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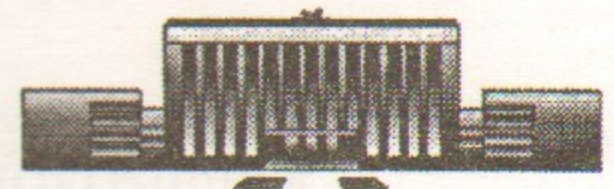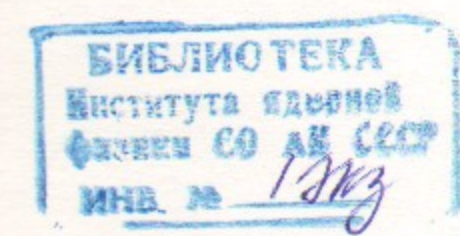V.N. Baier and V.M. Katkov

MULTI-PHOTON EFFETS IN ENERGY LOOSES SPECTRA

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Multi-photon effects in energy
losses spectra

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Abstract

Effect of radiation of many photons by a single electron traversing a target is discussed. When the summary energy of emitted photons (the energy losses spectrum) is measured only, the photon spectrum is distorted comparing with the photon spectrum in one interaction. Influence of this effect is discussed for the cases (1) bremsstrahlung (described by Bethe-Heitler formula), (2) the strong Landau-Pomeranchuk-Migdal effect and (3) transition radiation. Qualitative picture of the phenomenon is discussed in detail. Comparison with the recent SLAC experiment in relatively thick target (2.7% of the radiation length), where the effect of emission of many photons by a projectile is very essential, shows perfect agreement of the theory and data.

1 Introduction

A very successful series of experiments [1] - [3] was performed at SLAC during last years. In these experiments the cross section of bremsstrahlung of soft photons with energy from 200 KeV to 500 MeV from electrons with energy 8 GeV and 25 GeV is measured with an accuracy of the order of a few percent. Both LPM (the Landau-Pomeranchuk-Migdal effect, i.e. suppression of the bremsstrahlung from high energy electron due to a multiple scattering of an emitting electron in dense media [4] - [6]) and dielectric suppression (the effects of the polarization of a medium, [7]) are observed and investigated. These experiments were the challenge for the theory since in all the previous papers calculations (cited in [8]) are performed to logarithmic accuracy which is not enough for description of the new experiment. The contribution of the Coulomb corrections (at least for heavy elements) is larger then experimental errors and these corrections should be taken into account.

Very recently authors developed the new approach to the theory of LPM effect [8], where the cross section of bremsstrahlung process in the photon energies region where the influence of the LPM is very strong was calculated with term $\propto 1/L$, where L is characteristic logarithm of the problem, and with the Coulomb corrections taken into account. In the photon energy region, where the LPM effect is "turned off", the obtained cross section gives the exact Bethe-Heitler cross section (within power accuracy) with Coulomb corrections. This important feature was absent in the previous calculations. The contribution of an inelastic scattering of a projectile on atomic electrons is also included. The polarization of a medium is incorporated into this approach. The considerable contribution into the soft part of the measured

spectrum of radiation gives a photon emission on the boundaries of a target. We calculated this contribution taking into account the multiple scattering and polarization of a medium for the case when a target is much thicker than the formation length of the radiation. We considered also a case when a target is much thinner than the formation length. A case of an intermediate thickness of a target (between cases of a thick and a thin target) is analyzed later in [9].

It should be noted that in the experiments [1] - [3] the summary energy of all photons radiated by a single electron is measured. This means that besides mentioned above effects there is an additional "calorimetric" effect due to the multiple photon radiation. This effect is especially important in relatively thick used targets. Since the energy losses spectrum of an electron is actually measured, which is not coincide in this case with the spectrum of photons radiated in a single interaction, one have to consider the distribution function of electrons over energy after passage of a target. As it is known, this distribution function is the solution of the corresponding kinetic equation. However, the problem can be simplified essentially if one is interested in the soft part of the energy losses spectrum. Just this situation was in the experiments [1]-[3], since the measurements were performed in the region of the photon energies of one to five orders of magnitude lower than the electron energy.

In Sec. 2 of the present paper the general approach to the consideration of the soft part of the energy losses spectrum due to of many photons emission from a single electron is formulated without limitation of the target thickness. The case when the formation length is larger than the target thickness is analyzed also. Using this approach we calculated corrections (in a form of the reduction factor) to the spectrum of energy losses for the Bethe-Heitler case as well as for the situation when the LPM effect is strong and when the transition radiation is essential. In Sec. 3 we analyze basic properties of the phenomenon and apply the theory developed in [8] with multiphoton corrections to discussion of data [3]. We conclude that the theory is in a perfect agreement with data. In Appendix we calculate the radiation probability of arbitrary number of hard photons.

2 Spectral distribution of the energy losses

The probability of the successive radiation of n soft photons with energies $\omega_1, \omega_2, \ldots \omega_n$ by a particle with energy ε ($\omega_k \ll \varepsilon, k = 1, 2 \ldots n$) on the length l in the energy intervals $d\omega_1 d\omega_2 \ldots d\omega_n$ is given by expression (in this paper,

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we employ units such that $\hbar = c = 1$)

$$dw(\omega_1, \omega_2, \ldots \omega_n) = A \int_0^l dW(\omega_1) dl_1 \int_0^{l_1} dW(\omega_2) dl_2 \ldots \int_0^{l_{n-1}} dW(\omega_n) dl_n$$
$$= \frac{A}{n!} dw(\omega_1) dw(\omega_2) \ldots dw(\omega_n), \quad (2.1)$$

where A is the normalization constant, $dW(\omega)/d\omega$ is the differential probability of the photon radiation per unit length, $dw(\omega)/d\omega = l dW(\omega)/d\omega$ is the differential probability of the photon radiation per length l. If the probability $dw/d\omega$ doesn't depend on the particle energy ε then integrating (2.1) over all photon energies we obtain

$$w_n = \frac{A}{n!} w^n, \quad w = \int \frac{dw}{d\omega} d\omega. \quad (2.2)$$

The value A is defined by the condition that probability of all the possible events with radiation of any number of photons or without photon radiation is equal to unit.

$$\sum_{n=0}^{\infty} w_n = A \sum_{n=0}^{\infty} \frac{1}{n!} w^n = A \exp w = 1, \quad A = \exp(-w). \quad (2.3)$$

Using (2.1),(2.3) we can write the expression for the differential distribution of the energy losses in the form

$$\frac{1}{\omega} \frac{d\varepsilon}{d\omega} = \sum_{n=1}^{\infty} \frac{1}{n!} \exp(-w) \int \frac{dw}{d\omega_1} \frac{dw}{d\omega_2} \cdots \frac{dw}{d\omega_n} \delta \left(\sum_{k=1}^{n} \omega_k - \omega \right) d\omega_1 \ldots d\omega_n.$$
$$(2.4)$$

Here on on the right-hand side we have the sum of the probabilities of radiation of n photons with summary energy ω.

Using standard parametrization of δ-function

$$\delta \left(\sum_{k=1}^{n} \omega_k - \omega \right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp \left(is \left(\omega - \sum_{k=1}^{n} \omega_k \right) \right) ds$$

we obtain

$$\frac{d\varepsilon}{d\omega} = \frac{\omega}{2\pi} \int_{-\infty}^{\infty} \exp(is\omega - w) \sum_{1}^{\infty} \frac{1}{n!} \left(\int \frac{dw}{d\omega_1} \exp(-is\omega_1) d\omega_1 \right)^n ds$$

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$$= \frac{\omega}{2\pi} \int_{-\infty}^{\infty} \exp\left(is\omega\right) \exp\left\{ -\int_0^{\infty} \frac{dw}{d\omega_1} \left[1 - \exp(-is\omega_1)\right] d\omega_1 \right\} ds$$

$$= \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \exp\left(ix\right) \exp\left\{ -\int_0^{\infty} \frac{dw}{d\omega_1} \left[1 - \exp\left(-ix\frac{\omega_1}{\omega}\right)\right] d\omega_1 \right\} dx. \quad (2.5)$$

The formula (2.5) was derived by Landau [10] as solution of the kinetic equation under assumption that energy losses are much smaller than particle's energy (the paper [10] was devoted to the ionization losses). Let us notice the following. The energy losses are defined by the hard part of the radiation spectrum. In the soft part of the energy losses spectrum (2.5) the probability of radiation of one hard photon only is taken into account accurately. To calculate the probability of the emission of two and more hard photons one has to take into account step by step the recoil in previous acts of the photon emission. The probability of radiation of two and more hard photons is of the order $(l/L_{rad})^2$ and so on. Thus, the formula (2.5) is applicable for the thin targets and has accuracy l/L_{rad}. If we want to improve accuracy of the formula (2.5) and for the case of thick targets $l \geq L_{rad}$ one has to consider radiation of an arbitrary number of hard photons. This problem is solved in Appendix for the case when hard part of the radiation spectrum is described by the Bethe-Heitler formula. In this case the formula (2.5) acquires the additional factor. As a result we extend this formula on the case thick targets.

We will analyze first the interval of photon energies where Bethe-Heitler formula is valid. We write it in the form

$$\omega \frac{dw}{d\omega} = \frac{l}{L_{rad}} \left[\frac{4}{3}\left(1 - \frac{\omega}{\varepsilon}\right) + \frac{\omega^2}{\varepsilon^2} \right] \vartheta(\varepsilon - \omega) = \beta\left(1 - x_1 + \frac{3}{4}x_1^2\right) \vartheta(1 - x_1),$$
$$(2.6)$$

where l is the thickness of the target, L_{rad} is the radiation length,

$$\beta = \frac{4l}{3L_{rad}}, \quad x_1 = \frac{\omega}{\varepsilon}.$$

The integral in the curly brackets in (2.5) we split into two integrals with integration intervals $(0, \omega')$ and (ω', ε), where we choose ω' such that $\omega \ll \omega' \ll \varepsilon$ (note, that the radiation probability vanishes for $\omega > \varepsilon$).

We will begin with consideration of rather thin targets such that $l \ll L_{rad}$ (the case $l \sim L_{rad}$ will be discussed below). In this case in the integral over x in (2.5) contributes mainly $x \sim 1$ and the integral over the second interval

is

$$J_2 \simeq \int_{\omega'}^{\varepsilon} \frac{dw}{d\omega_1} \left[1 - \exp\left(-ix\frac{\omega_1}{\omega}\right)\right] d\omega_1 \simeq \int_{\omega'}^{\varepsilon} \frac{dw}{d\omega_1} d\omega_1 = \beta\left(\ln\frac{\varepsilon}{\omega'} - \frac{5}{8}\right),$$
$$(2.7)$$

The integral containing $\exp\left(-ix\frac{\omega_1}{\omega}\right)$ vanishes because of the fast oscillations of the integrand. The integration over the first interval gives

$$J_1 = \int_0^{\omega'} \frac{dw}{d\omega_1} \left[1 - \exp\left(-ix\frac{\omega_1}{\omega}\right)\right] d\omega_1 \simeq \beta\left[\ln\left(\frac{\omega'}{\omega}x\right) + C + i\frac{\pi}{2}\right], \quad (2.8)$$

where $C = 0.577\ldots$ is the Euler constant. So, the total integral over ω_1 in (2.5) is

$$J = J_1 + J_2 \simeq \beta\left(\ln x + \mu + i\frac{\pi}{2}\right), \quad \mu = \ln\frac{\varepsilon}{\omega} - \frac{5}{8} + C. \quad (2.9)$$

Substituting this result into (2.5) we have

$$\frac{d\varepsilon}{d\omega} = \frac{1}{\pi}\operatorname{Re}\int_0^{\infty} \exp\left[ix - \beta\left(\ln x + \mu + i\frac{\pi}{2}\right)\right] dx. \quad (2.10)$$

In this integral we turn the integration contour over x by angle $\frac{\pi}{2}$ and substitute $x \to ix$. As a result we find

$$\frac{d\varepsilon}{d\omega} = \frac{1}{\pi}\operatorname{Re} i \exp(-i\beta\pi)\exp(-\beta\mu)\int_0^{\infty} x^{-\beta}e^{-x} dx$$

$$= \beta\exp(-\beta\mu)\Gamma(1 - \beta)\frac{\sin\beta\pi}{\beta\pi} = \beta\frac{\exp(-\beta\mu)}{\Gamma(1 + \beta)}, \quad (2.11)$$

where $\Gamma(z)$ is the Euler gamma function. If we consider radiation of the one soft photon, we have from (2.5) $d\varepsilon/d\omega = \beta$. Thus, formula (2.11) gives additional "reduction factor" f_{BH} which characterizes the distortion of the soft Bethe-Heitler spectrum due to multiple photons radiation. With regard for factor $g(\beta)$ (see Appendix Eqs.(A.15), (A.16) we obtain the reduction factor valid for $\beta \geq 1$ while Eq.(2.11) is true for $\beta \ll 1$

$$f_{BH} = g(\beta)\frac{\exp(-\beta\mu)}{\Gamma(1 + \beta)} = \left(\frac{\omega}{\varepsilon}\right)^{\beta}(1 + \beta)^{1/4}\left(1 + \frac{\beta}{2}\right)^{3/4}. \quad (2.12)$$

In Fig.1 the function $f_{BH}(\omega)$ is given for electron with the energy $\varepsilon = 25$ GeV. One can see that for $\omega = 100$ MeV the value $f_{BH}(\omega)$ differs from unit in 5-6
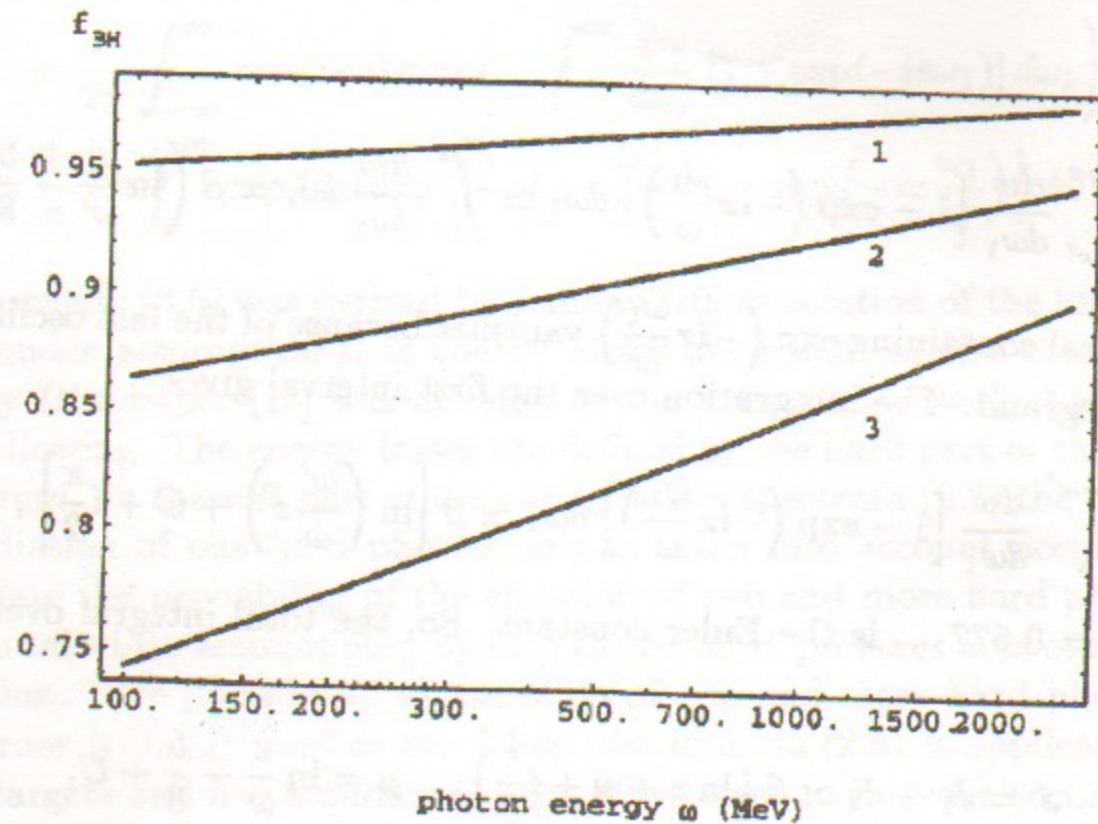
Figure 1: The reduction factor f_{BH} for energy of the initial electron $\varepsilon = 25$ GeV. The curves 1, 2 and 3 are respectively for $\beta = 0.01$, 0.03 and 0.06.

times greater than value β. Let us discuss this circumstance. The emission of accompanying photons with energy much lesser or of the order of ω (we consider in this figure the situation connected with SLAC experiment [1]-[3] where $\omega \ll \varepsilon$, $\beta \ll 1$) changes the spectral distribution on quantity order of β. However, if one photon with energy $\omega_r > \omega$ is emitted, at least, then photon with energy ω is not registered at all in the corresponding chanel of the calorimeter. Since mean number of photons with energy larger than ω is determined by the expression (see (2.1)-(2.5))

$$\sum_{n=0}^{\infty} n w_n = w_\omega = \int_\omega^\varepsilon \frac{dw}{d\omega} d\omega, \qquad (2.13)$$

the probability of the event when no photon with energy $\omega_r > \omega$ is radiated is defined by $\exp(-w_\omega)$. This is just the main factor in the expression for the reduction factor f_{BH} (2.12) while the difference begins with the terms $\sim \beta^2$. In the case, when radiation is described by the Bethe-Heitler formula the value w_ω increases as a logarithm with ω decrease ($w_\omega \simeq \beta \ln \varepsilon/\omega$) and for large ratio ε/ω the value w_ω is much larger than β. Thus, amplification of the effect is connected with large interval of the integration at evaluation

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of the radiation probability of a hard photon.

The Bethe-Heitler formula becomes inapplicable for the photon energies $\omega \leq \omega_c$, where LPM effect starts to manifest itself (see [8])

$$\omega_c = \frac{4\pi}{\alpha} \frac{\gamma^2}{L_{rad}}, \qquad (2.14)$$

where $\alpha = 1/137$, $\gamma = \varepsilon/m$. The expression for the photon spectrum for this case can be obtained from [8], Sec.2

$$\omega \frac{dw}{d\omega} = \beta\chi(\nu_0), \quad \chi(\nu_0) = 3 \operatorname{Im} \int_0^\infty \exp(-it) \left(\frac{1}{\sinh^2 z} - \frac{1}{z^2} \right) dt;$$

$$z = \nu t, \quad \nu^2 = i\nu_0^2, \quad \nu_0^2 = \frac{\omega_c}{\omega}. \qquad (2.15)$$

In this expression the terms $\sim 1/L$ (see [8]) are not taken into account, since the terms of this type can be neglected at calculation of the reduction factor (the terms of the order β/L). Using (2.15) we calculate now the total probability of radiation w. As it was done above, we divide the integration interval into two parts $(0, \omega')$ and (ω', ε) where $\omega_c \ll \omega' \ll \varepsilon$. In the interval (ω', ε) the Bethe-Heitler formula is valid and we have for w_1 formula (2.7). The integral over the second interval is

$$w_2 = \int_0^{\omega'} \frac{dw}{d\omega} d\omega = 3\beta \operatorname{Im} \int_0^{\omega'} \frac{d\omega}{\omega} \int_0^\infty \exp(-it) \left(\frac{1}{\sinh^2 z} - \frac{1}{z^2} \right) dt \quad (2.16)$$

$$= 6\beta \operatorname{Im} \int_0^\infty \Phi(z_0) \exp(-it) dt; \quad \Phi(z_0) = \int_{z_0}^\infty \frac{dz}{z} \left(\frac{1}{\sinh^2 z} - \frac{1}{z^2} \right),$$

$$z_0^2 = it^2 \frac{\omega_c}{\omega'}.$$

Performing integration by parts in the expression for $\Phi(z_0)$ we have

$$\Phi(z_0) = \ln z_0 \left(\frac{1}{z_0^2} - \frac{1}{\sinh^2 z_0} \right) + 2 \int_{z_0}^\infty \ln z \left(\frac{\cosh z}{\sinh^3 z} - \frac{1}{z^3} \right) dz. \qquad (2.17)$$

Into the integral over t in (2.17) the interval $t \leq 1$ contributes, so that $|z_0| \ll 1$. Because of this we can expand the first term on the right-hand side of formula (2.17) and put the lower limit of integration equal to zero in the second term and than turn the integration contour by angle $-\pi/4$. We find

$$\Phi(z_0) = \frac{\ln z_0}{3} + 2 \int_0^\infty \ln z \left(\frac{\cosh z}{\sinh^3 z} - \frac{1}{z^3} \right) dz. \qquad (2.18)$$

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Using the integrals

$$\int_0^\infty \exp(-it)\,dt = -i, \quad \int_0^\infty \ln z_0 \exp(-it)\,dt = i\left(\frac{1}{2}\ln\frac{\omega'}{\omega_c} + C\right), \quad (2.19)$$

we have

$$w_2 = \beta\left(\ln\frac{\omega'}{\omega_c} + 2C + C_1\right), \quad C_1 = 12\int_0^\infty \ln z\left(\frac{1}{z^3} - \frac{\cosh z}{\sinh^3 z}\right)dz \simeq 1.4294. \quad (2.20)$$

Adding to this result the probability of radiation w_1 we obtain for the total probability of radiation

$$w_c = w_1 + w_2 = \beta\left(\ln\frac{\varepsilon}{\omega_c} + C_2\right), \quad C_2 = 2C + C_1 - \frac{5}{8} \simeq 1.959. \quad (2.21)$$

We consider now the spectral distribution of the energy losses in the conditions of the strong LPM effect ($\omega \ll \omega_c$). In this case the contribution into the integral defining the function $\chi(\nu_0)$ in (2.15) gives an interval $|z| \leq 1$, $t \leq 1/\nu_0 = \sqrt{\omega/\omega_c} \ll 1$, so that $\exp(-it) \simeq 1$. We have as a result

$$\chi(\nu_0) = -3\,\mathrm{Im}\,\frac{1}{\nu} = \frac{3}{\sqrt{2}\nu_0}; \quad \frac{dw}{d\omega} = \frac{3}{\sqrt{2}}\frac{\beta}{\sqrt{\omega\omega_c}}. \quad (2.22)$$

Let us use this result in the integral in formula (2.5)

$$\int_0^\infty \frac{dw}{d\omega_1}\exp\left(-ix\frac{\omega_1}{\omega}\right)d\omega_1 = \frac{3}{\sqrt{2}}\beta\int_0^\infty \frac{d\omega_1}{\sqrt{\omega_1\omega_c}}\exp\left(-ix\frac{\omega_1}{\omega}\right)$$

$$= 3\beta\sqrt{\frac{\pi\omega}{2x\omega_c}}\exp\left(-i\frac{\pi}{4}\right) \equiv \varphi(x). \quad (2.23)$$

Substituting (2.21) and (2.23) into formula (2.5) and taking into account the factor $g(\beta)$ (A.15) we obtain for the energy losses

$$\frac{d\varepsilon}{d\omega} = \exp(-w_c)\frac{g(\beta)}{\pi}\,\mathrm{Re}\int_0^\infty \exp\left(ix + \varphi(x)\right)dx. \quad (2.24)$$

The main contribution to the last integral gives $x \sim 1$. Then $|\varphi(x)| \ll 1$ and one can expand the integrand in (2.24) in powers of $\varphi(x)$. Conserving the two first term of the expansion we have integrals

$$\mathrm{Re}\,\exp\left(-i\frac{\pi}{4}\right)\int_0^\infty \frac{dx}{\sqrt{x}}\exp(ix) = \sqrt{\pi}, \quad \mathrm{Re}\,\exp\left(-i\frac{\pi}{2}\right)\int_0^\infty \frac{dx}{x}\exp(ix) = \frac{\pi}{2}. \quad (2.25)$$

Using this integrals we find for the distribution of the spectral energy losses

$$\frac{d\varepsilon}{d\omega} = 3\beta\sqrt{\frac{\omega}{2\omega_c}}f_{LPM}, \quad f_{LPM} = g(\beta)\left(1 + \frac{3\pi}{2\sqrt{2}}\beta\sqrt{\frac{\omega}{\omega_c}}\right)\exp(-w_c), \quad (2.26)$$

where f_{LPM} is the reduction factor in the photon energy range where the LPM effect is essential. In Fig.2 the function $f_{LPM}(\omega)$ is shown for $\omega < 100$ MeV ($\varepsilon = 25$ GeV, $\omega_c = 228$ MeV). Taking in consideration that $\omega \ll \omega_c$ and using Eq.(2.22) we find (see also discussion after (2.12))

$$w_\omega = \int_\omega^\varepsilon \frac{dw}{d\omega}d\omega = w_c - \int_0^\omega \frac{dw}{d\omega}d\omega \simeq w_c - 3\sqrt{2}\beta\sqrt{\frac{\omega}{\omega_c}},$$

$$\exp(-w_\omega) \simeq \left(1 + 3\sqrt{2}\beta\sqrt{\frac{\omega}{\omega_c}}\right)\exp(-w_c). \quad (2.27)$$

In the last expression the coefficient $3\sqrt{2}$ entering into the term $\sim \beta$ differs from exact value in (2.26) on factor $4/\pi = 1.27$ only. It is seen in Fig.2 that the reduction factor f_{LPM} changes appreciably in the region of high photon energies solely. This is due to the fact that here the total probability of photon radiation is finite (in contract to the Bethe-Heitler formula) and the integral which defines this probability converges at $\omega \to 0$.

In the above analysis we neglected an influence of the polarization of a medium on the bremsstrahlung. This is correct if $\kappa \ll \nu_0$ (see (2.15) and [8], Sec.3) where

$$\kappa = 1 + \frac{\omega_p^2}{\omega^2} \equiv 1 + \kappa_0^2, \quad \omega_p = \gamma\omega_0, \quad \omega_0^2 = \frac{4\pi\alpha n}{m}, \quad (2.28)$$

where n is the electron density. We assume, for definiteness, that $\omega_p \ll \omega_c$. This is true in any case for a dense matter if electron energy $\varepsilon \geq 10$ GeV. In the opposite case when $\kappa \gg \nu_0 \gg 1$ there is an additional suppression of the bremsstrahlung (see [8], Sec.3):

$$\omega\frac{dw}{d\omega} \simeq \frac{3l}{4\kappa L_{rad}} = \frac{\beta}{\kappa} \quad (2.29)$$

and this contribution into reduction factor (2.26) can be neglected.

The main contribution into effect considered for the photon energies such that $\omega \ll \omega_p$ gives the transition radiation The differential probability of the transition radiation on the boundary of the vacuum and a medium is (neglecting interference between nearby edges, see, e.g. [11], Sec.14)

$$\omega\frac{dw_{tr}}{d\omega} = \frac{2\alpha}{\pi}\left[\left(1 + \frac{2}{\kappa-1}\right)\ln\kappa - 2\right] \equiv \eta J_{tr}, \quad (2.30)$$

f_{LPM} vertical axis with values 0.95, 0.9, 0.85, 0.8, 0.75, 0.7. Curves labeled 1, 2, 3. Horizontal axis: photon energy ω (MeV), values 1, 2, 5, 10., 20., 50., 100.
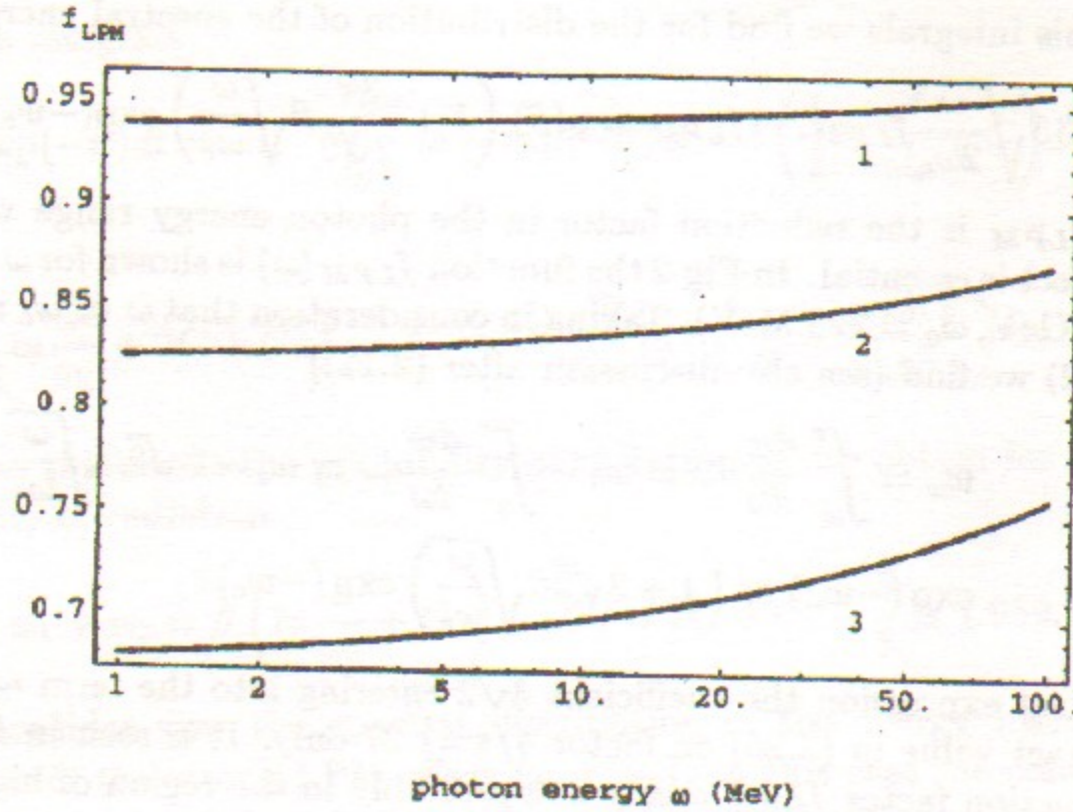
Figure 2: The reduction factor f_{LPM} for energy of the initial electron $\varepsilon = 25$ GeV in tungsten ($\omega_c = 228$ MeV). The curves 1, 2 and 3 are respectively for $\beta = 0.01, 0.03$ and 0.06.

where notations are introduced

$$\eta = \frac{2\alpha}{\pi}, \quad J_{tr} = \left(1 + \frac{2}{\kappa - 1}\right) \ln \kappa - 2. \quad (2.31)$$

Let us note, that for the transition radiation the given above derivation of the expression (2.5) is directly inapplicable. However, for application of formula (2.5) it is enough that acts of radiation of soft photons are statistically independent. We calculate now the integral over ω_1 in (2.5) splitting as above the integration interval over ω_1 into two parts: $(0, \omega')$ and (ω', ∞) where $\omega \ll \omega' \ll \omega_p$. In the upper interval we have

$$\int_{\omega'}^{\infty} J_{tr}(\omega_1) \left[1 - \exp\left(-ix\frac{\omega_1}{\omega}\right)\right] \frac{d\omega_1}{\omega_1} \simeq \int_{\omega'}^{\infty} J_{tr}(\omega_1) \frac{d\omega_1}{\omega_1} \simeq \left(\ln \frac{\omega_p}{\omega'} - 1\right)^2 + \frac{\pi^2}{12}. \quad (2.32)$$

In the second interval $(\omega_1 \leq \omega')$ we find

$$J_{tr}(\omega_1) \simeq -2\left(\ln \frac{\omega_1}{\omega_p} + 1\right), \quad \int_0^{\omega'} \frac{d\omega_1}{\omega_1}\left(\ln \frac{\omega_1}{\omega_p} + 1\right)\left[1 - \exp\left(-ix\frac{\omega_1}{\omega}\right)\right]$$

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$$= \int_0^b \frac{dz}{z}\left(\ln \frac{\omega}{x\omega_p} + 1 + \ln z\right)[1 - \exp(-iz)], \quad b = x\frac{\omega'}{\omega}. \quad (2.33)$$

Since the probability $dw(\omega_1)/d\omega_1$ in (2.5) is relatively small (this situation could changes drastically if one considers a pile consisting of many plates) and the region $x \sim 1$ contributes mainly into the integral over x we have $b \gg 1$, one can expand integral in (2.33) over $1/b$. Conserving the main term of this expansion and adding the result to formula (2.32) we obtain

$$\int_0^{\infty} J_{tr}(\omega_1)\left[1 - \exp\left(-ix\frac{\omega_1}{\omega}\right)\right]\frac{d\omega_1}{\omega_1} \simeq (\ln x + \sigma)(\ln x + \sigma + i\pi), \quad (2.34)$$

where

$$\sigma = \ln \frac{\omega_p}{\omega} + C - 1. \quad (2.35)$$

Substituting the expression obtained into formula (2.5) and taking into account the factor $g(\beta)$ (A.15) we have

$$\frac{d\varepsilon}{d\omega} = \frac{g(\beta)}{\pi} \exp(-w_c) \operatorname{Re} \int_0^{\infty} \exp\left[ix - \eta(\ln x + \sigma)(\ln x + \sigma + i\pi)\right] dx. \quad (2.36)$$

In the integral in (2.36) we turn the integration contour by the angle $\pi/2$. We find

$$\frac{d\varepsilon}{d\omega} = \frac{g(\beta)}{\pi} \exp(-w_c) \operatorname{Im} \exp\left[-\eta\left(\sigma^2 + 2i\pi\sigma - \frac{3\pi^2}{4}\right)\right]$$
$$\times \int_0^{\infty} \exp\left(-x - \eta \ln^2 x\right) x^{-2\eta(\sigma + i\pi)} dx. \quad (2.37)$$

Expanding here $\exp\left(-\eta \ln^2 x\right) \simeq 1 - \eta \ln^2 x$ $(\eta \ll 1)$ we have integrals

$$\int_0^{\infty} \exp(-x) x^{-2\eta(\sigma + i\pi)} dx = \Gamma(1 - 2\eta(\sigma + i\pi)),$$
$$\int_0^{\infty} \exp(-x) \ln^2 x\, x^{-2\eta(\sigma + i\pi)} dx = \Gamma''(1 - 2\eta(\sigma + i\pi)), \quad (2.38)$$

where $\Gamma''(z) = \dfrac{d^2\Gamma(z)}{dz^2}$. Substituting these integrals into (2.37) and taking into account that $\eta \ll 1$ (but it may be that $\eta\sigma \sim 1$) we obtain

$$\frac{d\varepsilon}{d\omega} = \frac{g(\beta)}{\pi}\exp(-w_c)\exp\left[-\eta\left(\sigma^2 - \frac{3\pi^2}{4} + \psi^2(1 - 2\eta\sigma) + \psi'(1 - 2\eta\sigma)\right)\right]$$
$$\times \sin\left[2\pi\eta(\sigma + \psi(1 - 2\eta\sigma))\right]\Gamma(1 - 2\eta\sigma), \quad (2.39)$$

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where $\psi(z) = d\ln\Gamma(z)/dz$. In the case $\eta\sigma \ll 1$ we have from (2.39), (2.35)

$$\frac{d\varepsilon}{d\omega} = 2\eta\left(\ln\frac{\omega_p}{\omega} - 1\right)g(\beta)\exp(-w_c)f_{tr},$$

$$f_{tr} = \exp\left[-\eta\left(\ln\frac{\omega_p}{\omega} - 1\right)^2\left(1 + \frac{\eta\pi^2}{3}\right) + \frac{\eta\pi^2}{4}\right]. \quad (2.40)$$

The case of a thin target, where the photon formation length

$$l_f = \frac{2\gamma^2}{\omega}(\kappa + \nu_0) \geq l, \quad (2.41)$$

l is the target thickness, should be analyzed separately. We consider situation when effects of the polarization of a medium are weak ($\kappa_0^2 < \nu_0$) In this case the scattering takes place on a target as a whole during the radiation process and the spectral probability $dw/d\omega$ in Eqs.(2.2)-(2.5) depends on the momentum transfer $q = \gamma\vartheta$ (see Sec.V of [8], q is measured in the electron mass)

$$\frac{dw_r}{d\omega} = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}F_2\left(\frac{q}{2}\right) \equiv \beta_1(q)\frac{d\omega}{\omega}, \quad (2.42)$$

$$F_2(x) = \frac{2x^2 + 1}{x\sqrt{1+x^2}}\left(\ln\left(x + \sqrt{1+x^2}\right) - 1\right),$$

where the function $F_2(x)$ is defined in Eq.(5.17) of [8]. On the final step one have to average $d\varepsilon_r/d\omega$ over q with the distribution function $F_s(\mathbf{q})$ (see Eq.(5.18) of [8]).

Since the expression for the spectral distribution (2.42) has the same form as in the Bethe-Heitler case, one can use Eqs.(2.8) and (2.11) for calculation of the reduction factor. As a result we have for the contribution of the thin target region

$$f_{th} = \frac{1}{\langle\beta_1(q)\rangle}\left\langle\beta_1(q)\exp\left[-\beta_1(q)\ln\frac{\omega_{th}}{\omega}\right]\right\rangle \simeq 1 - \frac{2\alpha}{\pi}\frac{\langle F_2^2\left(\frac{q}{2}\right)\rangle}{\langle F_2\left(\frac{q}{2}\right)\rangle}\ln\frac{\omega_{th}}{\omega}, \quad (2.43)$$

where ω_{th} is the boundary energy starting from which the target becomes thin. The lower limit of the applicability of Eq.(2.43) is determined from the condition $l_f = l$. The expression (2.43) depends essentially on the mean square of the momentum transfer q_s^2 associated with the target thickness. In the case $q_s^2 \ll 1$ when an influence of the multiple scattering is weak ($\vartheta_s^2 \ll 1/\gamma^2$)

$$F_s(q) = \frac{4Z^2\alpha^2}{(q^2 + \alpha_s^2)^2}\frac{nl}{m^2}, \quad \alpha_s = \frac{\lambda_c}{a_{s2}}, \quad (2.44)$$

where a_{s2} is the effective screening radius (see Eq.(2.19) in [8]). When the multiple scattering is strong ($q_s^2 \gg 1$, $\vartheta_s^2 \gg 1/\gamma^2$), the reduction factor can be calculated using the Gaussian distribution for $F_s(q)$

$$F_s(q) = \frac{1}{\pi q_s^2}\exp\left(-\frac{q^2}{q_s^2}\right), \quad (2.45)$$

$$q_s^2 = \frac{4\pi Z^2\alpha^2}{m^2}nl\int_0^{q_s^2+1}\frac{q^2 dq^2}{(q^2 + \alpha_s^2)^2} \simeq \frac{4\pi Z^2\alpha^2}{m^2}nl\ln\frac{q_s^2 + 1}{\alpha_s^2}.$$

In the first case ($q_s^2 \ll 1$) one has $\langle F_2^2\rangle \ll \langle F_2\rangle$ and one can neglect the correction to unit in (2.43). In the opposite case ($q_s^2 \gg 1$) the main contribution into entering mean values of F_2 gives values $q \gg 1$ where

$$F_2(q) \simeq \ln q^2 - 1 + \frac{2}{q^2}. \quad (2.46)$$

In this case the effect under consideration may be noticeable.

3 Basic properties of the multi-photon radiation. Comparison with experimental data

From the analysis in Sec. 2 one can find an approximate expression for the reduction factor in the general form. One can present the integral over ω_1 in (2.5) as

$$\int_0^\infty \frac{dw}{d\omega_1}\left[\exp\left(-ix\frac{\omega_1}{\omega}\right) - 1\right]d\omega_1 \simeq \quad (3.1)$$

$$-\int_\omega^\infty \frac{dw}{d\omega_1}d\omega_1 + \int_0^\omega \frac{dw}{d\omega_1}\left[\exp\left(-ix\frac{\omega_1}{\omega}\right) - 1\right]d\omega_1.$$

The second integral on the right-hand side of (3.1) (as well as terms omitted in the first integral) has an order $\omega dw/d\omega$. For the case $\beta \ll 1$ this value is small

$$\omega\frac{dw}{d\omega} \leq \frac{4l}{3L_{rad}} \equiv \beta \quad (\omega \geq \omega_p);$$

$$\omega\frac{dw}{d\omega} \simeq \frac{4\alpha}{\pi}\left(\ln\frac{\omega_p}{\omega} - 1\right) = 2\eta(\sigma - C), \quad \eta = \frac{2\alpha}{\pi} \quad (\omega \ll \omega_p). \quad (3.2)$$

Let us note that a contribution of the transition radiation can be enlarged n times if one makes a target as a collection of n plates conserving the total

thickness l provided that definite conditions are fulfilled for the plate thicknesses and gaps between plates. In that case $\eta \to n\eta$ and formulae of Sec.2 are valid if $n\eta \leq 1$.

Expanding the exponential in (2.5) with the second integral of (3.1) and integrating over x we have

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(ix)dx \int_0^{\omega} \frac{dw}{d\omega_1} \left[\exp\left(-ix\frac{\omega_1}{\omega} \right) - 1 \right] d\omega_1$$

$$= \int_0^{\omega} \frac{dw}{d\omega_1} \delta \left(1 - \frac{\omega_1}{\omega} \right) d\omega_1 = \omega \frac{dw}{d\omega}. \qquad (3.3)$$

Thus, the spectral distribution of the energy losses and reduction factor f have the following general form

$$\frac{d\varepsilon}{d\omega} = \omega \frac{dw}{d\omega} f, \quad f = \exp\left[-\int_{\omega}^{\infty} \frac{dw}{d\omega_1} d\omega_1 \right] \left(1 + O\left(\omega \frac{dw}{d\omega} \right) \right). \qquad (3.4)$$

The cross section of the bremsstrahlung process in the photon energies region where the influence of the LPM is very strong was calculated in [8] with term $\propto 1/L$, where L is characteristic logarithm of the problem, and with the Coulomb corrections taken into account. The polarization of a medium was included into the approach. We calculated also the contribution of a photon emission on the boundaries of a target which involves the transition radiation. The energy losses spectrum in the tungsten target with the thickness $l = 2.7\% L_{rad} (\beta = 0.036)$ for the initial electron energies $\varepsilon = 25$ GeV and $\varepsilon = 8$ GeV was calculated and shown in Fig.2 in [8]. The contribution of an inelastic scattering of a projectile on atomic electrons was not included into the numerical calculation (it is ~ 1 %), although it can be done using Eq.(2.46) of [8]. Here in Fig.3 we add to these results the multi-photon effects. The curves 1-5 is the same curve as in Fig.2 of [8], the curve T involves the reduction factor which we constructed as the interpolation of Eqs.(2.12), (2.26), (2.40) with accuracy up to 1%.

The curves T in Fig.3 (a) and (b) are the final theory prediction in the units $2\alpha/\pi$. Experimental data are taken from [3] and recalculated according with given in this paper procedure

$$\left(\frac{d\varepsilon}{d\omega} \right)_{exp} = \frac{l}{L_{rad}} \frac{N_{exp}}{k}. \qquad (3.5)$$

In our papers [8] and [9] we put k=0.09 according with the instruction given in paper [3]. However, in the recent review [12] it was definitely indicated that k=0.096 what is correspond to the definition (photon energies were histogrammed logarithmically, using 25 bins per decade of energy). So, in this paper we used k=0.096. Data recalculated with this coefficient are also given in Fig.3. It is seen that there is the perfect agreement of the curves T with data for both energies.

In our paper [9] we compared calculation for the gold target with the thickness $l = 0.7$ % L_{rad} and energy ε=25 GeV with data from [3]. In this case the reduction factor $f \simeq 0.94$ for photons with energy $\omega < 10$ MeV (plateau region). From the other side, use of the coefficient $k = 0.096$ in (3.5) instead of $k = 0.09$ lowers data upon 6%. As a result, the excellent agreement of the theory and data marked in [9] is not broken.

It is seen in Fig.3, as well as in Fig.2 [8] and in Fig.1 [9], that in the soft part of the measured spectrum the boundary photons emission dominates. The formulae describing this effect were derived in [8] and [9] and this is allowed to obtain description of the measured spectrum on the whole. The authors of [3] came across some problems with surface effects theory. Because of this an attempt was made to eliminate the boundary radiation by subtracting spectra from targets made of the same material, but different thicknesses (e.g. 5 % - 3 % of L_{rad} in uranium (U)) assuming that an interference between the two target edges is negligible (this is true for thick target [9]). However, the subtraction gives no direct quantitative information about boundary radiation because the effect of the multiple photon emission is rather strong (more than 30 % in the target with the thickness 5 % L_{rad}). One can conclude only that this difference should vanish (if thicknesses are not very different) in the region where the contribution of the boundary radiation is comparable with radiation inside a target. In the photon energy interval where the transition radiation dominates this difference is negative and it is proportional to the difference of the reduction factors. There is no data in the mentioned interval.

We would like to thank S. Klein who attract our attention to important role of many photons emission at the SLAC experiment.
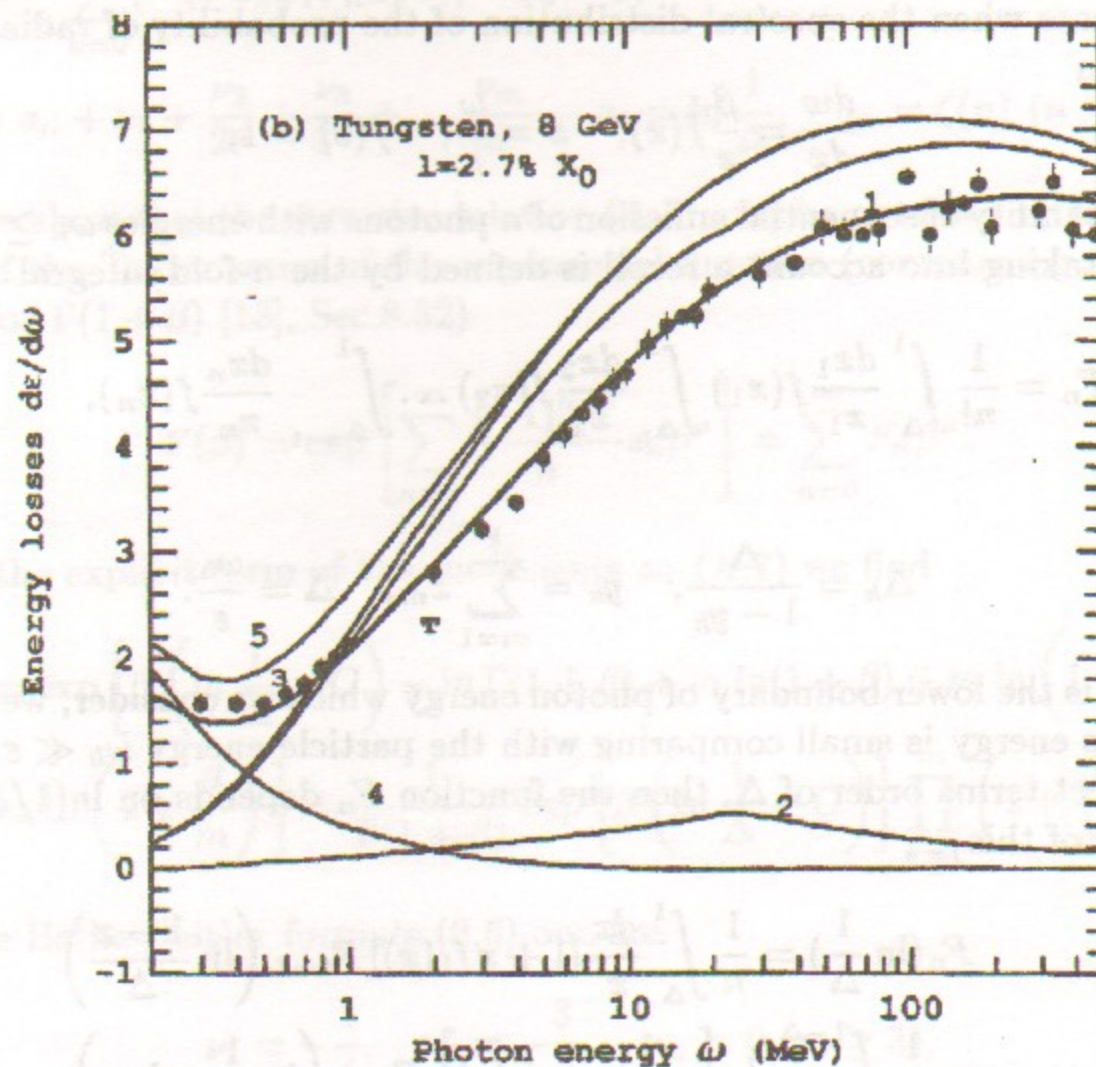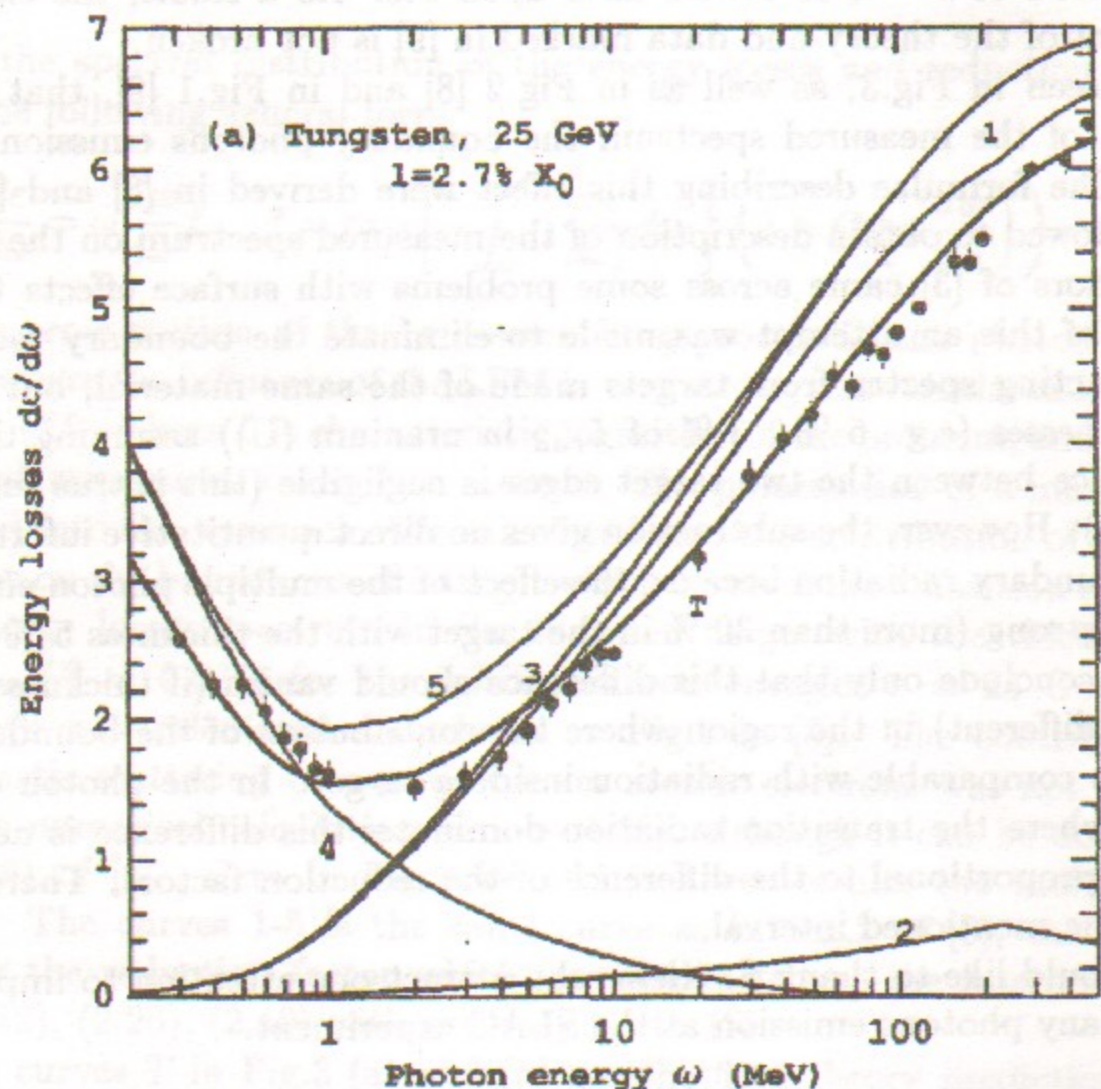
(a) Tungsten, 25 GeV
$l=2.7\% \ X_0$

Energy losses $d\varepsilon/d\omega$

Photon energy ω (MeV)

(b) Tungsten, 8 GeV
$l=2.7\% \ X_0$

Energy losses $d\varepsilon/d\omega$

Photon energy ω (MeV)

Figure 3: The energy losses $\frac{d\varepsilon}{d\omega}$ in tungsten with thickness $l = 0.088$ mm in units $\frac{2\alpha}{\pi}$, ((a) is for the initial electrons energy $\varepsilon = 25$ GeV and (b) is for $\varepsilon = 8$ GeV). The Coulomb corrections and the polarization of a medium are included: Curve 1 is the contribution of the main term describing LPM effect; curve 2 is the correction term; curve 3 is the sum of the previous contributions; curve 4 is the contribution of the boundary photons; curve 5 is the sum of the previous contributions; curve T is the final theory prediction with regard for the reduction factor (the multiphoton effects). Experimental data from Fig.9 of [3].

A Appendix

In the case when the spectral distribution of the probability of radiation has the form

$$\frac{dw}{dx} = \frac{\beta}{x} f(x), \quad x = \frac{\omega}{\varepsilon}, \quad f(0) = 1, \tag{A.1}$$

the probability of sequential emission of n photons with energies $\omega_k \geq \omega_b$ ($k = 1 \ldots n$) taking into account a recoil is defined by the n-fold integral

$$F_n = \frac{1}{n!} \int_\Delta^1 \frac{dx_1}{x_1} f(x_1) \int_{\Delta_1}^1 \frac{dx_2}{x_2} f(x_2) \ldots \int_{\Delta_{n-1}}^1 \frac{dx_n}{x_n} f(x_n), \tag{A.2}$$

where

$$\Delta_k = \frac{\Delta}{1 - y_k}, \quad y_k = \sum_{m=1}^k x_m, \quad \Delta = \frac{\omega_b}{\varepsilon}. \tag{A.3}$$

Here ω_b is the lower boundary of photon energy which we consider, we assume that this energy is small comparing with the particle energy $\omega_b \ll \varepsilon$. Below we neglect terms order of Δ, then the function F_n depends on $\ln(1/\Delta)$ only. Because of this

$$F_n(\ln \frac{1}{\Delta}) = \frac{1}{n} \int_\Delta^1 \frac{dx}{x} [1 + x f_1(x)] F_{n-1}\left(\ln \frac{1-x}{\Delta}\right)$$
$$= \frac{1}{n} \int_0^{1-\Delta} dz \left[\frac{1}{1-z} + f_2(z)\right] F_{n-1}\left(\ln \frac{1}{\Delta} + \ln z\right), \tag{A.4}$$

where $f(x) = 1 + x f_1(x)$, $f_2(z) = f_1(1-z)$.
We assume that $f_2(z)$ is a polynomial of degree m

$$f_2(z) = \nu_1 + \nu_2 z + \ldots \nu_m z^{m-1} \tag{A.5}$$

Using the integrals

$$\int_0^{1-\Delta} \frac{dz}{1-z} = \ln \frac{1}{\Delta},$$
$$\int_0^{1-\Delta} \frac{dz}{1-z} \ln^n z \simeq \int_0^1 \frac{dz}{1-z} \ln^n z = (-1)^n n! \, \zeta(n+1),$$
$$\int_0^1 dz z^m \ln^n z = (-1)^n \frac{n!}{(m+1)^{n+1}}, \tag{A.6}$$

we find the following recursive relation

$$F_n = \frac{1}{n} \sum_{k=0}^{n-1} (-1)^k s_{k+1} F_{n-k-1}, \quad F_0 = 1, \tag{A.7}$$

$$s_n = a_n + \nu_1 + \frac{\nu_2}{2^n} + \frac{\nu_3}{3^n} + \ldots \frac{\nu_m}{m^n}, \quad a_1 = \ln \frac{1}{\Delta}, \quad a_n = \zeta(n) \ (n \geq 2) \tag{A.7}$$

One can show that the recursive relation (A.7) may be obtained by rearrangement of the Taylor series of the exponential function (compare e.g. with the series for $\Gamma(1 + \beta)$ [13], Sec.8.32)

$$F(\beta) = \exp\left[\sum_{n=1}^\infty \frac{(-1)^{n-1}}{n} s_n \beta^n\right] = \sum_{n=0}^\infty F_n \beta^n. \tag{A.8}$$

Using the explicit form of the coefficients s_n (A.7) we find

$$F(\beta) = \exp\left[\beta\left(\ln \frac{1}{\Delta} - C\right) - \ln \Gamma(1 + \beta) + \nu_1 \ln(1 + \beta) + \nu_2 \ln\left(1 + \frac{\beta}{2}\right)\right.$$
$$\left.+ \ldots \nu_m \ln\left(1 + \frac{\beta}{m}\right)\right] = \frac{1}{\Gamma(1 + \beta)} \exp\left[\beta\left(\ln \frac{1}{\Delta} - C\right)\right] \prod_{k=1}^m \left(1 + \frac{\beta}{k}\right)^{\nu_k} \tag{A.9}$$

For the Bethe-Heitler formula (2.6) one has

$$\nu_1 = -\frac{1}{4}, \quad \nu_2 = -\frac{3}{4}, \quad \nu_m = 0 \ (m \geq 3), \tag{A.10}$$

so that

$$F_{BH}(\beta) = \exp\left[\beta\left(\ln \frac{1}{\Delta} - C\right)\right] \frac{1}{\Gamma(1 + \beta)(1 + \beta)^{1/4}(1 + \beta/2)^{3/4}}. \tag{A.11}$$

For radiation of the scalar particle one has $\nu_1 = -1$, $\nu_m = 0$ ($m \geq 2$) so that

$$F_s(\beta) = \exp\left[\beta\left(\ln \frac{1}{\Delta} - C\right)\right] \frac{1}{\Gamma(1 + \beta)(1 + \beta)}. \tag{A.12}$$

So the value A in Eq.(2.3) has the form

$$A(\beta) = F_{BH}^{-1}(\beta) = \exp\left[-\beta\left(\ln \frac{1}{\Delta} - C\right)\right] \Gamma(1 + \beta)(1 + \beta)^{1/4}(1 + \beta/2)^{3/4}. \tag{A.13}$$

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Just the normalization factor $A(\beta)$ (this is the probability of the passage of a projectile through a target without radiation) should enter in Eq.(2.5) instead of $\exp(-w)$. Thus, when we multiply the right-hand side of (2.5) by the factor

$$g(\beta) = \exp(w)) A(\beta),$$

we take into account accurately the emission of arbitrary number of the hard photons. This extends Eq.(2.5) on the case $\beta \geq 1$ if the radiation of the hard $(\omega \sim \varepsilon)$ photons is given by the Bethe-Heitler formula.
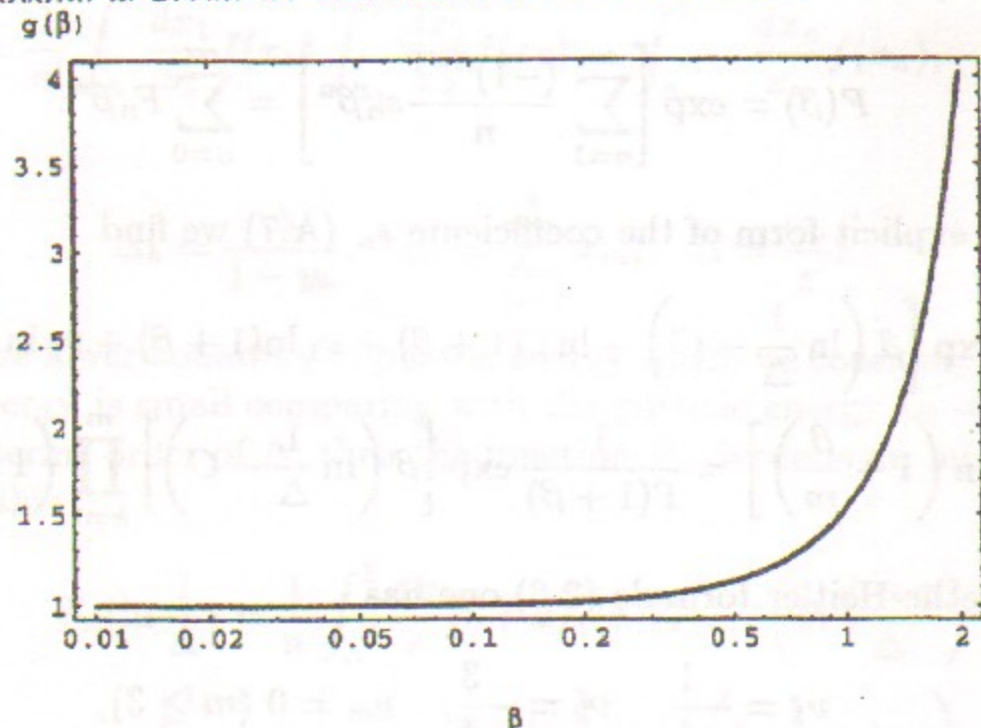

Figure 4: The function $g(\beta)$ Eq.(A.15).

Taking into account that the probability of radiation of the photon with the energy larger than ω_b is (see (2.7))

$$w(\Delta) = \beta \left(\ln \frac{1}{\Delta} - \frac{5}{8} \right), \tag{A.14}$$

we obtain following expression for the factor $g(\beta)$

$$g(\beta) = \exp(w(\Delta)) A(\beta) = \exp\left[\beta \left(C - \frac{5}{8} \right) \right] \Gamma(1+\beta)(1+\beta)^{1/4} (1+\beta/2)^{3/4}. \tag{A.15}$$

As one expected, the function $g(\beta)$ is independent of the boundary energy ω_b (Δ) and its expansion at $\beta \ll 1$ has the form $g(\beta) = 1 + O(\beta^2)$. The function $g(\beta)$ is shown in Fig.4. One can see that $g(\beta) \simeq 1$ for $\beta < 0.2$, a

noticeable growth begins at $\beta \sim 0.5$, and at $\beta \sim 1$ the function $g(\beta)$ increases very fast.

This factor enters into the generalization of formula (2.5) for the case of a thick target ($\beta \geq 1$)

$$\frac{d\varepsilon}{d\omega} == \frac{g(\beta)}{\pi} \operatorname{Re} \int_0^\infty \exp(ix) \exp\left\{ - \int_0^\infty \frac{dw}{d\omega_1} \left[1 - \exp\left(-ix \frac{\omega_1}{\omega} \right) \right] d\omega_1 \right\} dx \tag{A.16}$$

It should be noted that Eq.(2.5) was obtained as the solution of the kinetic equation for the case when the probability of radiation $dw/d\omega$ is independent of the particle energy ε. After the factor $g(\beta)$ is incorporated, Eq.(A.16) is the solution of the corresponding kinetic equation in the soft part of the energy losses $((1 + \beta)\omega \ll \varepsilon)$ in the case when the mentioned dependence of the particle energy ε is included.

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Multi-photon effects in energy looses spectra

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