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FLUCTUATIONS IN QUANTUM CHAOS

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Fluctuations in Quantum Chaos[1]

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Abstract

Various fluctuations in quantum systems with discrete spectrum are discussed, including recent unpublished results. Open questions and unexplained peculiarities of quantum chaotic fluctuations are formulated.

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1 Introduction: chaos as relaxation and fluctuations

The principal characteristics of statistical behavior are *relaxation* and *fluctuations.* The former means a monotonic, at average, approach to some steady state (statistical equilibrium) while the latter are associated with irregular oscillations during relaxation as well as around the steady state. The most surprising result of recent studies into the so-called quantum chaos is in that both properties do take place even in case of discrete quantum spectrum. Unlike in classical dynamics discrete spectrum is always the case for any quantum motion bounded in phase space. Statistical relaxation in discrete spectrum had been found already in the first numerical experiments with quantum chaos [1], and was well confirmed afterwards in many other papers (see, e.g., [2]). Particularly, the very existence of quantum diffusion, observed in Ref.[1], implies already a correlation decay and relaxation.

In the following we will focus on the other principal statistical property - the fluctuations in quantum chaos with discrete spectrum. As we shall see the quantum chaos exhibits a lot of various fluctuations justifying, thus, the term chaos even in discrete spectrum. The ultimate origin of these dynamical fluctuations is related to *decoherence*, both in space and time, of a typical quantum state associated with the classically chaotic system. In a more formal language, the decoherence results from the large number (in quasiclassical region) of incommensurate energies (frequencies) which control the dynamics. Indeed, the particular energy eigenvalue is, in general, a typical irrational number, and therefore has random digits thus producing a very irregular dynamics even though, in case of discrete spectrum, this dynamics is different from the asymptotic, classical-like, chaos (see, e.g., Ref.[4] for informal discussion).

Below we shall mainly discuss the statistical properties in *momentum* space which is relevant to the applications in nuclear, atomic and molecular physics. There exists a broad class of the so-called *dual* problems related to the conjugated *configurational* (coordinate) space which are most relevant to the solid-state physics. The interrelation between the two had been discovered in Refs.[6], and proved to be very fruitful for both fields of research (for further discussion see, e.g., Ref.[4]).

2 An illustrative model

Our main goal is to describe fluctuations in *dynamical* systems. To simplify the presentation we will use as an example a very 'simple' model called 'kicked rotator' [1] which is specified by a unitary operator (2.3), or quantum map. Even though it looks rather primitive many realistic physical models of both time-dependent as well as conservative systems can be approximately reduced to such, or to a similar, map (see, e.g., Ref.[3]). Remarkably, this 'simple' model still remains inexhausted, even in classical mechanics (see, e.g., Ref.[7] and below).

In the classical limit the model is described by the *standard map*

$$\overline{n} = n + k\cos\phi, \qquad \overline{\phi} = \phi + \overline{n}T \tag{2.1}$$

where n, ϕ are action–angle variables, k stands for the perturbation strength, and T is perturbation period.

Quantization of this map leads to the unitary operator over period T [1]:

$$\hat{U}_T = \exp\left(-i\frac{T\hat{n}^2}{2}\right)\cdot\exp\left(-ik\cdot\cos\hat{\phi}\right) \tag{2.2}$$

where $\hat{n} = -i\partial/\partial\phi$, and $\hbar = 1$.

The standard map (2.1) is defined on a cylinder ($-\infty < n < +\infty$) where the motion can be unbounded. To describe bounded motion of a conservative system it is more convenient to make use of another version of the standard map [5], namely, one on a torus with *finite* number of states L. In momentum representation ($\psi(n,\tau)$ where τ is the number of map's iterations) it is described by a finite unitary matrix U_{nm}:

$$\psi(n, \tau + T) = \sum_{m=-L_1}^{L_1} U_{nm}\,\psi(m,\tau)$$

where $L = 2L_1 + 1 \approx 2L_1$, $T/4\pi = M/2L$ is rational, and

$$U_{nm} = \frac{1}{L}\exp\left(i\frac{T}{4}(n^2 + m^2)\right)\times$$

$$\sum_{j=-L_1}^{L_1}\exp\left[-ik\cdot\cos\left(2\pi j/L\right) - 2\pi i(n-m)j/L\right] \tag{2.3}$$

There are three quantum parameters in this model: perturbation k, period T and the full number of states L, but only two classical combinations remain: perturbation $K = k\cdot T$ and classical size $M = TL/2\pi$ which is the number of resonances over the torus. Notice that the quantum dynamics is generally more rich than the classical one as the former depends on an extra parameter. This is, of course, another representation of Planck's constant which we have set $\hbar = 1$.

The quasiclassical region, where we expect quantum chaos, corresponds to $T \to 0$, $k \to \infty$, $L \to \infty$ while the classical parameters K and M are usually assumed to remain constant. The global classical chaos, particularly diffusion in n, occurs for $K > 1$. Notice that in quantum mechanics the transient pseudochaos requires an additional condition $k \gtrsim 1$ (see Ref.[1,20]).

3 The quantum steady state

The structure (particularly, fluctuations) in the steady state crucially depends on the so-called *ergodicity parameter* λ which, in our model, can be defined as

$$\lambda = \frac{D_0}{L} \sim \left(\frac{\tau_R}{\tau_e}\right)^{1/2} \sim \frac{k^2}{L} \sim \frac{K}{M} \cdot k \tag{3.1}$$

where D_0 stands for the classical diffusion rate, $\tau_e \sim L^2/D_0$ is a characteristic time of the classical relaxation to the ergodic steady state, and $\tau_R \sim D_0$ is the so-called relaxation time scale on which the quantum diffusion remains close to the classical one [4].

If $\lambda \gg 1$ the final steady state as well as all the eigenfunctions are ergodic (after Shnirelman [8]; see also Refs.[9] and [10]) that is the corresponding Wigner functions are close to the classical microcanonical distribution in phase space $\delta(H(n,\phi) - E)$. We call this region *quasiclassical asymptotics.* It can be reached, particularly, if the classical parameter K/M is kept fixed while the quantum parameter $k \to \infty$.

However, if $\lambda \ll 1$ all the eigenstates and the steady state are non-ergodic. It means that their structure remains essentially quantum, no matter how large is the quasiclassical parameter $k \to \infty$. We call this region *intermediate asymptotics* or *mesoscopic domain.* Particularly, it corresponds to $K > 1$ fixed, $k \to \infty$ and $M \to \infty$ while $\lambda \ll 1$ remains small. The mesoscopic domain we are speaking about refers to the momentum space and is a new conception [4] not commonly accepted as yet. For the dual problems in configurational space the mesoscopic phenomena are well known and currently under intensive studies (see, e.g., Refs.[11], [12] and [13]).

3.1 Ergodic states

A characteristic statistical property of ergodic eigenstates is the so-called *level repulsion*: the nearest level spacing distribution, or level fluctuation, is described by the Wigner – Dyson law:

$$p(s) \approx A\, s^\beta \exp(-B\, s^2) \tag{3.2}$$

where the repulsion parameter β takes the values $\beta = 1,\ 2,\ 4 \equiv \beta_e$ only, depending on system's symmetry. Quite often this distribution is still called universal (see, e.g., Ref.[14]) in spite of the fact that as we stressed several times [4] Eq.(3.2) is true in the case of quantum ergodicity only. Instead, we call Eq.(3.2) the *limiting statistics* because it corresponds to the ergodicity parameter $\lambda \to \infty$ (see also Ref.[14]).

As was mentioned already above the ergodic steady state is close to the classical one up to the quantum fluctuations. For example, in the standard map we would expect the temporal *rms* energy fluctuations to be of the order

$$\frac{\Delta E_s}{E_s} \sim \frac{1}{\sqrt{L}} \tag{3.3}$$

However, to our knowledge, nobody has checked this as yet numerically.[2]

[2] Recently, we have confirmed this in numerical experiments, indeed [31].

The spatial fluctuations in chaotic eigenfunctions are known to be close to Gaussian (Refs.[15] and [16]). Interestingly, a clear deviation from the Gaussian distribution was found due to finite dimensions of eigenfunctions in the Hilbert space [15].

3.2 Localized states

Statistical properties of localized states are much more rich and interesting since localization, which is an essential quantum effect, considerably modifies a classical–like ergodic structure. Instead of 'universal' fluctuations (3.2), the *intermediate statistics* was discovered to hold (Refs.[5] and [17]), namely

$$p(s) \approx A\, s^{\beta} \exp\left(-\frac{\pi^2}{16} \beta\, s^2 \, - \, sB \, + \, \frac{\pi\, \beta s}{4} \right), \qquad 0 \leq \beta \leq 4 \tag{3.4}$$

where now the value of the repulsion parameter β (cf. Eq.(3.2)) depends on the degree of localization. In particular, it can take the value $\beta = 0$ (Poisson statistics) which corresponds to the classically integrable case (or to extreme localization!). Moreover, the repulsion parameter β was found to be simply related to the localization parameter β_l [18] (see also Ref.[19]) which gives a measure of the size of eigenfunctions. There are many different quantities which characterize the size of a localized state. Below we will consider three of them:

- *Asymptotic localization length* l (for eigenstates) or l_s (for the steady state) from the approximate representation of both (at average) as $\varphi(n) \sim \exp{(-|n|/l)}$, $|n| \to \infty$. For the standard map $l \approx D_0/2$ and $l_s \approx 2l$[20].

- *Entropy localization length*[5]:

$$l_H = e^H\,, \qquad H = -\sum_n |\varphi_m(n)|^2 \ln |\varphi_m(n)|^2 \tag{3.5}$$

which characterizes the 'width" of the quantum state, and which is approximately proportional to l.

- *Participation ratio*:

$$\xi_m = \left(\sum_n |\varphi_m(n)|^4 \right)^{-1} \tag{3.6}$$

which is close but not identical to the previous length l_H.

Now, let us define the localization parameter β_l as

$$\beta_l = \frac{l_{<H>}}{l_e} \tag{3.7}$$

where brackets < > indicate averaging over all eigenfunctions, and where $l_e \geq l_{<H>}$ is the maximal entropy localization length corresponding to ergodic states. Then, it has been numerically found [18,19] that

$$\beta \approx \beta_e \cdot \beta_l \tag{3.8}$$

No explanation of this surprisingly simple relation has been given as yet.

The dependence $\beta_l(\lambda)$ on the ergodicity parameter λ, or *scaling*, can be approximated by a simple expression[5]

$$\beta_l(\lambda) \approx \frac{a\, \lambda}{1 \, + \, a\, \lambda}\,, \qquad a \approx 3\,, \qquad \lambda \lesssim 10 \tag{3.9}$$

Deviations for larger λ remain unclear, and may depend on the boundary conditions and/or $\psi(n)$ symmetry [21]. This scaling is not universal, and may be completely different depending on the model [19].

The intermediate statistics (3.4) is due to the quantum localization under the assumption of *complete chaos* in the classical limit. This is why observation of such statistics (with $\beta \approx 0.31$)

in a billiard with classically divided phase space, containing about $\delta \approx 17\%$ of regular (stable) component of the motion, was rather puzzling[22]. The point is that the presence of a stable component leads to a nonzero level spacing density $p(s) \to p_0 > 0$ as $s \to 0$ contrary to numerical results[22] which confirm Eq.(3.4) down to $s \approx 0.004$. In our opinion, this demonstrates that the effect of quantum localization can be dominant, in the *mesoscopic region*, even for $\delta > 0$. To clarify this puzzle we suggest to change the model[22] as follows. The boundary of Robnik's 2D billiard is given by the equation

$$| z + \varepsilon z^m | = 1 \tag{3.10}$$

where z is the complex coordinate in billiard's plane. For $m = 2$, used in Ref.[22], the requirement for complete classical chaos (large ε) contradicts with that of the diffusive evolution of particle's velocity direction (small ε) which is necessary for quantum localization (cf. Ref.[23]). To satisfy both conditions we need another parameter, for example $m \gg 1$, so that billiard's boundary becomes slightly wiggly which leads to diffusion in velocity. Another possibility is the everywhere slightly convex billiard.

Exponentially localized eigenfunctions $\varphi_m(n)$ show wild fluctuations which are not only very large in size[5] but, moreover, diffusively increasing in both directions of n [20]:

$$< (\eta_{mn} - < \eta_{mn} >)^2 > = D_\eta \, |\Delta n| \,, \qquad D_\eta = \frac{1}{l} = -\frac{< \eta_{mn} >}{|m - n|} \tag{3.11}$$

where $\eta_{mn} \sim \ln |\varphi_m(n)|$. Particularly, such fluctuations result in a surprising increase of the steady–state localization length $l_s \approx 2l$ as compared to that of eigenfunctions (l). Nevertheless, fluctuations of l itself vanish asymptotically as $|m - n| \to \infty$.

Namely, the *rms* dispersion

$$\frac{\Delta l}{l} = \sqrt{\frac{l}{|\Delta n|}} \to 0 \,, \qquad |\Delta n| \to \infty \tag{3.12}$$

This is not the case for the 'global' localization lengths l_H (3.5) or ξ (3.6) which give a measure of the extension of the eigenfunctions. It has been shown numerically that empirical fluctuations of entropy H (and, hence, those of l_H) are described surprisingly well by a simple expression [24]:

$$p(H) = \frac{1}{\cosh\left[\pi \left(H - < H \rangle\right)\right]} \,, \qquad \int p\, dH = 1 \tag{3.13}$$

This distribution, which has as yet no explanation, leads to the *rms* dispersion:

$$\frac{\Delta l_H}{l_H} \approx 0.66 \tag{3.14}$$

According to recent preliminary numerical data [25] the fluctuations of the steady–state localization length are qualitatively different:

$$\frac{\Delta \xi_s}{\xi_s} \sim \xi_s^{-0.25} \to 0 \,, \qquad k \to \infty \tag{3.15}$$

but equally unexplained. Unlike Eq.(3.14) these fluctuations are vanishing in the quasiclassical region. Another interesting feature of fluctuations (3.15) is an 'abnormal' (fractal?) exponent ≈ 0.25 instead of 0.5 for the 'normal' fluctuations. Possible fractal properties of the quantum steady state are confirmed by the temporal fluctuations of the steady–state energy [26] (cf. Eq.(3.3)):

$$\frac{\Delta E_s}{E_s} \sim l_s^{-0.3} \to 0 \,, \qquad k \to \infty \tag{3.16}$$

Notice that both exponents, in Eqs.(3.15) and (3.16), are equal within the accuracy of numerical experiments. A naive interpretation of these fluctuations would imply that $\psi_s(n)$ for the chaotic steady state represents a *finite* ensemble of ν statistically independent systems. In this case one expects $\Delta E_s/E_s \sim 1/\sqrt{\nu}$, and comparison with (3.16) leads to

$$\nu \sim l_s^{0.6} \tag{3.17}$$

Notice that formally the steady state is a pure quantum state, and describes a single quantum system.

4 Quantum statistical relaxation

Wild fluctuations show up also in the process of statistical relaxation to the quantum (nonergodic) steady state [27, 28]. The fluctuations still persist even after averaging over 10^4 runs [27]. No explanation of these fluctuations exists as yet. Using a special time averaging we have managed to get rid of them [31] and, thus, to compare our numerical results [28] with the theory based upon a phenomenological diffusion equation with the backscattering term (Refs. [4] and [29]).

In terms of the scaled variables $\tilde{\sigma}$ and $\tilde{n}$ the diffusion equation for the Green function g reads

$$\frac{\partial g(\tilde{n}, \tilde{\sigma})}{\partial \tilde{\sigma}} = \frac{1}{4}\frac{\partial^2 g}{\partial \tilde{n}^2} + B(\tilde{n})\frac{\partial g}{\partial \tilde{n}} \tag{4.1}$$

Here $g(\tilde{n}, 0) = \delta(\tilde{n} - \tilde{n}_0)$, and

$$\tilde{n} = \frac{n}{2D_0}, \qquad \tilde{\sigma} = \ln(1 + \tilde{\tau}), \qquad \tilde{\tau} = \frac{\tau}{2D_0}$$

The additional drift term with

$$B(\tilde{n}) = \mathrm{sign}(\tilde{n} - \tilde{n}_0) = \pm 1$$

describes the so–called quantum coherent backscattering which is the main cause of localization.

The solution of Eq.(4.1) can be expressed in terms of the Error functions [4]. In particular, if we consider the scaled unperturbed energy

$$\tilde{E}(\tilde{\tau}) = \frac{< n^2 >}{2E_s},$$

where $E_s = D_0^2/4$ is the energy of the quantum steady state, then the relaxation rate is given by:

$$\frac{d\tilde{E}}{d\tilde{\tau}} =$$

$$2\, e^{-\tilde{\sigma}} \left[\left(\tilde{\sigma} + \frac{1}{2}\right) \cdot \mathrm{erfc}(\sqrt{\tilde{\sigma}}) - \sqrt{\frac{\tilde{\sigma}}{\pi}}\, e^{-\tilde{\sigma}} \right] \rightarrow \frac{4}{\sqrt{\pi}\,\tilde{\tau}^2} \cdot (\ln \tilde{\tau})^{-3/2} \tag{4.2}$$

the latter expression showing up the asymptotics as $\tilde{\tau} \rightarrow \infty$. Numerical experiments agree with Eq.(4.2) only to logarithmic accuracy in τ. More precisely, our numerical results [28] lead to the asymptotic behaviour

$$\tilde{R} \rightarrow \frac{4}{\sqrt{\pi}\tilde{\tau}^2} \cdot (\ln \tilde{\tau})^{-1/2} \tag{4.3}$$

while in a different approach [27] the theoretical expression

$$\tilde{R} \rightarrow \frac{4}{\sqrt{\pi}\tilde{\tau}^2} \cdot \ln \tilde{\tau} \tag{4.4}$$

was obtained.

5 Transmission fluctuations

Recently a dynamical model for the conductance in disordered solids was introduced and studied [30]. To the best of our knowledge, this was the first dynamical model in this problem. The authors made use of the standard map for a dual problem to describe the electron motion in a quasi-1D lattice (the model for a thin wire). In this case the variable n describes electron's position on the lattice while conjugate ϕ is a quasimomentum (cf. Eq.(2.1) above). Another important difference from the dual dynamical problem in the momentum space is in that the model is now open with incoming (Q_{in}) and outgoing (Q_{out}) fluxes. The quantity of principal interest is *transmission* τ which was computed via solution of the classical diffusion equation [30]

$$\tau \equiv \frac{Q_{out}}{Q_{in}} = \frac{C(K)\,D_1}{(2C(K)-1)\,D_1 + \nu L} \to \frac{C(K)\,D_1}{\nu L}, \quad L \to \infty \tag{5.1}$$

Here L is the length of the lattice, $D_0 = C(K) \cdot D_1$ is the classical diffusion rate, $C(K) \sim 1$ is the dynamical correlation factor, $D_1 = k^2/2$ the asymptotic diffusion rate ($K \to \infty$), and $\nu = 2k/\pi$. If we apply this classical solution to quantum diffusion on the relaxation time scale (Section 4) ν has a meaning of the number of quantum scattering channels. Expression (5.1) for τ was derived from the classical map under the condition $k \ll L$ which allows for classical diffusion.

In the quantum case, the diffusive regime is characterized by the double inequality

$$k \ll L \ll k^2 \tag{5.2}$$

The right inequality means that localization length is larger than the sample size, so that the effect of quantum localization is weak and can be neglected in the first approximation. In this regime the so–called 'universal' mesoscopic fluctuations

$$< (\Delta\tau(E))^2 > \approx \frac{2}{15\,\nu^2} \tag{5.3}$$

have been found in the frames of a statistical theory of random

matrices. Numerical results in Ref.[30] confirm this relation once more but in a *dynamical* model.

The question we are going to discuss here: what is the dynamical origin of such fluctuations?

First of all we need to distinguish between the transmission $\tau(E)$ at a given energy E and the total transmission τ (averaged over energy) as derived from the classical diffusion equation [32]. While their averages are equal the fluctuations differ considerably. According to Ref.[32] for the total transmission

$$< (\Delta\tau)^2 > \approx \frac{\alpha^2}{L^2} \tag{5.4}$$

with $\alpha^2 \sim \pi^2/2$. We conjecture that these fluctuations are related to the fluctuations of the quantum diffusion rate D_q which are unknown as yet. Using the asymptotic relation (5.1) we obtain

$$\frac{< (\Delta D_q)^2 >}{D_0^2} = \left(\frac{4\alpha}{\pi C(K)}\right)^2 \cdot \frac{1}{k^2} \tag{5.5}$$

A possible physical interpretation of this result is in that the $k \times k$ scattering channels are statistically independent in case of the quantum chaos. At the first glance, it contradicts to the conclusion in Ref.[33] on the strong correlation of the fluctuations in different channels. However, the latter was found for the transmission at a fixed energy while Eq.(5.5) corresponds to the total transmission. It would be very interesting to directly check numerically the fluctuations of the quantum diffusion rate.[3]

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[3] Preliminary results of our recent numerical experiments [31] seem to confirm scaling (5.5) for the fluctuation of quantum diffusion rate.

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