method is stated in Sec.2. We give here an alternative presentation of approach developed in [11] in a more formal way. In Sec.3 we apply the method developed in [16], [17] to investigation of influence of the multiple scattering on the process of the coherent photon scattering. The general formulas describing this influence were derived and the asymptotic cases of the strong and weak effects are analyzed.

2 Coherent scattering of a photon in the Coulomb field

2.1 Formulation of approach

The coherent scattering of a photon in the Coulomb field (the photon with momentum $k_1 = (\omega_1, \mathbf{k}_1) \to k_2 = (\omega_2, \mathbf{k}_2)$) is represented by the electron loop in the Coulomb field, and $\omega_1 = \omega_2 = \omega$. The corresponding amplitude is

$$T = -\frac{2\pi\alpha}{\omega} i \int d^4x_1 \int d^4x_2 Tr \left[\hat{e}_1 \exp(ik_1x_1)\right.$$

$$\left. \times G(x_1, x_2)\hat{e}_2^* \exp(ik_2x_2) G(x_2, x_1)\right], \tag{2.1}$$

where $\hat{e} = e_\mu \gamma^\mu$, e_μ is the photon polarization vector, $G(x_1, x_2)$ is the electron Green function in the Coulomb field the standard representation of which is

$$G(x_2, x_1) = \left\{ \begin{array}{cc} - & i\sum_n \Psi_n^{(+)}(x_2)\overline{\Psi}_n^{(+)}(x_1), \quad t_2 > t_1 \\ & i\sum_n \Psi_n^{(-)}(x_2)\overline{\Psi}_n^{(-)}(x_1), \quad t_2 < t_1 \end{array} \right\}. \tag{2.2}$$

Here $\Psi_n^{(\pm)}(x_1)$ are the solution of Dirac equation in the Coulomb field, signs $(+)$ and $(-)$ relate respectively to positive and negative frequencies.

As well known, see e.g.[21], Sec.12, in the noncovariant perturbation theory, which we use here, in high-energy region ($\omega \gg m$) the diagram contributes into the amplitude T where the electron-positron pair is first created by the initial photon with 4-momentum k_1 and polarization e_1 in time t_1 and then annihilate in time $t_2 > t_1$ into final photon with 4-momentum k_2 and polarization e_2, while the contribution of the interval in which $t_2 < t_1$ is of the order m^2/ω^2. Such contribution is neglected in the scope of our method. Taking this into account and substituting the Green function into the amplitude (2.1) we find

$$T = \frac{2\pi\alpha}{\omega} i \sum_{n,m} \int dt_1 \int dt_2 \vartheta(t_2 - t_1) V_{nm}(\mathbf{e}_1, \mathbf{k}_1, t_1) V_{nm}^*(\mathbf{e}_2, \mathbf{k}_2, t_2), \tag{2.3}$$

where

$$V_{nm}(\mathbf{e},\mathbf{k},t) = \int d^3r\,\Psi_n^{(+)+}(x)\boldsymbol{\alpha}\mathbf{e}\exp(-ikx)\Psi_m^{(-)}(x), \qquad (2.4)$$

It is evident that $V_{nm}(\mathbf{e},\mathbf{k},t)$ (2.4) is the matrix element of pair creation by a photon in an external field. In the quasiclassical operator method (see [10], Sec.3) the wave functions in an external field can be presented in the form

$$\Psi_n^{(\pm)}(\mathbf{r},t) =< \mathbf{r}|\Psi^{(\pm)}(\mathbf{P},\mathcal{H})\exp\left[\mp i(\mathcal{H}\pm e\varphi)t\right]|n>, \quad \mathcal{H}=\sqrt{m^2+\mathbf{P}^2}, \qquad (2.5)$$

where $\varphi=\varphi(\mathbf{r})$ is the potential of an atom, $|n>$ is the state in the configuration space at the time $t=0$, the functions $\Psi^{(\pm)}(\mathbf{P},\mathcal{H})$ have form of wave functions for free particles in the momentum space ($\Psi^{(\pm)}(\mathbf{p},\varepsilon)$. We substitute (2.5) into (2.4) and take into account completeness of states $|\mathbf{r}>$ ($\int|\mathbf{r}><\mathbf{r}|d^3r=I$). After this we convey consistently in (2.4) the operator $\exp(i\mathbf{kr})$ to the right up to $|m>$, and then the operator $\exp\left[i(\mathcal{H}+e\varphi)t\right]$ from $<n|$ to $\exp\left[i(\mathcal{H}(\mathbf{k}-\mathbf{P})-e\varphi)t\right]$. As a result we obtain the combination of operators

$$\begin{aligned} V_{nm}(\mathbf{e},\mathbf{k},t) = \Big\langle n|\Psi^{(+)+}(\mathbf{P}(t))\boldsymbol{\alpha}\mathbf{e}\Psi^{(-)}(\mathbf{k}-\mathbf{P}(t)) \\ \times\exp(i(\mathcal{H}-e\varphi)t)\exp(i(\mathcal{H}(\mathbf{k}-\mathbf{P})+e\varphi)t)\exp(i\mathbf{kr})|m\Big\rangle \end{aligned} \qquad (2.6)$$

the disentanglement of which can be performed in the standard way (see [10], Eqs.(3.7)-(3.11)):

$$\begin{aligned} L_p(t) &= \exp\left[i(\mathcal{H}-e\varphi)t\right]\exp\left[i(\mathcal{H}(\mathbf{k}-\mathbf{P})+e\varphi)t\right] \\ &\simeq \exp(i\omega t)\mathrm{T}\exp\left[i\int_0^t \frac{kP(t)}{\omega-\mathcal{H}(t)}dt\right], \quad P=(\mathcal{H},\mathbf{P}), \end{aligned} \qquad (2.7)$$

where T is the operator of the chronological product, and the expression

$$R_p = \overline{\Psi}^{(+)}(\mathbf{P},\mathcal{H})\hat{e}\Psi^{(-)}(\mathbf{k}-\mathbf{P},\mathcal{H}(\mathbf{k}-\mathbf{P})) \qquad (2.8)$$

becomes the Heisenberg operator depending on time $\mathbf{P}=\mathbf{P}(t)$, $\mathcal{H}=\mathcal{H}(t)$. It can be expressed in terms of two-component spinors.

Substituting these results into Eq.(2.4) we find

$$V_{nm}(\mathbf{e},\mathbf{k},t) = \left\langle n|R_p(t)\mathrm{T}\exp\left[i\int_0^t \frac{kP(t)}{\omega-\mathcal{H}(t)}dt\right]\exp(i\mathbf{kr})|m\right\rangle,$$
$$R_p(t) = i\varphi_{\overline{s}}^{+}\left(A(t)-i\boldsymbol{\sigma}\mathbf{B}(t)\right)\varphi_s, \quad A=\frac{1}{2\mathcal{H}(\omega-\mathcal{H})}(\mathbf{e}(\mathbf{k}\times\mathbf{P})),$$
$$\mathbf{B}=\frac{1}{2\mathcal{H}(\omega-\mathcal{H})}\left[\mathbf{e}m\omega+(\mathbf{eP})(\mathbf{k}-2\mathbf{P})\right], \qquad (2.9)$$

where $\varphi_{\overline{s}}, \varphi_s$ are two-component spinors describing polarization of created positron and electron. Here the Coulomb gauge is used.

Substituting (2.9) into (2.3) and performing summation over states $|n>$ and $|m>$, as well as summation over spin states $\overline{s}, s$ we have the amplitude of the coherent scattering of a photon in the Coulomb field

$$T=\frac{2\pi\alpha}{\omega}i\int dt_1\int dt_2\vartheta(t_2-t_1)S_{21}$$
$$S_{21}=\mathrm{Tr}\left[m_2^{+}(t_2)\exp[-i\boldsymbol{\Delta}\mathbf{r}]m_1(t_1)\right], \quad \boldsymbol{\Delta}=\mathbf{k}_2-\mathbf{k}_1, \qquad (2.10)$$

where the operations Tr means sum of the diagonal matrix elements both in the configuration and the spin spaces,

$$m_{1,2}(t)=m(\mathbf{e}_{1,2},k_{1,2},t)=\left(A(t)-i\boldsymbol{\sigma}\mathbf{B}(t)\right)\mathrm{T}\exp\left[i\int_0^t\frac{k_{1,2}P(t)}{\omega-\mathcal{H}(t)}dt\right], \qquad (2.11)$$

functions $A(t), \mathbf{B}(t)$ are defined in (2.9). Let us remind that $\mathbf{P}(t)$ is the momentum operator in the Heisenberg picture:

$$\mathbf{P}(t)=\exp(-iHt)\mathbf{P}\exp(iHt), \quad H=\mathcal{H}+V, \qquad (2.12)$$

where $V=e\varphi$.

We present the evolution operator as

$$\exp(-iHt)=\exp(-i\mathcal{H}t)N(t), \quad N(t)=\exp(i\mathcal{H}t)\exp(-i(\mathcal{H}+V)t) \qquad (2.13)$$

Differentiating the last expression over time we find

$$\frac{dN(t)}{dt} = -i\exp(i\mathcal{H}t)V(\mathbf{r})\exp(-i(\mathcal{H}+V)t) = -iV(\mathbf{r}+\mathbf{v}t)N(t),$$

$$\mathbf{v} = \frac{\mathbf{P}}{\mathcal{H}}. \tag{2.14}$$

The solution of this differential equation for the initial condition $N(0)=1$ is

$$N(t) = \mathrm{T}\exp\left[-i\int_0^t V(\mathbf{r}+\mathbf{v}t')dt'\right], \tag{2.15}$$

If the formation length of photon scattering is much longer than the characteristic length of electron scattering ($\omega/m^2 \gg a$), one can present the dependence on time of the operator $\mathbf{P}(t)$ in (2.9) as (see e.g. [10], Sec.7.1)

$$\mathbf{P}(t) = \vartheta(-t)\mathbf{P}(-\infty) + \vartheta(t)\mathbf{P}(\infty), \quad \mathbf{P}(\pm\infty) = N^+(\pm\infty)\mathbf{P}(t)N(\pm\infty). \tag{2.16}$$

Let us introduce states convenient for further calculations

$$|i>= N^+(-\infty)|\mathbf{p}_i>, \quad |f>= N^+(\infty)|\mathbf{p}_f>,$$
$$\mathbf{P}|\mathbf{p}_i>= \mathbf{p}_i|\mathbf{p}_i>, \quad \mathbf{P}|\mathbf{p}_f>= \mathbf{p}_f|\mathbf{p}_f>. \tag{2.17}$$

The states $|i>$ and $|f>$ are the eigenvectors of the operators $\mathbf{P}(-\infty)$ and $\mathbf{P}(\infty)$ correspondingly

$$\mathbf{P}(-\infty)|i>= N^+(-\infty)\mathbf{P}N(-\infty)N^+(-\infty)|\mathbf{p}_i>= \mathbf{p}_i|i>$$
$$\mathbf{P}(\infty)|f>= N^+(\infty)\mathbf{P}N(\infty)N^+(\infty)|\mathbf{p}_f>= \mathbf{p}_f|f>. \tag{2.18}$$

The general expression for the amplitude of photon scattering (Eqs.(2.10) and (2.11)) is simplified significantly for small momentum transfers $\Delta = |\mathbf{\Delta}|$ ($\Delta \ll m,\ \Delta/\varepsilon \ll 1/\gamma$). With the accuracy $\sim \Delta/m$ we can put that the operator $\mathbf{P}$ commutes with $\exp[-i\mathbf{\Delta}\mathbf{r}]$ and substitute the vector $\mathbf{k}_2$ in the expression for $m_2(t_2)$ by the vector $\mathbf{k}_1$. Using (2.16) and the states $|i>$ and $|f>$ (2.18) for construction of the matrix element we have

$$\left\langle f|m(\mathbf{P}(t))\exp\left[-\frac{i}{2}\mathbf{\Delta}\mathbf{r}\right]|i\right\rangle = m_{fi}(\mathbf{e},\mathbf{p}(t))\left\langle f|\exp\left[-\frac{i}{2}\mathbf{\Delta}\mathbf{r}\right]|i\right\rangle,$$
$$m_{fi}(\mathbf{e},\mathbf{p}(t)) = \left[\vartheta(-t)m(\mathbf{e},\mathbf{p}_i) + \vartheta(t)m(\mathbf{e},\mathbf{p}_f)\right], \tag{2.19}$$

where we describe motion of particles of the created pair as a trajectory in "the form of an angle". Bearing in mind that the commutator

$$[r_i, v_j] = \frac{i}{\mathcal{H}}(\delta_{ij} - v_i v_j) \tag{2.20}$$

we can discard operator T in the expression for operator $N(t)$ (2.15) and with relativistic accuracy (i.e. with accuracy up to terms $\sim m/\omega$) present the operator $N(t)$ as

$$N(t) = \exp\left[-i\int_0^t V(\boldsymbol{\varrho}, z + t')dt'\right], \tag{2.21}$$

where the axis z is directed along the momentum of the initial photon $\mathbf{k}_1$, $\boldsymbol{\varrho}$ is two-dimensional vector transverse to axis z.

Using (2.17) and (2.21) we calculate the matrix element in (2.19)

$$\begin{aligned}\left\langle f\right| \exp\left[-\frac{i}{2}\boldsymbol{\Delta}\mathbf{r}\right] \left|i\right\rangle &= \int < f|\mathbf{r} >< \mathbf{r}|i > \exp\left[-\frac{i}{2}\boldsymbol{\Delta}\mathbf{r}\right] d^3r \\ &= \frac{i}{2\pi} f\left(\mathbf{q}_\perp - \frac{\boldsymbol{\Delta}_\perp}{2}\right)\delta\left(\mathbf{q}_\parallel - \frac{\boldsymbol{\Delta}_\parallel}{2}\right),\end{aligned} \tag{2.22}$$

where

$$\begin{aligned} < f|\mathbf{r} > &= < \mathbf{p}_f|N(\infty)|\mathbf{r} >= N(\infty) < \mathbf{p}_f|\mathbf{r} >, \\ < \mathbf{p}_f|\mathbf{r} > &= \frac{\exp(-i\mathbf{p}_f\mathbf{r})}{(2\pi)^{3/2}}, \\ < \mathbf{r}|i > &= < \mathbf{r}|N^+(-\infty)|\mathbf{p}_i >= N^+(-\infty) < \mathbf{r}|\mathbf{p}_i >, \\ < \mathbf{r}|\mathbf{p}_i > &= \frac{\exp(i\mathbf{p}_i\mathbf{r})}{(2\pi)^{3/2}}, \\ f(\mathbf{Q}) &= \frac{1}{2\pi i}\int \exp[i\mathbf{Q}\boldsymbol{\varrho} + i\chi(\boldsymbol{\varrho})]d^2\varrho, \\ \chi(\boldsymbol{\varrho}) &= -\int_{-\infty}^{\infty} V(\boldsymbol{\varrho}, z)dz. \end{aligned} \tag{2.23}$$

We introduced here vectors

$$\mathbf{p}_i \equiv \mathbf{p}, \quad \mathbf{p}_f = \mathbf{p} + \mathbf{q}, \tag{2.24}$$

the function $f(\mathbf{Q})$ is the scattering amplitude in the eikonal approximation.

Substituting Eq.(2.19) into T_{21} Eq.(2.10) we have

$$
\begin{aligned}
S_{21} &= \mathrm{Tr}\left[m_2^+(t_2)\exp[-i\mathbf{\Delta r}]m_1(t_1)\right] \\
&= \sum_{i,f}\mathrm{Tr}\left[m_{fi}^+(\mathbf{e}_2,\mathbf{p}(t_2))m_{fi}(\mathbf{e}_1,\mathbf{p}(t_1))\right] \\
&\quad\times\left\langle i\right|\exp\left[\frac{i}{2}\mathbf{\Delta r}\right]\left|f\right\rangle\left\langle f\right|\exp\left[-\frac{i}{2}\mathbf{\Delta r}\right]\left|i\right\rangle \\
&= \frac{\delta(\Delta_\parallel)}{(2\pi)^2}\int d^3p\ \mathrm{Tr}\left[m_{fi}^+(\mathbf{e}_2,\mathbf{p}(t_2))m_{fi}(\mathbf{e}_1,\mathbf{p}(t_1))\right] \\
&\quad\times\int d\mathbf{q}_\perp\ f\left(\mathbf{q}_\perp-\frac{\mathbf{\Delta}_\perp}{2}\right)f^*\left(\mathbf{q}_\perp+\frac{\mathbf{\Delta}_\perp}{2}\right),\quad q_\parallel=0. \qquad (2.25)
\end{aligned}
$$

2.2 Helicity amplitudes

In the above analysis we traced the transition to the expressions calculated on the trajectory of particle "in the form of an angle" in the momentum space. Precisely these trajectories determine the amplitude of the coherent photon scattering for $\omega/m^2 \gg a$ (see Eq.(2.10)):

$$
T_{21}=\frac{i\alpha}{(2\pi)^2}(2\pi\delta(\Delta_\parallel))\int\frac{d^3p}{\omega}\int d\sigma(\mathbf{q}_\perp,\mathbf{\Delta})R_{21}, \qquad (2.26)
$$

where

$$
\begin{aligned}
&d\sigma(\mathbf{q}_\perp,\mathbf{\Delta})=f\left(\mathbf{q}_\perp-\frac{\mathbf{\Delta}_\perp}{2}\right)f^*\left(\mathbf{q}_\perp+\frac{\mathbf{\Delta}_\perp}{2}\right)d\mathbf{q}_\perp, \\
&R_{21}=\int dt_1\int dt_2\vartheta(t_2-t_1)\mathrm{Tr}\left[m_{fi}^+(\mathbf{e}_2,\mathbf{p}(t_2))m_{fi}(\mathbf{e}_1,\mathbf{p}(t_1))\right] \\
&=\frac{1}{2}\int dt_1\int dt_2\vartheta(t_2-t_1)\mathcal{L}(\mathbf{e}_2,\mathbf{e}_1;\mathbf{v}_{2\perp},\mathbf{v}_{1\perp})\exp\left[-i\frac{\varepsilon}{\varepsilon'}\int_{t_1}^{t_2}k_1v(t)dt\right], \\
&\mathcal{L}(\mathbf{e}_2,\mathbf{e}_1;\mathbf{v}_{2\perp},\mathbf{v}_{1\perp})=\frac{\omega^2}{\varepsilon'^2}\Bigg[(\mathbf{e}_2^*\mathbf{n}\mathbf{v}_{2\perp})(\mathbf{e}_1\mathbf{n}\mathbf{v}_{1\perp})+\frac{m^2}{\varepsilon^2}(\mathbf{e}_2^*\mathbf{e}_1) \\
&+\frac{(\omega-2\varepsilon)^2}{\omega^2}(\mathbf{e}_2^*\mathbf{v}_{2\perp})(\mathbf{e}_1\mathbf{v}_{1\perp})\Bigg],\quad \mathbf{v}_{1,2\perp}=\mathbf{v}_\perp(t_{1,2}) \\
&v=\frac{p}{\varepsilon}=(1,\mathbf{v}),\quad \varepsilon'=\omega-\varepsilon,\quad \mathbf{n}=\frac{\mathbf{k}_1}{\omega},\quad (\mathbf{v}_{1,2\perp}\mathbf{n})=0, \qquad (2.27)
\end{aligned}
$$

where $(\mathbf{env})\equiv(\mathbf{e}(\mathbf{n}\times\mathbf{v}))$.

We calculate now R_{21} in this expression for the trajectory in "the form of an angle" (see (2.19)):

$$\mathbf{p}(t) = \mathbf{p}\ \vartheta(-t) + (\mathbf{p}+\mathbf{q})\vartheta(t). \tag{2.28}$$

We get for this case

$$\begin{aligned}
m^2 R_{21} &= (\mathbf{e}_2^*\mathbf{n}\mathbf{g})(\mathbf{e}_1\mathbf{n}\mathbf{g}) + \frac{(\omega-2\varepsilon)^2}{\omega^2}(\mathbf{e}_2^*\mathbf{g})(\mathbf{e}_1\mathbf{g}) + (\mathbf{e}_2^*\mathbf{e}_1)g_0^2;\\
\mathbf{g} &= \frac{\mathbf{p}_\perp+\mathbf{q}_\perp}{m^2+(\mathbf{p}_\perp+\mathbf{q}_\perp)^2} - \frac{\mathbf{p}_\perp}{m^2+\mathbf{p}_\perp^2},\\
g_0 &= \frac{m}{m^2+(\mathbf{p}_\perp+\mathbf{q}_\perp)^2} - \frac{m}{m^2+\mathbf{p}_\perp^2}.
\end{aligned} \tag{2.29}$$

The functions $\mathbf{g}$ and g_0 have very important properties:

$$\mathbf{g}(\mathbf{q}_\perp = 0) = 0, \quad g_0(\mathbf{q}_\perp = 0) = 0. \tag{2.30}$$

In the case of complete screening when the screening radius $a \ll \omega/m^2$ the amplitude of photon scattering T_{21} (2.26) is imaginary at any $\mathbf{\Delta}$. Taking into account that at $\mathbf{\Delta} = 0$ this amplitude is connected due to unitary relation with the known probability of pair photoproduction, see e.g. [21], [22], we will calculate the difference

$$\delta R_{21} = T_{21}(\mathbf{\Delta}) - T_{21}(0).$$

The interval $|\mathbf{q}_\perp| \leq |\mathbf{\Delta}| \ll m$ contributes into this difference and one can expand the functions $\mathbf{g}, g_0$ as a power series in $\mathbf{q}_\perp$:

$$\mathbf{g} \simeq \frac{\mathbf{q}_\perp}{m^2+\mathbf{p}_\perp^2} - \frac{2\mathbf{p}_\perp(\mathbf{q}_\perp\mathbf{p}_\perp)}{(m^2+\mathbf{p}_\perp^2)^2}, \quad g_0 \simeq -\frac{2m(\mathbf{q}_\perp\mathbf{p}_\perp)}{(m^2+\mathbf{p}_\perp^2)^2}. \tag{2.31}$$

We introduce the dimensionless variable $\mathbf{u} = \mathbf{p}_\perp/m$ and carry on averaging over angles of $\mathbf{u}$ (the azimuthal angle of the component of electron momentum in the plane which is perpendicular to the direction of initial photon $\mathbf{n}$) using formulas

$$\overline{u_i u_j} = \frac{u^2}{2}\delta_{ij}, \quad \overline{u_i u_j u_k u_l} = \frac{u^4}{8}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right), \tag{2.32}$$

where δ_{ij} is the two-dimensional Kronecker delta. Substituting Eq.(2.31) into (2.29) and using (2.32) we get

$$m^4\zeta^4 R_{21} = (\mathbf{e}_2^*\mathbf{n}\mathbf{q}_\perp)(\mathbf{e}_1\mathbf{n}\mathbf{q}_\perp) + \frac{(\varepsilon'-\varepsilon)^2}{\omega^2}(\mathbf{e}_2^*\mathbf{q}_\perp)(\mathbf{e}_1\mathbf{q}_\perp)$$
$$+\mathbf{q}_\perp^2(\mathbf{e}_2^*\mathbf{e}_1)\left[\left(1-\frac{2\varepsilon\varepsilon'}{\omega^2}\right)(\zeta^2-1)+\frac{4\varepsilon\varepsilon'}{\omega^2}(\zeta-1)\right],\ \zeta = 1+\mathbf{u}^2. \quad (2.33)$$

It is convenient to describe the process of photon scattering in terms of helicity amplitudes. We choose the polarization vectors with helicity λ

$$\mathbf{e}_\lambda = \frac{1}{\sqrt{2}}(\mathbf{e}_1 + i\lambda\mathbf{e}_2), \quad \mathbf{e}_1 = \boldsymbol{\nu} = \frac{\boldsymbol{\Delta}}{|\boldsymbol{\Delta}|}, \quad \mathbf{e}_2 = \mathbf{n}\times\boldsymbol{\nu}, \quad \lambda = \pm 1,$$
$$\mathbf{e}_\lambda\mathbf{e}_\lambda^* = 1, \quad \mathbf{e}_\lambda\mathbf{e}_{-\lambda}^* = 0, \quad \mathbf{e}_\lambda\times\mathbf{n} = i\lambda\mathbf{e}_\lambda, \quad (2.34)$$

where $\mathbf{n}$ is defined in Eq.(2.27). There are two independent helicity amplitudes:

$$M_{++} = M_{--}, \quad M_{+-} = M_{-+},$$

where the first subscript is the helicity of the initial photon and the second is the helicity of the final photon. When the initial photons are unpolarized the differential cross section of scattering summed over final photons polarization contains the combination

$$2[|M_{++}|^2 + |M_{+-}|^2]. \quad (2.35)$$

Substituting (2.34) into (2.33) we find

$$R_{++} = \frac{\mathbf{q}_\perp^2}{m^4\zeta^4}\left[\left(1-\frac{2\varepsilon\varepsilon'}{\omega^2}\right)(\zeta^2-1)+\frac{4\varepsilon\varepsilon'}{\omega^2}(\zeta-1)\right],$$
$$R_{+-} = -\frac{2\mathbf{q}_\perp^2}{m^4\zeta^4}\frac{\varepsilon\varepsilon'}{\omega^2}\cos 2\varphi, \quad \cos\varphi = \frac{\mathbf{q}\boldsymbol{\nu}}{q}. \quad (2.36)$$

We define the amplitude of the coherent photon scattering M_{21} as

$$T_{21} = \frac{\pi}{\omega}\delta(\Delta_\parallel)M_{21}, \quad (2.37)$$

then the cross section of the coherent photon scattering has a form

$$d\sigma_s = \frac{1}{2\pi\delta(\Delta_\parallel)}|T_{21}|^2\frac{d^3k_2}{(2\pi)^3} = \frac{1}{16\pi^2}|M_{21}|^2 d\Omega_2. \qquad (2.38)$$

Substituting (2.37), (2.33) and (2.36) into Eq.(2.26) we obtain for the helicity amplitudes

$$M_{\lambda\lambda'} = \frac{i\alpha}{2\pi}m^2\int_0^\omega d\varepsilon\int_1^\infty d\zeta\int d\sigma(\mathbf{q}_\perp, \mathbf{\Delta})R_{\lambda\lambda'}, \qquad (2.39)$$

where $\lambda\lambda'$ is $++$ or $+-$. For the chosen normalization of the helicity amplitudes the unitary relation with total cross section of pair photoproduction σ_p is

$$\frac{1}{\omega}\text{Im } M_{++} = \sigma(\gamma\to e^+e^-) \equiv \sigma_p. \qquad (2.40)$$

So we present the helicity amplitude M_{++} as

$$\begin{aligned} M_{++} &= \delta M_{++} + i\omega\sigma_p, \\ \delta M_{++} &= \frac{i\alpha}{2\pi}m^2\int_0^\omega d\varepsilon\int_1^\infty d\zeta\int(d\sigma(\mathbf{q}_\perp, \mathbf{\Delta}) - d\sigma(\mathbf{q}_\perp, 0))R_{++}. \end{aligned} \qquad (2.41)$$

Since the forward scattering amplitude ($\Delta = 0$) with helicity flip vanishes we have for amplitude M_{+-}

$$M_{+-} = \delta M_{+-} = \frac{i\alpha}{2\pi}m^2\int_0^\omega d\varepsilon\int_1^\infty d\zeta\int d\sigma(\mathbf{q}_\perp, \mathbf{\Delta})R_{+-}. \qquad (2.42)$$

Integrals over $\mathbf{q}_\perp$ in Eqs.(2.41) and (2.42) we denote $\overline{\delta\mathbf{q}_\perp^2}$ and $\overline{\mathbf{q}_\perp^2\cos 2\varphi}$. We calculate these integrals using the scattering amplitude $f(\mathbf{Q})$ in the eikonal approximation Eq.(2.23). We find

$$\begin{aligned} \overline{\delta\mathbf{q}^2} &\equiv Z_1 = \int\mathbf{q}_\perp^2(d\sigma(\mathbf{q}_\perp, \mathbf{\Delta}) - d\sigma(\mathbf{q}_\perp, 0)) \\ &= \int d\mathbf{q}_\perp\mathbf{q}_\perp^2\left[f^*\left(\mathbf{q}_\perp + \frac{\mathbf{\Delta}_\perp}{2}\right)f\left(\mathbf{q}_\perp - \frac{\mathbf{\Delta}_\perp}{2}\right) - f^*(\mathbf{q}_\perp)f(\mathbf{q}_\perp)\right] \\ &= 2\pi\int_0^\infty(\chi'(\varrho))^2(J_0(\Delta\varrho) - 1)\varrho d\varrho, \\ -\overline{\mathbf{q}^2\cos 2\varphi} &\equiv Z_2 = 2\pi\int_0^\infty(\chi'(\varrho))^2 J_2(\Delta\varrho)\varrho d\varrho. \end{aligned} \qquad (2.43)$$

For the screened Coulomb potential

$$U(r)=\frac{Z\alpha}{r}\exp(-r/a),\quad \chi'(\varrho)=\frac{2Z\alpha}{a}K_1\left(\frac{\varrho}{a}\right),\tag{2.44}$$

$$Z_1=-4\pi Z^2\alpha^2F_2\left(\frac{\Delta a}{2}\right),\quad F_2(z)=\frac{2z^2+1}{z\sqrt{1+z^2}}\ln\left(z+\sqrt{1+z^2}\right)-1,$$

$$Z_2=4\pi Z^2\alpha^2F_1\left(\frac{\Delta a}{2}\right),\ F_1(z)=1-\frac{1}{z\sqrt{1+z^2}}\ln\left(z+\sqrt{1+z^2}\right).$$

The functions $F_1(x)$ and $F_2(x)$ encounter in the radiation theory and in the theory of Landau-Pomeranchuk-Migdal (LPM) effect. In this derivation the following relations have been used

$$\int_0^\infty xK_1^2(x)J_2(\beta x)=-\frac{1}{\beta}\int_0^\infty(xK_1(x))^2\frac{d}{dx}\left(\frac{J_1(\beta x)}{x}\right)$$
$$=\frac{1}{2}-\frac{2}{\beta}\int_0^\infty K_0(x)K_1(x)xJ_1(\beta x)$$
$$=\frac{1}{2}+\frac{1}{\beta}\int_0^\infty\frac{d}{dx}(K_0(x))^2xJ_1(\beta x)=\frac{1}{2}-\int_0^\infty(K_0(x))^2J_0(\beta x),$$
$$\int_0^\infty xK_0^2(x)xdx=\frac{1}{2}.\tag{2.45}$$

The cross section of e^-e^+ pair photoproduction in the case of complete screening ($a\ll\omega/m^2$) to within terms $\sim m/\omega$ has a form (see e.g. Eq.(19.17) in [21])

$$\sigma_p=\frac{28Z^2\alpha^3}{9m^2}\left[\ln(ma)+\frac{1}{2}-f(Z\alpha)-\frac{1}{42}\right],\tag{2.46}$$

where

$$f(\xi)=\mathrm{Re}\left[\psi(1+i\xi)-\psi(1)\right]=\xi^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+\xi^2)},\tag{2.47}$$

here $\psi(x)$ is the logarithmic derivative of the gamma function. Substituting (2.43)-(2.46) into Eqs.(2.41) and (2.42) we obtain

$$\mathrm{Im}\ M_{\lambda\lambda'}=\frac{4Z^2\alpha^3\omega}{m^2}\int_0^1dx\int_0^1dy\ \mu_{\lambda\lambda'}f_{\lambda\lambda'},\tag{2.48}$$

where

$$\mu_{++} = 1 - 2x(1-x) + 4x(1-x)y(1-y), \quad \mu_{+-} = x(1-x)y^2;$$
$$f_{++} = -\frac{1}{2}F_2\left(\frac{\Delta a}{2}\right) + \ln(ma) + \frac{1}{2} - f(Z\alpha) - \frac{1}{42}$$
$$= \ln(ma) - \frac{2s^2+1}{2s\sqrt{1+s^2}}\ln\left(s+\sqrt{1+s^2}\right) - f(Z\alpha) + \frac{41}{42},$$
$$f_{+-} = F_1(s), \quad s = \frac{\Delta a}{2}. \qquad (2.49)$$

Here we passed to the variables $x = \varepsilon/\omega,\ y = 1/\zeta$.

2.3 The coherent photon scattering in different cases

The important property of Eq.(2.48) is that the dependence on the screening radius a originates in it from the Born approximation. In this approximation in the case of arbitrary screening the radius a enters only in the combination

$$\frac{1}{a^2} + q_\parallel^2, \quad q_\parallel = \frac{m^2\omega}{2\varepsilon\varepsilon'}(1+u^2) = \frac{q_m}{x(1-x)y}, \quad q_m = \frac{m^2}{2\omega}. \qquad (2.50)$$

Since in the course of derivation of Eq.(2.48) we did not integrate by parts over the variables x and y, we can extend Eq.(2.49) on the case of arbitrary screening making the substitution

$$\frac{1}{a} \to \sqrt{q_\parallel^2 + a^{-2}} \equiv q_{ef}, \quad s = \frac{\Delta}{2q_{ef}} \qquad (2.51)$$

Recently the amplitudes of the coherent photon scattering were derived in [11] for arbitrary interrelation between parameters in the Molière potential.

We calculate the real part of scattering amplitude using the dispersion relations method. We can apply this method directly since in the region of momentum transfer under consideration ($q_\perp \ll m$) the dependence of amplitude $T_{21} \propto M_{21}/\omega$ on $q_\parallel$ has the form (see Eqs.(2.10)-(2.15), (2.21), (2.24))

$$T(q_\parallel) = \int dt_1 \int dt_2 F(t_1,t_2)\vartheta(t_2-t_1)\exp[-iq_\parallel(t_2-t_1)]. \qquad (2.52)$$

Thus the real and imaginary part of the amplitude M_{21}/ω are connected by the Sokhotsky-Plemelj formulas

$$\text{Re } f(z) = \frac{\mathcal{P}}{\pi}\int_{-\infty}^{\infty} \frac{\text{Im } f(z')}{z'-z} dz' \tag{2.53}$$

Using this transformation and Eqs.(2.48), (2.49) we obtain

$$\text{Re } M_{\lambda\lambda'} = \frac{4Z^2\alpha^3\omega}{m^2}\int_0^1 dx \int_0^1 dy\ \mu_{\lambda\lambda'}\varphi_{\lambda\lambda'}, \tag{2.54}$$

where

$$\begin{aligned} \varphi_{\lambda\lambda'} &= \frac{1}{\pi}\int_0^{\infty}\left[f_{\lambda\lambda'}(z+q_\parallel) - f_{\lambda\lambda'}(|z-q_\parallel|)\right]\frac{dz}{z} \\ &= \frac{1}{\pi}\int_0^{\infty}\left[f_{\lambda\lambda'}(q_\parallel(z+1)) - f_{\lambda\lambda'}(q_\parallel|z-1|)\right]\frac{dz}{z} \end{aligned} \tag{2.55}$$

We consider now different limiting cases depending on interrelation between a and ω/m^2. In the case of complete screening ($a \ll \omega/m^2$) we can use directly Eqs.(2.48), (2.49). In this case the functions $f_{\lambda\lambda'}$ are independent of x and y and the corresponding integrals are

$$\int_0^1 dx \int_0^1 dy\ \mu_{++} = \frac{7}{9}, \quad \int_0^1 dx \int_0^1 dy\ \mu_{+-} = \frac{1}{18}. \tag{2.56}$$

For the scattering amplitudes we have

$$\begin{aligned} &\text{Im } M_{++} = \frac{28Z^2\alpha^3\omega}{9m^2}f_{++}, \quad \text{Im } M_{+-} = \frac{2Z^2\alpha^3\omega}{9m^2}f_{+-}, \\ &\text{Re } M_{++} = 0, \quad \text{Re } M_{+-} = 0 \end{aligned} \tag{2.57}$$

The results obtained are consistent with Eqs.(33) and (36) of [8] where calculation has been done for the Molière potential.

In the case $a \gg \omega/m^2$ (the screening radius is very large, or in other words we consider the photon scattering in the Coulomb field) we have, taking into account Eqs.(2.49), (2.51)

$$\begin{aligned} f_{++} &= \ln\frac{m}{q_\parallel} - \frac{2s_c^2+1}{s_c\sqrt{1+s_c^2}}\ln\left(s_c+\sqrt{1+s_c^2}\right) - f(Z\alpha) + \frac{41}{42}, \\ f_{+-} &= 1 - \frac{1}{s_c\sqrt{1+s_c^2}}\ln\left(s_c+\sqrt{1+s_c^2}\right), \\ s_c &= \frac{\Delta}{2q_\parallel} = \frac{\Delta\omega}{m^2}x(1-x)y. \end{aligned} \tag{2.58}$$

The expressions for amplitudes M_{++} and M_{+-} in this case are consistent with the results obtained in [5]. At $\Delta=0$ we find known result (see Eq.(8.1) in [5])

$$
\begin{aligned}
f_{++} &= \ln\left[\frac{2\omega}{m}x(1-x)y\right]-\frac{1}{2}-f(Z\alpha)+\frac{41}{42}, \qquad (2.59)\\
\varphi_{++} &= \frac{1}{\pi}\int_0^{\infty}\ln\frac{z+1}{|z-1|}\frac{dz}{z}=\frac{2}{\pi}\int_0^1\ln\frac{1+z}{1-z}\frac{dz}{z}=\frac{\pi}{2},\\
M_{++} &= i\frac{28Z^2\alpha^3\omega}{9m^2}\left[\ln\frac{2\omega}{m}-f(Z\alpha)-\frac{109}{42}-i\frac{\pi}{2}\right], \quad M_{+-}=0.
\end{aligned}
$$

At $\Delta=0$ and arbitrary interrelation between a and ω/m^2 we get

$$
f_{++}=\ln\frac{m}{q_{ef}}-\frac{1}{2}-f(Z\alpha)+\frac{41}{42}, \qquad (2.60)
$$
$$
\varphi_{++}=\frac{1}{2\pi}\int_0^{\infty}\ln\frac{\delta^2(z+1)^2+x^2(1-x)^2y^2}{\delta^2(z-1)^2+x^2(1-x)^2y^2}\frac{dz}{z}, \quad \delta=\frac{am^2}{2\omega}=\frac{m}{2\omega}\frac{a}{\lambda_c}.
$$

The photon scattering amplitude in this case for arbitrary value of parameter δ was found recently in [9].

3 Influence of the multiple scattering on the coherent scattering of a photon

3.1 Basic equations

When a photon is propagating in a medium it dissociates with probability $\propto\alpha$ into an electron-positron pair. The virtual electron and positron interact with a medium and can scatter on atoms. In this scattering the electron and positron interaction with the Coulomb field in the course of the coherent scattering of photon is involved also. There is a direct analogue with the Landau-Pomeranchuk-Migdal (LPM) effect: the influence of the multiple scattering on process of the bremsstrahlung and pair creation by a photon in a medium at high energy [12], [13]. However there is the difference: in the LPM effect the particles of electron-positron pair created by a photon are on the mass shell while in the process of the coherent scattering of photon this particles are off the mass shell, but in the high energy region (this is the only region where the influence of the multiple scattering is pronounced) the shift from the mass shell is relatively small. To include this scattering into consideration the amplitude R_{21} Eq.(2.26) should be averaged over all possible trajectories of electron and positron with the weight function $d\sigma(\mathbf{q}, \boldsymbol{\Delta})$. This operation can be performed with the aid of the distribution function averaged over the atomic positions of scatterers in the medium. This procedure was worked out in details in [17] (Sec.2), [16] (Sec.2). The amplitude of the coherent photon scattering for a photon propagating in a medium can be derived in the same way as Eqs.(2.4)-(2.6) of [17]:

$$M(\mathbf{e}_1, \mathbf{e}_2, \boldsymbol{\Delta}) = \frac{i\alpha}{(2\pi)^2 n_a} \int d^3p \exp\left(-i\frac{\varepsilon}{\varepsilon'}\tau\right) \int d\mathbf{v}' \int d\mathbf{r}' \mathcal{L}\left(\mathbf{e}_1, \mathbf{e}_2; \boldsymbol{\vartheta}', \boldsymbol{\vartheta}\right)$$

$$\times F\left(\mathbf{r}', \mathbf{v}', \tau; \mathbf{r}, \mathbf{v}\right) \exp\left[i\frac{\varepsilon'}{\varepsilon}\mathbf{k}_1(\mathbf{r}' - \mathbf{r})\right]. \qquad (3.1)$$

The distribution function $F\left(\mathbf{r}', \mathbf{v}', \tau; \mathbf{r}, \mathbf{v}\right)$ satisfies the kinetic equation

$$\frac{\partial F\left(\mathbf{r}', \mathbf{v}', \tau\right)}{\partial \tau} + \mathbf{v}' \frac{\partial F\left(\mathbf{r}', \mathbf{v}', \tau\right)}{\partial \mathbf{r}'}$$

$$= n_a \int d\sigma(\mathbf{v}', \mathbf{v}'', \boldsymbol{\Delta}) \left[F\left(\mathbf{r}', \mathbf{v}'', \tau\right) - F\left(\mathbf{r}', \mathbf{v}', \tau\right)\right], \qquad (3.2)$$

and the initial condition

$$F\left(\mathbf{r}', \mathbf{v}', 0; \mathbf{r}, \mathbf{v}\right) = \delta(\mathbf{r} - \mathbf{r}')\delta(\mathbf{v} - \mathbf{v}'),$$

here n_a is the number density of atoms in a medium and in the case of the screened Coulomb potential

$$d\sigma(\mathbf{v}', \mathbf{v}'', \boldsymbol{\Delta}) = d\sigma(\mathbf{q}, \boldsymbol{\Delta}), \quad \mathbf{q} = \varepsilon(\mathbf{v}' - \mathbf{v}). \qquad (3.3)$$

The combinations entering in Eq.(2.27) are

$$\int_{t_1}^{t_2} kv(t)dt = \omega\tau - \mathbf{k}_1(\mathbf{r}(t_2) - \mathbf{r}(t_1)) \to \omega\tau - \mathbf{k}_1(\mathbf{r}' - \mathbf{r}),$$

$$\mathcal{L}\left(\mathbf{e}_1, \mathbf{e}_2; \mathbf{v}_2, \mathbf{v}_1\right) \to \mathcal{L}\left(\mathbf{e}_1, \mathbf{e}_2; \boldsymbol{\vartheta}', \boldsymbol{\vartheta}\right). \qquad (3.4)$$

We can proceed with further calculation (integration over $\mathbf{v}'$, $\mathbf{r}'$) of (3.1) by analogy with the procedure used in Eqs.(2.7)-(2.18) of [17]. As a result we obtain for the differential cross section of the coherent photon scattering with the multiple scattering taken into account

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\left|M(\mathbf{e}_1, \mathbf{e}_2, \mathbf{\Delta})\right|^2, \tag{3.5}$$

where

$$M(\mathbf{e}_1, \mathbf{e}_2, \mathbf{\Delta}) = \frac{2i\alpha m^2\omega}{n_a}\int\frac{d\varepsilon}{\varepsilon\varepsilon'}\int_o^\infty dt\exp(-it)\Bigg[(\mathbf{e}_2^*\mathbf{e}_1)\varphi_0(0,t)$$
$$-i(\mathbf{e}_1\mathbf{n}\boldsymbol{\nabla})(\mathbf{e}_2^*\mathbf{n}\boldsymbol{\varphi}(0,t)) - i\frac{(\omega-2\varepsilon)^2}{\omega^2}(\mathbf{e}_1\boldsymbol{\nabla})(\mathbf{e}_2^*\boldsymbol{\varphi}(0,t))\Bigg]. \tag{3.6}$$

In derivation we have changed variables into $t, \boldsymbol{\varrho}$ as in Eq.(2.7) of [16]. The function $\varphi_\mu(\boldsymbol{\varrho}, t)$, $\varphi_\mu = (\varphi_0, \boldsymbol{\varphi})$ satisfies the equation

$$i\frac{\partial\varphi_\mu}{\partial t} = \mathcal{H}\varphi_\mu, \quad \mathcal{H} = \mathbf{p}^2 - iV(\boldsymbol{\varrho}, \mathbf{\Delta}), \quad \mathbf{p} = -i\boldsymbol{\nabla}_{\boldsymbol{\varrho}}, \tag{3.7}$$

$$V(\boldsymbol{\varrho}, \mathbf{\Delta}) = \frac{2\varepsilon\varepsilon' n_a}{\omega m^4}\int(1-\exp(i\mathbf{q}\boldsymbol{\varrho}))\, f\left(\mathbf{q}_\perp - \frac{\mathbf{\Delta}_\perp}{2m}\right) f^*\left(\mathbf{q}_\perp + \frac{\mathbf{\Delta}_\perp}{2m}\right) d^2q,$$

with the initial conditions

$$\varphi_0(\boldsymbol{\varrho}, 0) = \delta(\boldsymbol{\varrho}), \quad \boldsymbol{\varphi}(\boldsymbol{\varrho}, 0) = \mathbf{p}\delta(\boldsymbol{\varrho}). \tag{3.8}$$

The potential $V(\boldsymbol{\varrho}, 0) \equiv V(\boldsymbol{\varrho})$ was used in the theory of the LPM effect (see [16] (Sec.2)):

$$V(\boldsymbol{\varrho}) = Q\boldsymbol{\varrho}^2\left(L_1 + \ln\frac{4}{\boldsymbol{\varrho}^2} - 2C\right), \; Q = \frac{2\pi Z^2\alpha^2\varepsilon\varepsilon' n_a}{m^4\omega}, \; L_1 = \ln\frac{a_{s2}^2}{\lambda_c^2},$$

$$\frac{a_{s2}}{\lambda_c} = 183Z^{-1/3}\mathrm{e}^{-f}, \; f = f(Z\alpha) = (Z\alpha)^2\sum_{k=1}^{\infty}\frac{1}{k(k^2+(Z\alpha)^2)}, \tag{3.9}$$

where $C = 0.577216\ldots$ is Euler's constant. We can restrict ourselves to calculation of the difference of the potentials only:

$$\begin{aligned}\Delta V(\boldsymbol{\varrho}, \mathbf{\Delta}) &= V(\boldsymbol{\varrho}, \mathbf{\Delta}) - V(\boldsymbol{\varrho}) \\ &= \frac{2\varepsilon\varepsilon' n_a}{\omega m^4}\int(1-\exp(i\mathbf{q}\boldsymbol{\varrho}))\,(d\sigma(\mathbf{q}, \mathbf{\Delta}) - d\sigma(\mathbf{q}, 0))d^2q,\end{aligned} \tag{3.10}$$

where the cross section $d\sigma(\mathbf{q},\mathbf{\Delta})$ is defined in Eq.(2.26). In the above formulas $\boldsymbol{\varrho}$ is the two-dimensional space of the impact parameters measured in the Compton wavelength λ_c which is conjugate to the two-dimensional space of the transverse momentum transfers $\mathbf{q}$ measured in the electron mass m.

3.2 Amplitudes of coherent photon scattering under influence of the multiple scattering

When parameter $\Delta \ll m$ the main contribution into the integral in (3.11) gives the region $|\mathbf{q}| \leq \Delta/m \ll 1$. For $\varrho \leq 1$ one can expand the integrand in (3.11) in powers of $\mathbf{q}\boldsymbol{\varrho}$. Using Eq.(2.23) we get

$$\begin{aligned}
\Delta V(\boldsymbol{\varrho},\mathbf{\Delta}) &= \frac{\varepsilon\varepsilon' n_a}{\omega m^4}\int \frac{(\mathbf{x}\boldsymbol{\varrho})^2}{x^2}(\chi'(\mathbf{x}))^2(\exp(-i\boldsymbol{\delta}\mathbf{x})-1)d^2x,\\
\chi(\mathbf{x}) &= \int_{-\infty}^{\infty}\frac{Z\alpha}{r}\exp\left(-\frac{r}{a_s}\right) = 2Z\alpha K_0\left(x\frac{a_s}{\lambda_c}\right), \quad x=|\mathbf{x}|\\
\frac{a_s}{\lambda_c} &= 111Z^{-1/3} = \frac{a_{s2}}{\lambda_c}\exp\left(f-\frac{1}{2}\right), \quad \boldsymbol{\delta}=\frac{\mathbf{\Delta}}{m}, \qquad (3.11)
\end{aligned}$$

where $K_0(z)$ is the modified Bessel function. Integrating ΔV over azimuthal angle of the vector $\mathbf{x}$ we obtain

$$\begin{aligned}
\Delta V(\boldsymbol{\varrho},\mathbf{\Delta}) &= -2Q\boldsymbol{\varrho}^2\int_0^{\infty}K_1^2(x)\left(1-J_0(\beta x)+J_2(\beta x)\cos 2\varphi\right)x\,dx\\
&= -2Q\boldsymbol{\varrho}^2\int_0^{\infty}\left[K_0^2(x)\cos 2\varphi+K_1^2(x)\right]\left(1-J_0(\beta x)\right)x\,dx\\
&= Q\boldsymbol{\varrho}^2\left[F_1\left(\frac{\beta}{2}\right)\cos 2\varphi+F_2\left(\frac{\beta}{2}\right)\right], \quad \beta=\Delta a_s, \qquad (3.12)
\end{aligned}$$

where φ is the angle between the vectors $\boldsymbol{\varrho}$ and $\mathbf{\Delta}$, $J_n(z)$ is the Bessel function, the functions $F_1(z)$ and $F_2(z)$ are defined in Eq.(2.45). Some details of calculation are given after Eq.(2.45).

Summing $\Delta V(\boldsymbol{\varrho},\mathbf{\Delta})$ (3.12) and $V(\boldsymbol{\varrho})$ (3.9) we get

$$V(\boldsymbol{\varrho},\mathbf{\Delta}) = Q\boldsymbol{\varrho}^2\left(L_1+\ln\frac{4}{\boldsymbol{\varrho}^2}-2C-F_1\left(\frac{\beta}{2}\right)\cos 2\varphi-F_2\left(\frac{\beta}{2}\right)\right). \qquad (3.13)$$

Starting from Eq.(3.6) and advancing as in Sec.II (Eqs.(2.2)-(2.12)) of [17] we find

$$
\begin{aligned}
M_{++} &= \frac{2i\alpha m^2\omega}{n_a}\int\frac{d\varepsilon}{\varepsilon\varepsilon'}\left\langle 0|s_1\left(G_s^{-1}-G_0^{-1}\right)+s_2\mathbf{p}\left(G_s^{-1}-G_0^{-1}\right)\mathbf{p}|0\right\rangle,\\
M_{+-} &= \frac{2i\alpha m^2\omega}{n_a}\int\frac{d\varepsilon}{\varepsilon\varepsilon'}\left\langle 0|s_3\left(\mathbf{e}_-^*\mathbf{p}\right)\left(G_s^{-1}-G_0^{-1}\right)\left(\mathbf{e}_+\mathbf{p}\right)|0\right\rangle, \qquad (3.14)
\end{aligned}
$$

where

$$
s_1=1,\ s_2=\frac{\varepsilon^2+\varepsilon'^2}{\omega^2},\ s_3=\frac{2\varepsilon\varepsilon'}{\omega^2},\quad G_s=\mathcal{H}+1,\ G_0=\mathbf{p}^2+1, \quad (3.15)
$$

$\mathcal{H}$ is defined in Eq.(3.7). In derivation it is convenient to use properties of $\mathbf{e}_\lambda$ given in (2.34). Note, that coefficients s_1, s_2 coincide with the same coefficients in the theory of the LPM effect for pair creation by a photon [20]. As in [16] (Eq.(2.17)) and [20] (Eq.(2.5)) we split the potential $V(\boldsymbol{\varrho},\boldsymbol{\Delta})$ as

$$
\begin{aligned}
V(\boldsymbol{\varrho},\boldsymbol{\Delta}) &= V_c(\boldsymbol{\varrho},\boldsymbol{\Delta})+v(\boldsymbol{\varrho},\boldsymbol{\Delta}),\quad V_c(\boldsymbol{\varrho},\boldsymbol{\Delta})=q_p\boldsymbol{\varrho}^2,\quad q_p=QL_s,\\
L_s(\varrho_c) &= \ln\frac{a_{s2}^2}{\lambda_c^2\varrho_c^2}-F_2\left(\frac{\beta}{2}\right),\\
v(\boldsymbol{\varrho},\boldsymbol{\Delta}) &= -\frac{q_p\boldsymbol{\varrho}^2}{L_s}\left(2C+\ln\frac{\boldsymbol{\varrho}^2}{4\varrho_c^2}+F_1\left(\frac{\beta}{2}\right)\cos 2\varphi\right). \qquad (3.16)
\end{aligned}
$$

In accordance with such division of the potential we will write the photon scattering amplitudes in the form

$$
M_{++}=M_{++}^c+M_{++}^{(1)},\quad M_{+-}=M_{+-}^c+M_{+-}^{(1)}, \qquad (3.17)
$$

where M_{++}^c is the contribution of the potential $V_c(\boldsymbol{\varrho},\boldsymbol{\Delta})$ and $M_{++}^{(1)}$ is the first correction to the scattering amplitude due to the potential $v(\boldsymbol{\varrho},\boldsymbol{\Delta})$ (see [16], Eqs.(2.28) and (2.33), and [20] Eqs.(2.10) and (2.15)). Since dependence of the potential $V(\boldsymbol{\varrho},\boldsymbol{\Delta})$ on $\boldsymbol{\varrho}$ is the same as in [20] and the expression for M_{++} (3.14) formally coincides (up to external factor) with Eq.(2.2) in [20] the result for M_{++} can be taken from [20] (Eqs.(2.10) and (2.15)) with substitution $q\to q_p,\ L_c\to L_s$.

The combination of photon polarization vectors entering in the amplitude M^c_{+-} (3.14) gives in evaluation the expression $(\mathbf{e}^*_-\boldsymbol{\nabla})\,(\mathbf{e}_+\boldsymbol{\varrho})=0$. Substituting this combination in calculation of the first correction $M^{(1)}_{+-}$ (see Eqs.(2.31) and (2.32) of [16]) we obtain expression of type $(\mathbf{e}^*_-\boldsymbol{\varrho})\,(\mathbf{e}_+\boldsymbol{\varrho})=\boldsymbol{\varrho}^2\exp(2i\varphi)$. So in integration over the azimuthal angle φ only the term with $\cos 2\varphi$ survives in formula for $v(\boldsymbol{\varrho},\boldsymbol{\Delta})$. As a result we get for the main term

$$
\begin{aligned}
M^c_{++} &= \frac{\alpha m^2\omega}{2\pi n_a}\int\frac{d\varepsilon}{\varepsilon\varepsilon'}\Phi_s(\nu_s), \quad M^c_{+-}=0, \\
\Phi_s(\nu_s) &= \nu_s\int_0^\infty dt e^{-it}\left[s_1\left(\frac{1}{\sinh z}-\frac{1}{z}\right)-i\nu_s s_2\left(\frac{1}{\sinh^2 z}-\frac{1}{z^2}\right)\right] \\
&= s_1\left(\ln p-\psi\left(p+\frac{1}{2}\right)\right)+s_2\left(\psi(p)-\ln p+\frac{1}{2p}\right), \quad (3.18)
\end{aligned}
$$

where $\nu_s=2\sqrt{iq_s}, z=\nu_s t,\ p=i/(2\nu_s),\ \psi(x)$ is the logarithmic derivative of the gamma function.

The first corrections to the amplitudes are defined by

$$
\begin{aligned}
M^{(1)}_{++} &= -\frac{\alpha m^2\omega}{4\pi n_a L_s}\int\frac{d\varepsilon}{\varepsilon\varepsilon'}F_s(\nu_s), \\
M^{(1)}_{+-} &= \frac{\alpha m^2\omega}{4\pi n_a L_s}F_1\left(\frac{\beta}{2}\right)\int\frac{d\varepsilon}{\varepsilon\varepsilon'}F_3(\nu_s), \\
F_s(\nu_s) &= \int_0^\infty\frac{dze^{-it}}{\sinh^2 z}\left[s_1 f_1(z)-2is_2 f_2(z)\right], \\
f_2(z) &= \frac{\nu_s}{\sinh z}\left(f_1(z)-\frac{g(z)}{2}\right), \\
f_1(z) &= \left(\ln\varrho_c^2+\ln\frac{\nu}{i}-\ln\sinh z-C\right)g(z)-2\cosh zG(z), \\
F_3(\nu_s) &= i\nu_s s_3\int_0^\infty\frac{dze^{-it}}{\sinh^3 z}g(z)=\frac{s_3}{2}\left[1+\frac{1}{2p}-p\,\zeta(2,p)\right], \\
g(z) &= z\cosh z-\sinh z, \quad t=t_1+t_2,\ z=\nu_s t,\ p=\frac{i}{2\nu_s}, \\
G(z) &= \int_0^z(1-y\coth y)dy \\
&= z-\frac{z^2}{2}-\frac{\pi^2}{12}-z\ln\left(1-e^{-2z}\right)+\frac{1}{2}\mathrm{Li}_2\left(e^{-2z}\right), \quad (3.19)
\end{aligned}
$$

here $\mathrm{Li}_2(x)$ is the Euler dilogarithm, $\zeta(s,a)$ is the generalized Riemann zeta function. Use of the given representations of functions $F_3(\nu_s)$ and $G(z)$ simplifies the numerical calculation.

In the region of the weak effect of scattering ($|\nu_s| \ll 1,\ \varrho_c = 1$) the interval $z \ll 1$ contributes into the integrals (3.18), (3.19). In this region

$$\Phi_s(\nu) \simeq s_1\left(\frac{\nu^2}{6}+\frac{7\nu^4}{60}+\frac{31\nu^6}{126}\right)+s_2\left(\frac{\nu^2}{3}+\frac{2\nu^4}{15}+\frac{16\nu^6}{63}\right),$$

$$F_s(\nu) \simeq \frac{\nu^2}{9}\left(s_2-s_1\right), \quad F_3(\nu) \simeq \frac{\nu^2}{3}\left(1+\frac{4}{5}\nu^2+\frac{16}{7}\nu^4\right)s_3. \tag{3.20}$$

Substituting these expressions into (3.18), (3.19) and integrating over ε we obtain

$$\begin{aligned} M_{++} &= M_{++}^c + M_{++}^{(1)} = i\frac{14Z^2\alpha^3\omega}{9m^2}\left[L_{s1}\left(1+i\frac{59\omega}{175\omega_e}\frac{L_{s1}}{L_1}\right.\right. \\ &- \left.\left.\frac{3312}{2401}\left(\frac{\omega}{\omega_e}\frac{L_{s1}}{L_1}\right)^2\right)-\frac{1}{21}\right], \\ M_{+-} &= M_{+-}^{(1)} = i\frac{2Z^2\alpha^3\omega}{9m^2}F_1\left(\frac{\beta}{2}\right) \\ &\times \left(1+i\frac{16\omega}{25\omega_e}\frac{L_{s1}}{L_1}-\frac{384}{245}\left(\frac{\omega}{\omega_e}\frac{L_{s1}}{L_1}\right)^2\right), \end{aligned} \tag{3.21}$$

here

$$L_{s1} = \ln\frac{a_{s2}^2}{\lambda_c^2} - F_2\left(\frac{\beta}{2}\right), \quad \omega_e = \frac{m}{2\pi Z^2\alpha^2\lambda^3 n_a L_1},$$

$$L_{s1} - \frac{1}{21} = 2\left[\ln\frac{a_s}{\lambda_c} - \frac{1}{2}\left(F_2\left(\frac{\Delta a_s}{2}\right)+1\right) - f(Z\alpha) + \frac{41}{42}\right], \tag{3.22}$$

L_1 is defined in (3.9). The characteristic energy ω_e encountered in analysis of influence of the multiple scattering on the probability of pair photoproduction [20], in gold ω_e=10.5 TeV. The amplitudes (3.21) coincide with the formulas (2.49) and (2.57) if we neglect the terms $\propto \omega/\omega_e$ and $(\omega/\omega_e)^2$. The terms $\propto \omega/\omega_e$ define the real part of the scattering amplitudes while the terms $\propto (\omega/\omega_e)^2$ are the corrections to the imaginary part.

If the parameter $|\nu_s| > 1$ then the value of ϱ_c is defined by the equations (see (3.16))

$$4Q\varrho_c^4 L_s(\varrho_c) = 1, \quad L_s \simeq L_{s3} + \frac{1}{2}\ln\frac{4\varepsilon\varepsilon'}{\omega^2}, \quad L_{s3} = L_{s2} + \frac{1}{2}\ln L_{s2},$$
$$L_{s2} = L_{s1} + \frac{1}{2}\ln\frac{\omega}{\omega_e} = 2\ln(a_s\Delta_s) + 1 - F_2\left(\frac{\Delta a_s}{2}\right) - 2f(Z\alpha),$$
$$\Delta_s^4 = 2\pi Z^2\alpha^2\omega n_a, \quad \nu_s^2 = i\frac{\omega}{\omega_e}\frac{4\varepsilon\varepsilon'}{\omega^2}\left(L_{s3} + \frac{1}{2}\ln\frac{4\varepsilon\varepsilon'}{\omega^2}\right). \qquad (3.23)$$

We consider now the region of the strong effect of scattering ($|\nu_s| \gg 1$). We restrict to the main terms of the decomposition only, then we can substitute the exponential $\exp(-it)$ for 1 in integrand in Eqs.(3.18), (3.19). Performing the integration we find

$$\Phi_s(\nu_s) \simeq is_2\nu_s, \quad F_s(\nu_s) \simeq -is_2\nu_s\left(\ln 2 - C - i\frac{\pi}{4}\right), \quad F_3(\nu_s) \simeq \frac{i}{2}s_3\nu_s. \qquad (3.24)$$

Note, that the next terms of the decomposition can be obtained using the results of Appendix A of [20]. Substituting these expressions into Eqs.(3.17)-(3.19) we obtain

$$\begin{aligned} M_{++} &\simeq (i-1)\frac{3\alpha m^2}{4\sqrt{2}n_a}\sqrt{\frac{\omega}{\omega_e}\frac{L_{s3}}{L_1}}\left[1 - \frac{1}{4L_{s3}}\left(2C + \frac{1}{3} + i\frac{\pi}{2}\right)\right] \\ &= (i-1)\frac{3\pi Z^2\alpha^3\omega}{2\sqrt{2}\Delta_s^2}\sqrt{L_{s3}}\left[1 - \frac{1}{4L_{s3}}\left(2C + \frac{1}{3} + i\frac{\pi}{2}\right)\right], \\ M_{+-} &\simeq (i-1)\frac{\pi Z^2\alpha^3\omega}{8\sqrt{2}\Delta_s^2}\frac{1}{\sqrt{L_{s3}}}F_1\left(\frac{\beta}{2}\right). \end{aligned} \qquad (3.25)$$

It is seen from these equations that in the case of strong scattering the real and imaginary parts of the amplitudes are equal (if we neglect the term $\propto 1/L_{s3}$ in M_{++}). Moreover the amplitudes (3.25) don't depend on the electron mass m. In place of it we have the value Δ_s. Notice that the asymptotic expansions (3.25) are valid for $\Delta \ll \Delta_s$ ($\Delta_s > m$). In the interval $\Delta_s \gg \Delta \gg a_s^{-1}$ we have

$$F_2 \simeq 2\ln(\Delta a_s) - 1, \quad F_1 = 1, \quad L_{s2} = 2\left[\ln\frac{\Delta_s}{\Delta} + 1 - f(Z\alpha)\right]. \quad (3.26)$$

4 Conclusion

In this paper we considered the influence of a medium on the process of the coherent photon scattering illustrated in Fig.1 and Fig.2, where Im M_{++} and Re M_{++} as well as Im M_{+-} and Re M_{+-} are given as

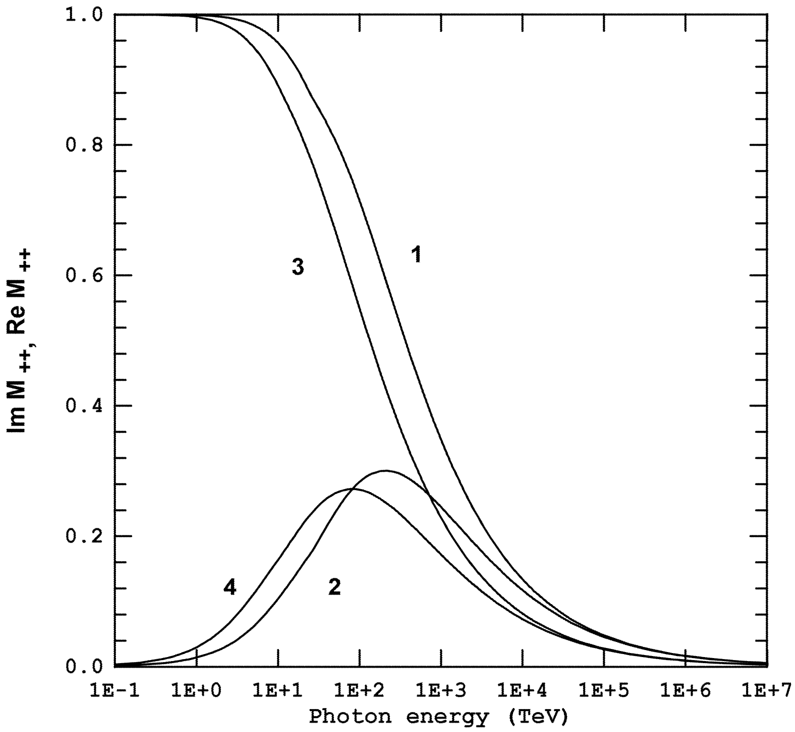

Figure 1: The amplitude M_{++} of the coherent photon scattering in gold under influence of the multiple scattering at the different momentum transfer to the photon Δ in terms of the amplitude ImM_{++} (2.57) calculated for the screened Coulomb potential.
– Curve 1 is ImM_{++} for Δ =0.4435 m.
– Curve 2 is ReM_{++} for Δ =0.4435 m.
– Curve 3 is ImM_{++} for Δ =0.0387 m.
– Curve 4 is ReM_{++} for Δ =0.0387 m.

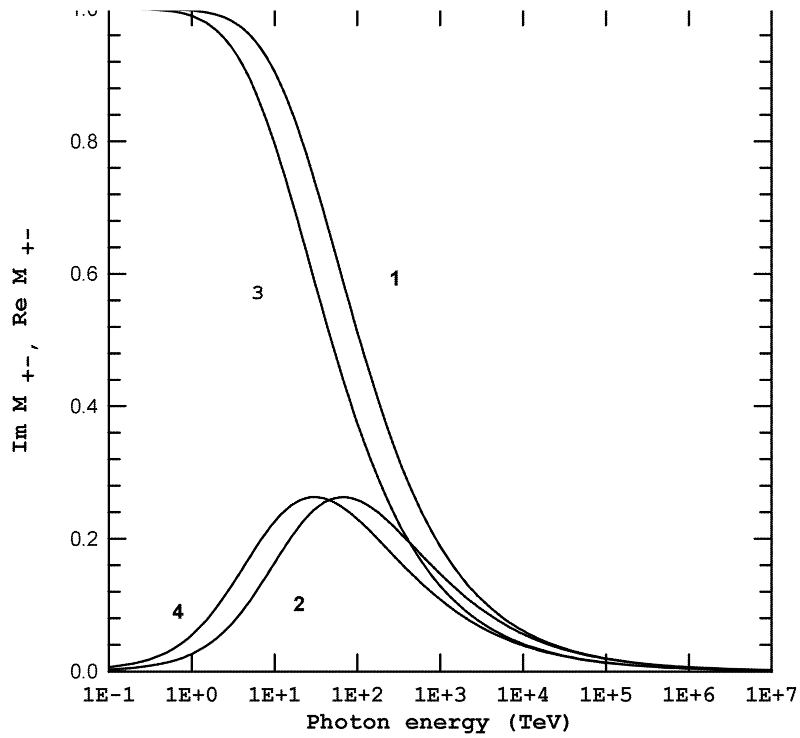

Figure 2: The same as in Fig.1 but for the amplitude M_{+-} in terms of the amplitude ImM_{+-} (2.57) calculated for the screened Coulomb potential.

a function of photon energy ω in gold. This influence is due to the multiple scattering of electron and positron of the virtual pair on the formation length of the process (see Eqs.(1.1) and (1.2))

$$l_f = \frac{\omega}{2(q_s^2 + m^2 + \Delta^2)}. \tag{4.1}$$

In the region $\Delta^2 \ll q_s^2 + m^2$ this formation length is independent of Δ and it value is coincide practically with the formation length of pair creation by a photon l_c [20]. There is some difference connected with the logarithmic dependence of ν_s^2 value on Δ^2:

$$|\nu_s^2| = \frac{q_s^2}{m^2} = \frac{4\pi Z^2\alpha^2}{m^2} n_a l_f \int_{q^2_{min}}^{q^2_{max}} \frac{dq^2}{q^2} \tag{4.2}$$

where $q^2_{max} = m^2 + q_s^2$ and $q^2_{min} = \Delta^2 + a^{-2}$, a is the screening radius of atom (1.2). This defines the weak (logarithmic) dependence of ImM_{++} on Δ in the region $\Delta < \sqrt{q_s^2 + m^2}$. This can be seen in Fig.1 where the curves 1 and 3 represent behavior of Im M_{++} for Δ=0.4435 m and Δ=0.0387 m respectively. For lower value of Δ the minimal momentum transfer q_{min} (1.2) diminishes thereby the interval of contributing the multiple scattering angles increases, so the multiple scattering affects the photon scattering amplitude at a lower energy (and smaller formation length). Because of this the curve 3 is shifted to the left respect the curve 1. Note that the curve 3 ($\Delta^{-1} \sim a_{s2}$) is very similar to the curve 2 in Fig.2 of [20] which represents the behavior of the probability of pair photoproduction in gold vs photon energy.

The new property of influence of a medium is the appearance of the real part of the coherent photon scattering amplitudes at high energy ω. In the region $\omega \ll \omega_e$ the value of ReM is small accordingly (3.21). In the asymptotic region $\omega \gg \omega_e$ we have -Re M = Im M according to (3.25). This property is seen clearly in Figs.1,2. So the value of -Re M is small at low and very high energies of photon. At intermediate energies the value of -Re M have the maximum at $\omega \simeq$220 TeV for Δ =0.4435 m and at $\omega \simeq$80 TeV for Δ =0.0387 m. In Fig.2 the same curves are shown for amplitude M_{+-}. These curves are very similar to curves in Fig.1. The curves in both figures are normalized to imaginary part of the corresponding amplitude in the absence of the multiple scattering. The ratio these imaginary parts one can find from Eqs.(3.21), (3.22):

$$r = \frac{\text{Im } M_{+-}}{\text{Im } M_{++}} = \frac{1}{14} F_1\left(\frac{\Delta a_s}{2}\right)$$
$$\times \left[\ln\frac{a_s}{\lambda_c} - \frac{1}{2}\left[F_2\left(\frac{\Delta a_s}{2}\right) + 1\right] - f(Z\alpha) + 1\right]^{-1}. \tag{4.3}$$

This ratio is $r = 0.04435$ for Δ=0.4435 m and $r = 0.003018$ for Δ=0.0387 m.

In numerical calculation of amplitude M_{++} we neglect corrections of the order $1/L_s$. These corrections are quite small, e.g. for Δ=0.0387 m they are of the order of a few percent.

We estimate now the integral (over $\mathbf{\Delta}$) cross section of the coherent photon scattering at $\omega \gg \omega_e$. From previous section (see Eqs.(3.25)-(3.26)) we have for module squared of the amplitude at $\Delta < \Delta_s$ (see also Introduction)

$$|M|^2 \simeq \frac{9\pi^2 Z^4\alpha^6\omega^2}{\Delta_s^4}\left(\ln\frac{\Delta_s}{\Delta}+1\right), \tag{4.4}$$

where value Δ_s is defined in (3.23). It is seen from this formula that in the considered region the differential probability of the coherent photon scattering depends weakly (logarithmically) on medium density n_a.

$$dW_{ph}(\Delta < \Delta_s) = \frac{1}{16\pi^2}|M|^2 n_a d\Omega = \frac{|M|^2 n_a}{16\pi^2\omega^2}d\mathbf{\Delta}. \tag{4.5}$$

In the interval $\Delta \gg \Delta_s$ the influence of the multiple scattering on the coherent scattering process is rather weak because the probability of transfer to a medium of the momentum $\geq |\mathbf{\Delta}|$ on the formation length $l_f = \omega/(2\Delta^2)$ appears to be small:

$$W_s(\Delta) = \frac{4\pi Z^2\alpha^2}{\Delta^2}n_a\frac{\omega}{2\Delta^2} = \frac{\Delta_s^4}{\Delta^4} \ll 1. \tag{4.6}$$

In this interval the amplitudes of the coherent photon scattering behave as $1/\Delta^2$ and it doesn't contribute to the integral cross section. So the interval $\Delta \leq \Delta_s$ contributes only. Taking into account Eqs.(4.4),(4.5) we have for estimate of the integral cross section of the coherent photon scattering

$$\sigma_{ph}(\omega \gg \omega_e) \sim \frac{\Delta_s^2}{16\pi\omega^2}|M(\Delta \sim \Delta_s)|^2 \sim \frac{Z^4\alpha^6}{\Delta_s^2} = \frac{Z^3\alpha^5}{\sqrt{2\pi\omega n_a}}. \tag{4.7}$$

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