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PROPAGATION OF POLARIZED HIGH ENERGY PHOTONS IN CRYSTALS

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photons in crystals

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Abstract

The density matrix is obtained for short-wavelength photons passing through a medium. For sufficiently large thicknesses of a target this matrix is expressed by the forward scattering amplitude of a photon. For multi-GeV photons scattering via virtual $e^+ e^-$ pairs is the most important process. Its amplitude in a crystal is calculated. The results obtained are used to find the optimal thicknesses, orientations and types of crystals for the circular - to - linear polarization conversion process.

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1 Introduction

High-energy circularly polarized photons, if available, would help in solving several important problems in high energy physics like that of spin-crisis (see e.g.[1] and literature cited there). If we start with linearly polarized multi-GeV photons, then they can be converted into circularly polarized ones provided that the suggestion of Cabibbo and collaborators [2] to use a specially chosen crystalline plate for this purpose is true. This idea is now under experimental investigation at CERN within NA59 project, where the linearly polarized photons are also produced by means of a crystalline target from unpolarized electrons passing through it. From theoretical point of view, the conversion process itself is very interesting since for multi-GeV photons it is caused by the polarization of vacuum in the presence of the periodical electric field of a crystal. This phenomenon has not been investigated so far.

To develop a description of the photon propagation, we start in Sec.2 from the Maxwell equations taking into account a current induced in a medium by the incident wave in a rather general form. When the wavelength of a photon is much shorter than any other characteristic distance scale of the problem, the parameters of a wave packet change slowly while it propagates in a medium. Using this fact,we obtain the solution to the Maxwell equations in the short-wavelength approximation, which is the analog of the eikonal approximation in the fast particle scattering theory. The approximation used becomes valid starting with a relatively small photon energy $\omega \gtrsim m$,

our solution with that of [3] obtained for an amorphous medium, we find that a matrix, describing a change of the polarization and intensity of a wave packet is deeply connected with the forward scattering amplitude. At least, it is the case for large thicknesses, when this change becomes really prominent. In the multi-GeV energy region of interest, the photon scattering via virtual $e^+ e^-$ pairs is the only process relevant to the problem. Its amplitude for a separate atom is well known (see [4] for the forward scattering amplitude). In crystals, along with this incoherent (amorphous like) contribution to the amplitude, the coherent one, caused by the periodicity of a lattice is present. The latter is calculated in Sec.3 by means of the so called quasiclassical operator method. The details of this method along with many applications can be found in [5]. We use the results obtained to consider the circular - to - linear polarization conversion process. Optimal (according to the criterion formulated in [6]) thicknesses and orientations are found for diamond, silicon and germanium crystals for photon energy $\omega \sim 100\, GeV$.

2 Propagation of short-wavelength photons

For the electric field $\mathbf{E}$ of a wave we obtain from the Maxwell equations

$$\Big(\frac{\partial^2}{\partial t^2} - \nabla^2 \Big) E_i(x) = 4\pi \int d^4x' \, R_{im}(x) \, \chi_{mj}(x, x') \, E_j(x') \,, \qquad (1)$$

where the operator

$$R_{im}(x) = \frac{\partial^2}{\partial x_i \partial x_m} - \delta_{im} \frac{\partial^2}{\partial t^2} \,.$$

When deriving eq.(1), the following relationship of the electric $\mathbf{E}$ and the induced $\mathbf{D}$ fields was used:

$$D_i(x) = E_i(x) + 4\pi \int d^4x' \, \chi_{ij}(x, x') \, E_j(x') \,, \qquad (2)$$

as well as the condition $\partial D_i(x) \,/\, \partial x_i = 0$. By definition, differentiating the integral in eq.(2) over t, we obtain $\mathbf{j}(x)$ being a density of the

difference $t - t'$, the equation for the vector potential $\mathbf{A}$ is essentially the same as eq.(1) for the electric field.

In the vacuum the solution to eq.(1), satisfying the condition $\partial E_i(x)/\partial x_i = 0$, reads

$$\mathbf{E}_0\,(x) \;=\; \int d\mathbf{k}\, g(\mathbf{k} - \mathbf{k}_0)\, \mathbf{e}^{\perp}(\mathbf{k})\, e^{-ikx}\,, \tag{3}$$

where x and k are 4-vectors: $x \equiv (t, \mathbf{r})$, $k \equiv (\omega\,, \mathbf{k}\,)$ with $\mathbf{k} = \omega\boldsymbol{\nu}\,, \boldsymbol{\nu}^2 = 1$, and $\mathbf{e}^{\perp}(\mathbf{k})$ being an arbitrary vector, perpendicular to $\mathbf{k}$. If we introduce $\delta_{ij}^{\perp} = \delta_{ij} - \nu_i\,\nu_j$, then $e_i^{\perp}(\mathbf{k}) = \delta_{ij}^{\perp}\, e_j$. We assume that the function g in eq.(3) vanishes except of the narrow region where $\mid \mathbf{k} - \mathbf{k}_0 \mid \ll \mid \mathbf{k}_0, \mid$. So, the incident wave is a wave packet propagating along $\mathbf{k}_0$. Let it encounter a medium at some boundary. We try to satisfy eq.(1) in the medium using the field $\mathbf{E}$ in the form of

$$E_i(x) \;=\; \int d\mathbf{k}\, g(\mathbf{k} - \mathbf{k}_0)\, F_{ij}(\mathbf{r}\,, \mathbf{k})\, e_j^{\perp}(\mathbf{k})\, e^{-ikx}, \tag{4}$$

with $F_{ij}(\mathbf{r}\,, \mathbf{k}) = \delta_{ij}$ on the boundary. Assuming that ω_0 is sufficiently large, we expect that the function $F_{ij}(\mathbf{r}\,, \mathbf{k})$ varies very slowly with respect to $\mathbf{r}$. Substituting eq.(4) into eq.(1) and neglecting the term $\nabla^2 F$ (keeping only $\mathbf{k}\partial/\partial\mathbf{r}\, F$), we obtain

$$\begin{aligned} 0 \;=\; \int d\mathbf{k}\, g(\mathbf{k} - \mathbf{k}_0) \Big\{ 2ie^{-ikx} k_l \frac{\partial F_{ij}(\mathbf{r}\,, \mathbf{k})}{\partial r_l} + \\ + 4\pi \int d^4x'\, R_{im}(x)\, \chi_{ml}(x, x')\, F_{lj}(\mathbf{r}'\,, \mathbf{k})\, e^{-ikx'} \Big\}\, e_j^{\perp}(\mathbf{k}). \end{aligned} \tag{5}$$

The integral $\int d^4x'$ in eq.(5) can be easily taken if we use the Fourier-transforms:

$$\begin{aligned} \chi_{ml}(x, x') \;&=\; \int \frac{d^4k_1 d^4k_2}{(2\pi)^8}\, e^{i(k_2x' - k_1x)} \chi_{ml}(k_1, k_2)\,, \\ F_{lj}(\mathbf{r}'\,, \mathbf{k}) \;&=\; \int \frac{d\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\mathbf{r}'} F_{lj}(\mathbf{q}\,, \mathbf{k})\,. \end{aligned} \tag{6}$$

$$\chi_{ml}(k_1,k_2) \;=\; (2\pi)^4 \sum_{\mathbf{Q}} c_{ml}(\mathbf{Q}\,,k_1)\delta(k_2 \;-\; k_1 \;-\; Q), \qquad (7)$$

where $Q \equiv (0,\mathbf{Q})$ is a reciprocal lattice vector. Using this representation, we obtain from eq.(5)

$$0 = \int d\mathbf{k}\, g(\mathbf{k} \;-\; \mathbf{k}_0)\, e^{-ikx} \Big\{ 2ik_l \frac{\partial F_{ij}(\mathbf{r}\,,\mathbf{k})}{\partial r_l} + \qquad (8)$$

$$+4\pi\omega^2\delta^{\perp}_{im} \int \frac{d\mathbf{q}}{(2\pi)^3} \sum_{\mathbf{Q}} c_{ml}(\mathbf{Q}\,,k+q-Q) e^{-i\mathbf{Qr}}\, F_{lj}(\mathbf{q},\,\mathbf{k})\, e^{i\mathbf{qr}} \Big\}\, e^{\perp}_j(\mathbf{k}).$$

The second argument of the function c_{ml} in eq.(8) is $k_1 \;=\; k+q-Q \;\equiv\; (\omega, \mathbf{k}+\mathbf{q}-\mathbf{Q})$. This function depends actually on $\omega_1 \;=\; \omega$ and $\mathbf{n}_1 \;=\; \mathbf{k}_1 \;/\; \mid \;\mathbf{k}_1 \;\mid \approx\; \boldsymbol{\nu} \;+\; \mathbf{s}^{\perp}$, where $\mathbf{s} \;=\; (\mathbf{q}-\mathbf{Q})/\omega$. We have already used the fact that $\mid \mathbf{s} \mid \ll 1$ neglecting it as compared to unity. In particularly, that is why we have $\delta^{\perp}_{im}$ in eq.(8) instead of $\delta_{im} \;-\; k_{1i}k_{1m}/\omega^2$. We shall see below that owing to this replacement the longitudinal components of $F_{lj}(\mathbf{r}\,,\mathbf{k})$ do not appear at any depth if they are absent on the boundary. However, when $\boldsymbol{\nu}$ is almost parallel to some crystal axis, the transverse (with respect to this axis) component of $\boldsymbol{\nu}$ should be compared with $\mid \mathbf{s} \mid$. We can rule out $\mathbf{q}-\mathbf{Q}$ from the second argument of the function c_{ml} when the angle of incidence ϑ_0 with respect to this axis (transverse component of $\boldsymbol{\nu}$) is sufficiently large: $\vartheta_0 \;\gg \mid \mathbf{s} \mid$. In what follows, we assume this condition to be fulfilled, bearing in mind that really it does not lead to serious limitations as a typical magnitude of $\mid \mathbf{s} \mid$ at $\omega \;\simeq\; 1\,GeV$ is 10^{-6}. The integral over $\mathbf{q}$ turns into $F_{lj}(\mathbf{r}\,,\mathbf{k})$ according to the definition eq.(6), when we change $c_{ml}(\mathbf{Q}\,,k+q-Q) \;\Rightarrow\; c_{ml}(\mathbf{Q}\,,k)$ in eq.(8). This means that the integral over $\mathbf{r}'$ in eq.(5) converges in the domain $\mid \mathbf{r} \;-\; \mathbf{r}' \mid \ll R$ where R is a characteristic scale for the noticeable change of the function $F_{lj}(\mathbf{r}')$. So that, we could substitute

$$0 = \int d\mathbf{k}\, g(\mathbf{k} - \mathbf{k}_0)\, e^{-ikx} \Big\{ 2ik_l \frac{\partial F_{ij}(\mathbf{r}\,,\mathbf{k})}{\partial r_l} + \\ + 4\pi\omega^2 \delta^{\perp}_{im} \sum_{\mathbf{Q}} c_{ml}(\mathbf{Q}\,,k) e^{-i\mathbf{Q}\mathbf{r}} F_{lj}(\mathbf{r}\,,\mathbf{k}) \Big\} e^{\perp}_j(\mathbf{k}). \tag{9}$$

Here we are interested in the transverse (with respect to $\boldsymbol{\nu}$) tensor F_{ij}. Let us analyze, whether such a form of the sought tensor is consistent with eq.(9) and boundary conditions. According to eq.(9), the left longitudinal components of this tensor $F^{||}_j = \nu_i F_{ij}(\mathbf{r}\,,\mathbf{k})$ are independent of the penetration depth. If we suppose that $F^{||}_j = 0$ on the boundary, then $F_{lj} \equiv \delta_{ln} F_{nj} = \delta^{\perp}_{ln} F_{nj}$ at any depth. In particularly, this implies that only transverse components of the tensor $c_{ml}(\mathbf{Q}\,,k)$ are present in eq.(9). The right longitudinal components of the sought tensor do not enter eq.(9) owing to the factor $e^{\perp}_j(\mathbf{k})$. So, all the tensors below are self-consistently assumed to be transverse. They can be presented as two-dimensional matrices. Remembering this, we obtain from eq.(9)

$$F(\mathbf{r}\,,\mathbf{k}) = \exp\Big\{ 2\pi i\omega z \sum_{\mathbf{Q}} c(\mathbf{Q}\,,k) e^{-i\mathbf{Q}\boldsymbol{\rho}} \int_0^1 dx\, e^{-ixzQ_{||}} \Big\}, \tag{10}$$

where $\boldsymbol{\rho} = \mathbf{r} - z\boldsymbol{\nu}\,, z = \boldsymbol{\nu}\mathbf{r}\,, Q_{||} = \boldsymbol{\nu}\mathbf{Q}$. For $Q_{||} = 0$ the integral in eq.(10) equals unity, while for $Q_{||} \neq 0$ it is of the order of $(Q_{||}z)^{-1}$. Noticeable effects appear when the main ($Q_{||} = 0$) term in the phase of eq.(10) is of the order of unity. As shown in the next Section, it happens for z of several centimeters when the contribution of terms with $Q_{||} \neq 0$ to this phase $\sim (Q_{||}z)^{-1} \ll 1$ and can be neglected. Since matrices $\sum_{\mathbf{Q}} c_{ml}(\mathbf{Q}\,,k) e^{-i\mathbf{Q}\mathbf{r}}$ are in general non-commutative at different z, the exact solution to eq.(9) is the z-ordered exponential function. However, corrections to eq.(10) due to z-ordering are connected with terms $Q_{||} \neq 0$ being as small ($(Q_{||}z)^{-1}$) as terms already neglected. It must be remembered also that (for the coherent yield)

to the sum in eq.(10). These means that for sufficiently thick crystals we should keep only terms with $Q_{||} = 0$ in the sum of eq.(10):

$$F(\mathbf{r}\,,\,\mathbf{k}) \,=\, \exp\Big\{\, 2\pi i\omega z \sum_{\mathbf{Q}_\perp} c(\mathbf{Q}_\perp\,,\,k) e^{-i\mathbf{Q}_\perp\boldsymbol{\rho}}\Big\}\,. \qquad (11)$$

The condition $Q_{||} = 0$ implies that $\mathbf{Q}_\perp = 0$ too, if $\boldsymbol{\nu}$ is not aligned on some crystal plane. More precisely, let ψ be the angle of $\boldsymbol{\nu}$ with respect to the plane. For $\psi \gg \psi_0 = d_{pl}/z$, with d_{pl} being the inter-planar distance, we can retain the single term $c(\mathbf{Q} = 0\,,k)$ of the sum in eq.(11). The angle ψ_0 is extremely small, even for very thin films. The integration over $\boldsymbol{\nu}$ in eq.(4) should smear out a dependence of the general result on $\boldsymbol{\rho}$. We are not interested here whether such a dependence is observable or not. In what follows, the sum $\sum_{\mathbf{Q}_\perp} c(\mathbf{Q}_\perp\,,k) e^{-i\mathbf{Q}_\perp\boldsymbol{\rho}}$ in eq.(11) will be replaced by the matrix $c(\mathbf{Q} = 0\,,k) \equiv c(0\,,k)$ which coincides with the average of this sum over $\boldsymbol{\rho}$. The matrix $c(0\,,k)$ has no singularities and is practically constant in the region $\psi < \psi_0$. So that, any directions are allowed for $\boldsymbol{\nu}$.

Keeping in the sum $\sum_{\mathbf{Q}_\perp} c(\mathbf{Q}_\perp\,,k) e^{-i\mathbf{Q}_\perp\boldsymbol{\rho}}$ the only term with $\mathbf{Q}_\perp = 0$, we disregard possible small-angle photon scattering. In this approximation there is no exchange between fractions of the wave-packet having different $\boldsymbol{\nu}$, i.e., these photons propagate independently. So that, for given direction, we can go over to the conventional idealization of a monochromatic plane wave. Then the electric field is obtained from eq.(11) and eq.(4) with the change in the latter $g(\mathbf{k} - \mathbf{k}_0) \Rightarrow \delta(\mathbf{k} - \mathbf{k}_0)$ and then $\mathbf{k}_0 \Rightarrow \mathbf{k}$

$$E(x) \,=\, e^{-i\omega t}\, e^{\,i\omega z n}\, e^{\perp}(\mathbf{k})\,, \qquad (12)$$

where the two-dimensional vectors $E(x)$ and $e^{\perp}(\mathbf{k})$ are correspondingly the electric field in a medium and the polarization vector of the incident wave. The quantity $n = \delta^{\perp}_{ij} + 2\pi c(0\,,k)_{ij}$ is a complex matrix representing, by definition, the index of refraction. It depends on ω and (in crystals) on $\boldsymbol{\nu}$. We introduce a matrix $\Pi = 4\pi\omega^2 c(0\,,k)$,

$$\rho\,(L)\;=\;\exp\left\{iL\frac{\Pi}{2\omega}\right\}\rho(0)\exp\left\{-iL\frac{\Pi^{\dagger}}{2\omega}\right\}\,, \qquad (13)$$

where $\rho(0)$ is the density matrix of the incident photon.

Comparing eq.(12) with the corresponding result of [3] obtained for an amorphous medium by the direct summation of fields coming from individual scatterers, we find that in this case $\Pi\;=\;4\pi f(0)\,N$. In this formula $f(0)$ is the forward scattering amplitude for an individual particle (the cross section $d\sigma/d\Omega\;=\mid f(\mathbf{\Delta})\mid^2$, $\mathbf{\Delta}=\;\mathbf{k}_2\;-\;\mathbf{k}_1$) and N is a number of particles per unit volume. So, to find the polarization operator Π, we have to calculate the forward scattering amplitude normalized in such a way, that it reproduces $4\pi f(0)$ for an individual particle. Moreover, using the amplitude (see Sec.3 for its normalization) $T(k_1,k_2)$ for an arbitrary momentum transfer $\mathbf{\Delta}$, we obtain for $\chi(k_1,k_2)$ defined in eq.(6)

$$\chi(k_1,k_2)\;=\;T(k_1,k_2)/(2\omega^2)\,. \qquad (14)$$

From this relation and eq.(7), we can extract $c_{ml}(\mathbf{Q}\,,k_1)$ and then, if needed, take into account terms with $\mathbf{Q}\;\neq\;0$ omitted at the transition from eq.(10) to eq.(12).

As any other 2×2 matrix, $\Pi/(2\omega)$ in eq.(13) can be presented in the form of $\Pi/(2\omega)\;\equiv\;a+\mathbf{b}\boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ are the Pauli matrices. Note that the eigenvalues of the matrix $a+\mathbf{b}\boldsymbol{\sigma}$ are $a\pm\sqrt{\mathbf{b}^2}$. Using a similar representation for the initial density matrix $\rho(0)\;=\;(I+\boldsymbol{\eta}\boldsymbol{\sigma}\,)/2$, with $\boldsymbol{\eta}$ being the initial Stokes vector, we find from eq.(13)

$$\rho\,(L)\;=\;\frac{1}{2}e^{-\gamma L}\Big\{\frac{1+\boldsymbol{\eta}\boldsymbol{\sigma}}{2}\,(\cosh\alpha+\cos\beta)+$$

$$+\,(\cosh\alpha-\cos\beta)\,\Big[\frac{1-\boldsymbol{\eta}\boldsymbol{\sigma}}{2}\left(\mathbf{g}_1^2+\mathbf{g}_2^2\right)+(\mathbf{g}_1\times\mathbf{g}_2\,,\boldsymbol{\sigma}-\boldsymbol{\eta})+$$

$$+\,(\mathbf{g}_1\boldsymbol{\eta})\,(\mathbf{g}_1\boldsymbol{\sigma})+(\mathbf{g}_2\boldsymbol{\eta})\,(\mathbf{g}_2\boldsymbol{\sigma})\,\Big]-[(\mathbf{g}_2\,,\boldsymbol{\sigma}+\boldsymbol{\eta})+(\mathbf{g}_1\times\boldsymbol{\eta}\,,\boldsymbol{\sigma})]\sin\beta-$$

$$-\,[(\mathbf{g}_1\,,\boldsymbol{\sigma}+\boldsymbol{\eta})-(\mathbf{g}_2\times\boldsymbol{\eta}\,,\boldsymbol{\sigma})]\sinh\alpha\Big\}\,, \qquad (15)$$

$i\mathbf{g}_2 = \mathbf{b}/\sqrt{\mathbf{b}^2}$. The real vectors $\mathbf{g}_1$ and $\mathbf{g}_2$ satisfy the conditions $\mathbf{g}_1\mathbf{g}_2 = 0$, $\mathbf{g}_1^2 - \mathbf{g}_2^2 = 1$, since $\mathbf{g}^2 = 1$. As $tr\,\rho(0) = 1$, the fraction of outgoing photons is given by $FRAC = tr\,\rho(L)$. The Stokes vector at the depth L is $\boldsymbol{\eta}(L) = tr\,\boldsymbol{\sigma}\rho(L)/FRAC$. In the explicit form:

$$\boldsymbol{\eta}(L) = \frac{\mathbf{S}}{P} \quad , \quad FRAC = e^{-\gamma L}\,P \quad , \quad \boldsymbol{\eta}^2(L) = 1 - \frac{1 - \boldsymbol{\eta}^2}{P^2} \quad , \quad (16)$$

where

$$P = \mathbf{g}_1^2 \cosh\alpha - \mathbf{g}_2^2 \cos\beta - (\mathbf{g}_1\,,\,\boldsymbol{\eta})\sinh\alpha - (\mathbf{g}_2\,,\,\boldsymbol{\eta})\sin\beta -$$

$$(\cosh\alpha - \cos\beta)(\mathbf{g}_1 \times \mathbf{g}_2\,,\,\boldsymbol{\eta})\,;$$

$$\mathbf{S} = \boldsymbol{\eta}\left(\mathbf{g}_1^2\cos\beta - \mathbf{g}_2^2\cosh\alpha\right) -$$

$$-\,[\mathbf{g}_1 - \mathbf{g}_2 \times \boldsymbol{\eta}]\sinh\alpha - [\mathbf{g}_2 + \mathbf{g}_1 \times \boldsymbol{\eta}]\sin\beta +$$

$$[\mathbf{g}_1 \times \mathbf{g}_2 + \mathbf{g}_1\,(\mathbf{g}_1\,,\,\boldsymbol{\eta}) + \mathbf{g}_2\,(\mathbf{g}_2\,,\,\boldsymbol{\eta})](\cosh\alpha - \cos\beta) \quad . \quad (17)$$

We emphasize that, according to eq.(16), the polarization degree $|\,\boldsymbol{\eta}(L)\,|$ is an increasing function of depth L except for $\boldsymbol{\eta}^2 = 1$ or $P^2 = 1$.

3 Scattering of photons in a crystal

We now assume that $\omega \gg m$. Generally speaking, the amplitude of the forward Compton scattering off electrons $f_C(0)$ may be of comparable size with that due to virtual $e^+\,e^-$ pairs (VP) even in GeV energy region (see discussion in [4]). However, we can neglect $f_C(0)$ in what follows since it is, first of all, the unit matrix multiplied by some scalar factor and, second, real. That is why its contributions to Π and $\Pi^\dagger$ in eq.(13) are cancelled. For the same reason we can omit the real part of the incoherent contribution to the forward scattering VP amplitude. Nevertheless, its imaginary part, which does not cancel in

ωW_p , with W_p being the probability per unit length of the $e^+ e^-$ pair production by a photon. Remember that the incoherent probability W_p in crystals is smaller than in corresponding amorphous media (see, e.g. [5]). So, we must calculate here only the coherent contribution to the VP amplitude.

The properly normalized transition amplitude of the photon with 4-momentum $k_{1\mu} = (\omega_1, \mathbf{k}_1)$ and polarization vector $e_{1\mu}$ into the photon with $k_{2\mu} = (\omega_2, \mathbf{k}_2)$, $e_{2\mu}$ reads

$$T(k_1\,,\,k_2) \;= \tag{18}$$

$$2i\alpha \int d^4x_1 d^4x_2\, Tr\left[G(x_2,x_1)\,\hat{e}_1\, e^{-ik_1x_1}\, G(x_1,x_2)\,\hat{e}_2^*\, e^{ik_2x_2}\right] ,$$

where $\alpha = 1/137$ is the fine structure constant. The electron Green function $G(x_1, x_2)$ can be expressed via solutions $\Psi_n^{(\pm)}(x)$ to the Dirac equation in the corresponding external field

$$iG(x_2,x_1) \;=\; \theta(t_2-t_1)\sum_n \Psi_n^{(+)}(x_2)\,\bar{\Psi}_n^{(+)}(x_1) \;-$$

$$-\,\theta(t_1-t_2)\sum_n \Psi_n^{(-)}(x_2)\,\bar{\Psi}_n^{(-)}(x_1)\,. \tag{19}$$

Recollect that (see discussion in [7]) for $\omega \gg m$ the contribution to the amplitude , in terms of the non-covariant perturbation theory, is given by the diagram where the pair production by the initial photon precedes the annihilation of this pair into the final photon. We find, keeping only terms proportional to $\theta(t_2 - t_1)$

$$T(k_1\,,\,k_2) \;=\; 2i\alpha \sum_{n,m} \int\; d^4x_1 d^4x_2\, \theta(t_2-t_1)\,\times$$

$$\times V_{nm}(x_1,e_1,k_1)\, V^+_{nm}(x_2,e_2,k_2)\;, \tag{20}$$

$$V_{nm}(x,e,k) \;=\; \bar{\Psi}_n^{(+)}(x)\,\hat{e}\,e^{-ikx}\,\Psi_m^{(-)}(x)\;.$$

potential

$$U(\mathbf{q}) \;=\; \int d\mathbf{r}\, e^{-i\mathbf{q}\mathbf{r}}\, U(\mathbf{r})$$

only small momenta $\mid \mathbf{q} \mid \ll m$ are present. For such potentials, the quasiclassical operator method can be applied in its standard form to calculate the amplitude $T(k_1\,,\,k_2)$. Then further transformations in eq.(20) are completely identical to those leading to the quasiclassical expression for the probability of e^+e^- pair production by a photon in an external field (see Sec.3 in [5]), because V_{nm} in eq.(20) is essentially the matrix element for this process. Notice that the amplitude $T(k_1\,,\,k_2)$ is the contraction of $e_{1\mu}e^*_{2\nu}$ with the sought tensor $T^{\mu\nu}(k_1\,,\,k_2)$. For the transverse (with respect to $\boldsymbol{\nu}_1 \,=\, \mathbf{k}_1/\omega_1$) components of this tensor we obtain from eq.(20)

$$T^{ij} = \frac{i\alpha}{(2\pi)^2}\delta(\omega_2-\omega_1)\int d\mathbf{r}\,\times$$

$$\times exp(-i\boldsymbol{\Delta}\mathbf{r})\int\limits_0^\infty d\tau \int\limits_0^\omega d\varepsilon\Big[\frac{\omega}{\varepsilon(\omega-\varepsilon)}\Big]^2\int d\mathbf{p}_\perp(0)e^{-iA}\,B^{ij},$$

$$A \;=\; \frac{\omega m^2}{2\varepsilon(\omega-\varepsilon)}\;\left[\tau \;+\; \int\limits_{-\tau/2}^{\tau/2}\frac{ds}{m^2}\mathbf{p}_\perp^2\,(s)\right]\;;$$

$$B^{ij} = \delta^{ij}_\perp\left[m^2+(\mathbf{p}_{1\perp}\mathbf{p}_{2\perp})\right]+\left(\frac{2\varepsilon\,-\,\omega}{\omega}\right)^2 p^i_{1\perp}\,p^j_{2\perp}\,-\,p^i_{2\perp}\,p^j_{1\perp}\,, \quad (21)$$

where $\mathbf{p}_{1,2} \;\equiv\; \mathbf{p}(\mp\tau/2)$ being the momentum of an electron on a classical trajectory at the corresponding time, $\boldsymbol{\Delta} \,=\, \mathbf{k}_2 \,-\, \mathbf{k}_1$. When the external field vanishes, the amplitude T must vanish as well. It assumes a subtraction which really affects only the term proportional to m^2 in B^{ij}. This subtraction will be performed in the explicit form below.

Generally speaking, for further calculations we have to know the dependence of $\mathbf{p}_\perp(t)$ on time, i.e. the corresponding mechanical problem should be solved. For arbitrary potential it cannot be done analytically. Fortunately, it is sufficient to know the alteration of the

is the characteristic size of the variable τ contributing to the integral. In the rectilinear trajectory approximation (RTA), we substitute into equations of motion $\mathbf{r} + \boldsymbol{\nu}_1 t$ instead of the exact solution $\mathbf{r}(t)$. We refer for details to [5] stating here that in crystals the amplitude T can be calculated by the use of RTA for arbitrary photon energy ($\omega \gg m$) and crystal orientation.

Representing the crystal potential $U(\mathbf{r})$ as a sum over vectors $\mathbf{q}$ of the reciprocal lattice

$$U(\mathbf{r}) = \sum_{\mathbf{q}} G(\mathbf{q})\, e^{i\mathbf{qr}} ,$$

we find within RTA for the transverse momentum

$$\mathbf{p}_\perp(t) = \mathbf{p}_\perp(0) + \boldsymbol{\delta}(t) \equiv \mathbf{p}_\perp(0) - \sum_{\mathbf{q}} G(\mathbf{q})\, e^{i\mathbf{qr}} \frac{\mathbf{q}_\perp}{q_\parallel} \left(e^{iq_\parallel t} - 1 \right), \quad (22)$$

where $q_\parallel = (\boldsymbol{\nu}_1 \mathbf{q})$. Substituting eq.(22) into eq.(21), we can take the Gaussian-type integral over $\mathbf{p}_\perp(0)$. Performing the subtraction at vanishing external field and going over to the variable $y = 1 - 2\varepsilon/\omega$, we find

$$T^{ij} = \frac{\alpha}{\pi} \delta(\omega_2 - \omega_1) \int d\mathbf{r}\, exp(-i\boldsymbol{\Delta}\mathbf{r}) \int_{-1}^{1} \frac{dy}{1-y^2} \times \quad (23)$$

$$\times \int_0^\infty \frac{d\tau}{\tau} \exp\left[-i\beta\tau\,(1+\psi(\tau))\right] B_1^{ij};$$

$$\psi(\tau) = \sum_{\mathbf{q},\mathbf{q}'} \frac{G(\mathbf{q})G(\mathbf{q}')}{m^2} \times$$

$$\times \frac{(\mathbf{q}_\perp \mathbf{q}'_\perp)}{q_\parallel q'_\parallel} e^{i(\mathbf{q}+\mathbf{q}',\mathbf{r})} \left[\frac{sin(q_\parallel + q'_\parallel)\tau/2}{(q_\parallel + q'_\parallel)\tau/2} - \frac{sinq_\parallel\tau/2}{q_\parallel\tau/2} \cdot \frac{sinq'_\parallel\tau/2}{q'_\parallel\tau/2} \right]$$

$$B_1^{ij} = y^2\, a_1^i\, a_2^j - a_2^i\, a_1^j + \delta_\perp^{ij} \left\{ (\mathbf{a}_1\mathbf{a}_2) + m^2 \left[(y^2-1) \cdot \tau \frac{d\psi}{d\tau} - \psi \right] \right\};$$

$$\mathbf{a}_{1,2} = \sum_{\mathbf{q}} G(\mathbf{q}) \cdot \frac{\mathbf{q}_\perp}{q_\parallel} e^{i\mathbf{qr}} \left[e^{\mp iq_\parallel\tau/2} - \frac{sinq_\parallel\tau/2}{q_\parallel\tau/2} \right], \quad \beta = \frac{2m^2}{\omega(1-y^2)} .$$

tions. It would be drastically simplified if we neglect the function $\psi(\tau)$ as compared to 1 in the phase of eq.(23). We suppose that the angle of incidence ϑ_0 with respect to some major crystal axis $\vartheta_0 \ll 1$, since precisely for such angles the strengthening of electromagnetic effects happens in crystals as compared with amorphous media. If $\boldsymbol{\nu}_1$ is far in azimuth from any major crystal plane, then $\mid q_{\parallel} \mid \sim \mid \mathbf{q}_{\perp}\vartheta_0 \mid$ and $\psi_{off}(\tau) \leq (V_0/m/\vartheta_0)^2$. In this estimate V_0 denotes a typical magnitude of $G(\mathbf{q})$ being of the order of the axial potential well depth. So, in this case we can omit $\psi(\tau)$ in the phase of eq.(23) for $\vartheta_0 \gg V_0/m$. If now $\boldsymbol{\nu}_1$ is aligned on some crystal plane, there is a subset of $\mathbf{q}$ ($\mathbf{q}'$) for which $q_{\parallel}$ ($q'_{\parallel}$) are extremely small or vanish. Those $\mathbf{q}$ are perpendicular to the plane. Expanding in $q_{\parallel}$ and $q'_{\parallel}$, we obtain for the corresponding contribution to the phase $\psi_{pl}(\tau) = (\tau/m)^2(dU_{pl}(x)/dx)^2/12$. Here x is the distance from the plane, $U_{pl}(x)$ being the planar potential. As long as $\psi(\tau) \leq 1$, we can substitute $\tau \sim \beta^{-1}$ into our estimate which turns into $\psi_{pl}(\tau) \ll \kappa^2(x)$. The magnitude of the strong field parameter

$$\kappa(x) = \frac{E_{pl}(x)}{E_0} \cdot \frac{\omega}{m} \qquad , \qquad E_0 = \frac{m^2}{e} = 1.32 \cdot 10^{16}\, V\,/\,\mathrm{cm}$$

can be estimated using the Table 1 of [8], where, in particularly, averaged over x values of $(E_{pl}(x)/E_0)^2$ are presented for (110) plane of several crystals. As a result, for commonly used crystals, $\psi_{pl}(\tau)$ is sufficiently small when ω is less than several TeV. Assuming that ω and ϑ_0 satisfy the formulated conditions, we, finally, rule out $\psi(\tau)$ from the phase of eq.(23).

Now the integration over $\mathbf{r}$ in eq.(23) can be performed

$$\int d\mathbf{r}\, exp\left[i(\mathbf{q} + \mathbf{q}' - \boldsymbol{\Delta}\ ,\ \mathbf{r})\right] = (2\pi)^3\, \delta(\mathbf{q} + \mathbf{q}' - \boldsymbol{\Delta})\,.$$

As a result, the tensor T^{ij} acquires the form

$$T^{ij}(k_1, k_2) = (2\pi)^3 \sum_{\mathbf{Q}} \pi^{ij}(\mathbf{Q}\,, k_1)\, \delta(k_2 - k_1 - Q),$$

eq.(14) we find that $c^{ij}(\mathbf{Q},\ k_1)$ in eq.(7) are $c^{ij}(\mathbf{Q},\ k_1) = \pi^{ij}(\mathbf{Q},\ k_1)/(4\pi\omega^2)$. Here we are interested only in $\pi^{ij}(0,\ k_1)$ being just the polarization operator Π which determines a development of the photon density matrix at large thicknesses according to eq.(13) . Taking elementary integrals over τ and y, we obtain from eq.(23)

$$\Pi^{ij} = \frac{\alpha\omega^2}{8m^2}\sum_{\mathbf{q}}\frac{\mid G(\mathbf{q})\mid^2}{m^2}\Big\{\mathbf{q}_\perp^2\delta_\perp^{ij}\left[id_1(\mu)+d_2(\mu)\right]+$$

$$+\left(\frac{1}{2}\mathbf{q}_\perp^2\delta_\perp^{ij}-q_\perp^i q_\perp^j\right)\left[id_3(\mu)-d_4(\mu)\right]\Big\}\ , \qquad (24)$$

where $\mu = 2m^2/(\omega\mid q_{||}\mid)$ and functions d_l are

$$d_1(x) = x^2\left[\left(1+x-\frac{1}{2}x^2\right)g(\sqrt{1-x})-(1+x)\sqrt{1-x}\right]\theta(1-x)\,,$$

$$d_3(x) = x^3\left(\frac{x}{2}g(\sqrt{1-x})+\sqrt{1-x}\right)\theta(1-x)\,,\quad g(x)=\ln\frac{1+x}{\mid 1-x\mid}\,,$$

$$d_4(x) = \frac{x^2}{\pi}\Big\{\left(\frac{x}{2}g(\sqrt{1+x})-\sqrt{1+x}\right)^2-$$

$$-\left(x\arctan\frac{1}{\sqrt{x-1}}-\sqrt{x-1}\right)^2\theta(x-1)+$$

$$+\left[\left(\frac{x}{2}g(\sqrt{1-x})+\sqrt{1-x}\right)^2-\left(\frac{\pi x}{2}\right)^2\right]\theta(1-x)\Big\}\,,$$

$$d_2(x) = d_4(x)+\frac{2x^2}{\pi}\Big\{\frac{1}{2}g(\sqrt{1+x})\left(\frac{x-1}{2}g(\sqrt{1+x})+\sqrt{1+x}\right)-$$

$$-3-\left[\frac{1}{2}g(\sqrt{1-x})\left(\frac{x+1}{2}g(\sqrt{1+x})-\sqrt{1-x}\right)+\right.$$

$$\left.+1-\frac{\pi^2(1+x)}{4}\right]\theta(1-x)+\left[\arctan\frac{1}{\sqrt{x-1}}\times\right.$$

$$\left.\times\left((x+1)\arctan\frac{1}{\sqrt{x-1}}+\sqrt{x-1}\right)-1\right]\theta(x-1)\Big\}\,, \qquad (25)$$

The photon energy ω enters eq.(24) in the combination μ except of the factor ω^2 in front of the sum. This sum is almost independent of ω near its maximum since the latter is given by $\mu \simeq 1$. As a result, the ω-dependence of the quantities α and β in eq.(15) and eq.(17) describing the behavior of the photon polarization is reduced to the factor ωL for optimal orientations. Correspondingly, the optimal thickness L_{opt} is roughly proportional to ω^{-1}. As mentioned above, there is a subset of $\mathbf{q}$ at perfect planar alignment for which $q_{||}$ vanishes ($\mu \to \infty$). The contribution of this subset to Π^{ij}, i.e. the yield of the corresponding planar potential obtained from eq.(24) reproduces well known results (see, e.g. [8] and literature cited there) derived within the Born approximation:

$$\frac{\alpha m^2}{45\pi} \langle \kappa^2(x) \rangle \left(4\, e_1^i e_1^j \, + \, 7\, e_2^i e_2^j \right),$$

where $\langle \ldots \rangle$ means averaging over the coordinate x, $\mathbf{e}_1$ is the unit vector perpendicular to the plane and $\mathbf{e}_2 = \boldsymbol{\nu}_1 \times \mathbf{e}_1$. What is lost when $\psi(\tau)$ has been omitted in the phase of eq.(23), are higher order corrections in crystal potential.

Using the explicit form of Π^{ij} eq.(24) and adding the incoherent yield,we can find the quantities presented in eq.(15), which describes the properties of a photon beam for any initial conditions dependent on the crystal thickness. As explained above, the incoherent yield is present in eq.(15) only in the absorption coefficient γ. We can use any basis to calculate Π^{ij}. Let this basis be formed by two real unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ satisfying $\mathbf{e}_1\mathbf{e}_2 = \boldsymbol{\nu}_1\mathbf{e}_1 = \boldsymbol{\nu}_1\mathbf{e}_2 = 0$. We should use the same basis to obtain the initial Stokes vector $\boldsymbol{\eta}$. Supposing that $\boldsymbol{\nu}_1$ is near some axis direction $\boldsymbol{\nu}_3$, we choose $\mathbf{e}_2$ in the plane (reaction plane) containing $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_3$. For the basis chosen, the circular polarization degree is given by the magnitude of the second Stokes parameter $\xi(L) = \mid \eta_2(L) \mid$. If we define the angle of incidence ϑ_0 as the angle between $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_3$, and φ_0 which is the angle between the reaction plane and the $(1\bar{1}0)$ plane, then the explicit form of the basis vectors reads

$$\mathbf{e}_1 \;=\; -\mathbf{e}_x \sin\varphi_0 \;+\; \mathbf{e}_y \cos\varphi_0 \,,$$

$$\mathbf{e}_2 \;=\; \boldsymbol{\nu}_3 \sin\vartheta_0 \;-\; (\mathbf{e}_x \cos\varphi_0 \;+\; \mathbf{e}_y \sin\varphi_0)\cos\vartheta_0 \,,$$

where $\mathbf{e}_x\mathbf{e}_y \;=\; \boldsymbol{\nu}_3\mathbf{e}_x \;=\; \boldsymbol{\nu}_3\mathbf{e}_y \;=\; 0$ and $\mathbf{e}_x$ is in the $(1\bar{1}0)$ plane. In this basis the quantities a and $\mathbf{b}$ presented in eq.(15) take the form

$$(a, \mathbf{b}) \;=\; \frac{\alpha\omega}{16m^2} \sum_{\mathbf{q}} \frac{\mid G(\mathbf{q}) \mid^2}{m^2} (A, \mathbf{B}) \,,$$

$$A \;=\; \mathbf{q}_\perp^2 \left[i d_1(\mu) + d_2(\mu) \right] \,,$$

$$B_1 \;=\; -(\mathbf{e}_1\mathbf{q}_\perp)(\mathbf{e}_2\mathbf{q}_\perp) \left[i d_3(\mu) - d_4(\mu) \right] \,,$$

$$B_2 \;=\; 0 \,, \quad B_3 \;=\; \frac{1}{2} \left[(\mathbf{e}_2\mathbf{q}_\perp)^2 \;-\; (\mathbf{e}_1\mathbf{q}_\perp)^2 \right] \left[i d_3(\mu) - d_4(\mu) \right] \,. \qquad (26)$$

Remember that the eigenvalues of the propagation matrix $\Pi/(2\omega)$ in eq.(13) are $a \,\pm\, \sqrt{\mathbf{b}^2}$. The corresponding eigenvectors $\mathbf{e}_\pm$ satisfying the normalization condition $\mid \mathbf{e}_\pm \mid^2 = \, 1$ are

$$\mathbf{e}_+ \;=\; \frac{\mathbf{e}_1 \;+\; r\mathbf{e}_2}{\sqrt{1 + \mid r \mid^2}} \quad , \quad \mathbf{e}_- \;=\; \frac{\mathbf{e}_2 \;-\; r\mathbf{e}_2}{\sqrt{1 + \mid r \mid^2}} \quad , \quad r \;=\; \frac{b_1}{b_3 + \sqrt{\mathbf{b}^2}} \,. \qquad (27)$$

In general, the eigenvectors $\mathbf{e}_\pm$ are complex. However, they become real when the quantity r does so. In particularly, when $\boldsymbol{\nu}_1$ is in the symmetry plane of a crystal like $(1\bar{1}0)$ plane ($\varphi_0 \;=\; 0$), r vanishes and $\mathbf{e}_\pm$ coincide with $\mathbf{e}_{1,2}$. Just such a case ($\varphi_0 \;=\; \pi/2$, $\boldsymbol{\nu}_3$ along $< 110 >$ axis) was the only orientation considered in [2] where a quantity accounting for the polarization conversion was calculated by the use of dispersion relations. In our notation it corresponds to the term in eq.(24) proportional to $d_4(\mu)$. We were unable to reproduce the factor in eq.(5) of [2], nevertheless, we emphasize that the expression in braces of the cited equation coincides with $d_4(\mu)/\mu^2$.

Consider now, as an example, fully linearly polarized ($\eta_2 \;=\; 0$, $\boldsymbol{\eta}^2 = 1$) initial photon beam. An efficiency of the polarization conversion process is determined not only by $\xi(L)$ but also by the fraction of surviving photons $FRAC(L)$. We use the criterion of such an efficiency

$$FOM(L) \;=\; 10 \cdot \xi(L)\sqrt{FRAC(L)} \quad ,$$

L being the crystal thickness. Recollect that $\xi(L)$ and $FRAC(L)$ depend also on ω and $\boldsymbol{\nu}_1$. For given orientation of a crystal, we still have a free parameter which is the angle ϕ of the initial polarization vector with respect to the basis vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. We do not claim here to the final analysis of the polarization conversion process, so that the yield for different orientations will be compared at the same initial condition. Namely, we set $\eta_1 = -1$ which corresponds to $\phi = \pi/4$ with respect to $\mathbf{e}_2$ ($\phi = 3\pi/4$ with respect to $\mathbf{e}_1$). Evidently, this is the best choice for the alignment on the symmetry plane of a crystal when the basis vectors are the eigenvectors of the matrix Π^{ij} as well.

In Fig.1 the maximum values of the figure of merit (FOM) are shown near $< 110 >$ axis of a diamond crystal for $\omega = 100$ GeV as a function of the orientation (angles ϑ_0 and φ_0). For each direction (ϑ_0, φ_0), the figure of merit was calculated first as a function of L, then its maximum value was found. Just these maximum values of FOM are plotted, so that different directions correspond usually to different optimal thicknesses L_{opt}. Owing to the crystal symmetry, there is no need to perform calculations for φ_0 beyond the interval chosen, since they will simply reproduce the results already obtained for corresponding φ_0 within the interval. For diamond crystal, the largest effect is achieved for $\vartheta_0 = 1.5$ mrad off the $< 110 >$ axis on the $(1\bar{1}0$ plane ($\varphi_0 = 0$), where (see Fig.1) $FOM \simeq 3.7$. However, in this case the size of $L_{opt} \simeq 4.7$ cm seems to be too large for practical use. Generally, a typical size of the optimal thickness is of a few cm. For the same axis of a silicon crystal (see Fig.2), the peak is at $\vartheta_0 = 2.3$ mrad and $\varphi_0 = 0$ with $FOM \simeq 2.6$. We emphasize rather narrow angular with of the peak in both directions. The optimal thickness in this case is $L_{opt} \simeq 8$ cm. We have performed the same kind of calculations for three major axes of diamond, silicon and germanium crystals. Comparing types of crystals, we see that the effect is the largest for diamond and the smallest for germanium, where $FOM \simeq 2.0$ can be obtained at $L_{opt} \simeq 2.8$ cm. What about

give a comparable but smaller yield.

For $\varphi_0 = 0$ the position of a peak is determined by the condition $\mu = 2m^2/(\omega \mid q_{\parallel} \mid) = 1$ for the smallest non-zero $\mid q_{\parallel} \mid$. This is connected with the threshold behavior of the functions $d_{1,3}(\mu)$ in eq.(24) at $\mu = 1$. From this condition we obtain for $fcc(d)$ structure near $< 110 >$ axis $\vartheta_0^{\max} = m^2 l_c/(\pi\omega)$, where l_c is a lattice constant. In particularly, for Si we have $\vartheta_0^{\max}(\mathrm{mrad}) \simeq 229/\omega(\mathrm{GeV})$. This fact is illustrated by Fig.3 where FOM (upper curves), $FRAC$ (lower curves), and the degree of circular polarization ξ are shown for $\varphi_0 = 0$ near $< 110 >$ axis of a 10 cm thick silicon crystal as functions of ω. Three sets of curves (from the left to the right) in Fig.3 correspond to the angles of incidence $\vartheta_0(mrad) = 2.29$, 2.08 and 1.91 respectively. All curves in Fig.3 have peaks exactly at the positions prescribed by the condition obtained above. For a given orientation (at fixed ϑ_0), rather narrow shape of these peaks does not allow us to handle with the same efficiency a photon beam having the wide energy spread.

The magnitude of $FOM(L)$ being proportional to $\xi(L)$ diminishes for partially polarized initial photon beam roughly proportionally to $\mid \boldsymbol{\eta} \mid < 1$ as compared to the fully polarized case ($\mid \boldsymbol{\eta} \mid = 1$). Depending on the orientation, more or less noticeable change of the polarization degree $\mid \boldsymbol{\eta}(L) \mid$ occurs for $\mid \boldsymbol{\eta} \mid < 1$. It can be seen in Figs.4,5 where the absolute values of three Stokes parameters and the polarization degree are presented as functions of the silicon crystal thickness L at $\omega = 100$ GeV. The calculations were carried out using eq.(16), eq.(17), and eq.(26) for $\eta_1 = -0.5$, $\eta_2 = \eta_3 = 0$. The angle of incidence $\vartheta_0 = 2.29$ mrad is the same for both figures. At $\varphi_0 = 0$ (Fig.4), $\eta_3(L)$ is small and $\mid \boldsymbol{\eta}(L) \mid$ (curve "tot") practically does not change due to the smallness of the parameter α (see eq.(15)) within the whole interval of L presented. The fracture of the curve (1) means only that $\eta_1(L)$ changes its sign at $L = 21$ cm. Since the polarization degree practically conserves and $\eta_3(L)$ can be neglected for this orientation, we could measure the linear polarization $\eta_1(L)$ to determine the circular polarization $\eta_2(L)$ appeared. However, the

seen in Fig.5 calculated at $\varphi_0 = 0.015$. Now the measurement of the linear polarization does not help in the determination of the circular one since the polarization degree is no more constant.

In conclusion, formulas derived describe the propagation of hard polarized photons through crystals. They are valid in a wide photon energy range for any orientation and any crystal type as long as the approximations used are correct. Our calculations show that the linear polarization of multi-GeV photons can be converted with an appropriate efficiency into the circular one using properly chosen single crystals. However, if we do not use theoretical results for some quantities involved (e.g. for the polarization degree), the only way to determine the circular polarization appeared in a crystal is the direct measurement of it.

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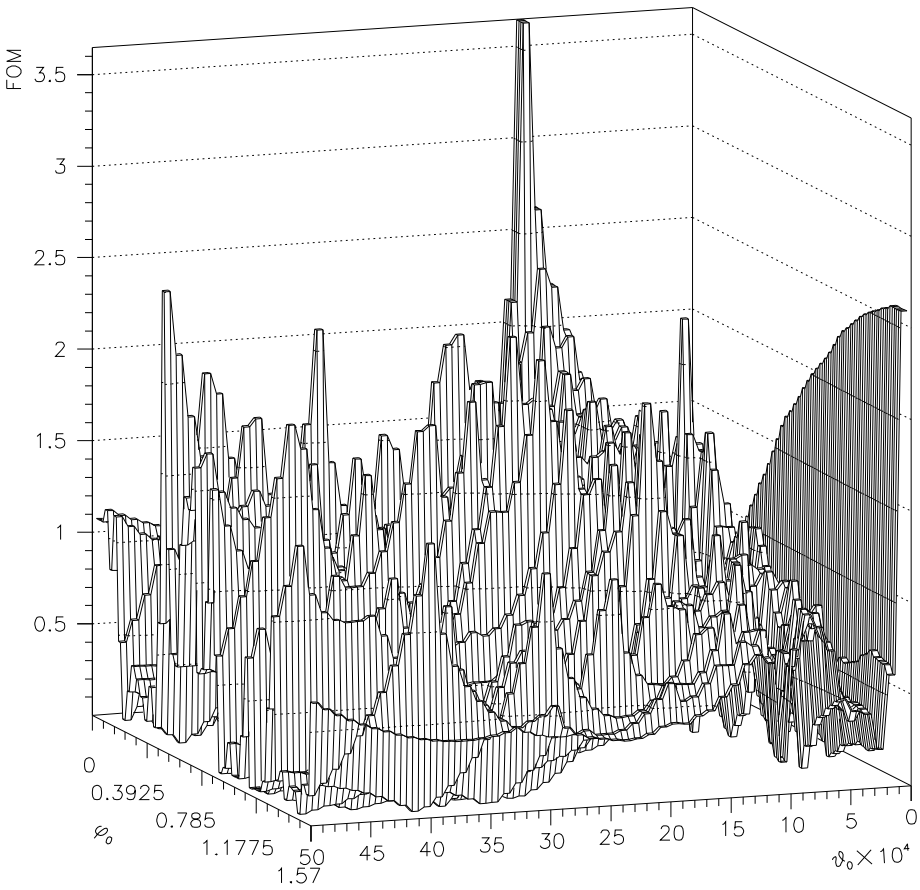

Figure 1: The maximum values of FOM (the figure of merit, see text) near $< 110 >$ axis of a diamond crystal for $\omega = 100$ GeV dependent on the angles of incidence ϑ_0 and φ_0.

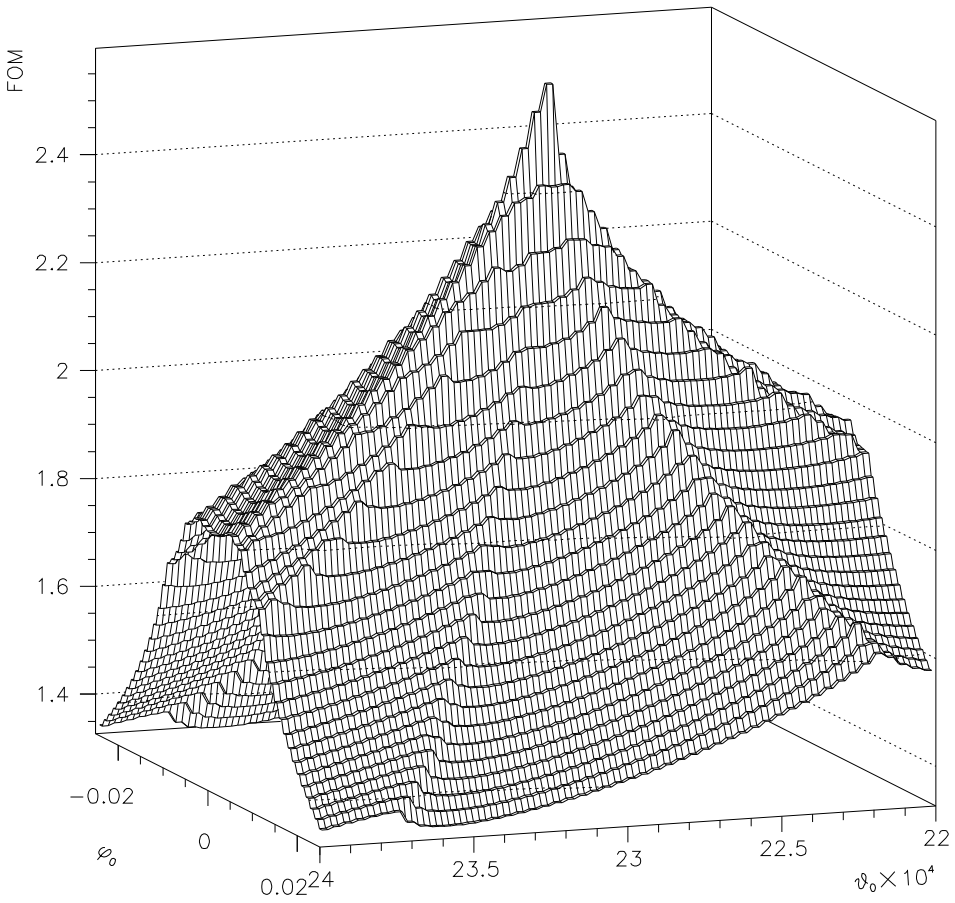

Figure 2: The same as in Fig.1 but for a silicon crystal in the neighborhood of the point $\vartheta_0 = 2.3 mrad$, $\varphi_0 = 0$.

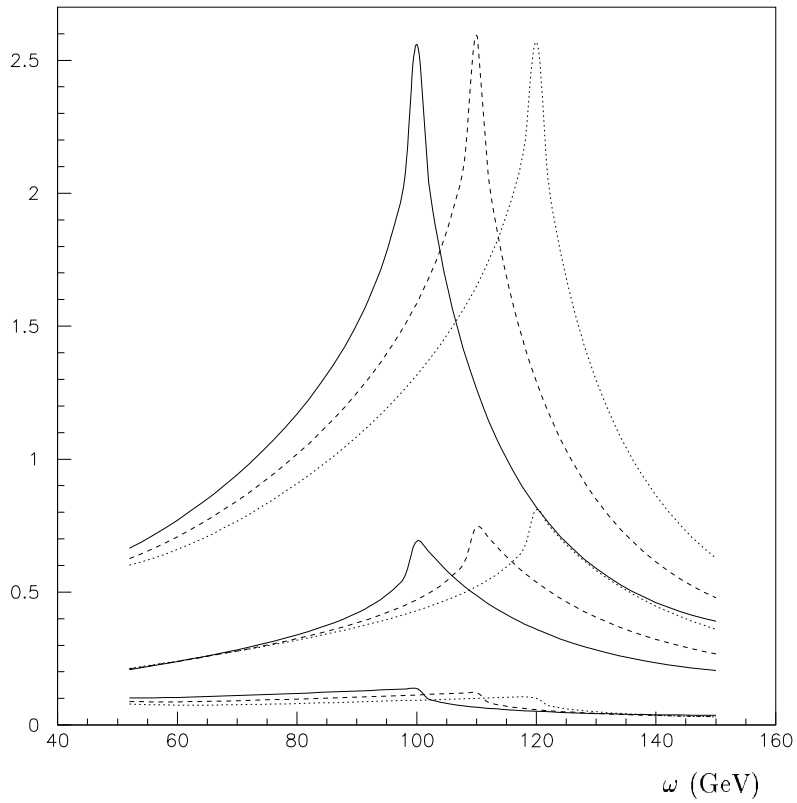

Figure 3: FOM (upper curves), $FRAC$ (lower curves), and the degree of circular polarization ξ for $\varphi_0 = 0$ near $< 110 >$ axis of a 10 cm thick silicon crystal as functions of ω (GeV). Three sets of curves (from the left to the right) correspond to the angles of incidence ϑ_0 (mrad) = 2.29, 2.08 and 1.91 respectively.

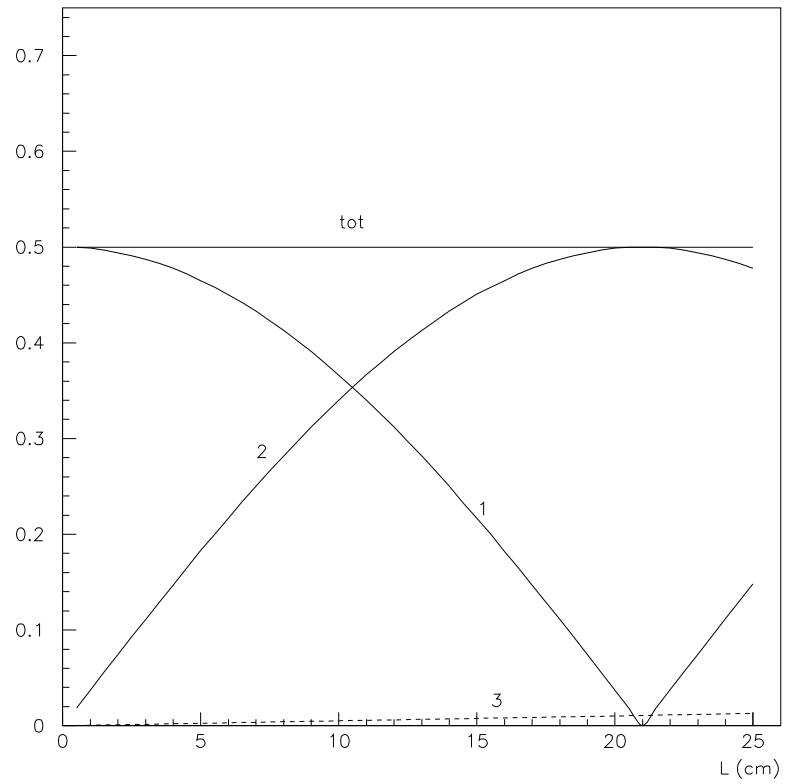

Figure 4: Absolute values of the Stokes parameters $\mid \eta_1(L) \mid$ (curve 1), $\mid \eta_2(L) \mid$ (curve 2), $\mid \eta_3(L) \mid$ (curve 3), and the polarization degree $\mid \boldsymbol{\eta}(L) \mid$ (curve "tot") depending on the thickness L of a silicon crystal at $\omega = 100$ GeV, $\eta_1 = -0.5$, $\eta_2 = \eta_3 = 0$, $\vartheta_0 = 2.29$ mrad, $\varphi_0 = 0$.

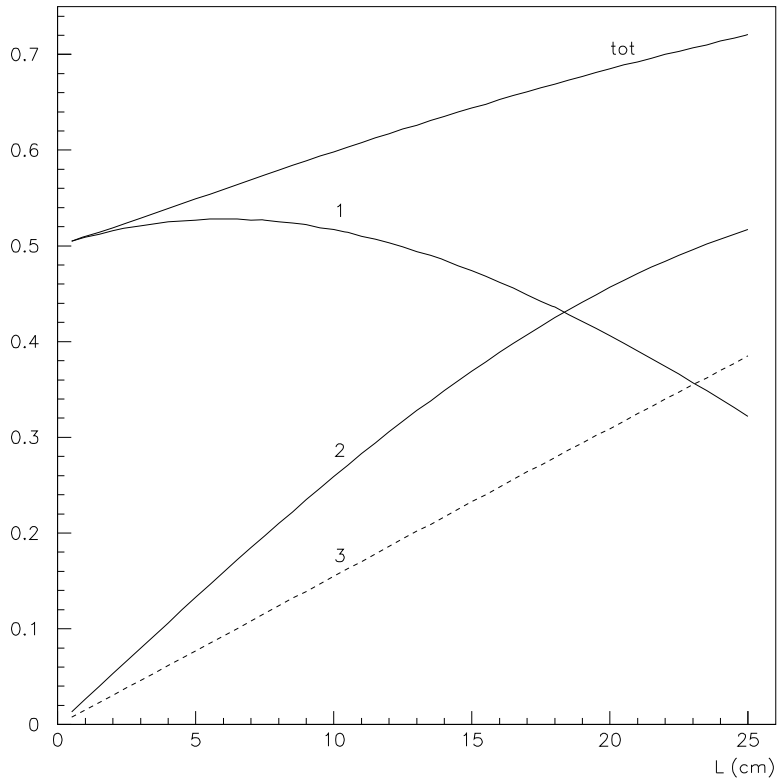

Figure 5: The same as in Fig.4 but for $\varphi_0 = 0.015$.