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CONSTRUCTIVE METHOD FOR CALCULATION OF PHOTON EMISSION AND PAIR PRODUCTION PROBABILITIES IN CRYSTALS

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Constructive Method for Calculation of Photon Emission and Pair Production Probabilities in Crystals

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Abstract

It is shown that at definite conditions the problem of calculation of the pair production and photon emission probabilities in aligned crystals can be solved analytically. The method developed allows one to describe the QED processes practically for any energies and crystal orientation. Possible applications are discussed.

1 Introduction

It is well known (see e.g. [1]) that QED-processes in crystals reveal energy and angular dependence with large magnitudes of effects especially at high energies. So, in contrast to amorphous targets crystal radiators and converters are tunable devices providing good angular resolution. To get the maximal gain one should choose specific optimal conditions. For example, when a crystal radiator is used in high energy photoproduction experiment, the hard part of photon spectra should be amplified while the soft part should be diminished to improve background conditions. On the other hand, the soft part increases faster than the hard one when electron momentum approaches some axial direction but far from this direction the sought enhancement turns out to be insufficient. Thus, one always deals with the multiparameter problem and to solve it contributions at different crystal orientations should be compared. To calculate these contributions, one should know the probabilities of basic QED-processes in rather wide energy and angular range.

In Sec. 2 new constructive method for the calculation of the probabilities of e^+e^- pair production by a photon and photon emission from electron (positron) valid for any crystal orientation and particle energy is suggested. This method can be applied to carry out the yield optimization for any problem dealing with crystals as radiators, converters or detectors. In Sec. 3 an example of such application is given: the possible advantages of a crystal radiator as compared to an amorphous one are considered for the conditions like those of Fermilab experiment dedicated to charm photoproduction studies.

2 Theoretical approach

In the framework of the operator quasiclassical method (see [2]) arbitrary characteristics of radiation and pair production processes are expressed in terms of classical particle trajectories. For example, the spectral distribution of the photon emission probability by the relativistic ($\gamma = \varepsilon/m \gg 1, \varepsilon$ and m are the electron energy and mass) electron or positron for unpolarized particles (eq.(2.1) in [1]) reads

$$dw_\gamma = \frac{i\alpha d\omega}{2\pi\gamma^2}\int\limits_{-\infty}^{\infty}\frac{dt d\tau}{\tau - i0}\left[1+\frac{1}{4}\beta\gamma^2(\mathbf{v}_2-\mathbf{v}_1)^2\right]\times$$

$$\times \exp\left\{-i\frac{\varepsilon\omega\tau}{2\varepsilon'\gamma^2}(1+\rho(\tau,t))\right\} \qquad (1)$$

here $\mathbf{v}(t)$ is the particle velocity, $\mathbf{v}_{1,2} \equiv \mathbf{v}(t_{1,2})$, $t_{1,2} = t \mp \tau/2$, ω is the photon energy, $\beta = \varepsilon/\varepsilon' + \varepsilon'/\varepsilon$, $\varepsilon' = \varepsilon - \omega$. The quantity $\rho(\tau,t)$

$$\rho(\tau,t) = \gamma^2\left\{\frac{1}{\tau}\int\limits_{t_1}^{t_2} dz\,\mathbf{v}^2(z) - \left(\frac{1}{\tau}\int\limits_{t_1}^{t_2} dz\,\mathbf{v}(z)\right)^2\right\} \qquad (2)$$

is the squared particle deflection angle Θ^2 measured in units of θ_γ^2 ($\theta_\gamma = 1/\gamma$ is the characteristic angle of photon emission) averaged over time τ which has meaning of the radiation formation time. In the limit $\tau \to \infty$, the quantity $\rho(\tau,t) \to \rho$ which is independent of t and has the same meaning as $\rho(\tau,t)$ but for the entire trajectory.

Let the angle of incidence ϑ_0 with respect to the chosen axis is small $\vartheta_0 \ll 1$, then the crystal potential can be represented as a sum of individual axial potentials arranged periodically in the transverse plane: $U(\mathbf{r}) = \sum_{\mathbf{q}} G(\mathbf{q})\exp(-i\mathbf{q}\mathbf{r}) \to \sum_{\mathbf{q}_\perp} G(\mathbf{q}_\perp)$ $\exp(-i\mathbf{q}_\perp\mathbf{r}_\perp)$. Here $\mathbf{a}_\perp$ means the component of a vector $\mathbf{a}$ transverse to the chosen axis. Since $\mathbf{v}_\perp \sim \vartheta_0 \ll 1$ and $v_\parallel \approx 1 - \frac{1}{2}(\gamma^{-2} + \mathbf{v}_\perp^2)$, one can substitute $\mathbf{v} \to \mathbf{v}_\perp$in eqs.(1) and (2). After that only transverse components of all the vectors are present and the subscript $\perp$ will be omitted in what follows when it does not lead to an ambiguity.

So at this stage a two-dimensional problem of classical motion is to be solved and it is well known that it can not be done analytically. The main goal of the present paper is to show that for sufficiently large ϑ_0 (in fact already for ϑ_0 of the order of U_0^{ax}/m or several axial critical angle $\theta_c^{ax} \sim (U_0^{ax}/\varepsilon)^{1/2}$, where U_0^{ax} is the depth of the axis potential well) the corresponding problem becomes one-dimensional and always can be solved analytically. Since the spectral (over the energy of one of the created particles) distribution of pair production probability dw_p obtained within quasiclassical method is very similar to dw_γ given by eq.(1), the pair production process can be described analytically under mentioned conditions as well. Actually (cf.eqs.(2.6) and (2.8) in [1]) the probability dw_p is obtained from eq.(1) by following substitutions: $d\omega/\varepsilon^2 \to d\varepsilon/\omega^2$, $\varepsilon' \to \omega - \varepsilon$, and $1 \to -1$ in [...]. Below the photon emission process will be discussed in detail, keeping in mind that one can always obtain corresponding expressions for the pair production process by means of simple substitution rules stated above.

Consider as an example the situation when the particle momentum is nearly aligned with the axis $\langle 001\rangle$ of a crystal having fcc(d) structure (C, Si, Ge). Numerical estimates will be given for Si, but qualitative features are the same for other crystals and axes. Let us choose two orthogonal unit vectors in the plane transverse to the $\langle 001\rangle$ axis : $\mathbf{e_1}$ lying in the $(1\bar{1}0)$ plane and $\mathbf{e_2}$ lying in the (110) plane. Taking into account the structure factor $S_{mnk}(G(\mathbf{q})\propto S_{mnk}$, see eqs.(2.9), (2.10), (2.11) in[1]) one can represent the reciprocal lattice vector $\mathbf{q}$ as $\mathbf{q}= 2\pi(\mathbf{e}_1 k + \mathbf{e}_2 m)/d_0$, where d_0 is the inter-planar distance for the $(1\bar{1}0)$ plane and k, m are arbitrary integers which do not vanish simultaneously ($\mathbf{q}^2 \neq 0$). Introducing also the azimuth angle of the average transverse particle velocity $\langle \mathbf{v}_\perp \rangle \simeq \vartheta_0(\mathbf{e_1}\cos\varphi_0 + \mathbf{e_2}\sin\varphi_0)$, one has $q_\parallel = (\mathbf{q}, \langle \mathbf{v}_\perp \rangle) = 2\pi\vartheta_0(k\cdot\cos\varphi_0 + m\cdot\sin\varphi_0)/d_0$. Then the angle ψ of the average particle velocity with respect to the corresponding plane is $\psi(m,k) \simeq q_\parallel/q = \vartheta_0(k\cdot\cos\varphi_0 + m\cdot\sin\varphi_0)/(k^2+m^2)^{1/2} \ll 1$. So, the angle ψ(m,k) $= \vartheta_0\cdot\sin(\varphi$ (m,k)), where φ (m,k) is the angle between one of the planes containing $\langle 001\rangle$ axis and the average particle velocity is small as long as $\vartheta_0 \ll 1$. From the other hand, the planes become apparent when average velocity along the plane is large enough. With respect to axes, the planar motion is over-barrier one and planar potential describes action of subsequent axes (let L be spacing between them) averaged over the fast frequency $\nu_f \simeq \vartheta_0\cdot\cos(\varphi(m,k))/L$ which should be much larger than the frequency of arising slow motion $\nu_{sl} = 1/T_{sl}$. Estimating $T_{sl} \leq d/\vartheta^{pl}$ (where d is the inter-planar distance and $\vartheta^{pl} = (2U^{pl}/\varepsilon)^{1/2}$, U^{pl} is the depth of the plane potential well), we can rewrite the condition $\nu_f \gg \nu_{sl}$ as $\vartheta_0\cdot\cos(\varphi(m,k)) \gg d_0\vartheta_0^{pl}/2d$, where d_0, ϑ_0^{pl} are the parameters of the $(1\bar{1}0)$ plane. Note that sums appearing in calculation of U^{pl} and of the corresponding contributions to the radiation converge rapidly. It will be shown below that the relative accuracy of the approach developed is about 5 percent what is provided when only terms with $(k^2+m^2)^{1/2} < 5$ are taken into account in a double sum over $\mathbf{q}$ (over m,k). It means that not very many planes actually contribute, they are not extremely weak and

the ratio $d_0/2d$ is not too large. Remind now(see [2, 3]) that for a given plane the parameter ρ is always less than $\rho_c = U^{pl}\varepsilon/m^2$ (our definition differs from that used in[1] by the factor 1/2). For the oscillator potential one has just above the plane barrier $\rho \simeq \rho_c/6.5$ and high above the barrier when $\psi \gg \vartheta^{pl}$, $\rho \simeq 2\rho_c(\vartheta^{pl}/\psi)^2/45$, so that for $\psi \sim U^{pl}$/m the parameter ρ is small if $\rho_c \gg 1$. Note also that the quantity $\sqrt{\rho}/T$ (T is the period of a planar motion) is independent of ψ in a wide ψ range practically beginning with $\psi \sim \vartheta^{pl}$($\sqrt{\rho}/T \simeq \vartheta^{pl}\sqrt{\rho_c}/5d$ for the oscillator potential) what will be used below. When ψ increases, the quantity $\sqrt{\rho}/T$ approaches the limiting value from above for positrons and from below for electrons.

Generally speaking, the range of φ_0 values is $-\pi \leq \varphi_0 \leq \pi$, but owing to the symmetry of the problem, results depend on the absolute value of φ_0 only. Moreover, in our example it is sufficient to consider the range $0 \leq \varphi_0 \leq \pi/4$ since the situation is the same for φ_0 and for $\pi/2 - \varphi_0$. This range is bounded by the $(1\bar{1}0)$ plane ((0,1) plane in (k,m) notation) and by (001) plane ((-1,1) plane in (k,m) notation). Besides these two strong planes ($U^{pl}(-1,1) \simeq 0.567U_0^{pl}$) there is only one plane of comparable strength, namely (-1,2) with $U^{pl}(-1,2) \simeq 0.248U_0^{pl}$ while all the others have $U^{pl} < 0.12U_0^{pl}$. As it was explained above, at sufficiently high energies when $\rho_c \gg 1$ the parameter ρ (and $\rho(\tau,t)$ in eq.(1)) is small for $\psi \geq U^{pl}$/m. For the given value of ψ, the corresponding range of φ_0 around the plane where the parameter ρ is not small narrows with growing ϑ_0. Beginning with some $\vartheta_0 = \theta_{sp}$, these domains corresponding to two different strong planes do not overlap already. In our example it happens first for $(0,1)$ and $(-1,1)$ planes at $\vartheta_0 \simeq 0.86\cdot 10^{-4}$ then for $(-1,1)$ and $(-1,2)$ planes at $\vartheta_0 \simeq 1.07\cdot 10^{-4}$ and, finally, for $(0,1)$ and $(-1,2)$ planes at $\vartheta_0 \simeq 1.14\cdot 10^{-4}$. Note that at $\vartheta_0 \simeq 1.14\cdot 10^{-4}$ the corresponding domains for weaker planes (-1.2) and (-1,3) also do not overlap. So, for $\vartheta_0 \geq \theta_{sp} = 1.14\cdot 10^{-4} \simeq 2.72\cdot U_0^{pl}$/m at any φ_0 only one plane could provide $\rho > 1$ while for any other plane the corresponding partial contribution to ρ is small and its influence on the motion can be considered in terms of a perturbation theory. In fact it means that for $\vartheta_0 \geq \theta_{sp}$ the motion becomes one-dimensional. One can make the corresponding expansion in eq.(1), keeping in the phase only the contribution of the plane providing $\rho > 1$. It should be emphasized that this picture is still valid in some interval of $\vartheta_0 < \theta_{sp}$, since for $\vartheta_0 = \theta_{sp}$ the corresponding domains are touching only and have $\rho \ll 1$ on their boundaries. Accurate calculations show that for any plane (-k,m) the quantity ρ which is the actual parameter of the problem varies from 0.098 to 0.125 for $\psi = U^{pl}/\mathrm{m}$ if $\rho_c \gg 1$, i.e. it becomes of the order of unity for $\psi \sim U^{pl}/3\mathrm{m}$. Then for the energy $\varepsilon > 100$ GeV we get two planes with $\rho > 1$ inside one domain when $\vartheta_0 < \theta_{br} \sim 1.25 \cdot U_0^{pl}/\mathrm{m}$. Hence, for such angles ϑ_0 our description is broken and the axis returns to the scene. From the other hand, for $\vartheta_0 < \theta_{br}$ the yield should be independent of ϑ_0 and φ_0. Therefore we hope that axial results may be obtained within our approach as the limiting case by averaging some yield over ϑ_0 in the interval $\theta_{br} \leq \vartheta_0 \leq \theta_{sp}$ and over φ_0 in the whole range $0 \leq \varphi_0 \leq \pi/4$. Unfortunately, up to now theoretical description of the radiation at the axial alignment is possible only for sufficiently high energies when $\rho_c^{ax} \gg 1$ and the so called constant field approximation (CFA) is valid. We have verified that in this case the limiting procedure suggested above reproduces with a good accuracy the results of the direct CFA calculation.

The obtained value of θ_{sp} can be expressed in terms of axial potential characteristics: $\theta_{sp} \simeq 0.69 U_0^{ax}/\mathrm{m}$. For moderate and low energies the consideration is practically the same, but now the angle ϑ^{pl} is the boundary ψ value instead of U^{pl}/m. Correspondingly one finds the value of $\vartheta_0 = \theta_{sp}^m$ when all the φ_0 region is splitted into non-overlapping domains : $\theta_{sp}^m \simeq 3.91\vartheta_0^{pl} \simeq 2.78\theta_c^{ax}$.

For $\varepsilon = 50$ GeV when $\rho_0 = 8.16$, one has $\theta_{sp} = \theta_{sp}^m$, so that our approach is certainly valid for $\vartheta_0 \geq \theta_{sp}$ in the region $\varepsilon \gtrsim 50$ GeV and for $\vartheta_0 \geq \theta_{sp}^m$ when the energy is lower than 50 GeV. These restrictions on ϑ_0 are compatible with the condition obtained above providing the existence of planes at least for planes giving the significant contribution. So, the approach is self-consistent. It is interesting that the period of motion near the plane providing $\rho > 1$ is always much larger than those connected with traversing any other plane. For example, even for $\psi = U_0^{pl}/3\mathrm{m}$ and $\vartheta_0 = \theta_{sp}$ when moving near the (0,1) plane, the ratio $T^{(0,1)}/T^{(-1,3)} \sim 5.5$ and increases at least linearly with increasing ϑ_0. So, the contributions to the motion are consistently separated within our approach into one slow with the comparatively large amplitude and all others being fast and having small amplitudes. We can now represent the transverse velocity as $\mathbf{v} \simeq \mathbf{v}^{slow} + \mathbf{v}^{fast}$ and carry out the corresponding expansion in eq.(1). In turn, we can use the rectilinear trajectory approximation by calculating $\mathbf{v}^{fast}$ (cf. eq(2.5) in [1]) :

$$\mathbf{v}^{fast}(t) = -\frac{1}{\varepsilon}\sum_{\mathbf{q}} G(\mathbf{q})\frac{\mathbf{q}}{q_{\parallel}}\exp\left[-i(q_{\parallel}t + \mathbf{q}\mathbf{r}_0)\right], \tag{3}$$

where $\mathbf{r}_0$ is the entry point and the probability will be averaged over it assuming uniform distribution within the area per one axis. Note that the velocity $\mathbf{v}^{slow}$ depends implicitly on $\mathbf{r}_0$ as well. After averaging over time t over an interval of the order of T_{fast} which does not affect the slow motion, we finally have:

$$dw_\gamma = \frac{i\alpha\, d\omega}{2\pi\gamma^2}\int_0^d \frac{dx_0}{d}\int_{-\infty}^{\infty}\frac{dt\, d\tau}{\tau - i0}\times$$

$$\exp\left\{-i\frac{\varepsilon\,\omega\,\tau}{2\varepsilon'\gamma^2}(1 + \rho^{slow}(\tau, t))\right\}(F_1 + F_2), \tag{4}$$

where

$$F_1 = 1 + \frac{1}{4}\beta\gamma^2(\mathbf{v}_2 - \mathbf{v}_1)^2, \; f(y) = \frac{\sin y}{y}, \; y = \frac{1}{2}\tau q_{\parallel}, \quad (5)$$

$$F_2 = \sum_{\mathbf{q}} \frac{|G(\mathbf{q})|^2 \mathbf{q}^2}{m^2 q_{\parallel}^2} \Big\{ \beta \sin y \left[\sin y + \frac{1}{q}(\mathbf{q}, \mathbf{v}_2 - \mathbf{v}_1)\Phi \right] - \frac{\varepsilon\omega\tau}{2\varepsilon'\gamma^2} F_1 \left[i(1 - f^2(y)) + |\Phi|^2 \right] \Big\},$$

$$\Phi = \frac{\varepsilon\omega}{q\varepsilon'} \int_{-\tau/2}^{\tau/2} dz(\mathbf{q}, \mathbf{v}(t+z)) \left[\exp(-izq_{\parallel}) - f(y) \right].$$

Here $\mathbf{v}(t) \equiv \mathbf{v}^{slow}(t, x_0)$ is actually one-dimensional vector directed perpendicular to the plane (x is the distance from this plane having the inter-planar distance d) providing slow motion. Integration over x_0 corresponds to the averaging over $\mathbf{r}_0$ mentioned above. The term $\propto F_1$ in eq.(4) represent the contribution to the radiation provided by the slow motion only. It can be calculated analytically for any analytical parametrization of the inter-planar potential. In the limiting cases of a non-relativistic motion ($\rho_c^{slow} \ll 1$) when the radiation has a dipole nature and of an ultra-relativistic motion ($\rho_c^{slow} \gg 1$) when the radiation has a magnetic bremsstrahlung nature, approximate methods can be used in the calculation. The intermediate case ($\rho_c^{slow} \sim 1$) was also investigated rather thoroughly (see e.g.[3]).

If we put $\mathbf{v}^{slow} = 0$ ($\rho_c^{slow} = 0$) in the second term $\propto F_2$ in eq.(4), then it will represent the results of the ordinary coherent bremsstrahlung (CBS) theory describing in fact the Compton scattering of equivalent photons provided by the periodic crystal field on a charge particle in the proper reference frame. The standard CBS theory is exhausted by this picture both qualitatively and quantitatively (see eqs.(4.8),(1.5) in [1]). In particular, due to the 4-momentum conservation in Compton effect, the energy

ω and the emission angle of a photon θ_{ph} are rigidly connected : $\omega/\varepsilon = s/(1 + s + (\gamma\theta_{ph})^2)$ and there is the boundary photon energy $\omega_b = \varepsilon s/(1+s)$ corresponding to $\theta_{ph} = 0$. Let the initial (equivalent) photon has the momentum q_ν, then the parameter s is $s = 2(qp)/m^2$ (in our case $s = 2\varepsilon \mid q_{\parallel} \mid /m^2$). The CBS formation time is $\tau_{coh}^{-1} \sim (1 - u/s) \mid q_{\parallel} \mid$ where $u = \omega/\varepsilon'$. It is about $\mid q_{\parallel} \mid^{-1}$ for small u (ω) and formally tends to infinity when ω tends to ω_b ($u \to s$). In deriving eq.(4), terms of the order of ρ_{fast}^2 are omitted and it determines the accuracy of our calculations. For example, at $\varphi_0 = 0$ (channeling along the $(1\bar{1}0)$ plane) we estimate the relative contribution of neglected terms of the second order to be about 5 percent. To the same time contributions of three planes to ρ_{fast} (first order), namely, of (-1,1),(-1,2) and (-1,3) planes give about 94 percent, the (-1,4) plane gives about 5 percent and others altogether about 1 percent. So, taking into account the contributions of high-index planes can not improve the accuracy.

The description in terms of the standard CBS theory holds as long as $\rho_c^{slow} \ll 1$ but beginning with $\rho_c^{slow} \sim 1$ the influence of the slow motion on CBS must be taken into account. This influence is caused by the deviation of the particle velocity during CBS formation time. If the angle of the deviation becomes larger than $\theta_\gamma = \gamma^{-1}$, the phase conditions are broken, what leads to a change of CBS. Let us consider first the case of large $\rho_c \gg 1$ (we omit index " slow" in what follows) which had been investigated in [4] theoretically and in [5] experimentally at $\varphi_0 = 0$ for a diamond. Far from the kinematic boundary where $(1 - u/s)$ is not too small, the values of $\tau \sim \tau_{coh} \sim \mid q_{\parallel} \mid^{-1}$ give the main contribution to the second term $\propto F_2$ in eq.(4) when $\omega_0\tau \ll 1$ ($\omega_0 = 2\pi/T$) and we can expand the velocity in eqs.(4),(5) : $\mathbf{v}(t+z) \simeq \mathbf{v}(t) + z\,\dot{\mathbf{v}}(t)$. Then $\rho(\tau, t) \simeq (\gamma\;\dot{v}\;\tau)^2/12 = (\mu\tau q_{\parallel})^2/3$ where as in [4] the parameter $\mu = \chi/s$ is introduced, and χ is the conventional parameter (see,e.g. [1, 2]) of CFA: $\chi = \gamma^2 \mid \dot{\mathbf{v}}(t) \mid /m$. It should be emphasized that in this photon energy range the expansion is valid for $\rho_c \sim 1$ as well. Carrying out the expansion and integrating over τ, we reproduce the term in eq.(8) of [4] $\propto \sum_q$ if we put $\rho = 0$ (in our notation ρ in [4] means ρ^{fast}) in that equation what is consistent with the accuracy of the approach developed. Let us now estimate the magnitude of the parameter μ for the strongest $(1\bar{1}0)$ plane. For the oscillator potential we obtain $\mu < U_0^{pl}/\pi m\vartheta_0$. Recalling the restriction on ϑ_0 obtained above, we conclude that the parameter μ is small even for the strongest plane and even for the smallest possible ϑ_0 value: $\mu \ll 1$. For small μ rather complicated formula (8) of [4] is essentially simplified. We can omit there the terms depending on z_+ and z_0 keeping only those which depend on $z_- = (2u/3\chi) \mid 1 - s/u \mid^{3/2}$. The smaller is z_-, the larger is a change of CBS. The quantity z_- is minimal ($z_- = 0$) for $s = u$ and becomes of the order of unity for $\mid 1 - s/u \mid \sim \mu^{2/3}$ what determines the range of photon energies where CBS is affected by the slow motion for $\rho_c \gg 1$. It can be easily understood in terms of formation times. In fact, we deal in this case with two competing mechanisms of the photon emission: CBS and magnetic bremsstrahlung. The formation time of the latter is $\tau_{mag} \sim T/\pi\sqrt{\rho} \cdot (3\chi/u)^{1/3}$ which, owing to the mentioned constancy of the ratio $T/\sqrt{\rho}$, can be rewrite as $\tau_{mag} \sim (3\chi/u)^{1/3}/(\mu \mid q_{\parallel} \mid)$. Equating τ_{mag} and τ_{coh} we find that it occurs just when $(1 - s/u) \sim \mu^{2/3}$ where the photon emission mechanism changes. Note that for $\rho_c \gg 1$ the photon emission process has a local nature since the formation time is always much less than the period of the slow motion and therefore the expansion in terms of $\omega_0\tau$ is valid for any photon energy as well as the corresponding formulae like eq.(8) of [4]. For $\rho_c \sim 1$ this picture is valid as long as $\omega_0\tau_{coh} \ll 1$. The latter condition is violated in the region $(1 - s/u) \sim \vartheta_0^{pl}/(2\pi\vartheta_0) \ll 1$ (recall that $\vartheta_0 > \theta_{sp}^m$)

which also is very narrow but to describe a change of CBS in this region of photon energies quantitatively, we should use the exact formulae (4). The same is true for $\rho_c \gg 1$ if the angle ψ is so large that $\rho \sim 1$.

3 Some applications and conclusion

In applications we usually deal with rather thick crystals where electromagnetic showers are developed. On-line calculation of the corresponding probabilities is extremely time consuming and we should calculate them beforehand. The set of the probabilities can be represented as the many-dimensional map where every point corresponds to some values of the variables $\omega, \varepsilon, \vartheta_0, \varphi_0$. Certainly, it is a discrete map. Such maps have been created first to estimate the possible gain from using a crystal radiator instead of an amorphous one in producing a wide band photon beam at the Fermilab Tevatron (see [6] describing the facility itself and what can be obtained using an amorphous radiator). Different crystals and axes were considered: < 001 > axis for C and Si; < 001 >, < 110 >, and < 111 > for Fe. The following steps were chosen: 20 GeV for ε, 0.02 for ω/ε, 0.1mrad for ϑ_0, $\pi/120$ for φ_0. Somewhat simplified version of the method presented in previous section was used in the calculation. In Figs.1,2 the intensity radiation spectra are shown extracted from the created map for C. The contributions from strong planes are clearly seen. They are especially prominent in the soft part of the spectra. In accordance with results of [4] the hardest peak corresponds also to the motion near the $(1\bar{1}0)$ plane (small φ_0). There are regions of φ_0 where the radiation is hard enough but its soft part is suppressed. As it should be, with the increase of ϑ_0 (cf. Fig.1 and Fig.2) the radiation becomes harder and regions where the soft part is suppressed are extended. So, it was a hope that if we choose the proper crystal orientation to locate rather wide in angular (ϑ_0, φ_0) space electron beam inside these preferable regions, the hard part of radiation will be increased while the soft one suppressed as compared to the amorphous radiator.The calculation of the photon yield were carried out for different crystal orientations and thicknesses. The comparison was made with the case of the amorphous $0.27X_0$ radiator used before (dash-dotted lines in Figs. 3,4). Note that in Fig.4 the amorphous yield is larger than for every crystal yield presented for $\omega \leq 15$ GeV what can be seen only in the logarithmic scale. The gain is larger for Fe crystal (Fig.3) than for Si crystal (Fig.4). The same relation is between C and Si crystals. However the spot size of the used electron beam on the radiator surface is about 4×4 cm and the pure technological preference of a Si crystal becomes evident.

We conclude that use of a Si radiator of the thickness about 1cm would increase the charm production rate by the factor of 2 or more. To the same time the soft part of photon spectra is suppressed as well as the hadronic background.

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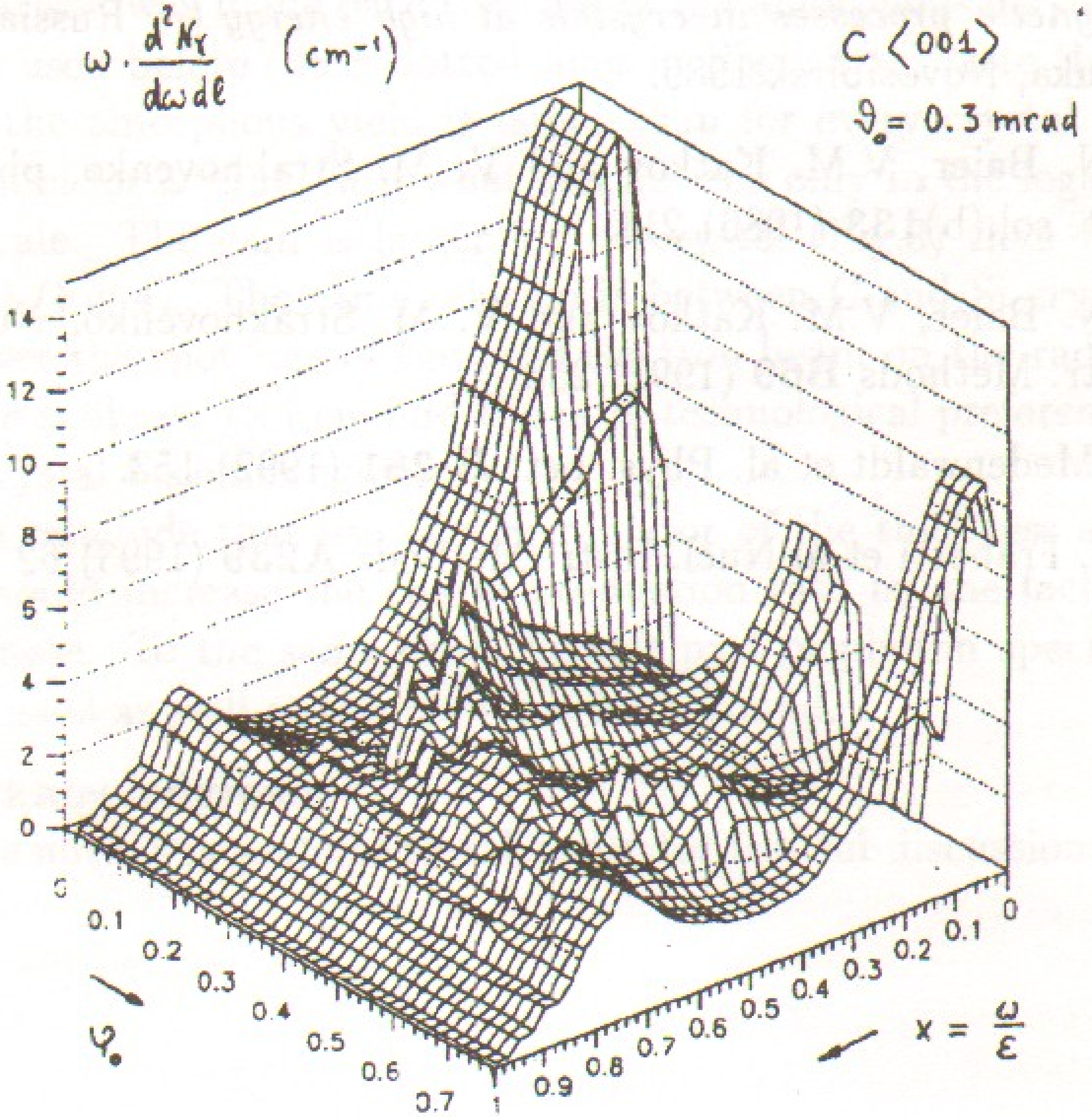

Figure 1: The intensity spectra in a diamond crystal for $\varepsilon = 250\ GeV$ and $\vartheta_0 = 0.3 mrad$.

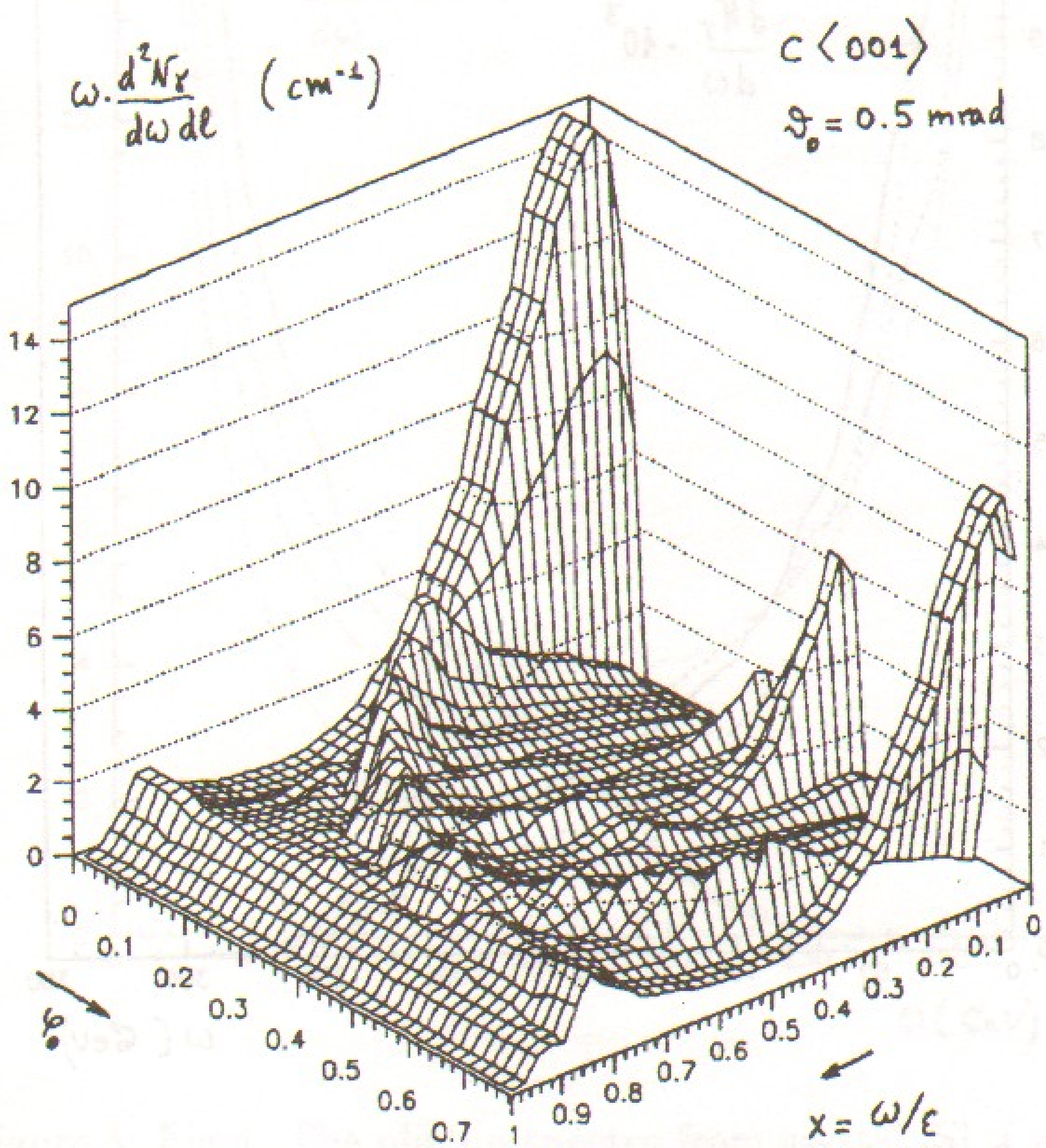

Figure 2: The same as in Fig.1 but for $\vartheta_0 = 0.5 mrad$.

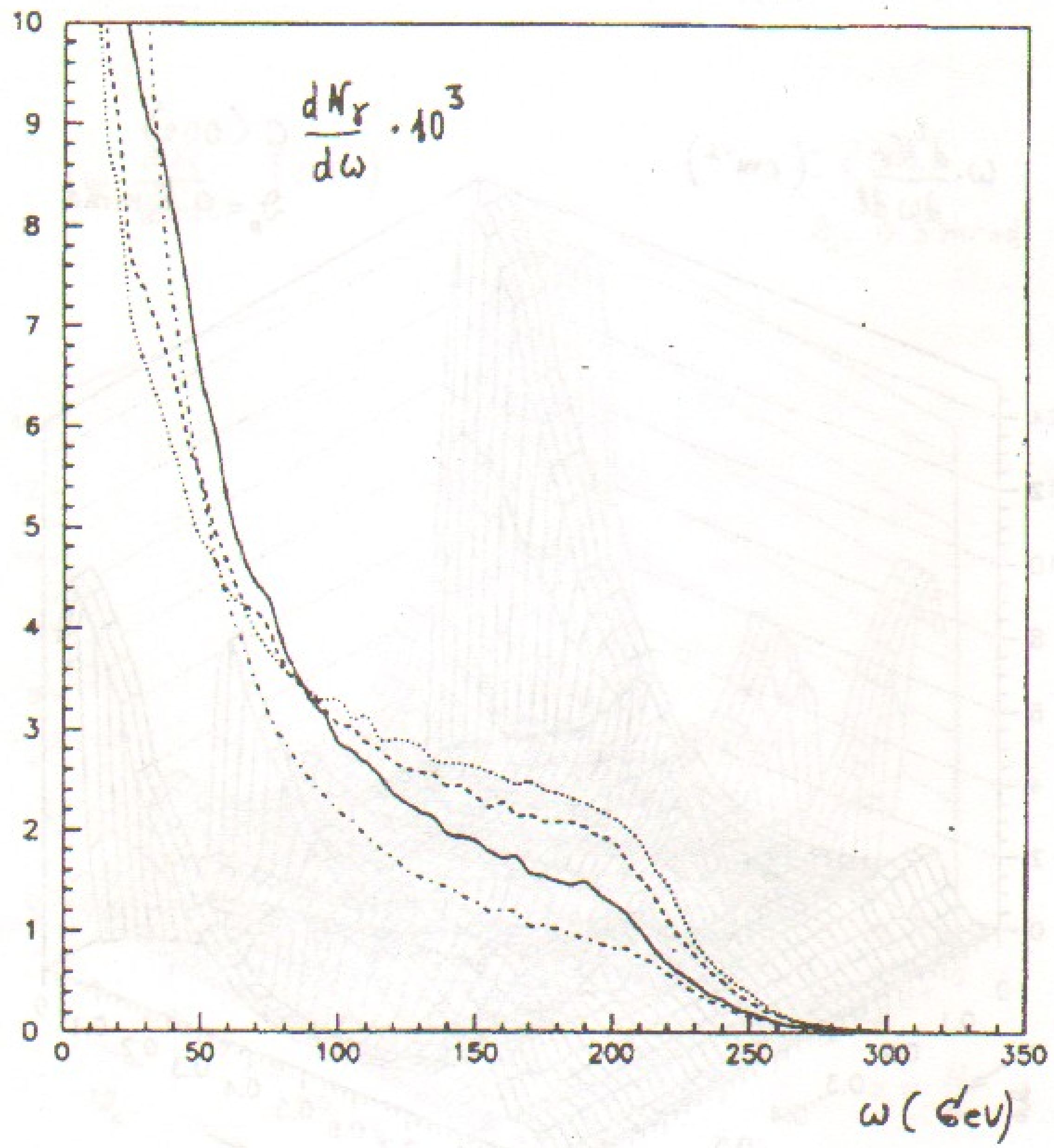

Figure 3: Fig.3. The photon spectra from crystal Fe (axis $< 110 >$) radiator of the thickness $L = 0.25$ cm. The beam is centered at $\varphi_0 = 0.1$ and $\vartheta_0 = 0.5$ mrad (solid curve), $\vartheta_0 = 0.9$ mrad (dashed curve), $\vartheta_0 = 1.3$ mrad (dotted curve). The dash-dotted curve represent the amorphous yield.

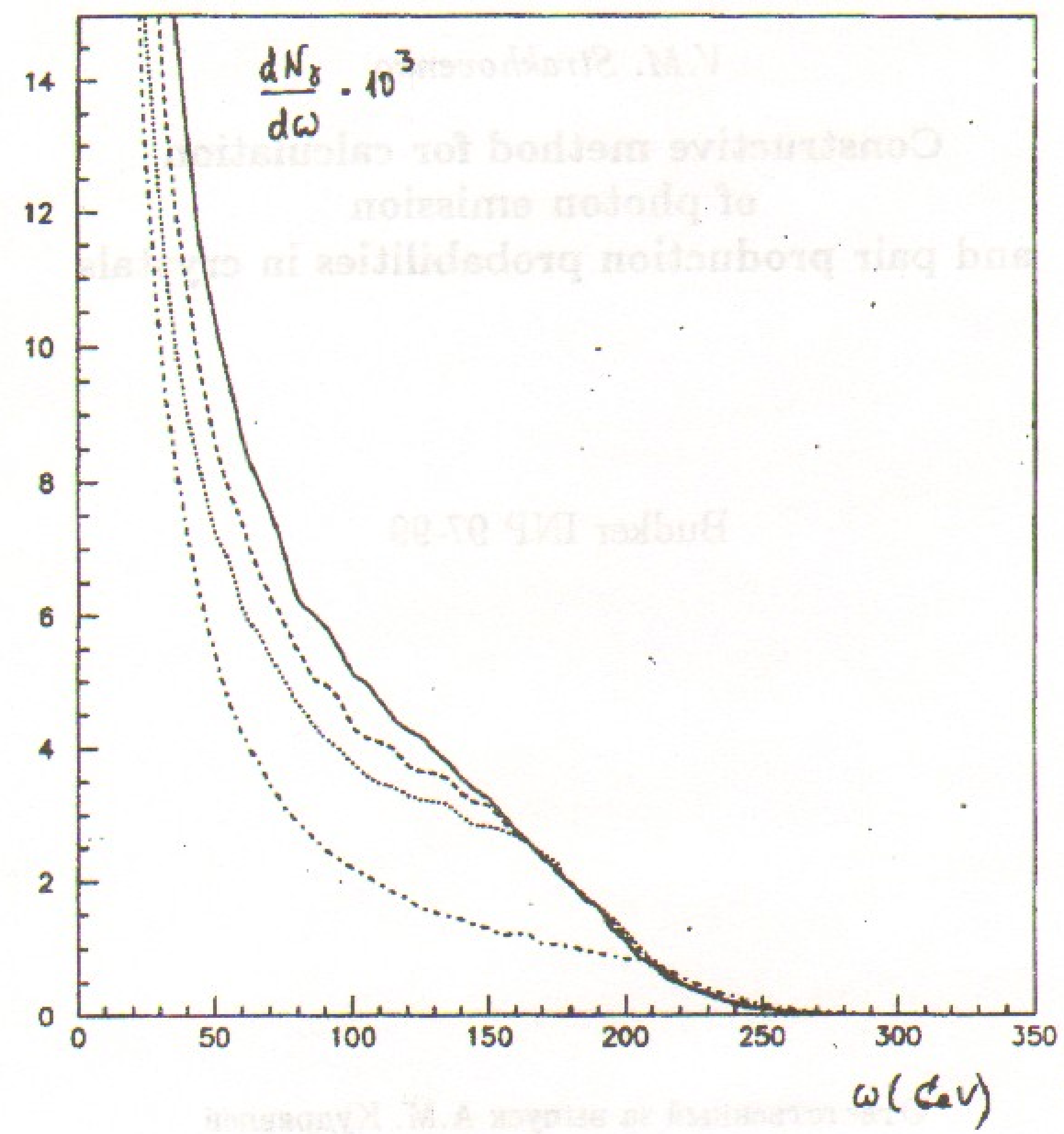

Figure 4: Fig.4. The photon spectra from crystal Si (axis $< 001 >$) radiator of the thickness $L = 1$ cm. The beam is centered at $\varphi_0 = 0.45$ and $\vartheta_0 = 0.4$ mrad (solid curve), $\vartheta_0 = 0.5$ mrad (dashed curve), $\vartheta_0 = 0.6$ mrad (dotted curve). The dash-dotted curve represent the amorphous yield.

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