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MECHANICAL STRESSES IN TARGET MATERIAL AND BEAM SWEEPING

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НОВОСИБИРСК

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Abstract

Considered is the mechanical stress arising in target material due to the energy deposition from a particle beam passing through the target. It is shown that the beam sweeping over the target with velocity below the sound velocity does not reduce the tension in target material responsible for the mechanical destruction.

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The production of secondary particle beams with high phase density is connected and in many cases restricted with the problem of target destruction caused by the high density energy deposition in target material. The beam sweeping over the target [1–3] was proposed for a case of very high specific energy deposition – comparable with the sublimation energy – as practically the only way to solve the problem of so called "mass depletion" – the decrease in target material density during the beam spill. To decide whether this method is efficient against the mechanical destruction of target, which takes place after repeated spills even at not so high specific energy deposition, one needs to analyze the pressure arising in target due to the beam passing through.

The equation of state for target material has a form (see [4]):

$$p = p_T + p_C = \Gamma\frac{\epsilon_T}{V} - \frac{d\epsilon_C}{dt}$$

$$\epsilon = \epsilon_T + \epsilon_C ,$$

where Γ is the Gruneisen coefficient, ϵ_T is the thermal constituent of specific internal energy and ϵ_C is the potential (the compressional) constituent, being a function of specific volume V only. The expression for $\epsilon_C(V)$ may be taken in an empiric form (see [5]):

$$\epsilon_C(V) = \frac{c_0^2}{2}\left(1 - \frac{V_0}{V}\right)^2$$

with c_0 standing for the sound velocity.

The compressional constituent of pressure p_C is negative when $V > V_0$ and if its absolute value is larger than p_T the full pressure appears to be negative as well. This means the tension arisen inside the target material, which, if exceeds the limit value, leads to mechanical destruction of material.

For pressure definition in the linear – acoustic – approximation, well applicable in the case under consideration, one obtains, using the hydrodynamic equations together with the equation of state, the second order wavelike differential equation:

$$\frac{\partial^2 p}{\partial t^2} - c_0^2\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right) = \frac{\Gamma}{V_0}\frac{\partial q(x,y,z,t)}{\partial t}, \qquad (1)$$

where $q(x,y,z,t)$ is a distribution function for the specific power deposition over the coordinates and time.

Let $q(x,y,z,t)$ be Gaussian with a half-width σ over the transverse coordinates x, y and homogeneous over both the longitudinal coordinate z and the time between the moments t_1 and t_2 of beam spill switch on and off. With sweeping of beam along one of transverse coordinates by a constant velocity u_0 the expression for $q(x,y,z,t)$ reads:

$$q(x,y,z,t) = q_0 \int_{-\infty}^{+\infty} E_{t_1,t_2}(t,\tau)\exp\left[-\frac{(x-u_0\tau)^2+y^2}{2\sigma^2}\right]d\tau,$$

where $E_{t_1,t_2}(t,\tau) = \int_{t_1}^{t_2}\delta(t-s)\delta(s-\tau)ds$ (filter operator).

The solution of equation (1) with such a right hand side expression has a form:

$$p(x,y,t) = \frac{\Gamma q_0 \sigma^2}{\pi V_0}\int_0^\infty \exp\left(-\frac{\xi^2\sigma^2}{2}\right)\times$$

$$\int_{max\in\{0,t\}}^{t+T} \cos\xi c_0\tau J_0\left(\xi\sqrt{(x^*+u_0\tau)^2+y^2}\right)d\tau d\xi, \qquad (2)$$

where $J_0(z)$ denotes the Bessel function. $x^* = x - u_0 t$ is the coordinate in relation to the beam axis, T stands for beam spill duration, $T = t_2 - t_1$, and t_2 is put equal 0.

Without sweeping ($u_0 = 0$) the pressure at the beam axis ($x = 0, y = 0$) during the beam spill, $-T < t < 0$, is defined as:

$$p(u_0 t, 0, t)_{-T<t<0} = \frac{\Gamma q_0 \sigma}{V_0 c_0}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!!}\left(\frac{c_0(T+t)}{\sigma}\right)^{2k+1}. \qquad (3)$$

It grows up to the maximum value $p(0,0,t)^{max}_{-T<t<0} \cong 0.77\frac{\Gamma q_0\sigma}{c_0 V_0}$ for a time t_m of the order of time of sound propagation across the energy deposition region, $t_m \cong 1.3\frac{\sigma}{c_0}$, and then decreases right down to zero if spill duration is big enough, $T >> \sigma/c_0$.

Such a behavior of pressure near the beam axis has an evident explanation: it grows in proportionality with the stored specific energy until the decompressive wave from the effective boundary of heated area achieves its center, while the compressive wave is radiated into the outer space. In a time of several t_m the equality between the p_T and $-p_C$ is established resulting in zero magnitude of $p(0,0,t)$. After the beam spill finish the equality is failed in favor of p_C and pressure near the beam axis becomes negative. The dependence of its absolute value on time here is similar to that in the spill beginning (see (3)). That is, the pressure achieves the minimum value $p(0,0,t)^{min}_{t>0,T\to\infty} \cong -0.77\frac{\Gamma q_0\sigma}{c_0 V_0}$ in the time t_m after the beam spill finish.

For not very large spill duration T, so that $p(0,0,t)_{-T<t<0}$ does not fall down zero, the negative pressure amplitude is less than that for $T\to\infty$ by the same specific power deposition q_0:

$$p(0,0,t)_{t>0} = \frac{\Gamma q_0\sigma}{V_0 c_0}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!!}\left[\left(\frac{c_0(T+t)}{\sigma}\right)^{2k+1} - \left(\frac{c_0 t}{\sigma}\right)^{2k+1}\right], \qquad (4)$$

but if the specific energy deposition $Q_0 = q_0 T$ is fixed, the amplitude of negative pressure is the more the shorter is the spill duration. In the limit $T << \sigma/c_0$ the minimum pressure is equal to: $p(0,0,t)^{min}_{t>0,T\to 0} \cong -0.285\frac{\Gamma Q_0}{V_0}$, which takes place at $t \cong 2.1\sigma/c_0$.

In the case of beam sweeping with velocity below the sound velocity, $u_0 < c_0$, the beam shift during the time $\sim \sigma/c_0$ is not big enough to provide with a significant gain in the stored specific energy. Meanwhile, in this case the wave of compression is formed downstream the sweeping beam and of decompression – upstream, moving with sweep velocity u_0. After large enough time from spill start the pressure in these waves is settled equal to:

$$p(x,0,t)_{\sigma/c_0<<t+T<T} = \text{sign } x^* \frac{\Gamma q_0 \sigma \sqrt{\pi/2}}{V_0\sqrt{c_0^2-u_0^2}} \times$$

$$\exp\left(-\frac{x^{*2}}{4\sigma^2}\right) \sum_{k=0}^{\infty} (-1)^k \left(\frac{u_0}{c_0+\sqrt{c_0^2-u_0^2}}\right)^{2k+1} I_{k+\frac{1}{2}}\left(\frac{x^{*2}}{4\sigma^2}\right) \quad (5)$$

in the plane of beam axis sweeping ($y = 0$). Here $I_\nu(z)$ is the Bessel function of imaginary argument and $x^* = x - u_0 t$. The maximum (minimum) of above pressure takes place near $|x - u_0 t| = 1.6\sigma$ being equal here to:

$$p(x,0,t)^{max(min)}_{\sigma/c_0<<t+T<T} \cong \text{sign } x^* \frac{0.9\,\Gamma q_0 u_0 \sigma}{V_0\sqrt{c_0^2-u_0^2}(c_0+\sqrt{c_0^2-u_0^2})} \times$$

$$\left[1 - 0.2\left(\frac{u_0}{c_0+\sqrt{c_0^2-u_0^2}}\right)^2 + \cdots\right].$$

In result the pressure near the beam axis ($x - u_0 t = 0, y = 0$), defined as:

$$p(u_0 t,0,t)_{-T<t<0} =$$

$$\frac{\Gamma q_0 \sigma}{V_0 c_0} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!!} \left(\frac{c_0(T+t)}{\sigma}\right)^{2k+1} F\left(-\frac{k}{2}, \frac{1-k}{2}; \frac{1}{2} - k; 1 - \frac{u_0^2}{c_0^2}\right) \quad (6)$$

with $F(\alpha,\beta;\gamma;z)$ being the hypergeometrical Gauss function, is not reduced as compared to the case without sweeping. It is practically independent of u_0 by $t < t_m$, but achieves its maximum – the larger

one the closer u_0 to c_0 – later, and decreases after it more slowly (see fig. 1).

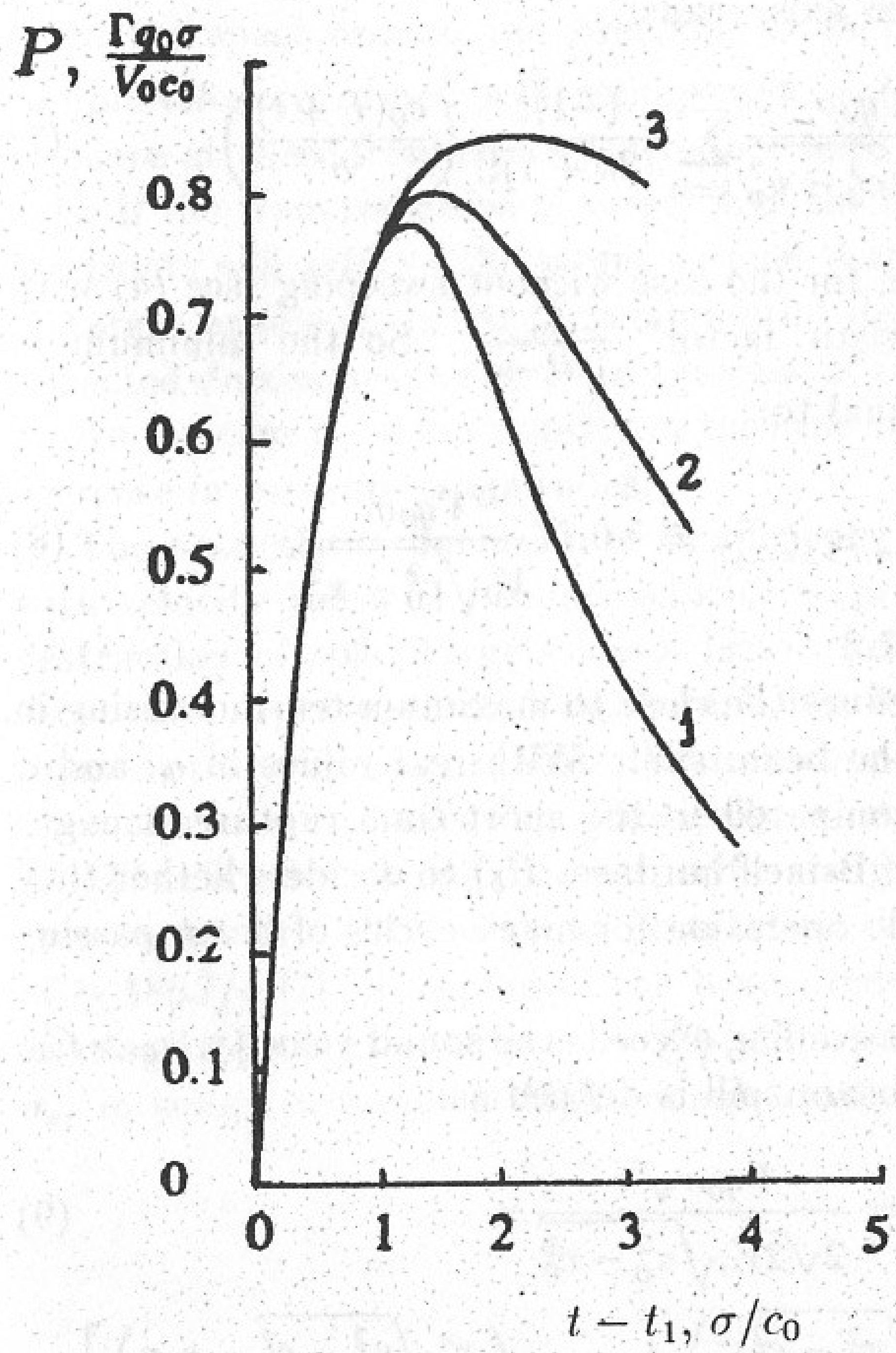

Figure 1: The pressure magnitude (in $\frac{\Gamma q_0 \sigma}{V_0 c_0}$ units) at the beam axis versus the time (in σ/c_0 units) during the beam spill for a case of: 1 – no beam sweeping; 2 – sweeping velocity equal to one half of the sound velocity, $u_0 = 0.5c_0$ and 3 – $u_0 = \sqrt{0.5}c_0$.

The negative pressure after beam spill finish is also not reduced due to the sweeping with $u_0 < c_0$. It is evidently the same for infinitely short spill duration, $T << \sigma/c_0$, and it is larger for larger T. At the limit $T >> \sigma/c_0$ the expression for pressure at $x = 0, y = 0$, that is at the final position of beam axis, reads:

$$p(0,0,t)_{t>0,T\to\infty} = -\frac{\Gamma q_0 \sigma}{V_0\sqrt{c_0^2 - u_0^2}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!!}\left(\frac{c_0(T+t)}{\sigma}\right)^{2k+1}. \quad (7)$$

It differs from expression for the case without sweeping (see (4) with $T \to \infty$) by a "relativistic factor" $\frac{c_0}{\sqrt{c_0^2-u_0^2}}$. So the minimum of $p(0,0,t)_{t>0,T>>\sigma/c_0 T}$ is equal to:

$$p(0,0,t)_{min,T>>\sigma/c_0 T} \cong -0.77\frac{\Gamma q_0 \sigma}{V_0\sqrt{c_0^2 - u_0^2}}, \quad (8)$$

taking place at $c_0 t/\sigma \cong 1.3$

The expression (8) defines the close to maximum tension arising in target material due to the beam spill. With real values of q_0 and σ substituted it is to be compared to the short time rupture strength σ_{vr} (being of the order of Brinell hardness H_b) to decide whether they are acceptable for reliable operation for many cycles of beam passing through the target.

When the velocity of sweeping exceeds the sound velocity, $u_0 > C_0$, the pressure during the beam spill is defined as:

$$p(x,y,t)_{\sigma/c_0<<t+T<T} = \frac{\Gamma q_0 \sigma\sqrt{\pi}}{2\sqrt{2}V_0\sqrt{u_0^2 - c_0^2}} \times \quad (9)$$

$$\left\{\exp\left(-\frac{(c_0 x^* + y\sqrt{u_0^2 - c_0^2})^2}{2\sigma^2 u_0^2}\right)\left[1 - \Phi\left(\frac{x^*\sqrt{u_0^2 - c_0^2} - c_0 y}{\sqrt{2}\sigma u_0}\right)\right] + \right.$$

$$\left.\exp\left(-\frac{(c_0 x^* - y\sqrt{u_0^2 - c_0^2})^2}{2\sigma^2 u_0^2}\right)\left[1 - \Phi\left(\frac{x^*\sqrt{u_0^2 - c_0^2} + c_0 y}{\sqrt{2}\sigma u_0}\right)\right]\right\},$$

where $\Phi(z)$ is the probability integral.

The pressure at the beam axis here does not decrease down zero after a large enough time from the beam spill start, but is stabilized at the level of $\frac{\Gamma q_0 \sigma\sqrt{\pi}}{V_0\sqrt{2}}\frac{1}{\sqrt{u_0^2-c_0^2}}$, which after beam spill finish exponentially decreases with the exponent $-\frac{c_0^2 t^2}{2\sigma^2}$.

In the space around the sweeping beam by $u_0 > c_0$ the fronts of compression are formed in directions $c_0(x - u_0 t) + |y|\sqrt{u_0^2 - c_0^2} = 0$. The pressure at these fronts does not reduce with distance from the beam axis. If the transverse size of target is of the order or less than $\sim c_0 T$, the fronts will achieve the target surface that can lead to destruction of target envelope, or to target material destruction caused by the refracted decompressive wave in the case of free target surface.

An increase in sweep velocity in the case of $u_0 > c_0$ results in direct decrease in pressure magnitudes.

The pessimistic conclusion of no efficiency of use of beam sweeping with velocity below the sound velocity to protect for the mechanical destruction of solid targets has a lucky exclusion in tungsten. The short time rupture strength σ_{vr} here, being very high at normal conditions – up to eight times higher than in copper, is strongly dependent on temperature, decreasing drastically when it is rising (see fig. 2) [6]. That is why the maximum specific energy deposition got in tungsten target in CERN without destruction does not exceed the value of $\sim 180 J/g$ [7]. Sweeping of the beam, reducing the temperature of heating in proportionality to $\sim \frac{2\sigma}{u_0 T}$, allows to keep the magnitude of σ_{vr} in tungsten target at high enough level.

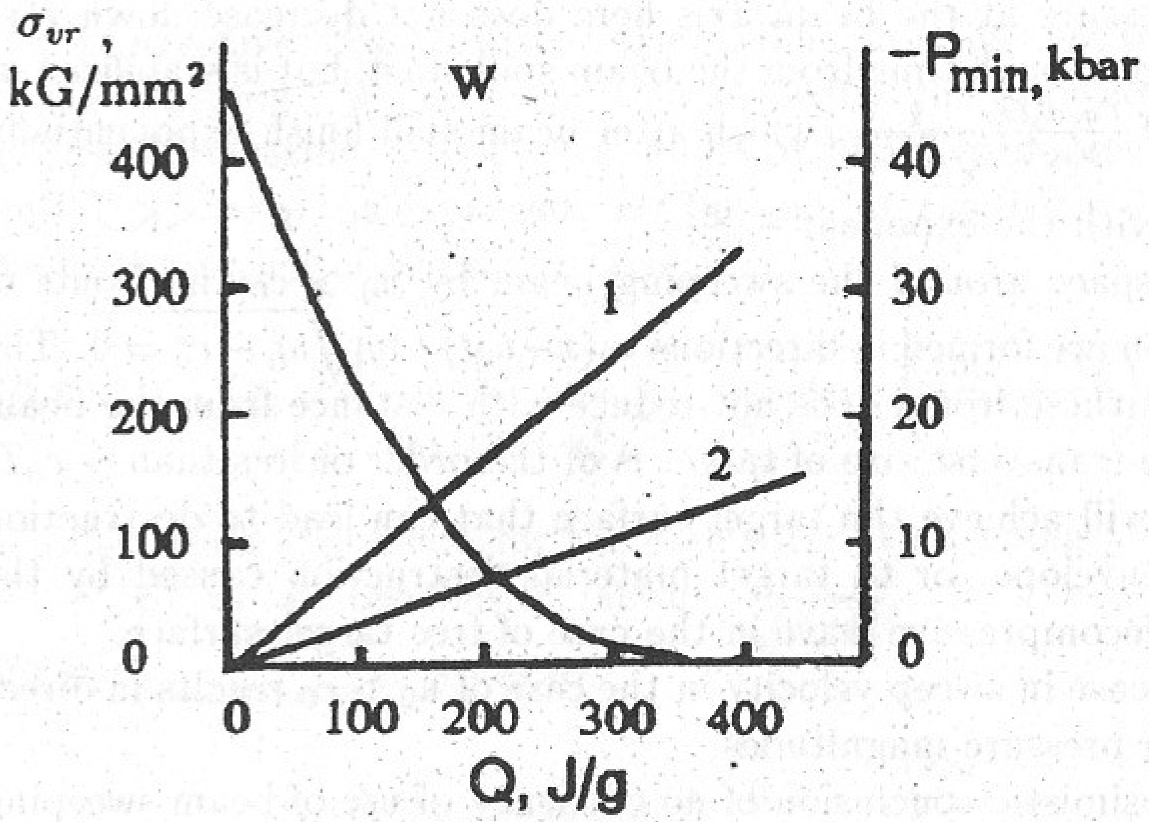

Figure 2: The short time rupture strength σ_{vr} in tungsten and maximum negative pressure $-p(0,0,t)^{min}_{t>0}$ versus the specific energy deposition at the axis of a beam with half-width of deposition region $\sigma = 0.7$ mm and spill duration T equal to: 1 – 0 and 2 – 1.67 μs.

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