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On Advantages of a Spatially-Encoded Fourier Spectrometer

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Abstract

A comparative analysis has been carried out for the main characteristics (geometric factor, resolving power, signal-to-noise ratio) of a spatially-encoded Fourier spectrometer (transforming transverse wave number into a coordinate), a classic "dynamic" Fourier spectrometer (transforming frequency into time), and dispersion spectrographs with record by single- and multi-element photodetectors. It is shown that at the same high geometric factor a spatially-encoded Fourier spectrometer (SFS) (in contrast to a classical Fourier spectrometer) retains the Fellgett advantage (a multiplex factor) even in the case of a noise determined by fluctuation of incident radiation. The SFS resolving power is determined by a number of detector elements and does not depend on the input aperture. A set of advantages of the SFS appears to be the best in comparison with the other instruments. Besides, it is resistant to the electromagnetic noise and vibration, and can operate in the "imagining spectroscopy" mode.

1 Introduction

In a classic Fourier spectrometer (Fig. 1,a) an interferogram that is a Fourier image of an initial spectrum is recorded by means of changing of an optical length of one of interferometer arms, in other words, the spectrum is encoded and decoded by the "time-frequency" transform[1]. The device of this type where a signal is recorded by a single-element photodetector we will call in further a dynamic Fourier spectrometer (DFS). It is well-known [1,2] that such a spectrometer is much more preferable (possess many advantages) in comparison with a dispersion one. The last years the spectroscopy techniques have considerably changed because of appearance of multi-element position-sensitive detectors (MPSD) with high resolution and personal computers. Dispersion spectrographs with MPSD have become widespread in practice (e.g., see the survey [3]), they are also called "optical multichannel analyzers" – OMA.

Appearance of the MPSD made possible practical realization of a well-known for rather long time (e.g., see [4,5]) scheme of a "spatially-encoded" Fourier spectrometer, where the transformation "wave number - coordinate" is performed, and an element changing an arm length (e.g. a moving mirror) is not necessary. In both devices a spectrum or interferogram are red not consecutively (element by element) but simultaneously, that is particularly important while recording high-

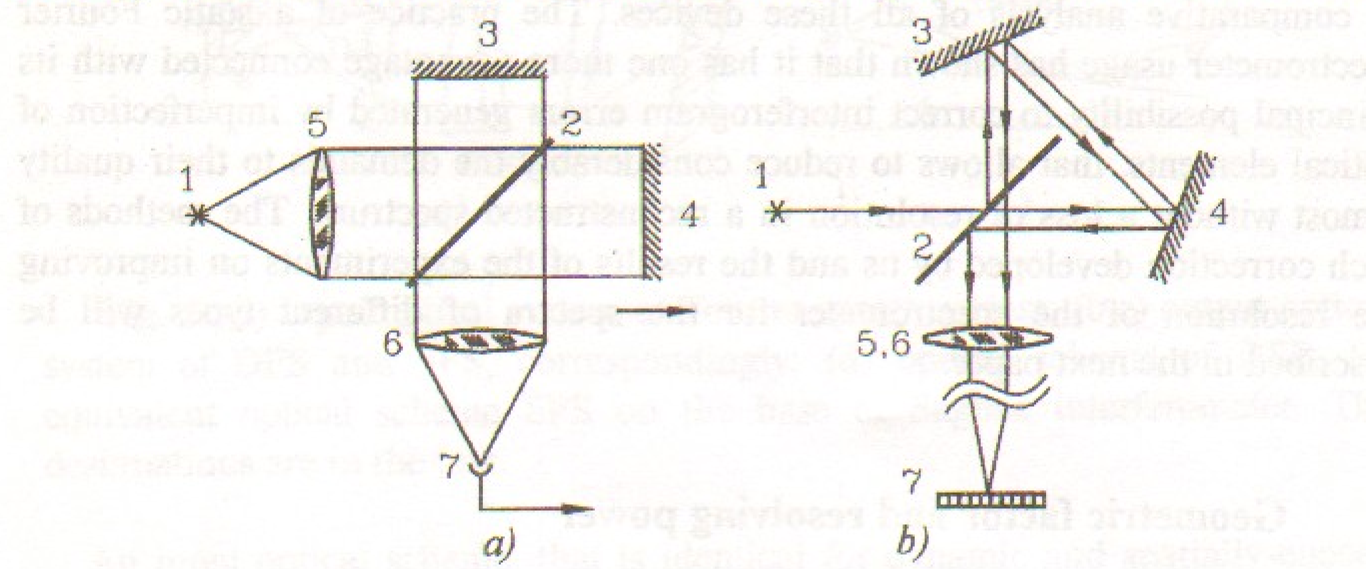

Fig. 1. Principal optical schemes of (a) a dynamic Fourier spectrometer based on the Michelson interferometer, and (b) a spatially encoded Fourier spectrometer on the base of the Sagnac interferometer: 1 - source; 2 - beam-splitter; 3,4 - mirrors; 5 - collimating lens; 6 - objective; 7 - detector.

[1] Present work have been performed in BINP and Novosibirsk State University.

speed processes.

The Fig. 1,b represents a principal scheme of SFS designed on the base of the Sagnac interferometer. In this scheme a single lens combines functions of both a collimating lens and an objective, and a source and a detector are in its front and back focal planes. If the mirror 4 is displaced at a distance *a* from its symmetrical position, two virtual sources are moved across the optical axis, and on the detector 7 interference patterns are formed as in the Young scheme[2]. To decrease the device sizes, an additional input lens may be used which images the source onto the plane between the mirrors. The SFS practical realizations are described, for instance in [6–13]. The static Fourier spectrometer is a perspective spectral device for measurement in UV, visible and near IR regions of the spectrum. Its merits are compactness, low sensitivity to vibration, possibility to function in the automatic mode, and also in the mode of "imaging spectroscopy". These properties allow to use it in both stationary and onboard equipment. For several years the first model of SFS [15] developed by us has been used in a students' laboratory of Novosibirsk University for to study the principles of Fourier optics and spectroscopy and demonstrated high reliability and educational value.

Since interest in static Fourier spectrometers increases evidently, it seems to be timely to compare the main characteristics of DFS, SFS and OMA: a geometric factor, a resolving power, a multiplex factor – that would permit to choose adequately the device type for concrete applications. This paper is devoted to comparative analysis of all these devices. The practice of a static Fourier spectrometer usage has shown that it has one more advantage connected with its principal possibility to correct interferogram errors generated by imperfection of optical elements, that allows to reduce considerably the demands to their quality almost without a loss of resolution in a reconstructed spectrum. The methods of such correction developed by us and the results of the experiments on improving the resolution of the spectrometer for the spectra of different types will be described in the next paper.

2 Geometric factor and resolving power

One of the main advantages of classic Fourier spectrometers is combination of a high resolving power and a high geometric factor (Jacquinot advantage), and in the cases when the noise is determined by a detector, they supply also the best

[2] "Symmetric position" of the mirror corresponds to the coincidence of both virtual sources, *i.e.* zero angle between interfering beams.

ratio "signal–noise" (Fellgett advantage or multiplex factor). A comparative analysis of these characteristics for dynamic Fourier spectrometer and dispersion spectrographs with single-element detectors was performed in the monographs [1, 2]. Transition to systems with multi-element detectors considerably changes the principle optical schemes of the devices; that demands some revision of the settled general ideas. At first we compare the geometrical factors of spatially-encoded and dynamic Fourier spectrometers and dispersion spectrographs while recording by means of single-element and multi-element detectors following [1, 2.]

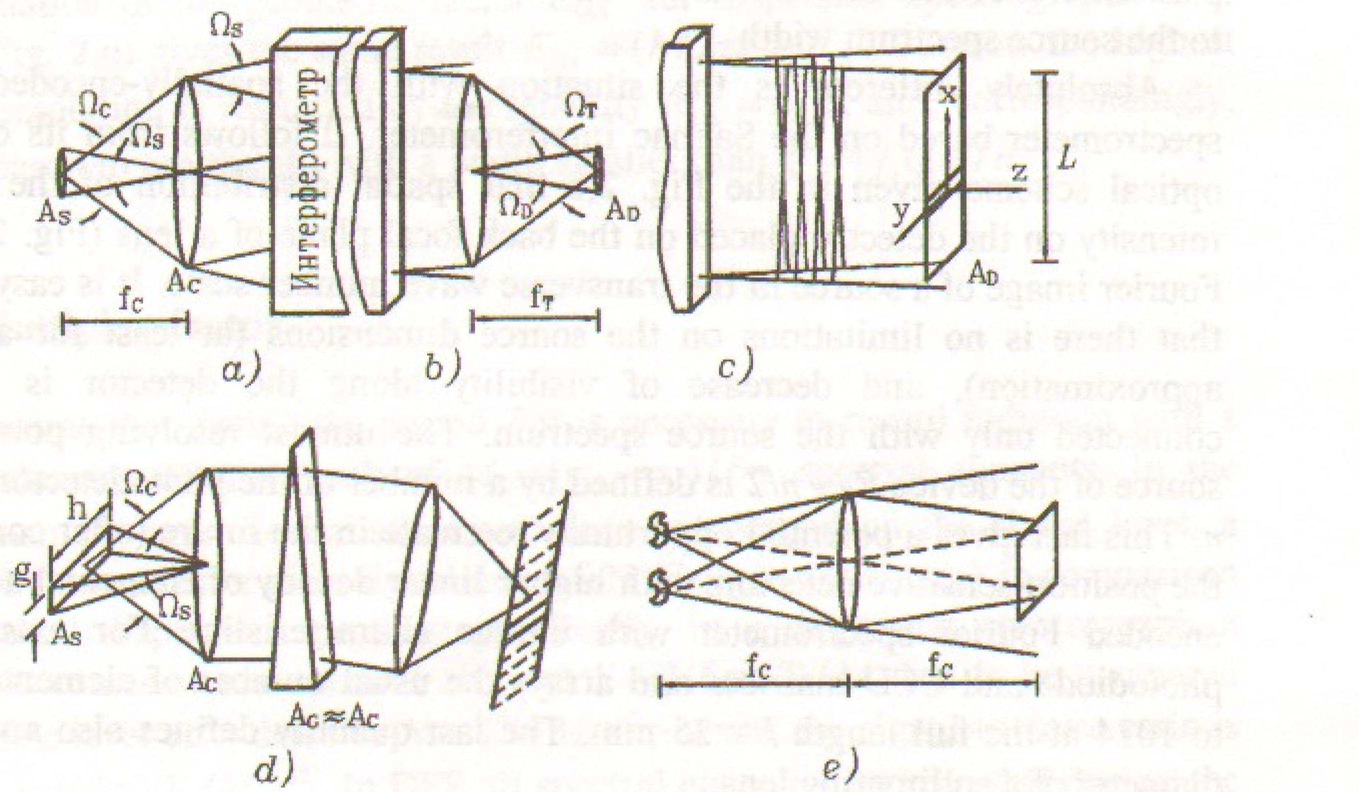

Fig. 2. (a) Input optical system of Fourier spectrometers; (b,c) output optical system of DFS and SFS, correspondingly; (d) optical scheme of DSP; (e) equivalent optical scheme SFS on the base of Sagnac interferometer. The designations are in the text.

An input optical scheme, that is identical for dynamic and spatially-encoded Fourier spectrometers, is shown in the Fig.2,a. The input characteristics of an optical device is a geometric factor that is equal to an area of an input aperture multiplied by a solid angle of a collimating lens $E = A_s\Omega_c = A_c\Omega_s$. A solid angle Ω_c is proportional to an aperture ratio of a device d_c^2/f_c^2, and $\Omega_s \approx \pi d^2/4f_c^2$, where d and d_c are diameters of an input aperture ("source") and a collimating lens, and f_c is a focal length of a collimator. In Michelson (dynamic) interferometer (Fig. 2,b) record is accomplished by a detector of an area A_d that is placed in the focus of an output lens (mirror). Let us introduce a wave number $\sigma = \lambda^{-1}$. The source dimensions are restricted (see [1]) by a path difference between axial and outer inclined rays which is permissible for a given resolving power $R_M \equiv \sigma / \delta\sigma = 8 f_c^2 / d^2$, that results in the well-known expression $R_M \Omega_s = 2\pi$. Hence, the geometric factor of the Michelson interferometer is equal to $E_M = 2\pi A_c / R_M$, so to improve its resolving power one can at the expense of decreasing of the geometric factor. The physical meaning of this limitation consists in the fact that with a displacement of a moving mirror it is impossible to distinguish the decrease of visibility of an interference signal, connected with the path difference between normal and inclined beams, and decrease of visibility due to the source spectrum width.

Absolutely different is the situation with the spatially-encoded Fourier spectrometer based on the Sagnac interferometer. It follows from its equivalent optical scheme given in the Fig. 2,e that spatial distribution of the radiation intensity on the detector placed on the back focal plane of a lens (Fig. 2,c) is the Fourier image of a source in the transverse wave number scale. It is easy to notice that there is no limitations on the source dimensions (at least for a paraxial approximation), and decrease of visibility along the detector is definitely connected only with the source spectrum. The utmost resolving power of the source of the device $R_S \sim n/2$ is defined by a number of the photodetector elements n. This fact gives a potential opportunity to create in the future (after constructing the position-sensitive detectors with higher linear density of elements) a spatially-encoded Fourier spectrometer with unique characteristics. For existing now photodiode and CCD-matrices and arrays the usual number of elements is equal to 1024 at the full length l = 25 mm. The last quantity defines also an adequate diameter of a collimating lens.

Let us assume that as a detector is used a matrix (or an array with a cylindrical lens) overlapping the whole beam aperture. For the same source and collimator the geometric factor is of course the same $E_S = E_M$. Hence, $R_S \Omega_s = \pi d^2 n / 8 f_c^2 = (R_M \Omega_s) \cdot d^2 n / 16 f_c^2$. It is simple to notice that the ratio of resolving powers of SFS and DFS is directly proportional to the number of elements of a photodetector and inversely proportional to the square of focal distance of a collimator. For compact devices f_c may be considerably small. Assuming d = 1 cm, f_c = 10 cm, at n = 1024 and the same Ω_s, we obtain $R_S \approx 2 R_M$, so the resolving power of the compact SFS is not smaller than one for DFS.

Let us turn to comparison with dispersion spectrographs (Fig. 2,d). The necessity to use an input slit leads to decrease of the geometric factor

$E_G = A_c \Omega_s = A_c g h / f_c^2$ at the similar dimensions and the same collimator. A slit width g is limited by a required resolution R_G. Taking into account that for a grating spectrograph $\sigma = (2md \cos\alpha \sin\psi)^{-1}$, where α - a half-angle between an incident and diffracted beams, d – a grating period, ψ – an inclination angle of a grating, we obtain $R_G = 2 f_c \mathrm{tg}\psi / g$. Substituting g, we get $E_G = (h / f_c) \cdot (2 A_c \mathrm{tg}\psi / R_G)$. Assuming $2\mathrm{tg}\psi \approx 1$, we transform the expression to the form $E_G \approx (h / 2\pi f_c) \cdot (2\pi A_c / R_G)$. Setting $R_G = R_M$, we obtain $E_G = (h / 2\pi f_c) \cdot E_M$, that is the geometric factor of dispersion spectrograph is considerably lower than that of Fourier spectrometer.

Calculation of the geometric factor E_{GA} for dispersion spectrographs with MPSD (Fig. 2,d) gives the same result $E_{GA} = (h / 2\pi f_c) \cdot E_M$. But now the highest possible resolution is restricted by the quantity $R_{GA} \approx n/2$, and, correspondingly, it is no reason to use the slit with a width smaller than $g = 4 f_c \mathrm{tg}\psi / n$.

3 Multiplex-factor

Let us assume that during the period T it is necessary to record radiation with a typical intensity $I(\delta\sigma)$ at each of $M = (\sigma_2 - \sigma_1)/\delta\sigma$ spectral elements. In the cases, when the noise is statistical and does not depend on the signal level, a classic DFS has a better ratio signal/noise S/N (Fellgett advantage) in comparison with a usual dispersion spectrograph. Really, in a dispersion spectrograph a signal that comes at one spectral element $S \sim I(\delta\sigma) \cdot T / M$ has to be compared with a noise level on a detector $N \sim (T / M)^{1/2}$, hence, the signal-to-noise ratio is $(S / N)_{G1} \sim I(\delta\sigma)(T / M)^{1/2}$. In DFS all spectral elements are recorded during the full period of time, and $(S / N)_{M1} \sim I(\delta\sigma) \cdot T^{1/2}$. So,

$$\frac{(S / N)_{M1}}{(S / N)_{G1}} \sim \sqrt{M} . \quad (1)$$

This advantage is lost if a detector noise is negligibly small in comparison with incident radiation fluctuations. For a dispersion device at one spectral element $(S / N)_{G2} \sim (TI(\delta\sigma) / M)^{1/2}$ is evident. In a dynamic Fourier spectrometer a detector records a signal from the whole spectrum during the full exposure time $\sim TMI(\delta\sigma)$. Correspondingly the noise is $N \sim (TMI(\delta\sigma))^{1/2}$. After the Fourier transform, according to the Parseval's theorem, the noise at the restored spectrum remains the same, whereas the signal intensity at one spectral element is

$S \sim TI(\delta\sigma)$. Hence, $(S/N)_{M2} \sim (TI(\delta\sigma)/M)^{1/2}$, and the Fellgett advantage disappears

$$\frac{(S/N)_{M2}}{(S/N)_{G2}} \sim 1. \qquad (2)$$

Now let us extend the above-stated (and well-known [1]) estimations to a spatially-encoded Fourier spectrometer and a dispersion spectrograph with MPSD, assuming that the number of sensitive elements n in order of magnitude corresponds the number of resolving elements M in the reconstructed spectrum. For the case, when the noise is determined by a detector ($N \sim T^{1/2}$), the signal-to-noise ratio for a single sensitive element in a dispersion spectrograph looks as $(S/N)_{GA1} \sim I(\delta\sigma)T^{1/2}$. For SFS with MPSD, taking into account that per one spectral element of a reconstructed spectrum there exists the signal $S \sim TI(\delta\sigma)M/n \approx TI(\delta\sigma)$, we also obtain $(S/N)_{SA1} \sim I(\delta\sigma)T^{1/2}$. If the detector noises are small, we obtain $(S/N)_{GA2} \sim (I(\delta\sigma)T)^{1/2}$ and $(S/N)_{SA2} \sim (I(\delta\sigma)T)^{1/2}$ by analogy. Thus,

$$\frac{(S/N)_{SA1}}{(S/N)_{GA1}} \sim 1, \quad \frac{(S/N)_{SA2}}{(S/N)_{GA2}} \sim 1. \qquad (3)$$

It is obvious, that while recording an interferogram and spectrum with the multi-element photodetector the Fellgett advantage exists for the both types of noise, if compare it with a single-element dispersion spectrograph.

The most interesting is, however, the comparison of the signal-to-noise ratio for dynamic and spatially-encoded Fourier spectrometers. Using the expressions obtained previously, for the noise determined by a detector, we obtain

$$\frac{(S/N)_{SA1}}{(S/N)_{M1}} \sim 1 \qquad (4)$$

whereas for the noise determined by intensity fluctuation, we have

$$\frac{(S/N)_{SA2}}{(S/N)_{M2}} \sim \frac{(I(\delta\sigma)T)^{1/2}}{(I(\delta\sigma)T/M)^{1/2}} = \sqrt{M} \qquad (5)$$

From the last expression it follows that a spatially-encoded Fourier spectrometer has better than a dynamic one the signal-to-noise ratio for the noise determined by photons statistic. Since modern CCD-matrixes have very low noise even at light detection in visible and ultraviolet ranges [16], the ratio (5) is a good argument for a wide application of spatial-encoded Fourier spectrometer. For better clearness all the results obtained previously are accumulated in the Table 1. The upper part of the table allows to compare the relative geometric factors of the instruments, and the low one (where the designation $I_1 = I(\delta\sigma)$ was introduced) demonstrates the presence (or lack) of a multiplex factor separately for the cases

when the main source of a noise is a detector itself (DN) and the noise is determined by statistical fluctuations of the radiation measured (SF).

Parameter	Spectrometer type	Noise	SE	MED
Geometric factor	DSD		$h/2\pi f_c \ll 1$	$h/2\pi f_c \ll 1$
	DFS		1	–
	SFS		–	1
Signal to noise ratio S/N (multiplex–factor)	DSD	DN	$I_1 \cdot (T/M)^{1/2}$	$I_1 \cdot (T)^{1/2}$
		SF	$(I_1 T/M)^{1/2}$	$(I_1 T)^{1/2}$
	DFS	DN	$I_1 \cdot (T)^{1/2}$	–
		SF	$(I_1 T/M)^{1/2}$	–
	SFS	DN	–	$I_1 \cdot (T)^{1/2}$
		SF	–	$(I_1 T)^{1/2}$

Table 1: Comparison of the characteristics of dispersion spectrograph (DSP), dynamic (DFS) and spatially-encoded (SFS) Fourier spectrometers with single-element (SE) and multi-element detectors (MED) at the noise determined by a detector (DN), and statistical fluctuations (SF).

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