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QUANTUM THEORY
OF TRANSITION RADIATION
AND TRANSITION PAIR CREATION

Quantum theory of transition radiation
and transition pair creation

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Abstract

Theory of the transition radiation and the transition pair creation is developed in the frame of QED. The spectral-angular distributions of probability of the transition radiation and of the transition pair creation are found. The total energy losses of an electron and the total probability of pair creation by a photon are calculated and analyzed. Features of radiation and pair creation processes in a superdence medium (typical for white dwarfs) are discussed.

1. The transition radiation arises at uniform and rectilinear motion of a charged particle when it intersects a boundary of two different media (in general case, when it moving in a nonuniform medium or near such medium). This phenomenon [1] was actively investigated during a few last decades (see, e.g. reviews [2], [3]) and widely used in transition radiation detectors. The existing theory of the transition radiation is based on the classical electrodynamics. The quantum theory of the transition radiation is of evident interest. An analysis in the frame of quantum electrodynamics indicates existence of the crossing process: electron-positron pair creation by a photon on a boundary of two different media. We shall call this process as the transition pair creation.

It turns out that the quasiclassical operator method developed by authors is adequate for consideration of the transition radiation. The probability of the process has a form (see [4], p.63, Eq.(2.27))

$$dw = \frac{e^2}{(2\pi)^2} \frac{d^3k}{\hbar\omega} \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_1 R^*(t_2)R(t_1) \exp\left[-\frac{i}{\varepsilon'} \int_{t_1}^{t_2} \left(kp - \frac{\hbar k^2}{2}\right) dt\right],$$

$$(1)$$

where $p = p^\mu = (\varepsilon, \mathbf{p})$ is the 4-momentum of the initial electron, $k = k^\mu = (\omega, \mathbf{k})$ is the 4-wave vector of the radiated photon, in a medium $|\mathbf{k}| = n\omega$, n is the refractive index, $\varepsilon' = \varepsilon - \hbar\omega$, the matrix element $R(t)$ is defined by the structure of a current, we employ units such that $c = 1$. Here we took into account the term k^2 in the exponent (this is result of the disentanglement of the operator expression, see Eq.(2.23) of [4]) which is essential in the considered case since we consider radiation in a medium (this term was skipped in our paper [5]) and we use the representation Eq.(2.24) of [4] because we are dealing with a nonuniform case. For electrons (spin 1/2 particle) one has

$$R(t) = \frac{m}{\sqrt{\varepsilon\varepsilon'}} \overline{u}_{s_f}(\mathbf{p}')\hat{e}^* u_{s_i}(\mathbf{p}) = \varphi_{s_f}^+ \left(A(t) + i\boldsymbol{\sigma}\mathbf{B}(t)\right) \varphi_{s_i},$$

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$$A(t) = \frac{\mathbf{e}^* \mathbf{p}(t)}{2\sqrt{\varepsilon\varepsilon'}} \left[\sqrt{\frac{\varepsilon' + m}{\varepsilon + m}} + \sqrt{\frac{\varepsilon + m}{\varepsilon' + m}} \right] \simeq \frac{1}{2} \left(1 + \frac{\varepsilon}{\varepsilon'} \right) \mathbf{e}^* \boldsymbol{\vartheta},$$

$$\mathbf{B(t)} = \frac{1}{2\sqrt{\varepsilon\varepsilon'}} \left[\sqrt{\frac{\varepsilon' + m}{\varepsilon + m}} \left(\mathbf{e}^* \times \mathbf{p}(t) \right) - \sqrt{\frac{\varepsilon + m}{\varepsilon' + m}} \left(\mathbf{e}^* \times (\mathbf{p}(t) - \hbar\mathbf{k}) \right) \right]$$

$$\simeq \frac{\hbar\omega}{2\varepsilon'} \left(\mathbf{e}^* \times \left(\frac{\mathbf{n}}{\gamma} - \boldsymbol{\vartheta} \right) \right), \tag{2}$$

here $\mathbf{e}$ is the vector of polarization of photon (the Coulomb gauge is used), the four-component spinors u_{s_f}, u_{s_i} and the two-component spinors φ_{s_f}, φ_{s_i} describe the initial (s_i) and the final (s_f) polarization of the electron, $\boldsymbol{\vartheta} = v^{-1}(\mathbf{v} - \mathbf{n}(\mathbf{n}\mathbf{v})) \simeq \mathbf{v}_\perp$, $\mathbf{v}_\perp$ is the component of particle velocity transverse to the vector $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$, $\gamma = \varepsilon/m$ is the Lorentz factor. The final expressions in (2) are given for radiation of ultrarelativistic electrons, they are written down with relativistic accuracy (terms $\sim 1/\gamma$ are neglected) and in the small angle approximation. For the rectilinear motion radiation arises because of variation of the refractive index $n(\omega)$, e.g. at intersection of a boundary of two different media. The main contribution gives the region of high frequencies where

$$n(\omega) = 1 - \frac{\omega_0^2}{2\omega^2}, \quad \omega_0^2 = \frac{4\pi e^2 N}{m}, \tag{3}$$

where N is the density of electrons in a medium, ω_0 is the plasma frequency.

2. Here we consider the transition radiation in the forward direction at the normal incidence of the relativistic particle on the boundary between vacuum and a medium. In this case the photon mass squared can be written as

$$\hbar^2 k^2 = (\hbar\omega_0)^2 g(t), \tag{4}$$

where the function $g(t)$ describes variation of the density of a medium on the projectile trajectory. The combination $R^*(t_2)R(t_1)$ in (1) can be presented in a form

$$R^*(t_2)R(t_1) =$$

$$= \frac{1}{4}\text{Tr}\left[(1 + \boldsymbol{\sigma}\boldsymbol{\zeta}_i)(A^*(t_2) - i\boldsymbol{\sigma}\mathbf{B}^*(t_2))(1 + \boldsymbol{\sigma}\boldsymbol{\zeta}_f)(A(t_1) + i\boldsymbol{\sigma}\mathbf{B}(t_1)) \right], \tag{5}$$

where $(1 + \boldsymbol{\sigma}\boldsymbol{\zeta})$ is two-dimensional polarization density matrix, we neglect here change of the electron spin during radiation process. If we are not interested in the initial and the final electron polarizations, then

$$\frac{1}{2} \sum_{s_i, s_f} R^*(t_2)R(t_1) = A^*(t_2)A(t_1) + \mathbf{B}^*(t_2)\mathbf{B}(t_1). \tag{6}$$

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Summing over the photon polarizations λ we have

$$\sum_{\lambda=1}^{2}\left(A^*(t_2)A(t_1)+\mathbf{B}^*(t_2)\mathbf{B}(t_1)\right)=\frac{1}{2\varepsilon'^2}\left[\frac{(\hbar\omega)^2}{\gamma^2}+(\varepsilon^2+\varepsilon'^2)\vartheta(t_2)\vartheta(t_1)\right].$$

(7)

This form of $R^*(t_2)R(t_1)$ was used in [6]. For the rectilinear trajectory

$$\sum_{\lambda=1}^{2}\left(A^*(t_2)A(t_1)+\mathbf{B}^*(t_2)\mathbf{B}(t_1)\right)=\frac{1}{2\varepsilon'^2}\left[r_1+r_2\vartheta^2\gamma^2\right],\qquad (8)$$

where ϑ is the angle between vectors $\mathbf{p}$ and $\mathbf{k}$,

$$r_1=\frac{(\hbar\omega)^2}{\varepsilon^2},\quad r_2=1+\frac{\varepsilon'^2}{\varepsilon^2}.$$

In the case considered one can expand

$$kp=\omega\varepsilon(1-\mathbf{nv})\simeq\omega\varepsilon\left(\frac{1}{2\gamma^2}+\frac{\vartheta^2}{2}+\frac{k^2}{2\omega^2}\right),$$

$$\hbar kp-\frac{(\hbar k)^2}{2}\simeq\frac{\hbar\omega\varepsilon}{2\gamma^2}\left[1+\gamma^2\vartheta^2+\frac{k^2}{m^2}\frac{\varepsilon(\varepsilon-\hbar\omega)}{\omega^2}\right].\qquad (9)$$

Substituting the results obtained ((6), (8) and (9)) into Eq.(1) we obtain the spectral-angular distribution of the probability of the transition radiation

$$\frac{dw}{d\hbar\omega\,dy}=\frac{e^2}{2\pi\hbar^2\omega}\left(r_1+r_2y\right)|M(y)|^2,\qquad (10)$$

where

$$M(y)=\int_{-\infty}^{\infty}\exp\left[-i\int_{0}^{x}(1+y+\varphi(t))dt\right]dx,\quad \varphi(t)=\frac{\omega_0^2}{\omega^2}\frac{\varepsilon\varepsilon'}{m^2}g(t)\equiv\varphi_0g(t).$$

(11)

Here we introduced the angular variable $y=\gamma^2\vartheta^2$ and substitution of the variable $\dfrac{\omega m^2}{2\varepsilon\varepsilon'}t\to t$ in the expression for $M(y)$ is performed.

An important case is the transition radiation on the boundary between vacuum and the medium. In this case $g(t)\to\vartheta(t)$ and we take integral over the angle

$$M(y)=i\left(\frac{1}{1+y}-\frac{1}{\kappa+y}\right),\quad \kappa=1+\varphi_0,$$

$$\frac{dw_{tr}}{d\hbar\omega}=\frac{e^2}{2\pi\hbar\omega}\left\{r_1\left[1+\frac{1}{\kappa}-\frac{2}{\kappa-1}\ln\kappa\right]+r_2\left[\left(1+\frac{2}{\kappa-1}\right)\ln\kappa-2\right]\right\}.\,(12)$$
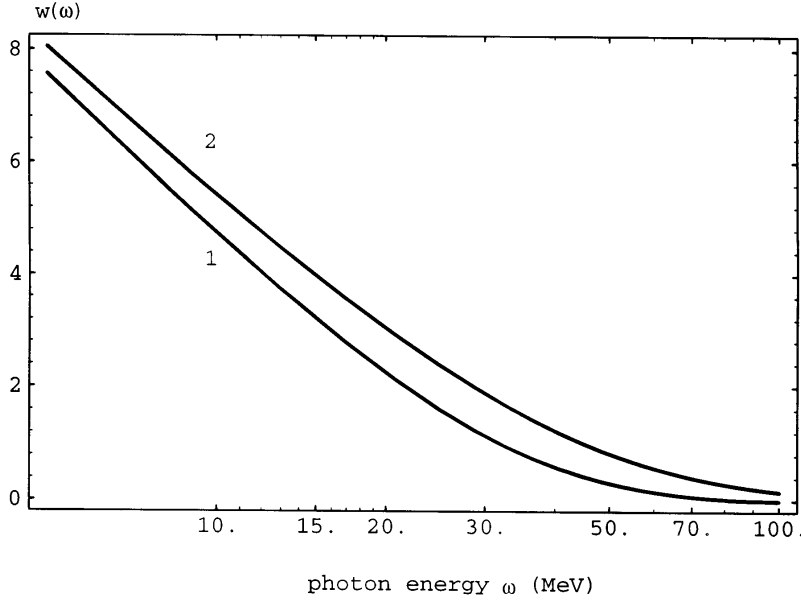
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Figure 1: The energy losses spectrum $w(\omega) \equiv \hbar\omega \dfrac{dw_{tr}}{d\hbar\omega}$ Eq.(12) in units $\dfrac{e^2}{2\pi}$ for the case $\hbar\omega_0 = 2m$, $\varepsilon = 100 \ MeV$ (curve 1) and the classical spectrum (curve 2).

The results obtained ((10)-(12)) is the quantum generalization of the theory of the transition radiation. In the classical limit $\hbar\omega \ll \varepsilon$ one has $r_1 \to 0$, $r_2 \to 2$, $\varphi_0 \to \dfrac{\omega_0^2}{\omega^2}\gamma^2 = \kappa_0^2$ and we have from (12) the known expression for the spectral distribution of radiated energy of the transition radiation on the boundary between vacuum and the medium $dE/d\omega \equiv \hbar\omega \ dw_{tr}/d\omega$ in the classical theory (see e.g. [2]-[3]). The spectrum (12) for the case $\hbar\omega_0 = 2m$, $\varepsilon = 100 \ MeV$ is shown in Fig.1 (curve 1), for comparison the classical spectrum (curve 2) is given. The classical probability is always larger than the exact one and remains finite at $\hbar\omega = \varepsilon$.

Process of the transition radiation in frame of the quantum theory was considered many years ago in [7]. The spectral distribution of the probability of the transition radiation $w(\omega)$ obtained in the cited paper differs from Eq.(12). It should be noted that Eq.(14) of [7] for $w(\omega)$ doesn't satisfy the

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symmetry relation with respect permutation $\varepsilon \leftrightarrow \varepsilon'$ in the crossing channel (after application the substitution law, see (19) below).

When $\varphi_0 = \dfrac{\omega_0^2}{\omega^2}\dfrac{\varepsilon\varepsilon'}{m^2} \ll 1$ the spectral energy losses are

$$\frac{dE}{d\omega} = \frac{e^2}{12\pi}\varphi_0^2\left[2r_1 + r_2\right] == \frac{e^2}{12\pi}\frac{\omega_0^4}{m^4}\frac{\varepsilon'^2(2(\hbar\omega)^2 + \varepsilon^2 + \varepsilon'^2)}{\omega^4}. \qquad (13)$$

So for $\hbar\omega_0 \ll m$ the hard part of the spectrum of the transition radiation (for $\hbar\omega \sim \varepsilon$) is suppressed as $(\hbar\omega_0)^4/m^4$ (for the case considered we have power suppression) tending to zero at the end of the spectrum as ε'^2.

Integrating (12) over photon energies we obtain after rather cumbersome calculation the total energy losses ΔE when the electron intersects the boundary between vacuum and the medium

$$\Delta E = \int_0^\varepsilon \hbar\omega\frac{dw_{tr}}{d\hbar\omega}d\hbar\omega$$

$$= \frac{e^2}{\pi\hbar}\varepsilon\left\{2a + \frac{4a}{3}(1 - 2a)\left[-2 + 4a\ln 4a + (1 + (1 - 2a)4a)J(a)\right]\right\}, \quad (14)$$

where $a = \dfrac{(\hbar\omega_0)^2}{4m^2} = \dfrac{\hbar^2 k^2}{4m^2}$,

$$J(a) = \begin{cases} \dfrac{1}{\sqrt{a(1-a)}}\left(\dfrac{\pi}{2} - \arctan\sqrt{\dfrac{a}{1-a}}\right), & a < 1; \\ 1, & a = 1; \\ \dfrac{1}{\sqrt{a(a-1)}}\ln\left(\sqrt{a} + \sqrt{a-1}\right), & a > 1. \end{cases}$$

In the limit $a \ll 1$ we have from (14)

$$\Delta E = \frac{e^2}{3}\omega_0\gamma\left(1 - \frac{3}{2\pi}\frac{\hbar\omega_0}{m}\right), \qquad (15)$$

here the first term is the known classical result and the second term is the first quantum correction. In the opposite limit $a \gg 1$ we find from (14) for the total energy losses ΔE

$$\Delta E = \frac{2e^2}{3\pi}\varepsilon\left(\ln 4a - \frac{1}{6}\right). \qquad (16)$$

The total energy losses ΔE when the electron intersects the boundary between vacuum and the medium are shown in Fig.2 vs $a = \dfrac{(\hbar\omega_0)^2}{4m^2}$.
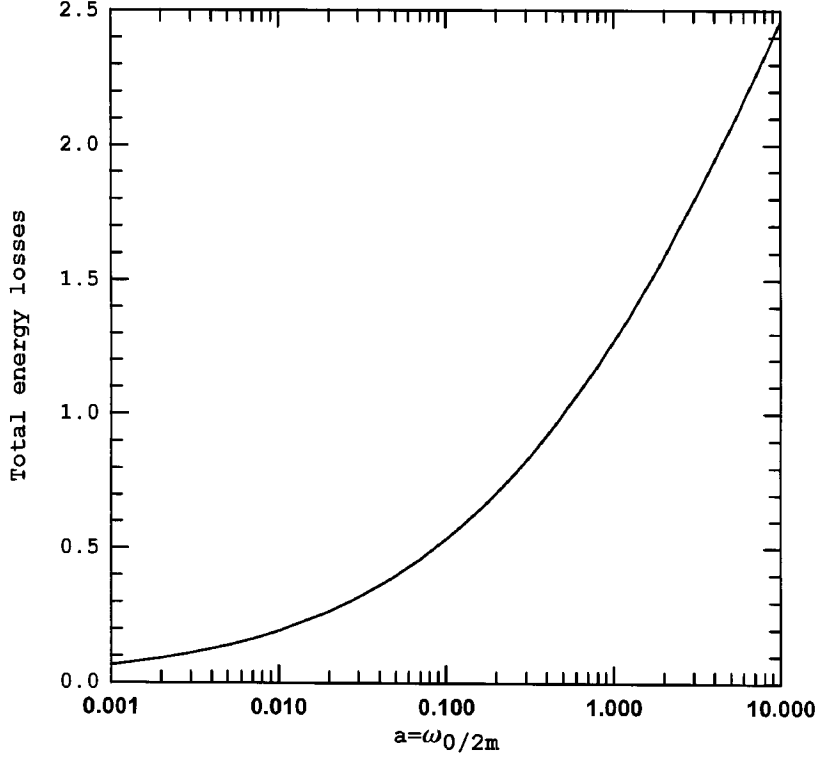
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Figure 2: The total energy losses ΔE Eq.(14) in units $\dfrac{e^2\varepsilon}{\pi\hbar}$ when the electron intersects the boundary between vacuum and a medium versus $a = \dfrac{(\hbar\omega_0)^2}{4m^2} = \dfrac{(\hbar k)^2}{4m^2}$.

For any matter on the Earth $\hbar\omega_0 < 100\ eV$ and it follows from above that photons of the transition radiation are soft ($\hbar\omega_{tr} \ll \varepsilon$). However for the matter with density $\varrho \sim 10^8 g/cm^3$ (white dwarfs) one has $\hbar\omega_0 \sim m$ and the region of photon energies $\hbar\omega_{tr} \sim \varepsilon$ is not suppressed. So the quantum theory of the transition radiation may have astrophysical applications.

The polarization of the transition radiation can be found from (6). We introduce two polarization vectors

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$$\mathbf{e}_1 = \frac{\mathbf{n} \times (\mathbf{s} \times \mathbf{n})}{|\mathbf{n} \times (\mathbf{s} \times \mathbf{n})|}, \quad \mathbf{e}_2 = \frac{\mathbf{s} \times \mathbf{n}}{|\mathbf{s} \times \mathbf{n}|}, \tag{17}$$

where $\mathbf{s} = \mathbf{v}/|\mathbf{v}|$. Substituting the amplitudes $A(t)$ and $\mathbf{B}(t)$ from Eq.(2) one obtains the Stokes's parameters

$$\xi_1 = \xi_2 = 0, \quad \xi_3 = \frac{2\varepsilon\varepsilon'\vartheta^2}{(\hbar\omega)^2/\gamma^2 + (\varepsilon^2 + \varepsilon'^2)\vartheta^2}. \tag{18}$$

In the classical limit one has $\hbar\omega \to 0$, $\varepsilon' \to \varepsilon$ and we arrive to the known result [2] that the transition radiation is completely linearly polarized ($\xi_3 = 1$) in the radiation plane.

3. The crossing process for the transition radiation is the transition pair creation: when a photon intersects the boundary of two different media (in the general case, when it is moving in a nonuniform medium) the photon mass squared $(\hbar k)^2 \neq 0$ changes and creation of the electron-positron pair becomes possible. The probability of the pair creation can be obtained from the probability of radiation with help of the substitution law:

$$d^3k \to d^3p/\hbar^3, \quad \omega \to -\omega, \quad \varepsilon \to -\varepsilon \tag{19}$$

Starting from (10) and (11) we have for the spectral-angular distribution of probability of the transition pair creation for the created electron

$$\frac{dw}{d\varepsilon dy} = \frac{e^2}{2\pi\hbar^2\omega}(s_1 + s_2 y)|M(y)|^2, \tag{20}$$

where

$$M(y) = \int_{-\infty}^{\infty} \exp\left[-i\int_0^x (1 + y - \varphi(t))dt\right] dx, \quad s_1 = 1,$$

$$s_2 = \frac{\varepsilon^2 + \varepsilon'^2}{(\hbar\omega)^2}, \quad \varepsilon + \varepsilon' = \hbar\omega, \tag{21}$$

here ε (ε') is the energy of the created electron (positron), $y = (\gamma\vartheta)^2$, ϑ is the angle between momentum of the initial photon and the momentum of the created electron, $\varphi(t)$ is defined in (11).

An important case is the transition pair creation on the boundary between vacuum and the medium. In this case $g(t) \to \vartheta(t)$ and we take integral over the angle

$$M(y) = i\left(\frac{1}{1+y} - \frac{1}{\chi + y}\right), \quad \chi = 1 - \varphi_0, \quad \varphi_0 = \frac{\omega_0^2}{\omega^2}\frac{\varepsilon\varepsilon'}{m^2}, \tag{22}$$

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$$\frac{dw_p}{d\varepsilon} = \frac{e^2}{2\pi\hbar^2\omega}\left\{ s_1\left[1 + \frac{1}{\chi} - \frac{2}{\chi - 1}\ln\chi\right] + s_2\left[\left(1 + \frac{2}{\chi - 1}\right)\ln\chi - 2\right]\right\}.$$

As one can expect the probability (22) is symmetrical with respect energies ε and ε'.

Integrating (22) over the electron energy we obtain the total probability of the transition pair creation

$$w_p = \int_0^\omega \frac{dw_p}{d\varepsilon}d\varepsilon = \frac{e^2}{2\pi\hbar}\left[\frac{5 + 2a - 4a^2}{3a(1 - a)}\sqrt{\frac{1 - a}{a}}\arctan\sqrt{\frac{a}{1 - a}} - \frac{5}{3a} - \frac{16}{9}\right],$$
(23)

where $a < 1$, a is defined in Eq.(14).

In the limit $a \ll 1$ one has from (23)

$$w_p = \frac{e^2}{2\pi\hbar}\left[\frac{8}{35}a^2 + \frac{256}{945}a^3 + O(a^4)\right].$$
(24)

This means that in this limit the pair creation probability is damped $\propto (\hbar\omega_0/m)^4$ (this is the only result obtained in [7] for pair creation, however with wrong coefficient).

The total probability of the transition pair creation w_p is shown in Fig.3 as the function of $a = (\hbar\omega_0)^2/4m^2 = (\hbar k)^2/4m^2$. It is seen that w_p grows very fast with a increase.

At $a \to 1$ Eq.(23) can be written as

$$w_p \simeq \frac{e^2}{4\hbar}\left[\frac{1}{\sqrt{1 - a}} - \frac{80}{9\pi}\right].$$
(25)

At $a = 1$ the value of w_p becomes divergent. We have to recall that w_p is the total probability of pair creation for the infinite time. If takes into account an absorption (or the imaginary part of the refraction index of the medium) the value of w_p will be finite.

At $a > 1((\hbar k)^2 > 4m^2)$ the photon becomes unstable since the channel of decay $\gamma \to e^+e^-$ pair will be open and just this process gives the contribution into the absorption. The corresponding expression for the probability the process per unit time one can obtain using Eqs.(20)-(21):

$$M(y) = 2\pi\delta\left(1 + y - a\frac{4\varepsilon\varepsilon'}{(\hbar\omega)^2}\right)$$

$$\frac{dw_p}{dtd\varepsilon} = \frac{e^2m^2}{2\hbar^2\varepsilon\varepsilon'}\int_0^\infty \left(1 + y\frac{\varepsilon^2 + \varepsilon'^2}{(\hbar\omega)^2}\right)\delta\left(1 + y - a\frac{4\varepsilon\varepsilon'}{(\hbar\omega)^2}\right)dy.$$
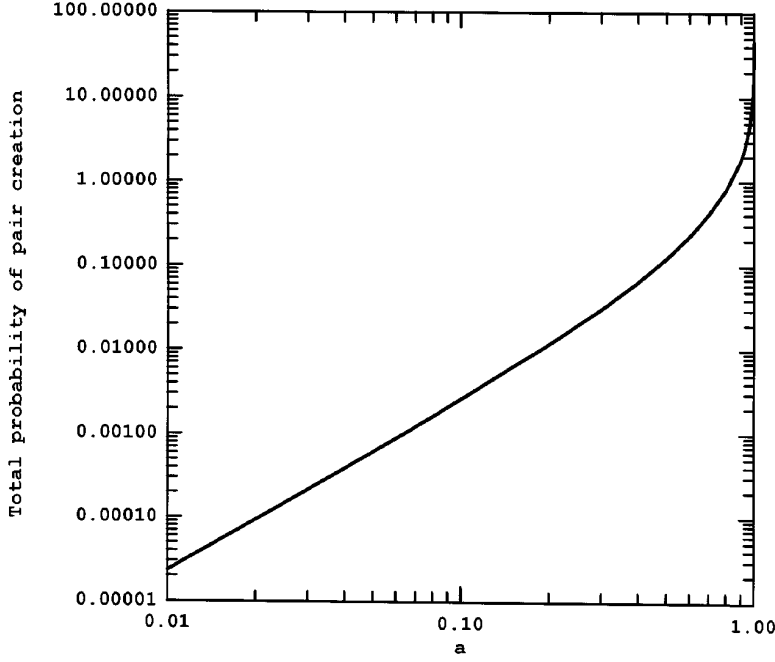(26)

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Figure 3: The total probability of the transition pair creation w_p Eq.(23) in units $\dfrac{e^2}{2\pi\hbar}$ as the function of $a = \dfrac{(\hbar\omega_0)^2}{4m^2} = \dfrac{(\hbar k)^2}{4m^2}$.

Here we returned to the standard time making the inverse substitution $t \to \dfrac{\omega m^2}{2\varepsilon\varepsilon'}t$ (see Eq.(11)). This expression can be obtained also if one considers large but finite part of the projectile trajectory in the medium. In this case the pole term (see (22)) dominates

$$M(y) \simeq \frac{i}{\chi + y}\left[\exp(i(\chi + y)T) - 1\right],$$

$$\int_0^\infty (s_1 + s_2 y)|M(y)|^2 dy \simeq \int_0^\infty (s_1 + s_2 y)\frac{1 - \cos(\chi + y)T}{(\chi + y)^2} \simeq (s_1 - s_2\chi)2\pi T;$$

$$\chi < 0, \quad -\chi T = -\chi\frac{m^2\omega}{2\varepsilon\varepsilon'}t \gg 1. \tag{27}$$

The same result will be found after integration over y in (26).

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Introducing the variable $x = \varepsilon/(\hbar\omega)$ and passing to the variable $z = 2x - 1$ we find

$$\frac{dw_p}{dt} = \frac{e^2 m^2}{\hbar^3 \omega} \int_0^\beta \left\{ 2 + (1 + z^2)\left[a(1 - z^2) - 1 \right] \right\} \frac{dz}{1 - z^2}$$

$$= \frac{e^2 m^2}{\hbar^3 \omega} \int_0^\beta \left[1 + a\left(1 + z^2 \right) \right] dz, \quad \beta = \sqrt{\frac{a - 1}{a}}. \tag{28}$$

From (28) we obtain the known expression for the probability per unit time of photon decay (or creation of the electron-positron pair by the virtual photon) for $(\hbar k)^2 > 4m^2$ in the medium (see, e.g. [8], Sec.113)

$$W \equiv \frac{dw_p}{dt} = \frac{2e^2 m^2}{3\hbar^3 \omega} \sqrt{\frac{a - 1}{a}}(2a + 1) = \frac{e^2}{3\hbar^3 \omega} \sqrt{\frac{(\hbar k)^2 - 4m^2}{(\hbar k)^2}} \left((\hbar k)^2 + 2m^2 \right) . \tag{29}$$

4. We consider now some features of the radiation and the pair creation processes in a superdense medium of the type which exists in white dwarfs. In the such medium the Landau-Pomeranchuk-Migdal (LPM) effect (suppression of the bremsstrahlung due to the multiple scattering of a projectile [9]) affects the bremsstrahlung process for any energy of radiated photon $\hbar\omega$ including the region where $\hbar\omega \sim \varepsilon$ for the initial energy $\varepsilon \geq 100$ MeV. Note that for heavy elements on the Earth this situation takes place starting from the initial electron energy of the order of a few TeV. For estimations we use the results of our papers [10], [5]. The function ν_0 is the important characteristics of the LPM effect which defines (for $\nu_0 > 1$) the mean square of the momentum transfer measured in the electron mass m on the formation length of radiation (see Eq.(2.36) of [10]):

$$\nu_0^2 = \frac{16\pi n Z^2 e^4 \varepsilon \varepsilon'}{m^4 \omega} \ln \frac{183 Z^{-1/3}}{\varrho_c}, \quad \nu_0^2(\varrho_c)\varrho_c^4 = 1, \tag{30}$$

where n is the number density of atoms of the medium, Z is the atomic number. When $\nu_0 \gg 1$ the standard (Bethe-Heitler) probability of bremsstrahlung is suppressed due to the LPM effect. For this case we introduce

$$\nu_0 = \xi_0 \sqrt{\frac{\varepsilon'}{\hbar\omega}}, \quad \xi_0 \simeq \sqrt{\frac{\varepsilon}{\varepsilon_0}} \left(1 + \frac{1}{8 \ln \left(183 Z^{-1/3} \right)} \ln \frac{\varepsilon}{\varepsilon_0} \right),$$

where $\varepsilon_0 = m \left(16\pi Z^2 \alpha^2 \lambda_c^3 n \ln \left(183 Z^{-1/3} \right) \right)^{-1}$, here $\alpha = e^2/\hbar = 1/137$, $\lambda_c = \hbar/m$. The accurate definition of ξ_0 follows from (30). For definiteness we

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consider $Z=26$ (iron), $\rho = 10^8 \ g/cm^3$ and $\varepsilon = 200$ MeV, then we have $\varepsilon_0 = 1.1$MeV, $\xi_0 \simeq 15.5$. It is apparent that for such value of ν_0 the energy losses of a projectile diminishes and the radiation length increases (see Eq.(2.43) of [10])

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dl} = L_{rad}^{-1} \simeq \frac{\alpha m^2}{2\pi\hbar\varepsilon}\int_0^\varepsilon \frac{\varepsilon^2 + \varepsilon'^2}{\varepsilon\varepsilon'}\frac{\nu_0}{\sqrt{2}}\frac{\hbar\omega \, d\hbar\omega}{\varepsilon^2}$$

$$= \frac{\xi_0\alpha m^2}{2\sqrt{2}\pi\hbar\varepsilon}\left[B\left(\frac{3}{2},\frac{1}{2}\right) + B\left(\frac{3}{2},\frac{5}{2}\right)\right] = \frac{9\xi_0\alpha m^2}{32\sqrt{2}\hbar\varepsilon} \simeq \frac{3\alpha m^2}{\hbar\varepsilon}, \quad (31)$$

where $B(x,y)$ is the Euler beta function. For chosen energy ε we have $L_{rad} \simeq 7 \cdot 10^{-7}$cm. At $\hbar\omega \sim \varepsilon$ value of ν_0 increases as $\sqrt{\gamma}$. The same behavior has $L_{rad} \propto \gamma/\xi_0 \propto \sqrt{\gamma}$. The formation length in this case is

$$l_f = \frac{2\gamma\hbar}{m(1+\nu_0)} = \frac{\gamma\hbar}{8m} \simeq 2 \cdot 10^{-9}cm. \qquad (32)$$

Note that under the selected conditions the value of $\varphi_0 = \dfrac{(\hbar\omega_0)^2}{m^2}\dfrac{\varepsilon\varepsilon'}{(\hbar\omega)^2}$ is rather small since $\hbar\omega_0 = 0.2$ MeV and one can neglect the polarization of the medium.

The differential probability of the pair creation can be found using the substitution law Eq.(19). The lifetime of a photon is

$$\frac{1}{\tau} = W_p = \frac{\alpha m^2}{2\pi\hbar^2\omega}\int_0^{\hbar\omega}\frac{\varepsilon^2 + \varepsilon'^2}{\varepsilon\varepsilon'}\frac{\nu_p}{\sqrt{2}}\frac{d\varepsilon}{\hbar\omega},$$

$$\nu_p = \xi_p\sqrt{\frac{\varepsilon\varepsilon'}{(\hbar\omega)^2}}, \quad \xi_p = \xi_0(\varepsilon \to \hbar\omega), \qquad (33)$$

here $\varepsilon' = \omega - \varepsilon$. For the used above parameters and $\omega = 200$ MeV we have

$$\frac{1}{\tau} = W_p \simeq \frac{\xi_p\alpha m^2}{\sqrt{2}\pi\hbar^2\omega}B\left(\frac{1}{2},\frac{5}{2}\right) = \frac{3\xi_p\alpha m^2}{8\sqrt{2}\hbar^2\omega} \simeq \frac{4\alpha m^2}{\hbar^2\omega}, \qquad (34)$$

$$\tau = \frac{3}{4}L_{rad} \simeq 5.3 \cdot 10^{-7} \ cm,$$

and the formation length of pair creation l_p for $\varepsilon = \omega/2$ is twice shorter than l_f ($l_p \simeq 10^{-9}cm$).

It is shown in [10] that when a projectile crosses boundary between vacuum and a medium it radiates boundary photons. The transition radiation

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can be considered as a particular mechanism of radiation of boundary photons. We consider the complete probability of the boundary radiation for $\nu_0 \gg 1$ (see Eq.(4.14) of [10]). Using this formula we have the contribution of boundary photons in the spectral distribution of energy losses

$$\frac{d\Delta E_b}{d\hbar\omega} =$$

$$= \frac{\alpha}{2\pi}\left\{r_1 + r_2\left[\ln\nu_0 - 1 - C - \ln 2 + \frac{\sqrt{2}}{\nu_0}\left(\frac{\pi^2}{24} + \ln\nu_0 + 1 - C + \frac{\pi}{4}\right)\right]\right\},$$

(35)

where we put $\kappa = 1$ (since $\varphi_0 \ll 1$). We present ν_0^2 Eq.(30) as $\nu_0 = \xi_0\sqrt{\varepsilon'/\omega}$. Because we consider situation when $\xi_0 \gg 1$ we can put ε the upper limit of the integration over ω since integrals are convergent. As a result we find for the energy losses due to boundary photons radiation

$$\Delta E_b = \frac{2\alpha\varepsilon}{3\pi}\left[\ln\xi_0 - \frac{9}{16} - C - \ln 2 + \frac{27\pi}{32\sqrt{2}\xi_0}\left(\frac{\pi^2}{24} + \ln\xi_0 + \frac{4}{27} - C + \frac{\pi}{4}\right)\right].$$

(36)

For the parameters used ($\xi_0 \simeq 15.5$)

$$\Delta E_b = 1.3\frac{2\alpha\varepsilon}{3\pi} \simeq 2 \cdot 10^{-3}\varepsilon$$

(37)

Now we turn to the boundary pair creation. Performing the substitutions Eq.(19) in Eq. (35) and integrating over the electron energy ε we obtain the total probability of boundary pair creation

$$w_p^b = \frac{\alpha}{3\pi}\left[\ln\xi_p - \frac{7}{12} - C - \ln 2+\right.$$

$$\left.\frac{9\pi}{4\sqrt{2}\xi_p}\left(\frac{\pi^2}{24} + + \ln\xi_p + \frac{5}{6} - C + \frac{\pi}{4} - 2\ln 2\right)\right],$$

(38)

here we define ν_p Eq.(33) as $\nu_p = \xi_p\sqrt{\varepsilon\varepsilon'/\omega^2}$. So we have for $\xi_p \simeq 15.5$

$$w_p^b = 1.8\frac{\alpha}{3\pi} \simeq 1.4 \cdot 10^{-3}.$$

Comparing the energy losses due to the boundary photon radiation (37) and due to the transition radiation (15) (for the parameters used $\hbar\omega_0 = 0.2$ MeV

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or
$a = 0.04 \ll 1$) we find

$$\frac{\Delta E_b}{\Delta E_{tr}} \simeq \frac{1.3}{\pi\sqrt{a}} \simeq 2.1, \tag{39}$$

so that ΔE_b is slightly larger than ΔE_{tr}. For the pair creation we have different situation: the probability of boundary pair creation w_p^b (38) is essentially larger than the probability of transition pair creation w_p (24):

$$\frac{w_p}{w_p^b} \simeq \frac{a^2}{5} \simeq 3 \cdot 10^{-4}. \tag{40}$$

Of course, one have to take into account that all the discussed boundary effects give visible contribution on the depth of the order of a few formation lengths.

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