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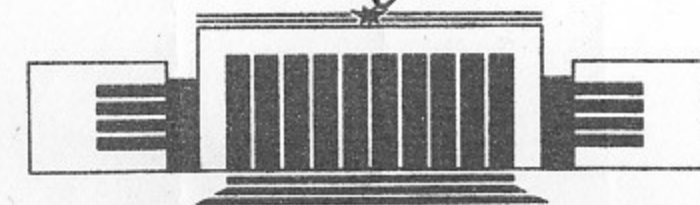


The State Scientific Center of Russia  
The Budker Institute of Nuclear Physics  
SB RAS

V.L. Chernyak

WHAT ARE THE RIGHT VALUES OF  $\bar{\Lambda}$   
AND THE HEAVY QUARK  
KINETIC ENERGY?

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# What are the right values of $\bar{\Lambda}$ and the heavy quark kinetic energy?

V.L. Chernyak

Budker Institute of Nuclear Physics SB RAS,  
and Novosibirsk State University  
630090 Novosibirsk, Russia

## Abstract

The values of two important parameters of the heavy quark effective theory,  $\bar{\Lambda}$  and  $\mu_\pi^2$  (the mean value of the heavy quark three momentum squared), have been determined recently in [2] from the precise CLEO data on the shape of the electron spectrum in semileptonic B meson decays. The values obtained in [2]:  $\bar{\Lambda} = (0.55 \pm 0.05) GeV$ ,  $\mu_\pi^2 = (0.35 \pm 0.05) GeV^2$ , disagree with the result obtained earlier in [1] from the D meson semileptonic width.

The purpose of this note is to show that the main reason for a disagreement is the secondary electron background present in the data, which influences strongly the extracted values of  $\bar{\Lambda}$  and  $\mu_\pi^2$ . We determine the amount of this background from the selfconsistency conditions, and subtract it out.

As a result, the values of  $\bar{\Lambda}$  and  $\mu_\pi^2$  become a factor two smaller and agree with those obtained from the D meson semileptonic decays.

E-mail: chernyak@inp.nsk.su

## 1.

The value of the charm quark (pole) mass,  $M_c$ , has been found in [1] from a calculation of the D meson semileptonic width. This is a good place for finding out a precise value of  $M_c$ , as the KM-factors  $V_{cs}$ ,  $V_{cd}$  are well known, the s-quark mass is reasonably well known and, in any case, plays a small role here, while the Born contribution behaves as  $\sim M_c^5$ . Moreover, all the main perturbative and power corrections to the Born term are also sensitive to a precise value of  $M_c$  and enter the answer with the same (negative) sign, which prevents accidental cancelations. So, the decay width depends on  $M_c$  as:  $\Gamma \sim M_c^{n_{eff}}$ , and  $n_{eff}$  is noticeably larger than 5.

The value:

$$M_c = [1650 \pm (35)_{theor} \pm (15)_{exp}] MeV \quad (1)$$

has been obtained in [1]. Being combined with the mass formula of the heavy quark effective theory ( $\bar{M}_D = (3M_{D^*} + M_D)/4$ ):

$$\bar{M}_D - M_c = \bar{\Lambda} + \frac{\mu_\pi^2}{2M_c}, \quad (2)$$

this gives a tight constraint on the combination of  $\bar{\Lambda}$  and  $\mu_\pi^2$  (the mean value of the c-quark three momentum squared) entering the right hand side of Eq.(2):

$$\bar{\Lambda} + \frac{\mu_\pi^2}{2\bar{M}_D} \left(1 + \frac{\bar{\Lambda}}{\bar{M}_D}\right) = [323 \pm (35)_{theor} \pm (15)_{exp}] MeV. \quad (3)$$



The first serious attempt has been undertaken recently in [2] to extract the values of  $\bar{\Lambda}$  and  $\mu_\pi^2$  from an independent source: using the precise CLEO data on the shape of the lepton spectrum in inclusive semileptonic  $B \rightarrow X l \bar{\nu}$  decays. Much larger values:

$$\bar{\Lambda} = 0.55 \pm 0.05 \text{ GeV}, \quad \mu_\pi^2 = 0.35 \pm 0.05 \text{ GeV}^2, \quad (4)$$

has been obtained in [2], in disagreement with Eq.(3).

The purpose of this note is to elucidate the reasons for a discrepancy and to present the results of a more careful treatment of CLEO data along the lines used in [2]. As a result, our values of  $\bar{\Lambda}$  and  $\mu_\pi^2$  extracted from the same data are a factor two smaller than in Eq.(4). The main reason for such a large difference originates from neglecting in [2] the secondary electron background present in the experimental data. Indeed, for the lepton energy interval used (see below) this background is small, about 1.5%. The matter is, however, that the parameters  $\bar{\Lambda}$  and  $\mu_\pi^2$  we are looking for enter the data as power corrections, and their effect is also a few per cent only. Therefore, the presence of the secondary electron background in the data used influences strongly the extracted values of  $\bar{\Lambda}$  and  $\mu_\pi^2$ . Besides, we account explicitly for the higher loops perturbative corrections (and this also decreases somewhat the value of  $\bar{\Lambda}$ ), and consider in more detail the role of third order corrections.

## 2.

Because the results presented in [2] are used heavily below, let us recall in short the line of approach and main definitions. The ratios are considered:

$$R_1 = \frac{\int_{1.5 \text{ GeV}} E_l \Gamma(E_l)}{\int_{1.5 \text{ GeV}} \Gamma(E_l)},$$

$$R_2 = \frac{\int_{1.7 \text{ GeV}} \Gamma(E_l)}{\int_{1.5 \text{ GeV}} \Gamma(E_l)}, \quad R_3 = \frac{\int_{1.8 \text{ GeV}} \Gamma(E_l)}{\int_{1.5 \text{ GeV}} \Gamma(E_l)}, \quad (5)$$

where  $\Gamma(E_l)$  is the differential distribution in the electron energy. The quantities like  $R_i$  are most suitable as the largest unknown factors  $M_b^5 |V_{cb}|^2$  cancel in ratios and, besides, the secondary electron background is small at  $E_l > 1.5 \text{ GeV}$ , while the role of power corrections we are looking for is enhanced.

The ratios like  $R_i$  are calculated then theoretically as series in powers of  $\Lambda_{QCD}/M_b$ , using the operator product expansions and the heavy quark effective theory. The second order corrections to the differential cross section have been found in [3][4], while the third order ones have been calculated recently in [5]. The results have the form (all numbers here and below are given in GeV units):

$$R_1^{theor} = 1.8061 - 10^{-2} [\delta_1 R_1 + \delta_2 R_1 + \delta_3 R_1],$$

$$\delta_1 R_1 = \left[ 5.82 \bar{\Lambda} - 8.22 \mu_\pi^2 + 4.67 \mu_G^2 + 1.25 \bar{\Lambda}^2 - 3.83 \bar{\Lambda} \mu_\pi^2 - \right. \\ \left. - 0.24 \bar{\Lambda} \mu_G^2 + 0.30 \bar{\Lambda}^3 \right],$$

$$\delta_2 R_1 = \frac{\alpha_s}{\pi} \left( 3.5 + \frac{7\bar{\Lambda}}{M_B} \right) \kappa_b^{(w)} - \left| 10 \frac{V_{ub}}{V_{cb}} \right|^2 \left( 1.33 - \frac{10.3\bar{\Lambda}}{M_B} \right) + \\ + \left( 0.41 - \frac{0.4\bar{\Lambda}}{M_B} \right) - \left( 0.62 + \frac{0.2\bar{\Lambda}}{M_B} \right),$$

$$\delta_3 R_1 = \left[ 5.11 \rho_1 + 1.11 \rho_2 + 2.15 \Delta_1 - 0.05 \Delta_2 + 2.04 \Delta_3 \right]. \quad (6)$$

$$R_2^{theor} = 0.6584 - 10^{-2} [\delta_1 R_2 + \delta_2 R_2 + \delta_3 R_2],$$

$$\delta_1 R_2 = \left[ 5.92 \bar{\Lambda} - 5.85 \mu_\pi^2 + 5.83 \mu_G^2 + 2.40 \bar{\Lambda}^2 - 4.73 \bar{\Lambda} \mu_\pi^2 + \right. \\ \left. + 1.69 \bar{\Lambda} \mu_G^2 + 1.0 \bar{\Lambda}^3 \right],$$



$$\delta_2 R_2 = \frac{\alpha_s}{\pi} \left( 3.9 + \frac{18\bar{\Lambda}}{M_B} \right) \kappa_b^{(w)} - \left| 10 \frac{V_{ub}}{V_{cb}} \right|^2 \left( 0.87 - \frac{3.8\bar{\Lambda}}{M_B} \right) + \left( 0.73 + \frac{0.5\bar{\Lambda}}{M_B} \right) - \left( 0.21 + \frac{0.3\bar{\Lambda}}{M_B} \right),$$

$$\delta_3 R_2 = \left[ 1.25 \rho_1 + 0.59 \rho_2 + 1.89 \Delta_1 + 0.39 \Delta_2 + 2.66 \Delta_3 \right]. \quad (7)$$

$$R_3^{theor} = 0.4878 - 10^{-2} \left[ \delta_1 R_3 + \delta_2 R_3 + \delta_3 R_3 \right],$$

$$\delta_1 R_3 = \left[ 8.85 \bar{\Lambda} - 8.78 \mu_\pi^2 + 8.10 \mu_G^2 + 3.57 \bar{\Lambda}^2 - 7.12 \bar{\Lambda} \mu_\pi^2 + 2.26 \bar{\Lambda} \mu_G^2 + 1.49 \bar{\Lambda}^3 \right],$$

$$\delta_2 R_3 = \frac{\alpha_s}{\pi} \left( 6.1 + \frac{26\bar{\Lambda}}{M_B} \right) \kappa_b^{(w)} - \left| 10 \frac{V_{ub}}{V_{cb}} \right|^2 \left( 1.28 - \frac{5.7\bar{\Lambda}}{M_B} \right) + \left( 0.88 + \frac{0.4\bar{\Lambda}}{M_B} \right) - \left( 0.39 + \frac{0.8\bar{\Lambda}}{M_B} \right),$$

$$\delta_3 R_3 = \left[ 1.63 \rho_1 + 1.0 \rho_2 + 2.73 \Delta_1 + 0.45 \Delta_2 + 3.64 \Delta_3 \right]. \quad (8)$$

The terms  $\delta_1 R_i$  and  $\delta_2 R_i$  in Eqs.(6-8) have been presented in [2],<sup>1</sup> and the terms entering  $\delta_3 R_i$  are easily calculated using the results for the second order terms and those from [5]. The terms  $\delta_1 R_i$  represent the first and second order corrections, and kinematical third order ones. The terms  $\delta_2 R_i$  represent strong and electromagnetic radiative corrections and Lorenz boost corrections. The terms  $\delta_3 R_i$  are "the dynamical" third order corrections. The nonperturbative parameters

<sup>1</sup> The Eq.(8) is not written down explicitly in [2] but has been kindly sent on request by Zoltan Ligeti to whom I am deeply grateful.

entering Eqs.(6-8) are defined as follows [6] (the nonrelativistic normalization of states is used,  $\bar{\Lambda}$  is defined by the matrix element of the light degrees of freedom part of the Hamiltonian in the infinite mass limit):

$$\langle B | \bar{b} \vec{\pi}^2 b | B \rangle = \left[ \mu_\pi^2 - \frac{(\Delta_1 - 0.5 \Delta_2)}{M_b} \right],$$

$$\langle B | \bar{b} (\vec{\sigma} g_s \vec{H}) b | B \rangle = \left[ -\mu_G^2 - \frac{(\Delta_3 - \Delta_4 - 0.5 \Delta_2)}{M_b} \right],$$

$$\langle B | \bar{b} \pi_\alpha \pi_\mu \pi_\beta b | B \rangle = \frac{\rho_1}{3} v_\mu (g_{\alpha\beta} - v_\alpha v_\beta),$$

$$\langle B | \bar{b} \pi_\alpha \pi_\mu \pi_\beta \gamma_\delta \gamma_5 b | B \rangle = \frac{i\rho_2}{6} v_\mu \epsilon_{\alpha\beta\nu\delta} v_\nu. \quad (9)$$

Here:  $\pi_\mu$  is the heavy quark momentum operator, the terms  $\Delta_i$  originate from the corrections to the B meson wave function and are naturally expressed through the corresponding two point correlators, while the terms  $\rho_{1,2}$  are the genuin local third order corrections. Being expressed in more visible terms they look as:

$$\rho_1 = -\frac{1}{2} \langle B | \bar{b} (g_s \vec{D} \vec{E}) b | B \rangle \simeq -2\pi\alpha_s \langle B | \bar{b} \frac{\lambda^a}{2} \gamma_\mu b \cdot \bar{q} \frac{\lambda^a}{2} \gamma_\mu q | B \rangle \simeq \frac{2}{9} \pi \alpha_s M_B f_B^2, \quad (10)$$

$$\rho_2 = \langle B | \bar{b} \vec{\sigma} (g_s \vec{E} \times \vec{\pi}) b | B \rangle. \quad (11)$$

In terms of the above parameters the meson masses look as ( $\bar{M}_B = (3 M_{B^*} + M_B)/4$ ):

$$\bar{M}_B = M_b + \bar{\Lambda} + \frac{\mu_\pi^2}{2 M_b} + \frac{(\rho_1 - \Delta_1 - \Delta_3)}{4 M_b^2}, \quad (12)$$



$$\frac{3}{2}(M_{B^*} - M_B)\overline{M}_B = \left[ \mu_G^2 - \frac{\Delta_0}{2M_b} \right], \quad (13)$$

$$\frac{3}{2}(M_{D^*} - M_D)\overline{M}_D \eta = \left[ \mu_G^2 - \frac{\Delta_0}{2M_c} \right] \quad (14)$$

$$\Delta_0 = (\Delta_2 + \Delta_4 + \rho_2 - 2\mu_G^2\overline{\Lambda}), \quad \eta = \left( \frac{\alpha_s(M_b)}{\alpha_s(M_c)} \right)^{\frac{9}{25}} \simeq 0.86. \quad (15)$$

As for other parameters entering Eqs.(6-8), we use:  $\alpha_s = \alpha_s(M_b) = 0.21$ ,  $|10 V_{ub}/V_{cb}| = 0.8$ , while the parameter  $\kappa_b^{(w)}$  describes the summary effect of the Borel resummed perturbation theory corrections. Its characteristic value for the B meson semileptonic decay is [7]:  $\kappa_b^{(w)} = 2.1$ . Besides, as the left hand sides of Eqs.(13,14) are known, we have:

$$\mu_G^2 \simeq 0.36, \quad \Delta_0 \simeq 0, \quad (\Delta_2 + \Delta_4 + \rho_2) \simeq 2\mu_G^2\overline{\Lambda}. \quad (16)$$

Finally, we use below the value:  $\rho_1 \simeq 0.012$ , which corresponds (see Eq.(10)) to  $f_B \simeq 0.12$  found in [1].

### 3.

The experimental values of the ratios  $R_i$  in Eqs.(6-8) are <sup>2</sup>:

$$\begin{aligned} R_1^{exp} = 1.7830 &= R_1^{theor}(1 - \sigma_1), & R_2^{exp} = 0.6108 &= R_2^{theor}(1 - \sigma_2), \\ R_3^{exp} = 0.4276 &= R_3^{theor}(1 - \sigma_3), \end{aligned} \quad (17)$$

where  $\sigma_i$  denote possible background contributions of secondary electrons. These can be found as follows. Equating the expressions of  $R_2^{theor}$  and  $R_3^{theor}$  from Eqs.(7,8) and Eq.(17), these can be rewritten

<sup>2</sup> The systematic errors are not considered and are expected to cancel to a large extent in the ratios Eq.(5), see [2], the statistical errors will be accounted for below.

as:

$$\begin{aligned} & \left[ \overline{\Lambda} - 0.85\mu_\pi^2 + 0.84\mu_G^2 + 0.35\overline{\Lambda}^2 - 0.69\mu_\pi^2\overline{\Lambda} + 0.24\mu_G^2\overline{\Lambda} + 0.15\overline{\Lambda}^3 \right. \\ & \quad \left. + 0.18\rho_1 + 0.09\rho_2 + 0.27\Delta_1 + 0.05\Delta_2 + 0.39\Delta_3 \right] = \\ & \quad = \left[ 0.616 - 8.86\sigma_2 \right], \end{aligned} \quad (18)$$

$$\begin{aligned} & \left[ \overline{\Lambda} - 0.87\mu_\pi^2 + 0.80\mu_G^2 + 0.35\overline{\Lambda}^2 - 0.70\mu_\pi^2\overline{\Lambda} + 0.22\mu_G^2\overline{\Lambda} + 0.15\overline{\Lambda}^3 \right. \\ & \quad \left. + 0.16\rho_1 + 0.10\rho_2 + 0.27\Delta_1 + 0.04\Delta_2 + 0.36\Delta_3 \right] = \\ & \quad = \left[ 0.541 - 4.21\sigma_3 \right], \end{aligned} \quad (19)$$

It is seen that the left hand sides are (nearly) equal. So, the right hand ones should be equal as well. Besides, it is clear that the secondary electron background is really small for  $E_l > 1.7 GeV$ , and originates mainly from the interval  $1.5 GeV < E_l < 1.7 GeV$ . So,  $\sigma_2$  and  $\sigma_3$  should be close to each other. Taking  $\sigma_2 \simeq \sigma_3$ , we obtain from Eqs.(18,19) as a first approximation:  $\sigma_2 \simeq \sigma_3 \simeq 1.6\%$ . We need also  $\sigma_1$ , which can be found now as follows.

Because we deal with the very end of the secondary electron spectrum, its form can be well approximated by a simplest stright line:

$$\frac{1}{\Gamma_0} \delta\Gamma(E_l) = C_0 (1.78 - E_l) \theta(1.78 - E_l),$$

$$\Gamma_0 = \int_{1.5 GeV} \Gamma^{theor}(E_l), \quad (20)$$

where  $\delta\Gamma(E_l)$  is the contribution to the differential cross section from secondary electrons. It is not difficult to obtain then:

$$\sigma_3 = 3.9\% \cdot C_0, \quad \sigma_2 = 3.4\% \cdot C_0, \quad \sigma_1 = 0.45\% \cdot C_0. \quad (21)$$

Choosing now the coefficient  $C_0 = 0.385$  from (see above)  $\sigma_3 = 1.5\%$ , we obtain:

$$\sigma_1 = 0.17\%, \quad \sigma_2 = 1.3\%, \quad \sigma_3 = 1.5\%. \quad (22)$$



As a check of the above values of the secondary electron background, we can estimate also the amount of this background for the  $E_l > 1.4 \text{ GeV}$  electrons and obtain  $\simeq 2.4 - 2.5\%$ , which compares well with the CLEO value  $(2.8 \pm 0.7)\%$  [8].

Let us emphasize, that the above found values of the secondary electron background are model independent as they are obtained solely from the selfconsistency requirements of the above written equations, i.e. requiring that  $R_2$  and  $R_3$  give the same result.<sup>3</sup> Let us repeat also that the background subtraction influences strongly the extracted values of  $\bar{\Lambda}$  and  $\mu_\pi^2$ . First, it is seen from Eqs.(18,19,22) that the background is not very small really by itself. Its role is strengthened additionally by the fact that the curves obtained from  $R_1$  and  $R_2$  (or from  $R_1$  and  $R_3$  which are the same now) intersect at a small angle, and so the position of the intersection point is sensitive to such corrections.

4.

Let us proceed now to some numerical results which follow from the above equations. As a zeroth approximation, we can neglect all third order corrections, both kinematical and dynamical ones, and obtain then from Eqs.(6,7,17,22) for the central values<sup>4</sup>:

$$\bar{\Lambda}^{(0)} = 0.310, \quad \mu_\pi^{2,(0)} = 0.175. \quad (23)$$

It is seen that the results for both  $\bar{\Lambda}$  and  $\mu_\pi^2$  are a factor two smaller in comparison with Eq.(4), and this is mainly due to a background subtraction.

Let us consider now in some detail a possible role of third order terms. As was noticed above, the terms  $\delta_1 R_{1,2}$  contain kinematical

<sup>3</sup>We see no reasons to question the validity of the quark-hadron duality for the  $E_l > 1.8 \text{ GeV}$  electrons and to trust it simultaneously for the  $E_l > 1.7 \text{ GeV}$  ones. Let us recall, that even with  $E_l > 1.8 \text{ GeV}$  we are summing over the hadron masses from  $M_D$  and up to  $\simeq 3.15 \text{ GeV}$ , covering thus a sufficiently large number of states.

<sup>4</sup> As  $R_2$  and  $R_3$  give identical results after the background is subtracted out, we deal with the  $(R_1, R_2)$  pair for a definiteness.

corrections:  $\mu_G^2 \bar{\Lambda}$ ,  $\mu_\pi^2 \bar{\Lambda}$  and  $\bar{\Lambda}^3$ , and nothing prevents us from accounting for these. Accounting also for  $\rho_1 \simeq 0.012$ , one obtains now:

$$\bar{\Lambda} = 0.280, \quad \mu_\pi^2 = 0.135, \quad (24)$$

( $\bar{\Lambda} = 0.265$ ,  $\mu_\pi^2 = 0.115$  with  $\rho_1 = 0$ ), which can be compared with the values:  $\bar{\Lambda} = 0.500$ ,  $\mu_\pi^2 = 0.270$ , obtained in [2] in a similar approximation.

The dynamical third order terms  $\Delta_i$  are unknown, of course. A hint on their possible values is given, however, by Eq.(16) which shows that, with the above used definitions, they are naturally positive and of a natural size:  $\sim \mu_G^2 \bar{\Lambda} \simeq 0.1$ , as one could expect beforehand.<sup>5</sup>

To illustrate their possible role we give below a number of examples (compare with Eq.(24)), taking various natural values for these parameters ( $\rho_1$  is always fixed at 0.012):

$$1) \Delta_1 = 0.1, \Delta_2 = \Delta_3 = \rho_2 = 0 : \bar{\Lambda} = 0.270, \mu_\pi^2 = 0.150,$$

$$2) \Delta_3 = 0.1, \Delta_2 = \Delta_3 = \rho_2 = 0 : \bar{\Lambda} = 0.240, \mu_\pi^2 = 0.125,$$

$$3) \Delta_2 = 0.2, \Delta_1 = \Delta_3 = \rho_2 = 0 : \bar{\Lambda} = 0.250, \mu_\pi^2 = 0.110,$$

$$4) \rho_2 = 0.2, \Delta_1 = \Delta_2 = \Delta_3 = 0 : \bar{\Lambda} = 0.295, \mu_\pi^2 = 0.170,$$

$$5) \Delta_2 = \rho_2 = 0.1, \Delta_1 = \Delta_3 = 0 : \bar{\Lambda} = 0.270, \mu_\pi^2 = 0.140. \quad (25)$$

It is seen that, in comparison with Eq.(24),  $\Delta_{1,2,3}, \rho_2 \neq 0$  give reasonably small corrections.

<sup>5</sup> Let us recall [1] that, analogously to the  $\langle (\bar{\sigma} \vec{\pi})^2 \rangle$  matrix element and unlike the quantum mechanics, there are no positiveness conditions for "the genuine nonperturbative terms"  $\Delta_i$ , in spite of that some bilocal correlators look positive definite. As usual, there are power divergent loop corrections in these correlators which should be subtracted out, and the terms  $\Delta_i$  represent "what is left". Clearly, "what is left" depends essentially on the subtraction scheme. We don't share the optimistic viewpoint that, i.e. with the upper cut off  $\mu \simeq 1 \text{ GeV}$ , "what is left" is much larger than the subtracted part. This latter, for instance, contributes typically  $\sim \alpha_s(\mu) \mu^3 / \pi \simeq 0.15$  to the correlator  $\langle \vec{\pi}^2, \vec{\pi}^2 \rangle$ , that is of the same size as  $\mu_G^2 \bar{\Lambda}$ , see Eq.(16).



Moreover, it is commonly believed that because  $\rho_2$  originates from the spin-orbital interaction, see Eq.(11), its real value is suppressed for the ground state B meson, so that it is more realistic that it is of the same order as  $\rho_1$ , rather than  $\sim 0.1 - 0.2$ .

Because  $\mu_G^2$  is considerably larger than  $\mu_\pi^2$ , one can expect also that the terms  $\Delta_3$  and  $\Delta_4$  are potentially the largest ones. But  $\Delta_3$  only decreases the answer (see Eq.(25)), while  $\Delta_4$  is already accounted for in Eqs.(6-8), as it is substituted by  $\Delta_4 = (2\mu_G^2\bar{\Lambda} - \Delta_2 - \rho_2)$  from Eq.(16), so that Eq.(24) corresponds really to the preferable case:  $\Delta_4 \simeq 0.2$ ,  $\Delta_1 \simeq \Delta_2 \simeq \Delta_3 \simeq \rho_2 \simeq 0$ .

In any case, varying third order terms within their natural limits we can see that their effect is small and is typically within the experimental statistical error bars (see below).

The effect due  $\kappa_b^{(w)} \neq 1$  in Eqs.(6-8) is also very mild. For instance, one obtains (with  $\Delta_{1,2,3} = \rho_2 = 0$ ,  $\rho_1 = 0.012$ , compare with Eq.(24)):  $\bar{\Lambda} = 0.330$ ,  $\mu_\pi^2 = 0.145$ , even with the unrealistic value:  $\kappa_b^{(w)} = 1$ . This shows that the value of  $\kappa_b^{(w)}$  influences mainly  $\bar{\Lambda}$ , while  $\mu_\pi^2$  stays nearly intact. This is as expected, as varying  $\kappa_b^{(w)}$  is equivalent, in essence, to the renormalon redefinition, i.e. changing the summation prescription for the divergent perturbation theory. And it is  $\bar{\Lambda}$  which is affected by the leading renormalon, while  $\mu_\pi^2$  is not. <sup>6</sup>

Finally, varying  $|V_{ub}/V_{cb}|$  also gives a small effect. One obtains:  $\bar{\Lambda} = 0.290$ ,  $\mu_\pi^2 = 0.115$  instead of Eq.(24) at  $|V_{ub}/V_{cb}| = 0.1$ .

## 5.

The statistical errors of CLEO data are given by the correlation matrix [2]:

$$V(R_1, R_2) = \begin{pmatrix} 1.64 \times 10^{-6} & 2.08 \times 10^{-6} \\ 2.08 \times 10^{-6} & 5.45 \times 10^{-6} \end{pmatrix} \quad (26)$$

<sup>6</sup> Really, one can expect the precise value of  $\kappa_b^{(w)}$  to be even a bit larger than the value 2.1 which we use and which corresponds to the total width. Indeed, as we deal with  $E_l > 1.5 GeV$ , the radiated gluons are softer and this will lead to a larger value of  $\kappa_b^{(w)}$ , in comparison with the total width.

Taking the case of Eq.(24) as a central point, one obtains the figure which is quite similar to Fig.1 in [2], but with the central point at  $\bar{\Lambda} = 280 MeV$ ,  $\mu_\pi^2 = 0.135 GeV^2$ . Eq.(3), which is obtained from the D meson semileptonic decays and is completely independent, gives the additional band. We don't even try here to write out "the right central values" of  $\bar{\Lambda}$  and  $\mu_\pi^2$  and, especially, "the right error bars" which follow from all the above described results. Rather, this is a problem for a specialist. As a typical example, we show only in Fig.1 the central lines of the case of Eq.(24), together with the central line of Eq.(3). (Two nearly coinciding lines are from  $R_2$  and  $R_3$ , those which intersects them at a small angle is from  $R_1$ , and those at a large one is from Eq.(3)).

Nevertheless, clearly, the final results of the form:

$$\bar{\Lambda} = (280 \pm 40) MeV, \quad \mu_\pi^2 = (0.14 \pm 0.03) GeV^2, \quad (27)$$

are definitely close to the "right" ones.

Most surprising is the small value of  $\mu_\pi^2$  which is more than three times smaller the widely used value  $\mu_\pi^2 \simeq 0.5 GeV^2$ , and is small even in comparison with the value  $0.25 GeV^2$  used in [1]. As a result, the difference of the quark (pole) masses looks now as:

$$M_b - M_c = (3370 \pm 10) MeV, \quad (28)$$

while the central values of  $M_c$  and  $M_b$  which follow from Eq.(27) are:

$$M_c = 1650 MeV, \quad M_b = 5020 MeV, \quad (29)$$

and agree with those obtained in [1]. As for  $|V_{cb}|$ , proceeding in the same way as in [1], one obtains the result:

$$|V_{cb}| \cdot 10^3 = (42.5 \pm 1) \left[ \frac{Br(B_d \rightarrow l\nu + X)}{11.0\%} \right]^{1/2} \left[ \frac{1.6 ps}{\tau(B_d)} \right]^{1/2} \quad (30)$$

which coincides practically with those obtained in [1], and only receives now more confidence.

Let us comment finally in short on a comparison of the above result, Eq. (27), with those obtained in [9] from a calculation of the hadron



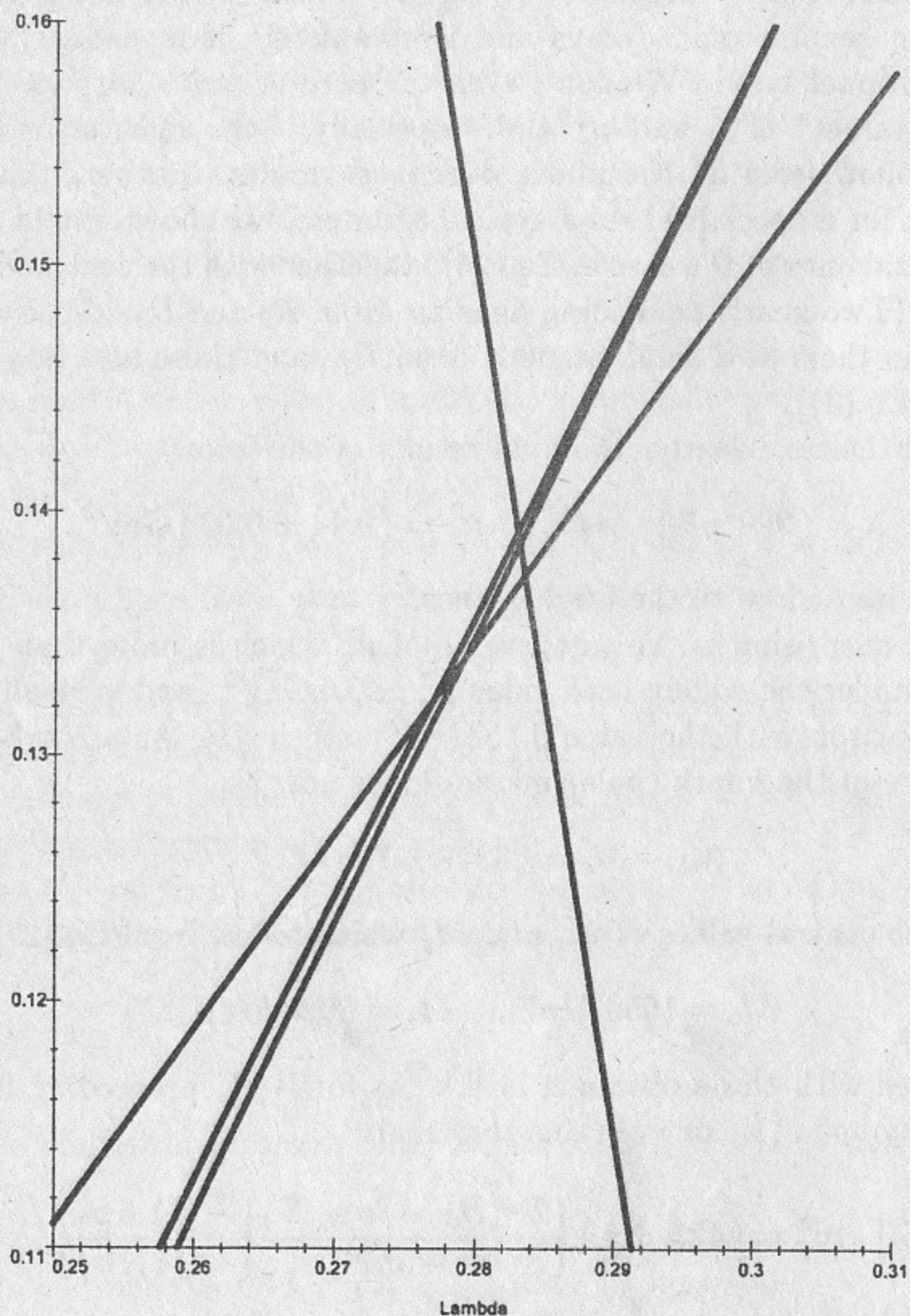


Figure 1:

invariant mass distributions in the B meson semileptonic decays. At present, the weak point of this approach is a poor accuracy of experimental data on a production of  $D^{**}$  states in B decays. The result:  $\bar{\Lambda} \simeq 450 \text{ MeV}$  [9] relies heavily on the OPAL data which gave highest production rates of the  $D^{**}$  states. At the same time, ALEPH, DELPHI and CLEO all indicate smaller production rates. It is not difficult to check that it is sufficient to diminish the OPAL central values on  $\sim 2\sigma$  to avoid disagreement with the above results, Eq.(27). Clearly, as the quality of the experimental data will improve, the results obtained within the approach used in [9] will become more reliable.

Some caution is needed, however, when comparing our results with those from [9]. These authors restrict themselves to two loop perturbative corrections. This corresponds to smaller value of  $\kappa_b^{(w)}$ , in comparison with those we use and which corresponds to a Borel resummed perturbation series. Being considered as a redefinition of the summation prescription for a divergent perturbative series, this will correspond to a redefinition of  $\bar{\Lambda}$ , so that their  $\bar{\Lambda}$  is a bit larger in comparison with our one.

**Note added:**

After this paper has been submitted as hep-ph/9604376, there appeared the revised version of ref. [2]. Not going into details, let us point out only that revised central values agree now with those obtained in this paper. (To see this one has to compare the above Eq.(27) with the values:  $\bar{\Lambda} = 350 \text{ MeV}$ ,  $\mu_\pi^2 = 0.15 \text{ GeV}^2$  obtained now in [2] (on account of third order corrections). The remained difference nearly disappears on account of higher loop perturbative corrections, i.e.  $\kappa_b^{(w)} \neq 1$ , and a difference in used values of  $\rho_1$ ).

Let us point out also that the statistical error bars have been increased considerably in the revised version of ref. [2] due to use of lower quality double tagged data. As we determine the background admixture from the selfconsistency conditions (see above), we use the smaller statistical error bars, Eq.(26), corresponding to higher quality single tagged data.



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## References

- [1] V.Chernyak, Nucl.Phys. **B457** (1995) 96.
- [2] M.Gremm, A.Kapustin, Z.Ligeti, M.Wise, CALT-68-2043, hep-ph/9603314.
- [3] I.I.Bigi, M.Shifman, N.G.Uraltsev, A.Vainshtein, Phys.Rev.Lett. **71** (1993) 496;  
B.Blok, L.Koyrakh, M.Shifman, A.I.Vainshtein, Phys.Rev.D**49** (1994) 3356.
- [4] A.V.Manohar, M.B.Wise, Phys.Rev.D**49** (1994) 1310;  
T.Mannel, Nucl.Phys. **B413** (1994) 396.
- [5] M.Gremm, A.Kapustin, CALT-68-2042, hep-ph/9603448.
- [6] T.Mannel, Phys.Rev. **D50** (1994) 428;  
I.I.Bigi, M.Shifman, N.G.Uraltsev, A.Vainshtein, Int.J.Mod.Phys. **A9** (1994)2467; Phys.Rev. **D52** (1995) 196.
- [7] M.Beneke, V.Braun, Phys.Lett. **B348** (1995) 513;  
P.Ball, M.Beneke, V.Braun, CERN-TH/95-65, hep-ph/9503492.
- [8] J.Bartels et.al., CLEO/CONF 93-19;  
B.Barish et.al., CLNS-95-1362, CLEO-95-17,  
Phys.Rev.Lett. **76** (1996) 1570.
- [9] A.F.Falk, M.Luke, M.Savage, UTPT-95-24, hep-ph/9511454.

*V.L. Chernyak*

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*В.Л. Черняк*

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