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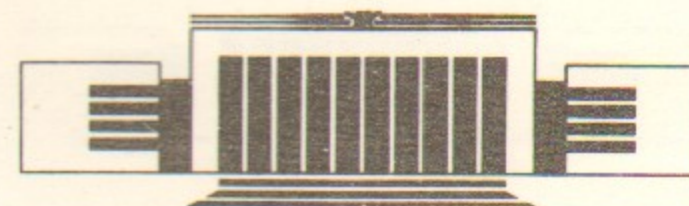
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NEW CONSTRAINTS FROM  
ELECTRIC DIPOLE MOMENTS  
ON PARAMETERS OF THE  
SUPERSYMMETRIC SO(10) MODEL



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НОВОСИБИРСК

# New Constraints From Electric Dipole Moments on Parameters of the Supersymmetric SO(10) Model

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## Abstract

We calculate the chromoelectric dipole moment (CEDM) of  $d$ - and  $s$ -quark in the supersymmetric SO(10) model. CEDM is more efficient than quark electric dipole moment (EDM), in inducing the neutron EDM. New, strict constraints on parameters of the supersymmetric SO(10) model follow in this way from the neutron dipole moment experiments. As strict bounds are derived from the upper limits on the dipole moment of  $^{199}\text{Hg}$ .

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1. The predictions of supersymmetric models for the neutron electric dipole moment can be comparable with the present experimental upper limit [1, 2]. Two different sources of CP-violation are possible here. First, the soft breaking potential contains the mass matrix  $m_{ij}^2$  of all scalars and the coupling matrix  $A_{ij}$  of trilinear terms. They are supposed to be defined by the structure of the hidden sector of some underlying supergravity and can contain imaginary CP-violating phases.

But the hidden sector can be flavour-blind and CP-invariant. It leads to the universality condition  $m_{ij}^2 = \delta_{ij}m_0^2$ ,  $A_{ij} = \delta_{ij}A_0$  at the Planck scale and to real soft breaking operators, in particular to  $\text{Im}A_0 = 0$ . In this situation the second source of CP-violation becomes essential. In the diagrams of the type 1a,b each quark-squark-gluino vertex contains the matrix of the rotation in the generation space of quarks with respect to squarks. These rotations are different for left- and right-handed particles, and the CP-violating part of the relative rotation induces the quark EDM. This, second case occurs only in those unified theories where all quarks in the given generation belong to the same representation of the unification group [3]. This is the case for the supersymmetric SO(10) (but not for MSSM and supersymmetric SU(5)).

The neutron EDM, as induced by the quark EDM, in the supersymmetric SO(10) was considered in Ref. [3]. The same model is discussed in the present note. But as distinct from Ref. [3], we concentrate on another CP-odd characteristic of a quark, its CEDM, and effects induced by it.

2. The quark CEDM is defined as the factor  $d^c$  in the effective operator

$$L_{eff} = \frac{1}{2} d^c \bar{q} \gamma_5 \sigma_{\mu\nu} \frac{\lambda^a}{2} q G_{\mu\nu}^a. \quad (1)$$

It is generated by diagrams 1a,b (obviously, only diagrams of the type 1a contribute to the quark EDM).

As usual, we choose the Yukawa coupling matrix  $\lambda^U$  of  $U$ -quarks ( $U = u, c, t$ ) real and diagonal. Then one can take into account in the renormalization group equations, written in Ref. [4], the top Yukawa coupling only. In this situation the  $U$ -quarks do not rotate in the generation space with respect to their superpartners. Therefore, in the case considered, of CP-invariant soft breaking operators, both EDMs and CEDMs of  $U$ -quarks are negligible.

The calculations of the  $d$ -quark EDM  $d$  and CEDM  $d^c$  are quite similar. If all squarks were degenerate as at the Planck scale, their contributions would cancel. However, due to large top Yukawa coupling, the third generation becomes considerably lighter already at the GUT scale [3, 5]. So, we take into account only contribution of the  $b$ -squark as the largest one. The result of both EDM and CEDM calculations can be conveniently presented as

$$d = e \frac{\alpha_s}{54\pi} \frac{v_d}{m_B^3} f\left(\frac{\tilde{m}}{m_B}\right) \text{Im} [(V_L)_{31} (V_R^*)_{31} (A^D \lambda^D + \mu \lambda^D \tan \beta)_{33}]; \quad (2)$$

$$d^c = g \frac{5\alpha_s}{72\pi} \frac{v_d}{m_B^3} f^c\left(\frac{\tilde{m}}{m_B}\right) \text{Im} [(V_L)_{31} (V_R^*)_{31} (A^D \lambda^D + \mu \lambda^D \tan \beta)_{33}]. \quad (3)$$

Here  $\mu$  is the constant of the  $\mu H_1 H_2$  superpotential,  $\tan \beta = v_u/v_d$  is the ratio of vacuum expectation values of  $H_2$  and  $H_1$ ;  $A^D$  is the  $3 \times 3$ -matrix in the trilinear soft breaking potential. We neglect the splitting between the masses  $m_B$  of left- and right-handed  $b$ -squarks. The dependence of the dipole moments on the gluino mass  $\tilde{m}$  is determined by the functions  $f$  and  $f^c$ :

$$f(x) = 6x \frac{1+5x^2}{(1-x^2)^3} + 24x^3 \frac{2+x^2}{(1-x^2)^4} \ln x; \quad (4)$$

$$f^c(x) = -12x \frac{11+x^2}{5(1-x^2)^3} - 12x \frac{9+16x^2-x^4}{5(1-x^2)^4} \ln x. \quad (5)$$

Here  $x = \tilde{m}/m_B$ , and both functions,  $f(x)$  and  $f^c(x)$  are normalized in such a way that  $f(1) = f^c(1) = 1$ .

$V_L$  and  $V_R$  are the matrices of the unitary rotations of left- and right-handed quarks with respect to their superpartners. They are related to the Yukawa coupling matrix  $\lambda^D$  as follows:

$$\lambda^D = \frac{1}{v_d} V_L^* \bar{M}^D V_R^T, \quad (6)$$

where  $\bar{M}^D$  is the real diagonal mass matrix of  $D$ -quarks ( $d, s, b$ ).

$V_L$  is nothing else but the Kobayashi-Maskawa matrix  $V$ . Meanwhile, in the MSSM and supersymmetric  $SU(5)$  model  $V_R$  is the unit matrix and the dipole moments vanish [3]. However, in the supersymmetric  $SO(10)$  model

$$V_R^* = VP^2,$$

where  $P$  is a diagonal phase matrix with two physical phases [6]. In this model the  $d$ -quark dipole moments, generally speaking, do not vanish. They can be written as

$$d = e \frac{\alpha_s}{54\pi} |V_{31}|^2 A'_b \sin \phi \frac{m_b}{m_B^2} f\left(\frac{\tilde{m}}{m_B}\right); \quad (7)$$

$$d^c = g_s \frac{5\alpha_s}{72\pi} |V_{31}|^2 A'_b \sin \phi \frac{m_b}{m_B^2} f^c\left(\frac{\tilde{m}}{m_B}\right). \quad (8)$$

In these expressions  $m_b$  is the  $b$ -quark mass,

$$A'_b = \frac{A_{33}^D + \mu \tan \beta}{m_B}.$$

The phase  $\phi$  is the sum over all phases present in (2) and (3):

$$\phi = 2(\phi_{31} - \phi_{33} + \tilde{\phi}_1 - \tilde{\phi}_3),$$

where  $\phi_{ij} = \arg V_{ij}$ ,  $P_{ii} = e^{i\tilde{\phi}_i}$ . At  $\tilde{m} = m_B$  formula (7) for the  $d$ -quark EDM coincides with the result obtained in Ref. [3].

The constraints put on the parameters of the supersymmetric  $SO(10)$  model by the neutron dipole moment, as induced by the  $d$ -quark EDM (7), were considered in Ref. [3]. We will discuss here the constraints following from our CEDM result (8).

But before of that we wish to mention the following circumstance. Let us consider, instead of the vertex part (Fig. 1), the corresponding mass operator (Fig. 2). This contribution to the  $CP$ -odd  $\gamma_5$ -mass of a quark, or to the induced  $\theta$ -term, is enormous. It exceeds by 6–7 orders of magnitude the upper limit on  $\theta$  [7, 8] following from the neutron EDM experiment. This situation is quite common to models of  $CP$ -violation. As common is the argument, according to which there should be some mechanism, for instance the Peccei-Quinn one, which makes the  $\theta$ -term harmless, which allows to transform it away. We will also adhere to this conservative point of view.

3. Coming back to the quark CEDM, to investigate its contribution to the observable effects, we have to bring the expression (8) down from the

scale of  $M \sim 300$  GeV. In particular, to substitute for  $m_b$  its "physical" value 4.5 GeV, we have to introduce the renormalization group (RG) factor

$$\left[ \frac{\alpha_s(M)}{\alpha_s(m_b)} \right]^{12/23}$$

Now, the QCD sum rule technique, used below to estimate the CEDM contribution to observable effects, refers to the hadronic scale of  $m \sim 1$  GeV and is applied directly to the operators of the type

$$g_s \bar{q} \gamma_5 \sigma_{\mu\nu} \frac{\lambda^a}{2} q G_{\mu\nu}^a,$$

which include  $g_s$  explicitly. This brings one more RG factor [9]

$$\left[ \frac{\alpha_s(M)}{\alpha_s(m)} \right]^{2/23}$$

On the other hand, as distinct from some other investigations, we see no special reasons to bring the explicit  $\alpha_s$  factor, entering the expression (8), down from the high-momenta scale  $M$ , where it is defined at least as well as at  $m \sim 1$  GeV.

The overall RG factor, introduced in this way into formula (8), is

$$\left[ \frac{\alpha_s(M)}{\alpha_s(m_b)} \right]^{12/23} \left[ \frac{\alpha_s(M)}{\alpha_s(m)} \right]^{2/23} = 0.57. \quad (9)$$

The values of the coupling constants, accepted here, are:

$$\alpha_s(M) = 0.11; \quad \alpha_s(m_b) = 0.26; \quad \alpha_s(m) = 0.43.$$

If, following [3], we assume for the estimates  $\tilde{m} = m_B$ , then at the same, as in Ref. [3], representative values of other parameters, the  $d$ -quark CEDM can be evaluated as follows:

$$d^c = 26 \cdot 10^{-26} \text{ cm} \left( \frac{|V_{td}|^2}{10^{-4}} \right) \left( \frac{A'}{1} \right) \left( \frac{\sin \phi}{0.5} \right) \left( \frac{250 \text{ GeV}}{m_B} \right)^2. \quad (10)$$

A serious problem is to find the CEDM contribution to the neutron dipole moment. The simplest way [10] to estimate this contribution is to assume, just by dimensional reasons, that  $d(n)/e$  is roughly equal to  $d^c(q)$  (obviously, the electric charge  $e$  should be singled out of  $d(n)$ , being a parameter unrelated to the nucleon structure).

In a more elaborate approach [10], the CEDM contribution to the neutron EDM is estimated in the chiral limit via diagram 3 (see Ref. [8]). The contribution of operator (1) to the  $CP$ -odd  $\pi NN$  constant  $\bar{g}_{\pi NN}$  is transformed by the PCAC technique:

$$\langle \pi^- p | g_s \bar{q} \gamma_5 \sigma_{\mu\nu} \frac{\lambda^a}{2} q G_{\mu\nu}^a | n \rangle = \frac{i}{f_\pi} \langle p | g_s \bar{u} \sigma_{\mu\nu} \frac{\lambda^a}{2} d G_{\mu\nu}^a | n \rangle. \quad (11)$$

QCD sum rule estimate gives for the last matrix element value close to  $-1.5 \text{ GeV}^2$ . Let us introduce now the ratio of the neutron dipole moment, as induced by a CEDM, to  $d^c$  itself:

$$\rho = \frac{d(n)/e}{d^c}. \quad (12)$$

Its value obtained in this, more elaborate approach,

$$\rho = 0.7,$$

is quite close indeed to unity. In our opinion, this good agreement with the above simple-minded result enhances the reliability of both estimates.

In this way at  $\rho = 0.7$  we obtain the following prediction for the neutron EDM:

$$d(n)/e = 18 \cdot 10^{-26} \text{ cm} \left( \frac{|V_{td}|^2}{10^{-4}} \right) \left( \frac{A'}{1} \right) \left( \frac{\sin \phi}{0.5} \right) \left( \frac{250 \text{ GeV}}{m_B} \right)^2. \quad (13)$$

It should be compared with the the experimental upper limit [1, 2]

$$d(n)/e < 7 \cdot 10^{-26} \text{ cm}. \quad (14)$$

The prediction (13) for the neutron dipole moment, as induced by the quark CEDM, is 4 times larger than the contribution to  $d(n)$  from the quark EDM [3]. Correspondingly, it constrains stronger the parameters of the supersymmetric  $SO(10)$  model.

4. Essentially larger contribution to the neutron EDM is induced by the CEDM  $d^c(s)$  of the  $s$ -quark. The expression for  $d^c(s)$  differs from (10) in two respects. First, the concrete expression for the phase  $\phi$  changes. But what is more essential, the mixing between the second and third generations is essentially larger than the mixing between the first and third ones:

$$|V_{ts}|^2 \simeq 17 \cdot 10^{-4}.$$

(Let us mention that in other models the advantage of the  $s$ -quark contribution is the large mass ratio  $m_s/m_d$  [11].)

On the other hand, for the  $s$ -quark, the ratio

$$\rho_s = \frac{d(n)/e}{d^c(s)}. \quad (15)$$

should be much smaller than unity. Indeed, according to the QCD sum rule calculations of [12], it is about 0.1. One should mention that other estimates [13, 14] predict for the ratio (15) a value an order of magnitude smaller.

Then, how reliable is the estimate  $\rho_s = 0.1$ ? There are strong indications now that the admixture of the  $\bar{s}s$  pairs in nucleons is quite considerable. In particular, it refers to the spin content of a nucleon. And though these indications refer to operators different from  $\bar{s}\gamma_5\sigma_{\mu\nu}(\lambda^a/2)sG_{\mu\nu}^a$ , they give serious reasons to believe that the estimate

$$\rho_s = 0.1 \quad (16)$$

is just a conservative one.

The central point of the contribution of the  $s$ -quark CEDM to the neutron dipole moment, resulting at  $\rho_s = 0.1$ ,

$$d(n)/e = 43 \cdot 10^{-26} \text{ cm} \left(\frac{A'}{1}\right) \left(\frac{\sin\phi_s}{0.5}\right) \left(\frac{250\text{GeV}}{m_B}\right)^2. \quad (17)$$

is 6 times larger than the experimental upper limit (14).

5. Let us compare at last the predictions of the supersymmetric  $SO(10)$  model with the result of the atomic experiment [15]. The measurements of atomic EDM of the mercury isotope  $^{199}\text{Hg}$  have resulted in

$$d(^{199}\text{Hg})/e < 9 \cdot 10^{-28} \text{ cm}. \quad (18)$$

According to calculations of Ref. [16], it corresponds to the upper limit on the  $d$ -quark CEDM

$$d^c < 2.4 \cdot 10^{-26} \text{ cm} \quad (19)$$

The central point of the prediction (10) exceeds this upper limit by an order of magnitude.

The analysis carried out in the present paper demonstrates that very special assumptions concerning the parameters of the supersymmetric  $SO(10)$  model (such as large mass  $m_B$  of the  $b$ -squark, small  $CP$ -violating angle  $\phi$ ,

etc) are necessary to reconcile the predictions of this model with the experimental upper limits on the electric dipole moments of neutron and  $^{199}\text{Hg}$ .

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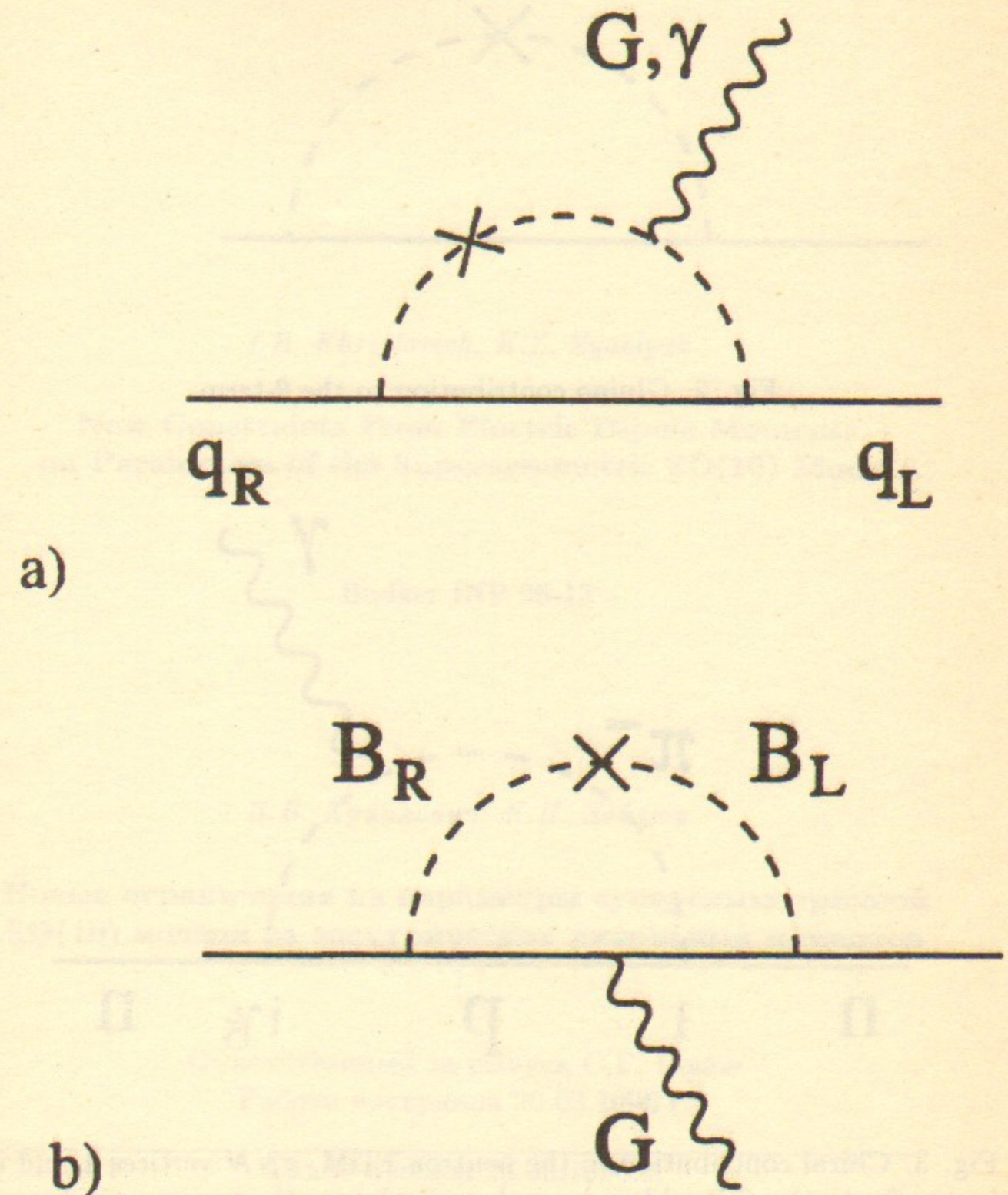


Fig. 1. Gluino contributions to quark dipole moments.  $B_{L(R)}$  denotes the left(right)-handed  $b$ -squark.

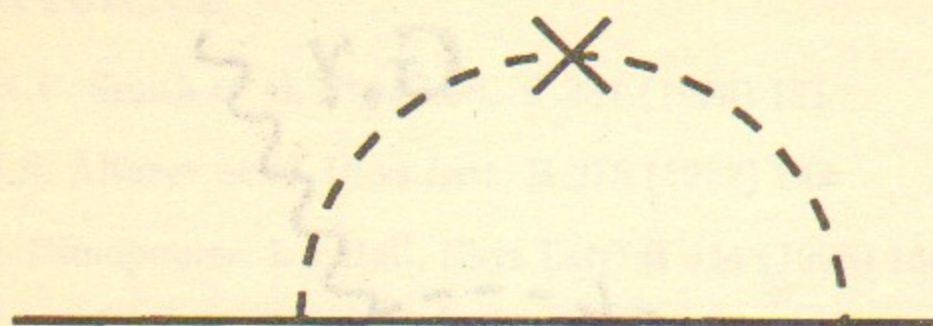


Fig. 2. Gluino contribution to the  $\theta$ -term.

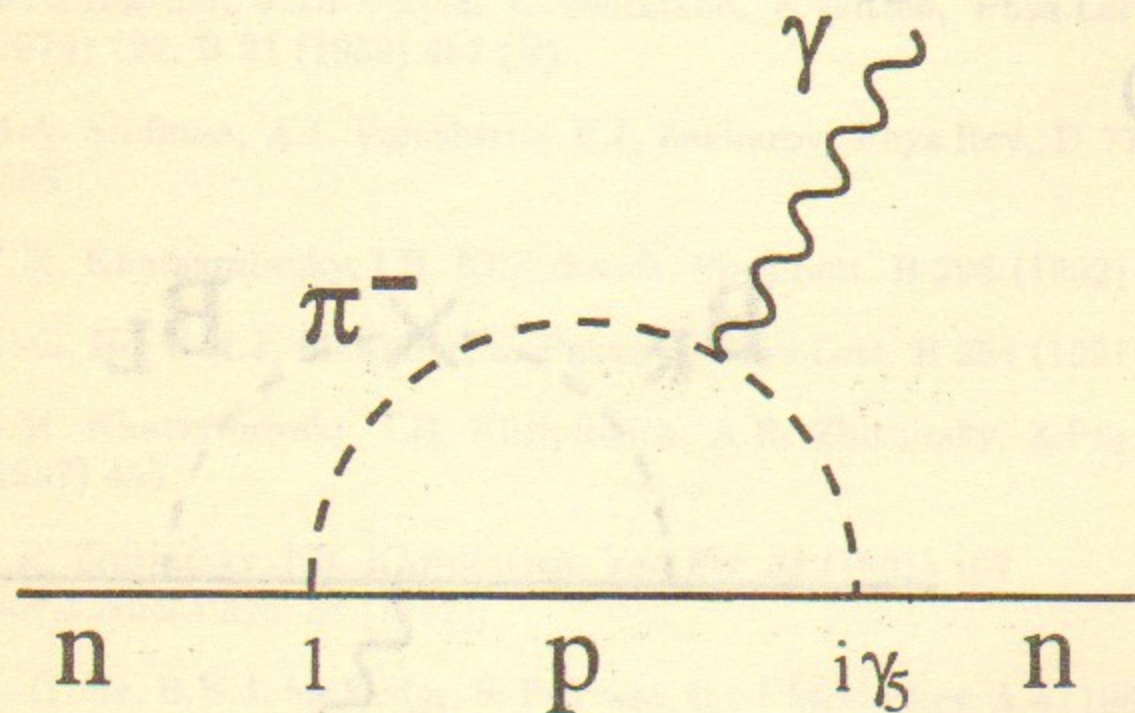


Fig. 3. Chiral contribution to the neutron EDM.  $\pi NN$  vertices 1 and  $i\gamma_5$  refer to the  $CP$ -odd and usual strong interactions, respectively.

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*И.Б. Хриплович, К.Н. Зяблюк*

**Новые ограничения на параметры суперсимметричной  
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