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BACKREACTION IN SPINOR QED AND DECOHERENCE FUNCTIONAL

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НОВОСИБИРСК

Backreaction in spinor QED and decoherence functional

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Аннотация

Using the Schwinger-Keldysh (closed time path or CTP) and Feynman-Vernon influence functional formalisms we obtain an expression for the influence functional in terms of Bogoliubov coefficients for the case of spinor quantum electrodynamics. Then we derive a CTP effective action in semiclassical approximation and its cumulant expansion. Using it we obtain a equation for the description of the charged particle creation in electric field and of backreaction of charged quantum fields and their fluctuations on time evolution of this electric field. Also an intimate connection between CTP effective action and decoherence functional will allow us to analyze how macroscopic electromagnetic fields are "measured" through interaction with charges and thereby rendered classical.

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1 Introduction

This paper is an extension of our previous work [1] to the case of spinor QED. We will study the quantum non-equilibrium effects of pair creation in strong electric fields. Backreaction of pair creation on electric field was recently discussed by Cooper, Mottola at all [2]. They derived the semiclassical Maxwell equation, carry out its renormalization and numerically solve it for some initial conditions in 1+1 dimensions.Their numerical results clearly exhibits the decay of the electric field because of screening by the produced particles. We wish to make a step further and derive a Langevin equation, taking into account a noise from quantum matter fields. To this purpose we will use some mixture of Schwinger-Keldysh (CTP) and Feynman-Vernon influence functional formalisms. It is important to note here that phenomenological equations of motion with noise term can also be derived using decoherence functional formulation of quantum mechanics. This was done for some model quantum systems in [3, 4].

This paper is organized as follows: In Sec.2 we give a brief review of CTP functional formalism, mainly to introduce notations. All essential details can be found in [5, 6, 7, 8]. In Sec.3 we will obtain an expression for the influence functional in terms of Bogoliubov coefficients for the case of quantum electrodynamics with spin 1/2 charged particles. Then in Sec.4 we will obtain a CTP effective action in semiclassical approximation and its cumulant expansion. It is the main result of this paper. We will apply it for study of two interesting problems. First this result will allow us to analyze the backreaction of created charged particles on electric field and to derive a Langevin equation, which take into account a noise from quantum matter fields. Then in Sec.5 we will use an intimate connection between CTP effective action and decoherence functional [9, 5] to analyze how macroscopic electromagnetic fields are "measured" through interaction with charges and thereby rendered classical.

2 The Closed Time Path Functional Formalism in Quantum Field Theory

Usually in quantum field theory our interest is in obtaining the amplitudes of transition from in-states to the out-states. But in many cases, mainly in statistical physics, we are concerned with expectation values of physical quantities at finite time. To solve such initial value problems Schwinger has invented close time path (CTP) formalism.

Let us consider the expectation value of an arbitrary operator A:

$$< A > (t) = Tr\ \rho(t)\ A \tag{1}$$

Here ρ is the density matrix that describes the (mixed) state of the system. The density matrix does not necessarily have to commute with the Hamiltonian, in which case it describes a non-equilibrium state. Using relation $\rho(t) = U(t,0)\,\rho(0)\,U^{-1}(t.0)$ where $U(t,0)$ is the evolution operator, inserting the identity operator $1 = U(t,T)\ U(T,t)$ we obtain

$$\begin{aligned} < A > (t) = Tr\ \rho(0)\ U^{-1}(t,0)\ A\,U(t,0) = \\ Tr\ \rho(0)\ U(0,T)\ U(T,t)\ A\,U(t,0) \end{aligned} \tag{2}$$

Equation (2) can be pictured as describing the evolution of the system from 0 to t, inserting the operator A , evolving further to some large time T (in practice, $T \to \infty$), and then backwards from T to 0. The insertion of operator may be achieved by introducing external sources coupled to the particular operator. This suggests the definition of the CTP generating functional

$$Z[J_+,J_-] \equiv \exp iW[J_+,J_-] = Tr\ \rho(0)\,U(0,T,J_-)\,U(T,0,J_+) \tag{3}$$

In the path integral representation we have

$$Z[J_+,J_-] = \int D\phi_1 D\phi_2 D\phi\ \langle\phi_1|\rho|\phi_2\rangle \int_{\phi_1}^{\phi} D\phi_+ \times$$

$$\times \int_{\phi_2}^{\phi} D\phi_-\ \exp i \int_0^T dt\,\{L[\phi_+] - L[\phi_-] + J_+\phi_+ - J_-\phi_-\} \tag{4}$$

The expectation values can be obtained as

$$\bar\phi_+ = \frac{\delta W}{\delta J_+} \quad , \ \bar\phi_- = -\frac{\delta W}{\delta J_-} \tag{5}$$

Then the CTP effective action is

$$\Gamma_{CTP}[\bar\phi_+,\bar\phi_-] = W[J_+,J_-] - J_+\bar\phi_+ + J_-\ \bar\phi_- \tag{6}$$

The equations of motion are

$$\frac{\delta \Gamma_{CTP}}{\delta \phi_+} = -J_+ \ , \quad \frac{\delta \Gamma_{CTP}}{\delta \phi_-} = J_- \tag{7}$$

The physical situations correspond to solutions of the homogeneous equations at$\phi_+ = \phi_-$. Then equations are real and causal.

To apply this formalism to our situation we should substitute the ϕ field by the pair ψ and σ. We will be interested in expectation values of ψ only, so we do not couple the σ field to an external source. Also we assume that the initial density matrix factorizes $\rho = \rho_\psi \ \rho_\sigma$. Then we have

$$\begin{aligned} Z[J_+, J_-] = & \int D\psi_1 D\sigma_1 D\psi_2 D\sigma_2 \ \langle \psi_1 | \rho_\psi | \psi_2 \rangle \ \langle \sigma_1 | \rho_\sigma | \sigma_2 \rangle \\ & \int D\psi D\sigma \int_{\psi_1}^{\psi} D\psi_+ \int_{\sigma_1}^{\sigma} D\sigma_+ \int_{\psi_2}^{\psi} D\psi_- \int_{\sigma_2}^{\sigma} D\sigma_- \\ & \exp i \int_0^T dt \ \{ \ L_\psi[\psi_+] - L_\psi[\psi_-] + J_+\psi_+ - J_-\psi_- + \\ & L_\sigma[\sigma_+] - L_\sigma[\sigma_-] + L_{int}[\psi_+, \sigma_+] - L_{int}[\psi_-, \sigma_-] \ \} = \\ & \int D\psi_1 D\psi_2 \ \langle \psi_1 | \rho_\psi | \psi_2 \rangle \int D\psi \int_{\psi_1}^{\psi} D\psi_+ \int_{\psi_2}^{\psi} D\psi_- \\ & \exp i \int_0^T dt \ \{ \ L_\psi[\psi_+] - L_\psi[\psi_-] + J_+\psi_+ - J_-\psi_- \ \} \ \Phi[\psi_+ \psi_-] \end{aligned} \tag{8}$$

where $\Phi[\psi_+ , \psi_-]$ is the so called influence functional

$$\begin{aligned} \Phi[\psi_+ , \psi_-] \equiv & \exp \ i S_{IF}[\psi_+ , \psi_-] = \int D\sigma_1 D\sigma_2 \ \langle \sigma_1 | \rho_\sigma | \sigma_2 \rangle \int_{\sigma_1}^{\sigma} D\sigma_+ \int_{\sigma_2}^{\sigma} D\sigma_- \\ & \exp \ i \int_0^T dt \ \{ L_\sigma[\sigma_+] - L_\sigma[\sigma_-] + L_{int}[\psi_+ , \sigma_+] - L_{int}[\psi_- , \sigma_-] \} = \\ & Tr \ [U(T, 0; \psi_+) \ \rho_\sigma(0) \ U^{-1}(T, 0; \psi_-)] \end{aligned} \tag{9}$$

It is now easy to show, using (8) and (9), that in semiclassical approximation CTP effective action has the form

$$\Gamma_{CTP}[\psi_+ , \psi_-] = S[\psi_+] - S[\psi_-] + S_{IF}[\psi_+ , \psi_-] \tag{10}$$

From this relation one may derive the semiclassical equations of motion for the expectation values of the ψ field. It is worth noting that

$$\Gamma_{CTP}[\psi_+ , \psi_-] = -\Gamma^*_{CTP}[\psi_- , \psi_+] \quad \text{and} \quad \Gamma_{CTP}[\psi , \psi] \equiv 0. \tag{11}$$

3 Influence functional for spinor QED

In this section we wish to find the influence functional in terms of Bogoliubov coefficients (as in [10]). The influence functional have now the form

$$\Phi[A', A] = Tr \ [U(T, 0; A') \ \rho_\psi(0) \ U^{-1}(T, 0; A)] \tag{12}$$

To obtain $U(T, 0; A)$ we will use the Heisenberg equation of motion

$$i \dot{\Psi} = \left[\vec{\alpha} \left(-i \frac{\partial}{\partial \vec{x}} - e\vec{A} \right) + m\beta \right] \Psi \tag{13}$$

We will choose $\vec{A} = (0, 0, A(t))$ and

$$\Psi(\vec{x}, t) = \sum_{\vec{p}, \sigma} \frac{1}{\sqrt{2\omega_0(\vec{p}) V}} \left\{ b_{\vec{p}\sigma}(t) \, u_{\vec{p}\sigma} + d^+_{-\vec{p}\sigma}(t) \, v_{-\vec{p}\sigma} \right\} e^{i \vec{p} \vec{x}} \tag{14}$$

where $b_{\vec{p}\sigma}$ and $d^+_{-\vec{p}\sigma}$ are usual annihilation and creation operators for particles and antiparticles respectively. In what follows we will set volume $V = 1$. Here $\omega_0(\vec{p}) \equiv \omega(\vec{p}, 0)$ with

$$\omega^2(\vec{p}, t) = \vec{p}_\perp^2 + (p^3 - e A(t))^2 + m^2 \tag{15}$$

Bispinors $u_{\vec{p}\sigma}$ and $v_{-\vec{p}\sigma}$ satisfy

$$\begin{aligned} \left[\vec{\alpha} \left(\vec{p} - e\vec{A}(0) \right) + m\beta \right] u_{\vec{p}\sigma} &= \omega_0(\vec{p}) \, u_{\vec{p}\sigma} \\ \left[\vec{\alpha} \left(\vec{p} - e\vec{A}(0) \right) + m\beta \right] v_{-\vec{p}\sigma} &= -\omega_0(\vec{p}) \, v_{-\vec{p}\sigma} \end{aligned} \tag{16}$$

and we choose them in following form

$$\begin{aligned} u_{\vec{p}\sigma} &= \frac{1}{\sqrt{2(\omega_0(\vec{p}) - \Pi^3(0))}} \left(\hat{\Pi}(0) + m \right) \chi_\sigma \\ v_{-\vec{p}\sigma} &= \frac{1}{\sqrt{2(\omega_0(\vec{p}) + \Pi^3(0))}} \left(-\hat{\tilde{\Pi}}(0) + m \right) \chi_\sigma \\ \text{where } \Pi^\mu(t) &\equiv \left(\omega(\vec{p}, t), \vec{p} - e\vec{A}(t) \right) = \left(\omega(\vec{p}, t), \vec{\Pi}(t) \right), \\ \tilde{\Pi}^\mu(t) &\equiv \left(\omega(\vec{p}, t), -\vec{\Pi}(t) \right) \text{ and } \hat{\Pi} = \Pi_\mu \gamma^\mu \\ \chi_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}&, \chi_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \alpha^3 \chi_1 = \chi_1; \alpha^3 \chi_2 = \chi_2 \end{aligned} \tag{17}$$

Then the solution of (13) for creation and annihilation operators has the following form

$$b_{\vec{p}\sigma}(t) = \alpha_{\vec{p}}(t)\, b_{\vec{p}\sigma}(0) + \beta^*_{\vec{p}}(t)\, d^+_{-\vec{p}\sigma}(0);$$

$$d^+_{-\vec{p}\sigma}(t) = -\beta_{\vec{p}}(t)\, b_{\vec{p}\sigma}(0) + \alpha^*_{\vec{p}}(t)\, d^+_{-\vec{p}\sigma}(0) \tag{18}$$

where the Bogoliubov coefficients $\alpha_{\vec{p}}(t)$ and $\beta_{\vec{p}}(t)$ obey to a system of ordinary first order differential equations

$$\dot{\alpha}_{\vec{p}}(t) = i\, h_{\vec{p}}(t)\, \alpha_{\vec{p}}(t) + i\, g_{\vec{p}}(t)\, \beta_{\vec{p}}(t) \;;\; \dot{\beta}_{\vec{p}}(t) = i\, g_{\vec{p}}(t)\, \alpha_{\vec{p}}(t) - i\, h_{\vec{p}}(t)\, \beta_{\vec{p}}(t) \tag{19}$$

with

$$\begin{aligned} h_{\vec{p}}(t) &= -\tfrac{1}{\omega_0(\vec{p})}\left(\vec{\Pi}(t)\,\vec{\Pi}(0) + m^2\right) = -\tfrac{m^2+\vec{p}^2_\perp+\Pi^3(t)\Pi^3(0)}{\omega_0(\vec{p})} \;; \\ g_{\vec{p}}(t) &= \tfrac{1}{\omega_0(\vec{p})}\sqrt{\omega_0^2(\vec{p}) - (\Pi^3(0))^2}\;\left(\Pi^3(t) - \Pi^3(0)\right) \end{aligned} \tag{20}$$

Using last equations we can show that $\alpha_{\vec{p}}(t)$ and $\beta_{\vec{p}}(t)$ are expressed via auxiliary functions $f_{\vec{p}}(t)$ and $\varphi_{\vec{p}}(t)$ by following relations

$$\begin{aligned} \alpha_{\vec{p}}(t) &= \sqrt{\tfrac{\omega_0(\vec{p})-\Pi^3(0)}{2\omega_0^2(\vec{p})}}\left\{i\tfrac{d}{dt}f_{\vec{p}}(t) + f_{\vec{p}}(t)\left[\omega_0(\vec{p}) - \Pi^3(t) + \Pi^3(0)\right]\right\} \\ \beta_{\vec{p}}(t) &= \sqrt{\tfrac{\omega_0(\vec{p})+\Pi^3(0)}{2\omega_0^2(\vec{p})}}\left\{i\tfrac{d}{dt}f_{\vec{p}}(t) - f_{\vec{p}}(t)\left[\omega_0(\vec{p}) + \Pi^3(t) - \Pi^3(0)\right]\right\} \\ \alpha_{\vec{p}}(t) &= \sqrt{\tfrac{\omega_0(\vec{p})+\Pi^3(0)}{2\omega_0^2(\vec{p})}}\left\{i\tfrac{d}{dt}\varphi^*_{\vec{p}}(t) + \varphi^*_{\vec{p}}(t)\left[\omega_0(\vec{p}) + \Pi^3(t) - \Pi^3(0)\right]\right\} \\ \beta_{\vec{p}}(t) &= \sqrt{\tfrac{\omega_0(\vec{p})-\Pi^3(0)}{2\omega_0^2(\vec{p})}}\left\{-i\tfrac{d}{dt}\varphi^*_{\vec{p}}(t) + \varphi^*_{\vec{p}}(t)\left[\omega_0(\vec{p}) - \Pi^3(t) + \Pi^3(0)\right]\right\} \end{aligned} \tag{21}$$

with the $f_{\vec{p}}(t)$ and $\varphi_{\vec{p}}(t)$ satisfying to equations

$$\begin{aligned} &\tfrac{d^2}{dt^2}f_{\vec{p}}(t) + \left[\omega^2(\vec{p},t) - ie\,\dot{A}\right] f_{\vec{p}}(t) = 0;\; f_{\vec{p}}(t) = \tfrac{e^{-i\omega_0(\vec{p})t}}{\sqrt{2(\omega_0(\vec{p})-\Pi^3(0))}} \text{ at } t\to 0 \\ &\tfrac{d^2}{dt^2}\varphi_{\vec{p}}(t) + \left[\omega^2(\vec{p},t) - ie\,\dot{A}\right] \varphi_{\vec{p}}(t) = 0;\; \varphi_{\vec{p}}(t) = \tfrac{e^{i\omega_0(\vec{p})t}}{\sqrt{2(\omega_0(\vec{p})+\Pi^3(0))}} \text{ at } t\to 0 \end{aligned} \tag{22}$$

Equations (19-22) enable one to consider the particle creation in a homogeneous electric field $E(t)$ having an arbitrary time dependence. For example the famous Schwinger results [11], describing pair creation in a constant electric field and the results, obtained earlier for fields with fixed time dependance (see review [12]) can be found using this equations . Note that the number of created particles in $\vec{p}$ th mode is given by

$$n_{\vec{p}} = |\beta_{\vec{p}}|^2 \tag{23}$$

Now we have enough formulae to find $U(t,0)$ using relations

$$\begin{aligned} b_{\vec{p}\sigma}(t) &= U^+ b_{\vec{p}\sigma} U = \alpha_{\vec{p}}\, b_{\vec{p}\sigma} + \beta^*_{\vec{p}}\, d^+_{-\vec{p}\sigma} \;; \\ d^+_{-\vec{p}\sigma}(t) &= U^+ d^+_{-\vec{p}\sigma} U = -\beta_{\vec{p}}\, b_{\vec{p}\sigma} + \alpha^*_{\vec{p}}\, d^+_{-\vec{p}\sigma} \end{aligned} \tag{24}$$

here we take for brevity $b_{\vec{p}\sigma} \equiv b_{\vec{p}\sigma}(0)\,; d^+_{-\vec{p}\sigma} \equiv d^+_{-\vec{p}\sigma}(0)$.
We will skip the details of calculations and express our result in the following form $U = \prod_{\vec{p},\sigma} U_{\vec{p}\sigma}$ where $U_{\vec{p}\sigma}$ is (we drop the mode label)

$$U = S(r,\phi)\, R(\theta) \tag{25}$$

where

$$S(r,\phi) = \exp\left[r\left(e^{2i\phi}\, b^+ d^+ + e^{-2i\phi}\, b\, d\right)\right] \;;\; R(\theta) = \exp\left[i\,\theta\,\left(d^+ d + b^+ b - 1\right)\right] \tag{26}$$

S and R are called two-mode squeeze and rotation operators respectively. The parameters r, ϕ, θ are determined from the equations

$$\alpha = e^{i\theta}\cos r\,;\; \beta = e^{i\theta - 2i\phi}\sin r \tag{27}$$

This expression for $U(t,0)$ may be useful in many situations. We may ,for example, rather easily describe time evolution of density matrix $\rho(t)$ from arbitrary initial $\rho(0)$, more interesting initial states are : vacuum state, thermal equilibrium and coherent states. Recently the time evolution of density matrix at finite temperature was considered in [13] using the functional Schrodinger representation. In this paper we will deal only with vacuum initial state. Applying (12) and (25) we find that the influence functional with vacuum as an initial state is given by

$$\Phi[A', A] = \prod_{\vec{p}}\left(\alpha'^*_{\vec{p}}\,\alpha_{\vec{p}} + \beta'^*_{\vec{p}}\,\beta_{\vec{p}}\right)^2 \tag{28}$$

4 Semiclassical CTP effective action and Langevin equation

We may now obtain for Γ_{CTP} :

$$\Gamma_{CTP}[A', A] = S[A'] - S[A] + S_{IF}[A', A]$$

where $\quad S_{IF}[A', A] = -i\ln\Phi[A', A] = -2i\sum_{\vec{p}}\ln\left[\alpha'^*_{\vec{p}}\,\alpha_{\vec{p}} + \beta'^*_{\vec{p}}\,\beta_{\vec{p}}\right] \quad (29)$

It is useful to introduce new variables as

$$\Xi = \frac{1}{2}(A' + A); \quad \Delta = A' - A \tag{30}$$

and define [10]

$$C_n(t_1, ..., t_n; \Xi_{t_1,0}, ..., \Xi_{t_n,0}] \equiv \frac{1}{i^{n-1}} \frac{\delta^n S_{IF}[A', A]}{\delta\Delta(t_1) \ldots \delta\Delta(t_n)}\bigg|_{\Delta=0} \tag{31}$$

The notation of $C_1(t_1; \Xi_{t_1,0}]$ means C_1 is a function of t_1 and also a functional of Ξ between endpoints t_1 and 0. By virtue of (11) the C_n's are real quantities.

Now we can write S_{IF} as a functional Taylor series which have sence for all Δ and Ξ

$$\begin{aligned} S_{IF}[A', A] = &\int_0^\infty d\tau_1 \Delta(\tau_1)\, C_1(t_1; \Xi_{t_1,0}] + \\ &+ \frac{i}{2}\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \Delta(\tau_1)\,\Delta(\tau_2)\, C_2(\tau_1, \tau_2; \Xi_{\tau_1,0}, \Xi_{\tau_2,0}] + \\ &\ldots + \frac{i^{n-1}}{n!}\int_0^\infty d\tau_1 \ldots d\tau_n \Delta(\tau_1) \ldots \Delta(\tau_n)\, C_n(\tau_1, \ldots, \tau_n; \Xi_{\tau_1,0}, \ldots, \Xi_{\tau_n,0}] + \ldots \end{aligned} \tag{32}$$

Following Feynman's procedure of deriving Langevin equation [14] we rewrite[1]

$$\begin{aligned} &\exp\Big\{ -\frac{1}{2}\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \Delta(\tau_1)\,\Delta(\tau_2)\, C_2(\tau_1, \tau_2; \Xi_{\tau_1,0}, \Xi_{\tau_2,0}) + \ldots \\ &+ \frac{i^n}{n!}\int_0^\infty d\tau_1 \ldots d\tau_n \Delta(\tau_1) \ldots \Delta(\tau_n)\, C_n(\tau_1, \ldots, \tau_n; \Xi_{\tau_1,0}, \ldots, \Xi_{\tau_n,0}) + \ldots \Big\} = \\ &= \int D\xi\, P[\xi, \Xi] \exp\Big\{ i\int_0^\infty d\tau \Delta(\tau)\, \xi(\tau) \Big\} \end{aligned} \tag{33}$$

The left hand side of (33) is interpreted now as a characteristic functional of a stochastic process $\xi(\tau)$. The probability density functional $P[\xi, \Xi]$ of $\xi(\tau)$ can be obtained from a given influence functional by inverting the functional fourier transform. The C_n's are now cumulants of colour noise $\xi(\tau)$.

We can use now $\Gamma_{CTP}[A', A]$ to obtain the semiclassical Langevin equations of motion using (7) in following form

[1] Here we closely follow the reasoning of Hu and Matacz [10].

$$\frac{\delta\Gamma_{CTP}[A', A]}{\delta\Delta(\tau)}\bigg|_{A'=A} = 0 \tag{34}$$

From (34) we obtain the Langevin equation

$$\ddot{A}(t) = C_1(t; A_{t,0}] + \xi(t) \tag{35}$$

In our case we have from (29) and (31)

$$\begin{aligned} C_1(t; \Xi_{t,0}] = &-2e\int \frac{d^3\vec{p}}{(2\pi)^3\omega_0(\vec{p})}\Big[(p^3 - eA(0)) \left(|\alpha_{\vec{p}}(t)|^2 - |\beta_{\vec{p}}(t)|^2\right) - \\ &-\sqrt{m^2 + \vec{p}_\perp^2}\left(\alpha^*_{\vec{p}}(t)\,\beta_{\vec{p}}(t) + \alpha_{\vec{p}}(t)\,\beta^*_{\vec{p}}(t)\right)\Big] = \\ &= -2e\int \frac{d^3\vec{p}}{(2\pi)^3\omega_0(\vec{p})}\sqrt{m^2 + \vec{p}_\perp^2}\left(|f_{\vec{p}}(t)|^2 - |\varphi_{\vec{p}}(t)|^2\right) \\ C_2(\tau_1\tau_2; \Xi_{\tau_1,0}\Xi_{\tau_2,0}] = &\langle \xi(\tau_1)\,\xi(\tau_2)\rangle = \\ = \sum_{\vec{p}} &\left[\frac{2e\Pi^3(0)}{\omega_0(\vec{p})}\alpha_{\vec{p}}(\tau_1)\,\beta_{\vec{p}}(\tau_1) + \frac{e}{\omega_0(\vec{p})}\sqrt{m^2 + \vec{p}_\perp^2}\left(\alpha_{\vec{p}}(\tau_1)^2 - \beta_{\vec{p}}(\tau_1)^2\right)\right] \times \\ &\times\Big[\frac{2e\Pi^3(0)}{\omega_0(\vec{p})}\alpha^*_{\vec{p}}(\tau_2)\,\beta^*_{\vec{p}}(\tau_2) + \frac{e}{\omega_0(\vec{p})}\sqrt{m^2 + \vec{p}_\perp^2} \\ &\left(\alpha^*_{\vec{p}}(\tau_2)^2 - \beta^*_{\vec{p}}(\tau_2)^2\right)\Big] + (\tau_1 \longleftrightarrow \tau_2) \end{aligned} \tag{36}$$

It is easy to show that without noise term Eq.(35) is equal to the semiclassical Maxwell equation, obtained in [2].Then renormalization of (35) can be carried out as in [2] .

So we obtain the finite renormalized Langevin equation that describe the process of pair production in a spatially homogeneous electric field and the backreaction from this pairs on time evolution of the electric field. The solution of the Langevin equation is beyond the scope of the present paper.We plan to consider this solution in future.

5 Decoherence in spinor QED

In this section we will show ,using the results of previous sections, how the programme of decoherence [3, 15] can be applied in the context of quantum electrodynamics in some detail. We then analyze an example where macroscopic electromagnetic fields are "measured" through interaction with charges and thereby rendered classical. This example was discussed recently by Kiefer for scalar QED [16] using different point of view.

In the consistent or decoherent histories formulation of quantum mechanics [17, 18, 19]the complete description of a coupled ψ, σ system is given in terms of fine-grained histories $\psi(t), \sigma(t)$. Let us take as a coarse-graining procedure of summing over the σ field. In other words the σ field play in our case the role of environment. Then the interference effects between coarse-grained histories are measured by the decoherence functional $D[\psi, \psi']$. It was shown in [9, 5, 20] that the decoherence functional, which is the fundamental object of the decoherent histories formulation, is connected with CTP effective action by following relation

$$D[\psi, \psi'] = e^{i\,\Gamma_{CTP}[\psi\,,\psi']} \quad (37)$$

The coarse-grained history $\psi(t)$ can be described classically if and only if the decoherence functional is approximately diagonal , that is, $D[\psi, \psi'] \simeq 0$ whenever $\psi \neq \psi'$. Now after this very brief discussion of the some aspects of decoherence,we will proceed to discuss an example where macroscopic field strengths decohere through their interaction with charges.We wish to consider, as an example, a macroscopic superposition of two electric fields, one pointing upwards, and the other pointing downwards. In the case of spinor QED we obtain from (29,32) using dimensional considerations and omitting unessential phase factor that

$$D[A, A'] = e^{i\,\Gamma_{CTP}[A',A]} \sim \exp\left\{-\frac{Ve^2E^2}{m}\sum_{k=0}^{\infty} a_k \left(\frac{E}{E_0}\right)^{2k}\right\} \quad (38)$$

Here a_k 's are numbers and $E_0 = m^2/e \sim 10^{16}$ V/cm. It is clear that,for example, at $E \approx 10^7$ V/cm (when $E/E_0 \sim 10^{-9}$) we can neglect all terms with $k \geq 1$. Now after an easy calculation we obtain

$$D[A, A'] \sim \exp\left\{-\frac{3V\,e^2\,E^2}{2^7\pi m}\right\} \quad (39)$$

Note that the interaction with the charge states leads to an exponential suppression factor of the corresponding interference terms for the field; in the infrared limit of $V \to \infty$ one finds exact decoherence.

Thus the programme of decoherence [18] may successfully be applied in the context of quantum field theory using the concepts and methods of nonequilibrium statistical field theory.

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