

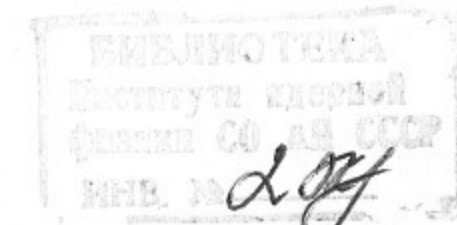
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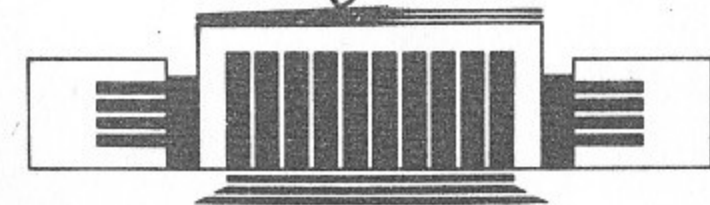
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TWO-LOOP ANOMALOUS DIMENSIONS  
OF HEAVY BARYON CURRENTS  
IN HEAVY QUARK  
EFFECTIVE THEORY



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Two-loop anomalous dimensions  
of heavy baryon currents  
in heavy quark effective theory

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Abstract

We present results on the two-loop anomalous dimensions of the heavy baryon HQET currents  $J = (q^T C \Gamma \tau q) \Gamma' Q$  with arbitrary Dirac matrices  $\Gamma$  and  $\Gamma'$ . From our general result we obtain the two-loop anomalous dimensions for currents with quantum numbers of the ground state heavy baryons  $\Lambda_Q$ ,  $\Sigma_Q$  and  $\Sigma_Q^*$ . As a by-product of our calculation and as an additional check we rederive the known two-loop anomalous dimensions of mesonic scalar, pseudoscalar, vector, axial vector and tensor currents ( $J = \bar{q} \Gamma q$ ) in massless QCD as well as in HQET.

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## 1 Introduction

The investigation of the renormalization-group (RG) properties of composite operators in QCD is an important ingredient in the numerous applications of QCD in particle physics. In the present paper we shall consider some aspects of the RG properties of heavy baryonic currents in the framework of the Heavy Quark Effective Theory (HQET) [1, 2], in particular we consider the anomalous dimensions of the currents associated with the ground-state heavy baryons. It is well known that the heavy mass dependence of operators and matrix elements in QCD can be very well organized by employing an  $1/m_Q$ -expansion provided by HQET. Here we work to lowest order in the heavy mass expansion, i.e. we treat the heavy quark fields as effective static fields. While this approximation should be a good starting point to understand the properties of heavy hadrons containing one heavy quark ( $b$  or  $c$ ) it leads to significant simplifications in the calculation of the Wilson coefficients that appear in the Operator Product Expansion (OPE) and in the calculation of RG coefficients such as the anomalous dimension coefficient dealt with in this paper.

The anomalous dimensions of composite operators are basic ingre-

dients in all applications of the OPE method in QCD such as QCD sum rules [3] and related approaches. In this paper we undertake to calculate the two-loop anomalous dimensions of heavy baryon currents in the static approximation. The anomalous dimensions of the baryonic currents are one of the ingredients needed in the analysis of the correlators of two baryonic currents via their short distance expansion. The ensuing QCD sum rules allow one to compute the mass of heavy baryons and their residues in terms of basic non-perturbative QCD parameters.

Another important application where the anomalous dimensions enter is the determination of the universal non-perturbative baryonic Isgur-Wise function  $\xi(\omega)$  [4] from three-point QCD sum rules. Of particular importance is the calculation of the slope of the Isgur-Wise function at zero recoil. Knowledge of the two-loop anomalous dimensions is important when one discusses the matching of HQET baryonic currents with the corresponding currents in full QCD. Furthermore, the anomalous dimensions are needed for the calculation of some basic QCD matrix elements that appear in QCD sum rules, as for example the expectation value of the kinetic energy  $\mu_\pi^2$ , the expectation value of the chromomagnetic energy  $\mu_G^2$ , the spin-dependent axial current matrix element  $\mu_s^2$ , the couplings of heavy baryons with pions and photons, and the magnetic and axial moments of heavy baryons.

The general structure of heavy baryon currents has the form (see e.g. [6] and refs. therein)

$$J = [q^{iT} C \Gamma \tau q^j] \Gamma' Q^k \epsilon_{ijk}. \quad (1)$$

Here the index  $T$  means transposition,  $C$  is the charge conjugation matrix with the properties  $C \gamma_\mu^T C^{-1} = -\gamma_\mu$  and  $C \gamma_5^T C^{-1} = \gamma_5$ ,  $i, j, k$  are colour indices and  $\tau$  is a matrix in flavour space. The effective static field of the heavy quark is denoted by  $Q$ .

The currents of the  $\Lambda_Q$  and the heavy quark spin doublet  $\{\Sigma_Q, \Sigma_Q^*\}$  are associated with the spin-parity quantum numbers  $j^P = 0^+$  and  $j^P = 1^+$  for the light diquark system, respectively. For each of the ground state baryon currents there are two independent current components  $J_1$  and  $J_2$ . They are given by [5, 6]

$$\begin{aligned} J_{\Lambda 1} &= [q^{iT} C \tau \gamma_5 q^j] Q^k \epsilon_{ijk}, & J_{\Lambda 2} &= [q^{iT} C \tau \gamma_5 \gamma_0 q^j] Q^k \epsilon_{ijk}, \\ J_{\Sigma 1} &= [q^{iT} C \tau \vec{\gamma} q^j] \cdot \vec{\gamma} \gamma_5 Q^k \epsilon_{ijk}, & J_{\Sigma 2} &= [q^{iT} C \tau \gamma_0 \vec{\gamma} q^j] \cdot \vec{\gamma} \gamma_5 Q^k \epsilon_{ijk}, \\ \vec{J}_{\Sigma^* 1} &= [q^{iT} C \tau \vec{\gamma} q^j] Q^k \epsilon_{ijk} + \frac{1}{3} \vec{\gamma} [q^{iT} C \tau \vec{\gamma} q^j] \cdot \vec{\gamma} Q^k \epsilon_{ijk}, & (2) \\ \vec{J}_{\Sigma^* 2} &= [q^{iT} C \tau \gamma_0 \vec{\gamma} q^j] Q^k \epsilon_{ijk} + \frac{1}{3} \vec{\gamma} [q^{iT} C \tau \gamma_0 \vec{\gamma} q^j] \cdot \vec{\gamma} Q^k \epsilon_{ijk}, \end{aligned}$$

where  $\vec{J}_{\Sigma^* 1}$  and  $\vec{J}_{\Sigma^* 2}$  satisfy the spin-3/2 condition  $\vec{\gamma} \vec{J}_{\Sigma^* i} = 0$  ( $i = 1, 2$ ). The flavour matrix  $\tau$  is antisymmetric for  $\Lambda_Q$  and symmetric for the heavy quark spin doublet  $\{\Sigma_Q, \Sigma_Q^*\}$ . The currents written down in Eq. (2) are rest frame currents. The corresponding expressions in a general frame moving with velocity four-vector  $v^\mu$  can be obtained by the substitutions  $\gamma_0 \rightarrow \not{v}$  and  $\vec{\gamma} \rightarrow \vec{\gamma}_\perp = \vec{\gamma} - \not{v} v^\mu$ .

The one-loop renormalization of these currents was considered in [6]. The renormalization of the corresponding light baryonic currents to one-loop order was presented in [7] and to two-loop order in [8]. In this paper we determine the two-loop anomalous dimensions of the heavy baryon currents. We evaluate the numerous two-loop diagrams with the help of REDUCE and MATHEMATICA packages written by us, which are based on the recurrence-relations in QCD and HQET presented in [9, 10, 11, 12].

The paper is organized as follows. In Sec. 2 we furnish some general remarks on anomalous dimensions, discuss the one-loop renormalization of the general heavy baryon currents (1) and give results on the one-loop anomalous dimensions in a general covariant gauge. In Sec. 3 we discuss general features of the two-loop renormalization procedure and present our general two-loop results, where we specify to the Feynman gauge. In Sec. 3 we also derive a powerful consistency check on our calculation, and we compare our results with previously published two-loop results in the mesonic sector. In Sec. 4 we consider applications of our general results to the ground-state heavy baryon currents (2). We work in the  $\overline{\text{MS}}$ -renormalization scheme throughout. As concerns the treatment of  $\gamma_5$  we give our results for two different

schemes, namely the 't Hooft-Veltman  $\gamma_5$ -scheme and the naive anti-commuting  $\gamma_5$ -scheme. Finally, we show how the results in the two  $\gamma_5$ -schemes become related through a finite renormalization prescription as has been demonstrated previously in a different context by Trueman and Larin [13, 14]. Sec. 5 contains our conclusions.

## 2 Renormalization procedure and one loop result

The renormalization procedure consists in the renormalization of all bare UV-divergent operators  $O_0$  in terms of a renormalization factor  $Z$  according to

$$O_0 = ZO. \quad (3)$$

Using dimensional regularization in  $D = 4 - 2\epsilon$  dimensions and the  $\overline{\text{MS}}$ -scheme [15], the factor  $Z$  is a polynomial in inverse powers of  $\epsilon$ . The  $Z$ -factor is constructed in such a way that the operator  $O$  is finite in physical four-dimensional space. After renormalization, the operator  $O$  can be seen to depend on the subtraction scale (normalization point)  $\mu$ . The anomalous dimension of the operator  $O$  is then defined by

$$\gamma = \frac{d \ln Z(\alpha(\mu), a; \epsilon)}{d \ln(\mu)}, \quad (4)$$

where  $a$  is the renormalized gauge parameter in the general covariant gauge (with a gluon propagator proportional to  $-g_{\mu\nu} + (1-a)k_\mu k_\nu/k^2$ ) and  $\alpha(\mu)$  is the renormalized coupling constant in four-dimensional space. The  $Z$ -factor and thereby the anomalous dimension  $\gamma$  is of course specific to the operator which it renormalizes. Thus they would carry an additional index which we suppress for the moment. The unrenormalized gauge parameter  $a_0$  and the unrenormalized coupling constant  $\alpha_0$  defined in  $D$  dimensions are related to the corresponding renormalized quantities  $a$  and  $\alpha(\mu)$  in four dimensions by

$$\alpha_0 = \alpha(\mu)\mu^{2\epsilon} Z_\alpha(\alpha(\mu), a; \epsilon), \quad a_0 = a Z_3(\alpha(\mu), a; \epsilon). \quad (5)$$

One-loop  $\overline{\text{MS}}$ -results for the factors  $Z_\alpha$  and  $Z_3$  have been given e.g. in [16]. They read

$$Z_\alpha = 1 - \frac{\alpha_s}{4\pi\epsilon} \left[ \frac{11}{3} C_A - \frac{4}{3} T_F N_F \right], \quad (6)$$

$$Z_3 = 1 + \frac{\alpha_s}{4\pi\epsilon} \left[ \frac{13 - 3a}{6} C_A - \frac{4}{3} T_F N_F \right]. \quad (7)$$

Note that there is a sign difference in the second  $O(\alpha_s)$  terms in Eqs. (6) and (7) relative to that in [16] due to a difference in the sign convention for  $\epsilon$ . The factors  $Z_\alpha$  and  $Z_3$  determine two universal RG functions  $\beta$  and  $\delta$  which denote the anomalous dimensions of the coupling and gauge parameters, i.e.

$$\beta(\alpha(\mu), a) = - \frac{d \ln Z_\alpha(\alpha(\mu), a; \epsilon)}{d \ln(\mu)}$$

$$\text{and } \delta(\alpha(\mu), a) = - \frac{d \ln Z_3(\alpha(\mu), a; \epsilon)}{d \ln(\mu)}. \quad (8)$$

In perturbation theory the renormalization factor or function  $Z(\alpha(\mu), a; \epsilon)$  will be a double series in the renormalized coupling constant  $\alpha_s = \alpha(\mu)$  and in inverse powers of  $\epsilon$ . We therefore write the double series expansion

$$Z = \sum_{m=1}^{\infty} \sum_{k=1}^m \left( \frac{\alpha_s}{4\pi} \right)^m \frac{1}{\epsilon^k} Z_{m,k} = \sum_{k=1}^{\infty} \frac{1}{\epsilon^k} Z_k. \quad (9)$$

In contradistinction to this the anomalous dimension  $\gamma$ , as well as the functions  $\beta$  and  $\delta$ , are only expanded in powers of the coupling constant  $\alpha_s$  since they are finite quantities for  $D \rightarrow 4$ . One has

$$\gamma = \sum_{m=1}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^m \gamma^{(m)}, \quad \beta = \sum_{m=1}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^m \beta^{(m)},$$

$$\delta = \sum_{m=1}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^m \delta^{(m)}. \quad (10)$$

Using these functions and differentiating  $Z(\alpha(\mu), a; \epsilon)$  with respect to  $\ln(\mu)$  one can compare coefficients at different orders of  $1/\epsilon^k$  [16]. For  $k = 0$  one obtains

$$\gamma = -2 \frac{\partial Z_1}{\partial \ln \alpha_s} \quad (11)$$

and for  $k > 0$

$$-2 \frac{\partial Z_{k+1}}{\partial \ln \alpha_s} = \left( \gamma - \beta \frac{\partial}{\partial \ln \alpha_s} - \delta \frac{\partial}{\partial \ln a} \right) Z_k. \quad (12)$$

The relations (11) and (12) allow one to determine the anomalous dimension on the one hand and, on the other hand, provide for a connection between the coefficients  $Z_{m,k}$  at different powers of  $1/\epsilon$  which allows one to check on the consistency of the two-loop calculations. For example, the first order contributions in  $1/\epsilon$  determine the one- and two-loop anomalous dimensions

$$\gamma^{(1)} = -2Z_{1,1} \quad \text{and} \quad \gamma^{(2)} = -4Z_{2,1}. \quad (13)$$

For the two-loop coefficients, Eq. (12) leads to the consistency condition

$$-4Z_{2,2} = \left( \gamma^{(1)} - \beta^{(1)} - \delta^{(1)} \frac{\partial}{\partial \ln a} \right) Z_{1,1}. \quad (14)$$

## 2.1 One-loop results

Let us first consider the one-loop renormalization of the effective heavy baryon currents (1). The bare light quark and bare effective heavy quark fields are related to the renormalized fields by

$$q_0 = Z_q^{1/2} q, \quad Q_0 = Z_Q^{1/2} Q. \quad (15)$$

In the  $\overline{\text{MS}}$ -scheme with  $D = 4 - 2\epsilon$  space-dimensions we have

$$Z_q = 1 - a_0 \frac{g_0^2 C_F}{(4\pi)^2 \epsilon}, \quad Z_Q = 1 + (3 - a_0) \frac{g_0^2 C_F}{(4\pi)^2 \epsilon}, \quad (16)$$

where  $g_0$  is the bare coupling in QCD and  $a_0$  is the bare gauge parameter. We use the usual definitions for  $SU(N)$ , i.e.  $C_F = (N_c^2 - 1)/2N_c$ ,

$C_A = N_c$ ,  $C_B = (N_c + 1)/2N_c$ , and  $T_F = 1/2$  for  $N_c = 3$ ,  $N_F$  being the number of light quarks. The bare current is renormalized by the factor  $Z_J$ , i.e.

$$J_0 = (q_0^T C \Gamma \tau q_0) \Gamma' Q_0 = Z_q Z_Q^{1/2} Z_V J = Z_J J. \quad (17)$$

As can be seen from Eq. (17), the anomalous dimension of the full current  $J$  is composed of three parts associated with the renormalization of the light and heavy quark fields, and the renormalization of the vertex. One thus has

$$\gamma_J = 2\gamma_q + \gamma_Q + \gamma_V. \quad (18)$$

The one-loop anomalous dimensions  $\gamma_q^{(1)}$  and  $\gamma_Q^{(1)}$  can be read off Eqs. (15) and (16) and are

$$\gamma_q^{(1)} = C_F a, \quad \gamma_Q^{(1)} = C_F (a - 3). \quad (19)$$

As concerns the corrections to the vertex  $(q_0^T C \Gamma \tau q_0) \Gamma' Q_0$ , one obtains

$$Z_V = 1 + \frac{\alpha_s C_B}{4\pi \epsilon} ((n-2)^2 + 3a - 1). \quad (20)$$

The one-loop vertex  $Z$ -factor has been calculated in the general covariant gauge  $a \neq 1$  since the consistency condition in Eq. (14) requires knowledge of the dependence of  $Z_{1,1}$  on the gauge parameter  $a$ . In deriving Eq. (20) we have parameterized the general light-side Dirac structure  $\Gamma$  in terms of antisymmetrized products of  $n$  Dirac matrices,  $\Gamma = \gamma^{[\mu_1 \dots \mu_n]}$ . For the time being we assume that there is no  $\gamma_5$  in the Dirac string  $\Gamma$  and postpone the discussion on how to define  $\gamma_5$  in  $D \neq 4$  dimensions to the end of the paper when we present explicit values for the anomalous dimensions. The above form for the Dirac string  $\Gamma$  introduces a  $n$ - and  $s$ -dependence in the vertex corrections due to the identities

$$\gamma_\alpha \Gamma \gamma_\alpha = h \Gamma = (-1)^n (D - 2n) \Gamma, \quad \gamma_0 \Gamma \gamma_0 = (-1)^n s \Gamma. \quad (21)$$

Using the results of Eq. (13) we immediately obtain the one-loop anomalous dimensions of the irreducible vertex

$$\gamma_V^{(1)} = -2C_B((n-2)^2 + 3a - 1). \quad (22)$$

According to Eqs. (18) and (19), the one-loop anomalous dimension of the baryonic current is given by

$$\gamma_J = \frac{\alpha_s}{4\pi} \left( -2C_B((n-2)^2 + 3a - 1) + 3C_F(a - 1) \right) + O(\alpha_s^2). \quad (23)$$

Note that the dependence on the gauge parameter  $a$  drops out of Eq. (23) when substituting  $C_F = 4/3$  and  $C_B = 2/3$ . This is a necessary requirement since the anomalous dimension is a gauge-independent quantity. When one specifies to the Feynman gauge ( $a = 1$ ), the results in Eqs. (20), (22) and (23) agree with the corresponding results presented in [6].

### 3 Two-loop renormalization

In this section we consider the two-loop renormalization of the heavy baryon current (1) in the  $\overline{\text{MS}}$ -scheme where we now work in the Feynman gauge to simplify the calculation. First of all, the two-loop anomalous dimensions of the quark fields are well known and can be taken e.g. from [11, 17, 18, 19, 20]. They are given by

$$\begin{aligned} \gamma_q^{(2)} &= C_F \left( \frac{17}{2}C_A - 2T_F N_F - \frac{3}{2}C_F \right), \\ \gamma_Q^{(2)} &= C_F \left( -\frac{38}{3}C_A + \frac{16}{3}T_F N_F \right). \end{aligned} \quad (24)$$

In order to determine the vertex renormalization factor  $Z_V$  that renormalizes the bare proper vertex according to  $\mathbb{V}_0 = Z_V \mathbb{V}$  we need to compute the bare proper vertex  $\mathbb{V}_0$  in one-loop and two-loop order. The Dirac structure of the unrenormalized and renormalized vertices  $\mathbb{V}_0$  and  $\mathbb{V}$  is given by the Born-term Dirac structure of the baryonic currents, i.e. by  $\Gamma \otimes \Gamma'$  in the notation of Eq. (1) (all heavy baryon

currents are renormalized multiplicatively in the effective theory!). We can thus write  $\mathbb{V}_0 = V_0 \Gamma \otimes \Gamma'$  and  $\mathbb{V} = V \Gamma \otimes \Gamma'$ , where  $V$  is finite. By requiring  $V = V_0/Z_V$  to be finite one can calculate  $Z_V$  in terms of  $V_0$ . For this purpose one first evaluates the one- and two-loop diagrams in terms of the bare coupling constant  $g_0$  and, in case of the one-loop diagrams, in terms of the bare gauge parameter  $a_0$ . One then substitutes for these in terms of the renormalized  $g_s$  ( $g_s^2 = 4\pi\alpha_s$ ) and  $a$  with the help of the Eqs. (5), (6) and (7). The resulting expression for the bare proper vertex  $V_0$  may again be presented as a double series in  $\alpha_s$  and  $\epsilon$ ,

$$V_0 = \sum_{m=0}^{\infty} \sum_{k=0}^m \left( \frac{\alpha_s}{4\pi} \right)^m \frac{1}{\epsilon^k} V_{m,k}. \quad (25)$$

Thus the coefficients  $V_{m,k}$  and  $Z_{m,k}$  become related. For example, for the first few coefficients one has the relations

$$Z_{1,1} = V_{1,1}, \quad Z_{2,2} = V_{2,2}, \quad Z_{2,1} = V_{2,1} - V_{1,1}V_{1,0}. \quad (26)$$

Note that in the relation for the  $Z$ -factor  $Z_{2,1}$  one also has to include one-loop contributions.

Three different contributions to the two-loop vertex corrections  $V_0$  may be identified. First one has the set of two-loop graphs where the two-loop contributions are associated with one of the heavy-light subsystems. Then there is the subset of two-loop graphs that are associated with the light-light system. Finally there are the irreducible contributions where the two loops connect all three quark lines. Hence we write

$$V_0 = 2V_0^{(hl)} + V_0^{(ll)} + V_0^{(ir)}, \quad (27)$$

where the factor of two on the r.h.s. accounts for the fact that there are two possible heavy-light configurations. The three types of contributions will be considered in turn in the following subsections.

#### 3.1 The heavy-light system

In this subsection we consider composite operators ( $qQ$ ) with one massless quark field  $q$  and one effective static heavy quark field  $Q$ . There

are 11 different two-loop diagrams contributing to the composite operator ( $qQ$ ) which are shown in Fig. 1. We choose the momentum of the light quark to be zero and regularize all IR-divergencies by taking the heavy quark off its mass-shell. The bare proper vertex  $V_0^{(hl)}$  of the heavy-light system is calculated to two-loop order in the Feynman gauge ( $a = 1$ ) using the algorithm developed in [11]. The result including the Born-term and one-loop contributions can be presented as

$$V_0^{(hl)} = 1 + \lambda_I C_0 b_0^{(hl)} + \sum_{i=1}^{11} \lambda_I^2 C_i b_i^{(hl)}, \quad (28)$$

where we use the notation  $\lambda_I := (g_0^2/(4\pi)^{D/2})(-2\omega)^{D-4}$ . The contributions of the various diagrams in Fig. 1 are separately identified according to  $i = 0$  for the one-loop contribution and  $i = 1, \dots, 11$  for the two-loop contributions. The associated colour factors are denoted by  $C_i$ ,

$$\begin{aligned} C_0 &= C_B, & C_1 &= C_B^2, & C_2 &= C_B(C_B - \frac{1}{2}C_A), \\ C_3 &= C_4 = C_B C_F, & C_5 &= C_6 = C_B(C_F - \frac{1}{2}C_A), \end{aligned} \quad (29)$$

$$C_7 = C_B N_F T_F, \quad C_8 = C_{11} = C_B C_A, \quad C_9 = C_{10} = \frac{1}{2} C_B C_A.$$

One should keep in mind that the colour factors have to be calculated for the completely antisymmetric baryonic colour configuration  $q^i q^j Q^k \varepsilon_{ijk}$ . Because of the difference in colour structure it is not a priori clear how the baryonic heavy-light two-loop results can be related to the mesonic case with colour structure  $\bar{q}_i Q^j \delta_j^i$ . However, it is not difficult to see that the corresponding colour-singlet colour factors for the mesonic case can be obtained from Eq. (29) by the substitution  $C_B \rightarrow C_F$ . Note, though, that this argument cannot be turned around. The contributions to the baryonic anomalous dimensions cannot be read off from the mesonic ones.

The coefficients  $b_i^{(hl)}$  are listed in Appendix A. The coefficients  $V_{n,k}^{(hl)}$  are then given by

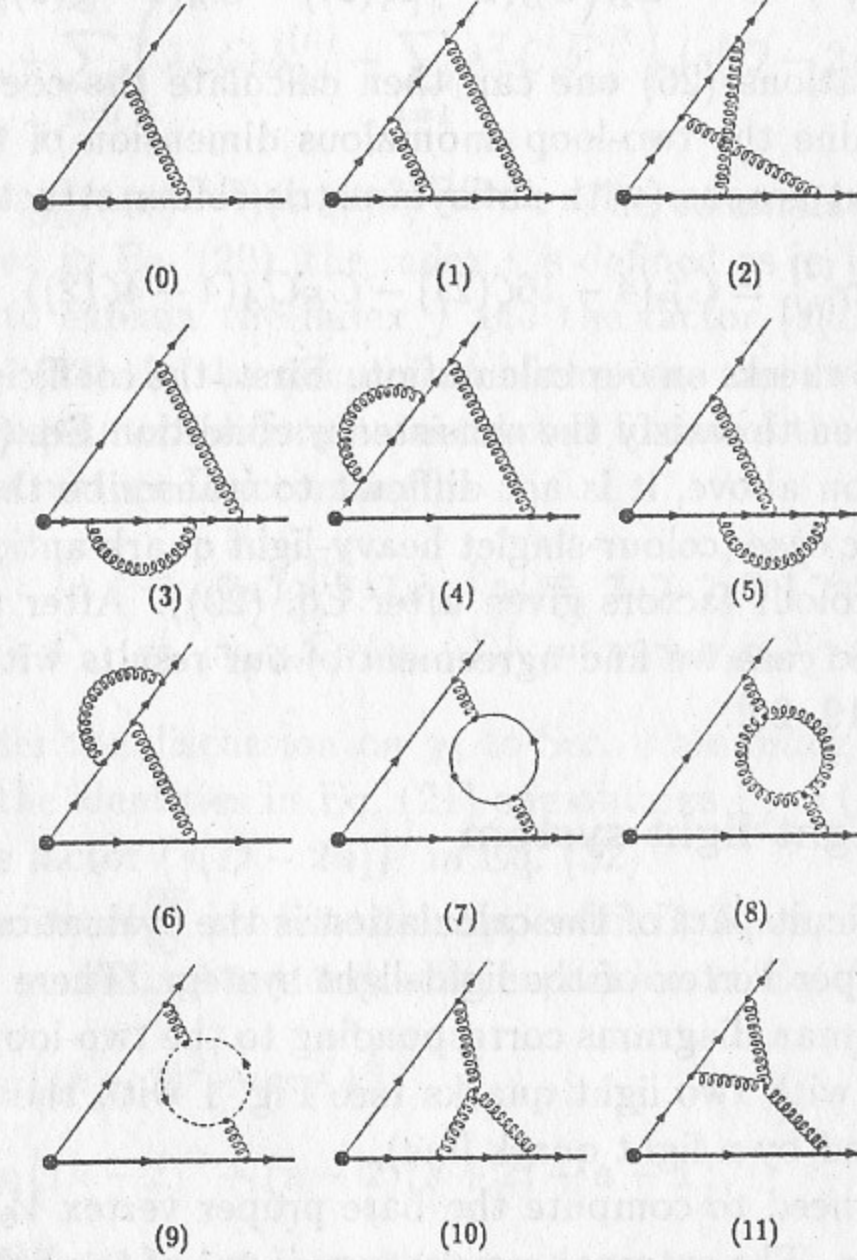


Figure 1

One-loop (diagram  $i = 0$ ) and two-loop (diagrams  $i = 1-11$ ) heavy-light vertex corrections. Light lines: light quarks; heavy lines: static heavy quark; curly lines: gluons; dashed lines: ghost contributions.

$$\begin{aligned}
V_{1,1}^{(hl)} &= C_B a, & V_{1,0}^{(hl)} &= 0, \\
V_{2,2}^{(hl)} &= C_B \left( \frac{1}{2} C_B - C_A \right), \\
V_{2,1}^{(hl)} &= -C_B \left( C_B (1 - 4\zeta(2)) - C_A (1 - \zeta(2)) \right).
\end{aligned} \tag{30}$$

Using the relations (26) one can then calculate the coefficients  $Z_{n,k}$  which determine the two-loop anomalous dimension of the subset of the heavy-light graphs (with antisymmetric colour structure),

$$\gamma_{(hl)}^{(2)} = C_B^2 (4 - 16\zeta(2)) - C_B C_A (4 - 4\zeta(2)). \tag{31}$$

There are two checks on our calculation. First, the coefficients  $Z_{2,2}$  and  $Z_{1,1}$  can be seen to satisfy the consistency condition Eq. (14). Second, as remarked on above, it is not difficult to transcribe the calculation to the mesonic case (colour-singlet heavy-light quark-antiquark system  $\bar{q}_i Q^j \delta_j^i$  with colour factors given after Eq. (29)). After transcription to the mesonic case we find agreement of our results with the results given in [11, 19, 20].

### 3.2 The light-light system

The most difficult part of the calculation is the evaluation of the two-loop bare proper vertex of the light-light system. There are again 11 different Feynman diagrams corresponding to the two-loop corrections of the vertex with two light quarks (see Fig. 1 with the heavy quark line substituted by a light quark line).

Again we need to compute the bare proper vertex  $V_0$  at one- and two-loop order. The external momentum of one of the light quark legs is set to zero. Using the power counting method, one can convince oneself that this procedure does not introduce any unwanted infrared singularities. All infrared singularities are regularized by choosing a nonzero momentum for the second light quark leg. On the other hand, setting one external quark momentum to zero, the ultraviolet properties of the diagrams are not affected. The advantage is that all diagrams now become two-point functions, which can be treated by using

the algorithm presented in [9]. In the first stage the one- and two-loop integrals are evaluated independently of the specific Dirac structure of the Dirac string  $\Gamma$ . In the second stage one specifies  $\Gamma$  and the general result can then be written down using the identities (21). The results of the calculation are again presented in coefficient form according to

$$V_0^{(ll)} = 1 + \sum_{j=0}^4 \left( \lambda_G C_0 b_{0,j}^{(ll)} + \sum_{i=1}^{11} \lambda_G^2 C_i b_{i,j}^{(ll)} \right) (s(D-2n))^j, \tag{32}$$

where  $\lambda_G := (g_0^2/(4\pi)^{D/2})(-1/p^2)^{D/2-2}$ . The colour factors  $C_i$  have been calculated in Eq. (29), the index  $i$  is defined as in Eq. (28).

We need to explain the index  $j$  and the factor  $(s(D-2n))^j$  appearing in Eq. (32). In the calculation of the one- and two-loop contributions one encounters different contracted forms of the general light Dirac string structure  $\Gamma$  according to

$$\begin{aligned}
\Gamma_0 &= \Gamma, & \Gamma_1 &= \gamma_\mu \gamma_0 \Gamma \gamma_0 \gamma_\mu, & \Gamma_3 &= \gamma_\mu \gamma_\nu \gamma_\rho \gamma_0 \Gamma \gamma_0 \gamma_\rho \gamma_\nu \gamma_\mu, \\
\Gamma_2 &= \gamma_\mu \gamma_\nu \Gamma \gamma_\nu \gamma_\mu, & \Gamma_4 &= \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \Gamma \gamma_\sigma \gamma_\rho \gamma_\nu \gamma_\mu.
\end{aligned} \tag{33}$$

Again, we defer the discussion on  $\gamma_5$  to Sec. 4 assuming  $\Gamma$  to contain no  $\gamma_5$ . From the identities in Eq. (21) one obtains  $\Gamma_j = (s(D-2n))^j \Gamma$  leading to the factor  $(s(D-2n))^j$  in Eq. (32).

The coefficients  $b_{i,j}^{(ll)}$  are listed in Appendix B. Then, organizing the contributions according to the double expansion in Eq. (25) one has

$$V_{1,1}^{(ll)} = C_B \left( (n-2)^2 + a - 1 \right), \tag{34}$$

$$V_{1,0}^{(ll)} = C_B \left( (n-2)^2 + (n-2)(s+2) + a - 1 \right), \tag{35}$$

$$\begin{aligned}
V_{2,1}^{(ll)} &= \frac{C_B}{36} \left( 9C_B(13n^4 - 96n^3 + 248n^2 - 256n + 88 + 4(n-2)^3 s) \right. \\
&\quad \left. - 72C_F(n-3)(n-1) - C_A(18n^2 - 144n^3 + 289n^2 \right. \\
&\quad \left. + 128n - 444) - 4N_F T_F(n^2 - 16n + 24) \right), \tag{36}
\end{aligned}$$

$$\begin{aligned}
V_{2,2}^{(ll)} &= \frac{C_B}{6} \left( 3C_B(n-2)^4 - C_A(11n^2 \right. \\
&\quad \left. - 44n + 39) + 4N_F T_F(n-3)(n-1) \right). \tag{37}
\end{aligned}$$



As in Sec. 3.1, the one-loop results have been calculated in the general covariant gauge whereas the two-loop results are given only for the Feynman gauge.

The result for the two-loop anomalous dimension of the proper light-light vertex is then given by

$$\begin{aligned} \gamma_{(ll)}^{(2)} = & \frac{C_B}{9} \left( -9C_B(5n^4 - 40n^3 + 104n^2 - 96n + 24 + 4(n-2)^3 s) \right. \\ & + 72C_F(n-3)(n-1) + C_A(18n^2 - 144n^3 + 289n^2 \\ & \left. + 128n - 444) + 4N_F T_F(n^2 - 16n + 24) \right). \end{aligned} \quad (38)$$

We have checked that the coefficients  $Z_{2,2}^{(ll)}$  and  $Z_{1,1}^{(ll)}$  satisfy the consistency condition Eq. (14) for any value of  $n$ .

A further check of our calculation is provided by the colour singlet projection of the proper vertex for the light-light mesonic configuration  $(\bar{q}^i q_j \delta_i^j)$  which can be obtained from our result with the substitutions  $q^T C \rightarrow \bar{q}$  and  $C_B \rightarrow C_F$ . Collecting the  $C_F$ -contributions one has

$$\begin{aligned} \gamma_{(ll)}^{(2)} = & \frac{C_F}{9} \left( -9C_F n(n-4)(5n^2 - 20n + 16) \right. \\ & + C_A(18n^4 - 144n^3 + 289n^2 + 128n - 444) \\ & \left. + 4N_F T_F(n^2 - 16n + 24) \right). \end{aligned} \quad (39)$$

After appending the anomalous dimensions  $2\gamma_q^{(2)}$  of the light quark fields one can then compare with previously published results in the mesonic sector. For  $n=0$  Eq. (39) coincides with the known anomalous dimension of the scalar current first written down by Tarrach [21]. For  $n=1$  one reproduces the well-known vanishing of the anomalous dimension of the vector current. For  $n=3$  one obtains agreement with the axial current case derived by Trueman [13], and, finally, for  $n=4$  with the two-loop anomalous dimensions of the pseudo-scalar current (with the 't Hooft-Veltman convention for  $\gamma_5$  to be discussed later on) which was calculated by Larin [14]. The anomalous dimension of the tensor current (with  $\Gamma = \sigma_{\mu\nu}$ ;  $n=2$ ) was recently obtained by Broadhurst and Grozin [22] in an on-shell QCD calculation, who

had also checked on the previously published results by considering a general Dirac structure for  $\Gamma$ . Again there is agreement.

### 3.3 The heavy-light-light irreducible vertex

Next one needs to calculate the three-quark irreducible proper vertex  $V_0^{(ir)}$ . In this case there are altogether 7 diagrams. The first four of them are shown in Fig. 2, the other three can be obtained from the first three by reflection on the heavy quark line which amounts to an exchange of the two light quark legs.

To start with, each quark leg carries a momentum  $p_i$  and thus every diagram depends on three external momenta. In diagrams (1), (2) and (4) of Fig. 2 we can safely set  $p_1 = p_2 = 0$  without introducing any infrared singularities, since they are naturally regularized by the off-shell energy  $\omega$  of the heavy quark. In diagram (3) we again set  $p_2 = 0$ . Power counting arguments imply that the contribution of diagram (3) contains a term proportional to  $\ln(p_1^2)$ . We thus have to keep  $p_1$  finite. The integrals in diagram (3) cannot entirely be reduced to two-point two-loop integrals, as is the case for diagrams (1), (2) and (4). In fact the general structure of the two-loop diagram (3) is given by

$$Ir_3 = T_{\mu\nu\rho} \int \frac{dk dl}{(2\pi)^{2D}} \frac{(l-p_1)^\mu l^\rho}{(l-p_1)^2 l^2 k^2 (\omega + kv)} \cdot \frac{(k-l)^\nu}{(k-l)^2}, \quad (40)$$

where  $T_{\mu\nu\rho}$  represents a string of Dirac matrices. The simplest way to evaluate this integral is to add and subtract an integral which has the same structure as the integral shown in Eq. (40) but with the simplified propagator  $k^\nu/k^2$  instead of  $(k-l)^\nu/(k-l)^2$ . The simplified integral can be easily calculated even for  $p_1 \neq 0$  because it factorizes into two one-loop type integrals. The advantage is that the difference of the two integrals with propagators  $(k-l)^\nu/(k-l)^2$  and  $k^\nu/k^2$  is IR-finite as  $p_1 \rightarrow 0$  and thus one can set  $p_1 = 0$  when evaluating the difference integral. The latter integral can then be solved by the standard methods. The results of our calculation are again presented

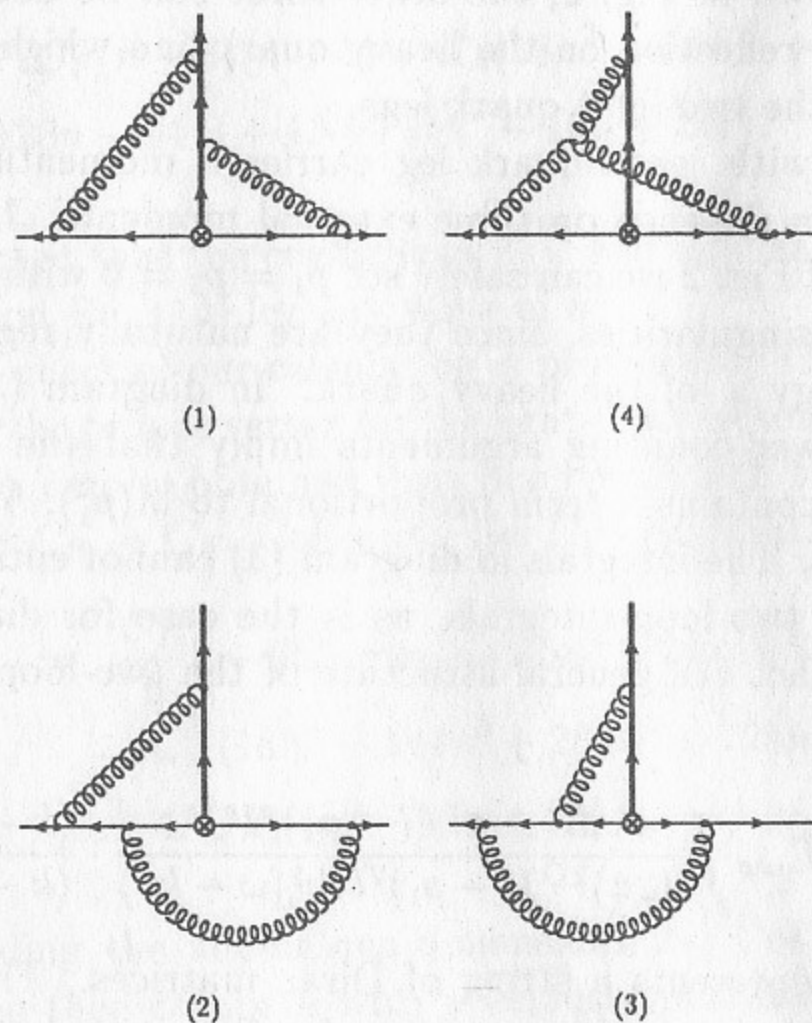


Figure 2

Two-loop heavy-light-light irreducible vertex correction. Light lines: light quarks; heavy lines: static heavy quark; curly lines: gluons. The remaining four diagrams are obtained by reflection on the heavy quark line.

in coefficient form,

$$V_0^{(ir)} = \sum_{j=0}^4 \left( \sum_{i=1}^7 \lambda_G^2 C_B^2 b_{i,j}^{(ir)} \right) (s(D-2n))^j. \quad (41)$$

For the definition of the index  $j$  we refer to Eqs. (32) and (33). The index  $i$  identifies the Feynman diagrams shown in Fig. 2 and the reflected diagrams. The coefficients  $b_{i,j}^{(ir)}$  are listed in the Appendix C. For the  $Z$ -factors and for the anomalous dimension we obtain

$$\begin{aligned} Z_{22}^{(ir)} &= C_B^2 (2n^2 - 8n + 9), \\ \gamma_{(ir)}^{(2)} &= -4Z_{21}^{(ir)} = -2C_B^2 (9n - 10 + 6s)(n - 2). \end{aligned} \quad (42)$$

It is interesting to note that the unrenormalized irreducible two-loop vertex contains a  $1/\epsilon^2$ -contribution even though there is no corresponding one-loop contribution to the irreducible three-quark vertex. The coefficient of the  $1/\epsilon^2$ -contribution is related to the  $1/\epsilon$ -pole in the total one-loop result by the non-linearity in the interplay between Eq. (13) and Eq. (14). We have added this comment because this interplay of one- and two-loop contributions is a feature specific to the baryonic case and does not occur in the mesonic case. The result in Eq. (42) completes our calculations.

#### 4 Anomalous dimensions of $\Lambda_Q$ and $\Sigma_Q$ baryons

We are now in the position to present our results for the anomalous dimension of the baryonic currents in Eqs. (2). We first sum up the results for the light-light, the heavy-light and the irreducible heavy-light-light vertex and obtain

$$\begin{aligned} \gamma_V^{(2)}(n, s) &= \frac{C_B}{9} \left( -9C_B(5n^4 - 40n^3 + 106n^2 \right. \\ &\quad \left. - 104n + 24 + 4(n-2)s + 32\zeta(2)) \right. \\ &\quad \left. + C_A(18n^4 - 144n^3 + 289n^2 + 128n - 516 + 72\zeta(2)) \right. \\ &\quad \left. + 72C_F(n-3)(n-1) + 4N_F T_F(n^2 - 16n + 24) \right). \end{aligned} \quad (43)$$

Table 1: Specific values of the pair  $(n, s)$  for particular cases of the light-side Dirac structure  $\Gamma$ .

$\Gamma$	$n$	$s$	particles
$\gamma_5^{\text{AC}}$	0	+1	$\Lambda_1$
$\gamma_5^{\text{AC}}\gamma_0$	1	-1	$\Lambda_2$
$\vec{\gamma}$	1	+1	$\Sigma_1, \Sigma_1^*$
$\gamma_0\vec{\gamma}$	2	-1	$\Sigma_2, \Sigma_2^*$
$\gamma_5^{\text{HV}}$	4	-1	$\Lambda_1$
$\gamma_5^{\text{HV}}\gamma_0$	3	+1	$\Lambda_2$

To obtain the full two-loop result for the anomalous dimension one has to add the anomalous dimensions of the heavy and light quark fields from Eq. (24) according to Eq. (18).

Up to this point we have steered clear of the  $\gamma_5$ -issue because we did not want to interrupt the general flow of arguments in the preceding sections. When one wants to apply the general result Eq. (43) to the  $\Lambda_Q$  case in Eq. (2) one faces the notorious problem of how to generalize  $\gamma_5$  to  $D \neq 4$  dimensions (the  $\gamma_5$  in the  $\Sigma_Q$ -type currents is harmless since it is attached to the heavy quark line). We shall not commit ourselves to any definite  $\gamma_5$ -scheme but give our results for the two generic  $\gamma_5$ -schemes that are being used in the literature, namely the naive  $\gamma_5$ -scheme with an anticommuting  $\gamma_5$  (for a general exposition of the naive  $\gamma_5$ -scheme see [23]) and the 't Hooft-Veltman-Breitenlohner-Maison (HV)  $\gamma_5$ -scheme [15]. In the naive  $\gamma_5$ -scheme one can simply anticommute the  $\gamma_5$  in the  $\Lambda_Q$ -currents to the end of the light fermionic string and then apply the general result Eq. (33). For the total anomalous dimension  $\gamma_j$  up to two loop order one then obtains (see Table 1 for the specific value of the pair  $(n, s)$ )

$$\gamma_{\Lambda 1} = -8 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9} (16\zeta(2) + 40N_F - 796) \left( \frac{\alpha_s}{4\pi} \right)^2, \quad (44)$$

$$\gamma_{\Lambda 2} = -4 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9} (16\zeta(2) + 20N_F - 322) \left( \frac{\alpha_s}{4\pi} \right)^2, \quad (45)$$

$$\gamma_{\Sigma 1} = -4 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9} (16\zeta(2) + 20N_F - 290) \left( \frac{\alpha_s}{4\pi} \right)^2, \quad (46)$$

$$\gamma_{\Sigma 2} = -\frac{8}{3} \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{27} (48\zeta(2) + 8N_F + 324) \left( \frac{\alpha_s}{4\pi} \right)^2. \quad (47)$$

The anomalous dimensions of the spin-3/2 currents  $\vec{J}_{\Sigma^*1}$  and  $\vec{J}_{\Sigma^*2}$  coincide with those of the spin-1/2 currents  $J_{\Sigma 1}$  and  $J_{\Sigma 2}$ , resp., because the respective light-side Dirac structures are identical.

In the HV  $\gamma_5$ -scheme the anomalous dimensions of  $J_{\Sigma 1}$  and  $J_{\Sigma 2}$  remain unchanged as remarked on before. Because one has

$$\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 = \frac{1}{4!} \varepsilon_{\mu_1\mu_2\mu_3\mu_4} \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4} \quad (48)$$

in the HV  $\gamma_5$ -scheme, this particular definition of  $\gamma_5$  is naturally included in the general representation of the  $\gamma$ -matrix string  $\Gamma$  used before in Eq. (21) and Eq. (43). In the HV  $\gamma_5$ -scheme one obtains

$$\gamma_{\Lambda 1} = -8 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9} (16\zeta(2) - 24N_F + 260) \left( \frac{\alpha_s}{4\pi} \right)^2, \quad (49)$$

$$\gamma_{\Lambda 2} = -4 \left( \frac{\alpha_s}{4\pi} \right) + \frac{1}{9} (16\zeta(2) - 12N_F + 206) \left( \frac{\alpha_s}{4\pi} \right)^2. \quad (50)$$

Comparing Eqs. (44) and (45) with Eqs. (49) and (50) one observes that the  $\zeta(2)$  contributions are not affected by the choice of  $\gamma_5$ , but there are big differences in the remaining contributions between the two  $\gamma_5$ -schemes. Needless to say that physical matrix elements do not depend on the choice of  $\gamma_5$ .

With the availability of the full two-loop result one can check on the "quick and easy"-prescription advanced in [22] to estimate the magnitude of two-loop corrections. This prescription goes by the name of "naive non-abelianization". The idea is to replace  $N_F$  by  $N_F - 33/2$  in the easily computed light-quark loop contribution. This prescription is based on the observation that the first term of the  $\beta$ -function is a large number and hence the dominant part of any corrections is connected with the term proportional to  $\beta^{(1)}$ . Thus one can hope to arrive at a good estimate of a non-abelian result by replacing the

abelian  $\beta^{(1)}$ -function by its non-abelian counterpart. We find that this prescription gives a good estimate of the full result with an accuracy of 15–30% as can easily be checked. An exception is the case  $J_{\Sigma 2}$  (and thereby  $\vec{J}_{\Sigma^* 2}$ ) where the “naive non-abelianization” estimate is completely off the mark. But note that in the latter case the anomalous dimension is an order of magnitude smaller than in the other cases.

As mentioned before, the anomalous dimensions of the baryonic currents  $J^{HV}$  and  $J^{AC}$  are different at two loop order. However, the currents can be related to each other by finite renormalization [13, 14],

$$J^{AC}(\mu) = Z_{\Gamma} J^{HV}(\mu). \quad (51)$$

The finite coefficients  $Z_{\Gamma}$  depend on the Dirac structure  $\Gamma$ , which occurs in the currents  $J = \bar{q}\Gamma q$  and  $J = (q^T C \Gamma q)Q$  in the mesonic case and baryonic case, respectively. The coefficients  $Z_{\Gamma}$  may be determined by comparing the renormalized matrix elements in Eq. (51) or by calculating the logarithmic derivative of Eq. (51),

$$\frac{d \ln Z_{\Gamma}}{d \ln \mu} = \gamma_{J^{HV}} - \gamma_{J^{AC}} = O_{\Gamma}. \quad (52)$$

Let us introduce a factor  $A_{\Gamma}$  through the relation  $Z_{\Gamma} = 1 + A_{\Gamma}(\alpha_s(\mu)/4\pi)$ , where the coefficient  $A_{\Gamma}$  can be calculated from

$$A_{\Gamma} \beta(\alpha_s) \left( \frac{\alpha_s(\mu)}{4\pi} \right) = -O_{\Gamma}, \quad (53)$$

where  $\beta(\alpha_s)$  was defined in Eq. (8). Using the results for the two-loop anomalous dimensions derived before one obtains

$$\begin{aligned} O_{\gamma_5} &= \frac{16}{3} C_B (11C_A - 4N_F T_F) \left( \frac{\alpha_s}{4\pi} \right)^2, \\ O_{\gamma_5 \gamma_i} = O_{\gamma_5 \gamma_0} &= \frac{8}{3} C_B (11C_A - 4N_F T_F) \left( \frac{\alpha_s}{4\pi} \right)^2, \\ O_{\gamma_5 \gamma_i \gamma_j} = O_{\gamma_5 \gamma_i \gamma_0} &= 0. \end{aligned} \quad (54)$$

For completeness we have also included the Dirac structures  $\Gamma = \gamma_5 \gamma_i$ ,  $\gamma_5 \gamma_i \gamma_j$  and  $\gamma_5 \gamma_i \gamma_0$  in our discussion even though they do not correspond

to the  $s$ -wave baryon states discussed in this paper. By calculating  $A_{\Gamma}$  according to Eq. (53) one finally obtains

$$\begin{aligned} Z_{\gamma_5} &= 1 - 8C_B \left( \frac{\alpha_s}{4\pi} \right), \\ Z_{\gamma_5 \gamma_i} = Z_{\gamma_5 \gamma_0} &= 1 - 4C_B \left( \frac{\alpha_s}{4\pi} \right), \\ Z_{\gamma_5 \gamma_i \gamma_j} = Z_{\gamma_5 \gamma_i \gamma_0} &= 1. \end{aligned} \quad (55)$$

Note that the finite renormalization coefficients for the baryonic currents differ from those in the mesonic case discussed in [13, 14] by the replacement  $C_B \rightarrow C_F$ .

## 5 Conclusion

We have calculated the anomalous dimensions of static heavy baryon currents at two-loop order. The anomalous dimensions are basic ingredients in the analysis of QCD sum rules in the heavy baryon sector. In order to improve on the accuracy of the analysis of the two-point and three-point sum rules in the literature one needs to avail of the next-to-leading order (NLO) radiative corrections to these sum rules. Part of the NLO corrections are determined by the two-loop anomalous dimensions of the baryon currents which have been calculated in this paper. We hope to return to the subject of the NLO baryonic sum rule corrections in the near future.

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## Appendix A

In this appendix we list the coefficient functions of the heavy-light bare proper vertex  $V_0^{(hl)}$  up to two-loop order. We use the abbreviations  $E_1 = \Gamma(1 - \epsilon)\Gamma(1 + 2\epsilon)$  and  $E_2 = \Gamma(1 - \epsilon)^2\Gamma(1 + 4\epsilon)$ , where  $\Gamma(x)$  is Euler's Gamma-function.

$$b_0^{(hl)} = \frac{-2aE_1}{D-4} \quad (\text{A1})$$

$$b_1^{(hl)} = \frac{4E_1^2}{(D-4)^3} - \frac{2(3D^2 - 24D + 44)E_2}{(D-6)(D-4)^3(D-3)} \quad (\text{A2})$$

$$b_2^{(hl)} = \frac{4E_1^2}{(D-4)^3(D-3)} - \frac{4(D-5)(D-2)E_2}{(D-6)(D-4)^3(D-3)} \quad (\text{A3})$$

$$b_3^{(hl)} = \frac{4E_2}{(D-4)^2(D-3)} \quad (\text{A4})$$

$$b_4^{(hl)} = \frac{2(D-2)E_2}{(D-6)(D-4)^2(D-3)} \quad (\text{A5})$$

$$b_5^{(hl)} = \frac{-8E_1^2}{(D-4)^2(D-3)} + \frac{4E_2}{(D-4)^2(D-3)} \quad (\text{A6})$$

$$b_6^{(hl)} = \frac{-2(D-2)E_2}{(D-6)(D-4)^2(D-3)} \quad (\text{A7})$$

$$b_7^{(hl)} = 0 \quad (\text{A8})$$

$$b_8^{(hl)} = \frac{E_2}{(D-6)(D-4)^2(D-3)} \quad (\text{A9})$$

$$b_9^{(hl)} = \frac{-E_2}{(D-6)(D-4)^2(D-3)} \quad (\text{A10})$$

$$b_{10}^{(hl)} = \frac{-4E_1^2}{(D-4)^2(D-3)} - \frac{2DE_2}{(D-6)(D-4)^2(D-3)} \quad (\text{A11})$$

$$b_{11}^{(hl)} = \frac{-4(D-1)E_2}{(D-6)(D-4)^2(D-3)} \quad (\text{A12})$$

## Appendix B

In this appendix we list the coefficient functions of the light-light bare proper vertex  $V_0^{(ll)}$  up to two-loop order using the abbreviations  $Q_1 = \Gamma(1 - \epsilon)^2\Gamma(1 + \epsilon)/\Gamma(1 - 2\epsilon)$  and  $Q_2 = \Gamma(1 - \epsilon)^3\Gamma(1 + 2\epsilon)/\Gamma(1 - 3\epsilon)$ .  $\Gamma(x)$  is Euler's Gamma-function.

$$b_{0,0}^{(ll)} = (1-a) \frac{(D-2)Q_1}{(D-4)(D-3)} \quad (\text{B1})$$

$$b_{0,1}^{(ll)} = \frac{-Q_1}{2(D-3)}$$

$$b_{0,2}^{(ll)} = \frac{-Q_1}{2(D-4)(D-3)}$$

$$b_{1,1}^{(ll)} = \frac{4(D-2)Q_1^2}{(D-4)^3(D-3)(D-1)} - \frac{4(D-2)(D^3 - 9D^2 + 20D - 4)Q_2}{(D-6)(D-4)^3(D-3)(D-1)(3D-10)}$$

$$b_{1,2}^{(ll)} = \frac{(D^3 - 12D^2 + 52D - 72)Q_1^2}{2(D-4)^3(D-3)^2(D-1)} - \frac{(D^4 - 13D^3 + 66D^2 - 168D + 176)Q_2}{(D-6)(D-4)^3(D-3)(D-1)(3D-10)}$$

$$b_{1,3}^{(ll)} = \frac{Q_1^2}{(D-4)(D-3)^2(D-1)} - \frac{2Q_2}{(D-4)(D-1)(3D-10)(3D-8)} \quad (\text{B2})$$

$$b_{1,4}^{(ll)} = \frac{Q_1^2}{2(D-4)^2(D-3)^2(D-1)}$$

$$\begin{aligned}
& -\frac{Q_2}{(D-4)^2(D-1)(3D-10)(3D-8)} \\
b_{2,0}^{(II)} &= \frac{2(D-2)(D^2-12D+24)Q_1^2}{(D-4)^3(D-3)^2(D-1)} \\
& -\frac{8(D-2)(D^3-8D^2+12D+8)Q_2}{(D-6)(D-4)^3(D-3)(D-1)(3D-10)} \\
b_{2,1}^{(II)} &= \frac{4(3D^2-18D+26)Q_1^2}{(D-4)^3(D-3)^2(D-1)} \\
& +\frac{4(5D^5-71D^4+350D^3-680D^2+296D+320)Q_2}{(D-6)(D-4)^3(D-3)(D-1)(3D-10)(3D-8)} \\
b_{2,2}^{(II)} &= \frac{(D^2-18D+40)Q_1^2}{(D-4)^3(D-3)^2(D-1)} \\
& +\frac{4(4D^4-43D^3+124D^2-40D-160)Q_2}{(D-6)(D-4)^3(D-3)(D-1)(3D-10)(3D-8)} \\
b_{2,3}^{(II)} &= \frac{2Q_1^2}{(D-4)^2(D-3)^2(D-1)} \\
& -\frac{4DQ_2}{(D-4)^2(D-1)(3D-10)(3D-8)} \quad (B3) \\
b_{2,4}^{(II)} &= \frac{Q_1^2}{(D-4)^3(D-3)^2(D-1)} \\
& -\frac{2DQ_2}{(D-4)^3(D-1)(3D-10)(3D-8)} \\
b_{3,1}^{(II)} &= \frac{-4(D-2)Q_2}{(D-4)(3D-10)(3D-8)} \\
b_{3,2}^{(II)} &= \frac{-2(D-2)Q_2}{(D-4)^2(3D-10)(3D-8)} \quad (B4) \\
b_{4,1}^{(II)} &= \frac{4(D-2)^2Q_2}{(D-6)(D-4)(D-3)(3D-10)(3D-8)} \\
b_{4,2}^{(II)} &= \frac{2(D-2)^2Q_2}{(D-6)(D-4)^2(D-3)(3D-10)(3D-8)} \quad (B5)
\end{aligned}$$

$$\begin{aligned}
b_{5,0}^{(II)} &= -\frac{(D-2)(D^3-16D^2+68D-88)Q_1^2}{(D-4)^3(D-3)^2(D-1)} \\
& -\frac{2(D-2)(D^2-8)Q_2}{(D-4)^3(D-1)(3D-10)} \\
b_{5,1}^{(II)} &= \frac{(D-6)(D-2)Q_1^2}{(D-4)(D-3)^2(D-1)} \\
& +\frac{2(D^2+6D-12)Q_2}{(D-4)(D-1)(3D-10)(3D-8)} \quad (B6) \\
b_{5,2}^{(II)} &= \frac{(D-6)(D-2)Q_1^2}{(D-4)^3(D-3)(D-1)} \\
& -\frac{2(D^3-17D^2+62D-56)Q_2}{(D-4)^3(D-1)(3D-10)(3D-8)} \\
b_{6,0}^{(II)} &= \frac{2(D-2)^2Q_2}{(D-6)(D-4)(D-3)(3D-10)} \\
b_{6,1}^{(II)} &= \frac{-4(D-2)^2Q_2}{(D-6)(D-4)(3D-10)(3D-8)} \quad (B7) \\
b_{6,2}^{(II)} &= \frac{-2(D-2)^2Q_2}{(D-6)(D-4)^2(3D-10)(3D-8)} \\
b_{7,0}^{(II)} &= \frac{-8(D-2)^2Q_2}{(D-6)(D-4)^2(D-3)(D-1)(3D-10)} \\
b_{7,1}^{(II)} &= \frac{16(D-2)^2Q_2}{(D-6)(D-4)(D-3)(D-1)(3D-10)(3D-8)} \quad (B8) \\
b_{7,2}^{(II)} &= \frac{8(D-2)^2Q_2}{(D-6)(D-4)^2(D-3)(D-1)(3D-10)(3D-8)} \\
b_{8,0}^{(II)} &= \frac{(D-2)(7D-6)Q_2}{(D-6)(D-4)^2(D-3)(D-1)(3D-10)} \\
b_{8,1}^{(II)} &= \frac{-2(D-2)(6D-5)Q_2}{(D-6)(D-4)(D-3)(D-1)(3D-10)(3D-8)} \quad (B9)
\end{aligned}$$

$$b_{8,2}^{(II)} = \frac{-(D-2)(6D-5)Q_2}{(D-6)(D-4)^2(D-3)(D-1)(3D-10)(3D-8)}$$

$$b_{9,0}^{(II)} = \frac{-(D-2)^2Q_2}{(D-6)(D-4)^2(D-3)(D-1)(3D-10)}$$

$$b_{9,1}^{(II)} = \frac{-2(D-2)Q_2}{(D-6)(D-4)(D-3)(D-1)(3D-10)(3D-8)} \quad (B10)$$

$$b_{9,2}^{(II)} = \frac{-(D-2)Q_2}{(D-6)(D-4)^2(D-3)(D-1)(3D-10)(3D-8)}$$

$$b_{10,0}^{(II)} = \frac{(D-2)^2(D^2-12D+30)Q_1^2}{(D-4)^3(D-3)^2(D-1)} - \frac{2(D-2)(D^4-17D^3+106D^2-268D+216)Q_2}{(D-6)(D-4)^3(D-3)(D-1)(3D-10)}$$

$$b_{10,1}^{(II)} = \frac{(5D^3-48D^2+138D-116)Q_1^2}{(D-4)^3(D-3)^2(D-1)} - \frac{2(11D^5-195D^4+1314D^3-4148D^2+6072D-3264)Q_2}{(D-6)(D-4)^3(D-3)(D-1)(3D-10)(3D-8)} \quad (B11)$$

$$b_{10,2}^{(II)} = \frac{2(2D^2-9D+8)Q_1^2}{(D-4)^3(D-3)^2(D-1)} - \frac{4(5D^3-34D^2+71D-44)Q_2}{(D-4)^3(D-3)(D-1)(3D-10)(3D-8)}$$

$$b_{11,1}^{(II)} = \frac{-8(D-2)(D-1)Q_2}{(D-6)(D-4)(D-3)(3D-10)(3D-8)}$$

$$b_{11,2}^{(II)} = \frac{-4(D-2)(D-1)Q_2}{(D-6)(D-4)^2(D-3)(3D-10)(3D-8)} \quad (B12)$$

## Appendix C

In this appendix we present the two-loop coefficient functions of the irreducible heavy-light-light proper vertex  $V_0^{(ir)}$ .

$$b_{1,0}^{(ir)} = b_{5,0}^{(ir)} = \frac{2E_2}{(D-4)^2} \quad b_{2,1}^{(ir)} = b_{6,1}^{(ir)} = \frac{-E_2}{(D-6)(D-4)(D-3)} \quad (C1)$$

$$b_{2,2}^{(ir)} = b_{6,2}^{(ir)} = \frac{-E_2}{(D-6)(D-4)^2(D-3)} \quad b_{4,j}^{(ir)} = 0 \quad (C2)$$

$$b_{3,1}^{(ir)} = b_{7,1}^{(ir)} = \frac{-(D-4)E_2}{2(D-8)(D-6)(D-3)} + \frac{E_1Q_1}{(D-4)(D-3)} \quad (C3)$$

$$b_{3,2}^{(ir)} = b_{7,2}^{(ir)} = \frac{(D^2-8D+8)E_2}{2(D-8)(D-6)(D-4)^2(D-3)} + \frac{E_1Q_1}{(D-4)^2(D-3)} \quad (C4)$$

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