The State Scientific Center of Russia
The Budker Institute of Nuclear Physics
SB RAS

K. Melnikov, O.I. Yakovlev

FINAL STATE INTERACTION IN THE PRODUCTION OF HEAVY UNSTABLE PARTICLES

Budker INP 96-9

НОВОСИБИРСК

Final State Interaction in the Production of Heavy Unstable Particles

K. Melnikov, O.I. Yakovlev***

Institut fur Physik (THEP), Johannes Gutenberg Universität,
Staudinger weg 7, D55099, Mainz, Germany

Abstract

We make an attempt to discuss in detail the effects originating from the final state interaction in the processes involving production of unstable elementary particles and their subsequent decay. Two complementary scenarios are considered: the single resonance production and the production of two resonances. We argue that part of the corrections due to the final state interaction can be connected with the Coulomb phases of the involved charge particles; the presence of the unstable particle in the problem makes the Coulomb phase "visible". It is shown how corrections due to the final state interaction disappear when one proceeds to the total cross-sections. We derive one-loop non-factorizable radiative corrections to the lowest order matrix element of both single and double resonance production. We discuss how the infrared limit of the theories with the unstable particles is modified. In conclusion we briefly discuss our results in the context of the forthcoming experiments on the W^+W^- and the $t\bar{t}$ production at LEP 2 and NLC.

*e-mail:melnikov@dipmza.physik.uni-mainz.de
**e-mail: yakovlev@dipmza.physik.uni-mainz.de

1 Introduction

Processes involving production of unstable fundamental particles such as top quark, electroweak massive gauge bosons, Higgs boson are now at the frontier of both theoretical and experimental high-energy physics. Investigation of such processes can provide a valuable information about fundamental parameters (mass, width) of the heavy unstable particles. Therefore the theory has to give reliable predictions to meet expected experimental precision. However it must be recognized that the accuracy of the theoretical description of the processes involving unstable particles and the planning accuracy of the measurements have been never previously combined. There exists a number of recent theoretical inventions which all are connected with the accurate description of the unstable particle in the vicinity of its pole: the S–matrix approach to the Z-pole [1] and the extension of this scheme to a more general cases [2], colour rearrangement phenomena [3], etc.

Currently, the main source of our experience in the field is the Z-boson physics. However, as it has been mentioned in [2] and will be quite clear from the forthcoming discussion, Z-pole description is distinguished due to the following points:

1. Z-boson is neutral;
2. Main results have been obtained for the process $e^+e^- \to Z \to f\bar{f}$, i.e. for the production of the Z-boson in the s-channel.

These features greatly simplify precise description of the Z-boson production cross-section.

The basics of the theoretical approach to the processes with the unstable particles can be described as follows: when two unstable particles are produced all Feynman graphs can be divided in two classes: the first one includes the graphs without interactions between decay products of different unstable particles, while the second includes such graphs in which two decay processes are not independent. Generally speaking the graphs of the second type give a correction of the relative order $\alpha\Gamma/M_l$, where α is an appropriate (depending on the process) coupling constant and M_l is the characteristic scale for the momentum flow inside the loop. For example the graphs with the Z–boson exchange between decay products of unstable particles provide $M_l \approx M_Z$. Hence their contribution is negligible. On the other hand for the graphs with photon or gluon exchange the scale M_l is of the order of the width of the unstable particle Γ. Consequently such graphs have no additional suppression in comparison with the factorizable ones [1]. It is not difficult to convince oneself that the dominant contribution comes from the soft photon or gluon region [4-6]. This contribution is a non-trivial function of the width of the unstable particle. The reason is that the soft massless particles probe the mass-shell limit of the theory and it is the width of the particle that changes the mass-shell behaviour of the resonance in comparison with the stable particle. Hence in this problem (similar to the case of the threshold production of the unstable particles [7]) one have to use resummed propagator of the unstable particle.

The other connected physical problem which has attracted serious attention in the past years is the photon or gluon radiation of the unstable particles [8]. It turns out that in the specific kinematical configuration the radiation is completely insensitive to the decay. The reason for this can be found in the conservation of the charged or colored currents.

As is well known the soft physics generally "suffers" from the cancellation between real and virtual corrections. It is therefore quite desirable to clarify how and when this cancellation occurs when unstable particles are involved and to what extent one can study real and virtual corrections independently.

Recently some progress has been achieved in the understanding of these problems. It has been argued [4-6] that non-factorizable corrections do not contribute to inclusive quantities, e.g. to the total cross-sections, when both virtual and real ones are taken into account. This can be viewed as the ex-

[1] As for gluon exchange, this also verifies the use of the perturbative QCD for the calculation of these contributions in the reactions with the top quark(s), since $M_{loop} \sim \Gamma_t$ sets the scale on which α_s must be evaluated for these non-factorizable contributions.

tension of the Bloch-Nordsieck-Lee- Nauenberg-Kinoshita cancellation for the processes involving unstable particles [5].[2] However our analyses shows that it is not only usual real–virtual cancellation but something more involved. This fact becomes clear if one studies the influence of the non-factorizable radiative corrections on differential distributions. These distributions (say in the invariant mass of the decay products of a resonance) are a possible tool to investigate fundamental parameters of the unstable particle (for recent discussion of the W–boson and the top physics see refs.[9], [10] and references there in). It turns out that differential distributions are affected by this non-factorizable interactions (to the best of our knowledge this fact has been first noted in the ref. [11] for the specific case of the top threshold production).

In what follows we evaluate $O(\alpha, \alpha_s)$ non-factorizable resonance radiative corrections to the differential distributions in the invariant mass of the unstable particle(s). If the integration over invariant masses is performed, these radiative corrections disappear [5], [6]. Calculating radiative corrections to the differential cross-section we are able to clarify the physical origin of this "inclusive zero".

However, it turns out that *the shape* of the differential distributions and *the position of the maximum* of the differential distribution in the invariant mass of the resonance which could be naively identified with the pole mass of the unstable particle [3] are affected in the energy region slightly above threshold of two resonances. For the particular case of two W-bosons this region is approximately $170 - 190$ GeV being almost the same as the LEP2 energy region.

Absolutely deliberately we do not consider the real threshold region (i.e $\sqrt{s} - 2M \sim O(\Gamma)$). In this region new physical phenomena appear (bound state formation, etc.) and our analyses would be more complicated there. Our idea is to get the most clean laboratory for the effects which are completely connected with the unstable nature of an appropriate particle. Threshold region represents a special case and has to be discussed separately (for the top threshold production see [11], [10]).

Let us note that through out the paper we use the Breit-Wigner with a constant width as the propagator for the unstable particle. As we are concerned with the corrections of the order $O(\alpha)$ to the lowest order result

[2] Note that in general this cancellation differs from the cancellation known from the Z–pole physics. This fact clearly follows from the ref. [4]. Indeed, in describing Z–pole we deal only with the initial–final non–factorizable interaction which is much more simple. In general case there is also final–final state interaction which brings some new features to the problem.

[3] We remind that the lowest order differential distribution is the common Breit-Wigner.

we can safely use the lowest order propagators for the unstable particles since all modifications show up only in higher orders.

Subsequent part of the paper is organised as follows: next section is devoted to the investigation of the single resonance production; more involved scenario with the production of two resonances is discussed in the section 3 where all basic formulae are presented. In the section 4 we analyse our results in a more informal way. Conclusion of the whole work is given in the section 5. A number of helpful formulae are presented in the Appendix.

2 Simple model

For simplicity we start with the model describing scalar particles which interact with the "photon" field. Suppose one of this particles (we call it W) can decay to two other (electron and neutrino for simplicity). Our W particle is produced by some neutral current (virtual photon) together with the other stable particle (B- particle). In such a model W and B have opposite electric charges.

In what follows we discuss reaction $\gamma^* \to W^+B^- \to e^+\nu B^-$ taking into account $O(\alpha)$ non-factorizable radiative corrections[4].

We consider this process in the center of mass frame of the virtual photon. Then it carries the total energy $\sqrt{s}$ and the zero three momentum. We are interested in the distribution over the invariant mass of the W particle. In order to describe this distribution we introduce a parameter:

$$\delta_W = \frac{p_W^2 - M_W^2}{M_W} \tag{1}$$

where M_W is the pole mass of the W and p_W^2 is the invariant mass of the final $e\nu$ system. The Born graph is shown in the fig.1. Above the production threshold of the W particle the Born graph has the resonant propagator forcing produced W to be almost on shell. Non-factorizable virtual corrections are also shown in the fig.1. Let us first discuss the graph with the B e interaction. Since we are interested in the corrections of the order $O(\alpha)$ we have to get the resonance denominator from this graph. Consequently the loop momentum must be small in order not to shift the W-particle propagator far from the pole. From this it is clear that the only loop momentum region which can provide such "resonance" correction is the soft region, where one

[4] In some sense this case corresponds to the process $t \to W^+b$.

can use soft-photon approximation (cf. ref. [4]). In the soft photon approximation the amplitude of this process reads (we use the Feynman gauge through out the paper):

$$M_a = -\frac{4\pi i\alpha M_0}{D(p_W)} \int \frac{d^4k}{(2\pi)^4} \frac{4p_B \cdot p_e}{(k^2 + i\epsilon)(2p_B k + i\epsilon)(2p_e k - i\epsilon)} D(p_W - k). \tag{2}$$

Here M_0 is the Born amplitude and

$$D(p_W) = \frac{1}{{p_W}^2 - {M_W}^2 + iM_W\Gamma_W} \tag{3}$$

is the propagator of the W particle with the finite width included explicitly.

To perform the integration over k we first integrate over zero component of this four vector. There are four poles in the complex plane of the k_0 variable. Let us integrate over the lower half plane of the k_0–complex plane. Then two poles have to be taken into account: one from the B particle propagator (" the particle pole") and the other one from the virtual photon propagator (" the photon pole"). It can be seen that in contrast to the soft photon approximation in the QED with stable particles, in our case the contribution due to the virtual photon pole does not cancel corresponding soft photon emission immediately. However, their difference appears to be pure imaginary and hence does not influence differential distributions (for more detailed discussion see section 4.2). Keeping this in mind we take into account the B-particle pole only.

The contribution due to the B particle pole reads:

$$M_a = -4\pi\alpha \, \frac{4p_B \cdot p_e}{4E_eE_B} \frac{M_0}{D(p_W)} \, I, \tag{4}$$

$$I = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\left((\mathbf{v}_B\mathbf{k})^2 - \mathbf{k}^2\right)\left(\mathbf{k}\mathbf{v}_B - \mathbf{k}\mathbf{v}_e - i\epsilon\right)} D(p_W - k)|_{k_0 = \mathbf{k}\mathbf{v}_B}.$$

Here the quantity $\mathbf{v}_i$ is the three velocity of the i-th particle. Using momentum conservation we get for the W propagator:

$$D(p_W - k)|_{k_0=\mathbf{k}\mathbf{v}_B} = \frac{1}{M_W(\delta_W + i\Gamma_W - 2\sqrt{s/M_W^2}\,\mathbf{k}\mathbf{v}_B)}. \tag{5}$$

In order to compute residual integral over k it is useful to exponentiate the propagators, introducing two different "times". The amplitude reads then:

$$M_a = \frac{-4\pi\alpha 4p_B \cdot p_e M_0}{4E_eE_BM_W D(p_W)} \int d\tau dt \exp\{i\tau(\delta_W + i\Gamma)\} \int \frac{d^3k}{(2\pi)^3} \frac{\exp\{i\mathbf{k}\mathbf{r}(t,\tau)\}}{(\mathbf{v}_B\mathbf{k})^2 - \mathbf{k}^2}$$

where $\mathbf{r}(t,\tau)$ stands for:

$$\mathbf{r}(t,\tau) = (\mathbf{v}_e - \mathbf{v}_B)t - 2\sqrt{\frac{s}{{M_W}^2}}\mathbf{v}_B\tau.$$

The integral over $\mathbf{k}$ is recognized to be retarded Coulomb potential of the particle moving with the velocity $\mathbf{v}_B$ and hence the result of $\mathbf{k}$ integration can be found in the text-books on classical electrodynamics:

$$\phi(r) = -4\pi \int \frac{d^3k}{(2\pi)^3}\frac{\exp\{i\mathbf{kr}\}}{(\mathbf{v}_B\mathbf{k})^2 - \mathbf{k}^2} = \frac{1}{\sqrt{(\mathbf{rn}_B)^2 + (1-\mathbf{v}_B{}^2)\mathbf{r}_\perp{}^2}}. \qquad (6)$$

Here $\mathbf{n}_B$ is the unit vector parallel to the B-particle velocity and $\mathbf{r}_\perp$ is the component of the vector $\mathbf{r}$ transverse to the vector $\mathbf{n}_B$. The final expression which can be obtained in this way is:

$$M_a = \alpha\frac{4p_B\cdot p_e}{4E_eE_B}\frac{M_0}{M_W D(p_W)}\int d\tau dt\phi(\mathbf{r}(t,\tau))\exp\{i\tau(\delta_W + i\Gamma_W)\}. \qquad (7)$$

Let us note that the way we proceed is quite similar to the eikonal approximation for the high-energy scattering. It is well known in that case and can be proved in ours that the leading contributions from the eikonal graphs to the amplitude can be summed up . The result is the Coulomb phase [14] of the wave function of the charged particle. Usually the Coulomb phase is not important due to its pure imaginary nature. We shall see that in our example this is not the case and that *the residual contribution from the Coulomb phase survives in the final result.*

Integrating the last equation over t and τ we neglect the terms which are pure imaginary and hence do not contribute to the differential cross-section at the $O(\alpha)$ order. We get then:

$$M_a = -\alpha\frac{1-\mathbf{v}_e\mathbf{v}_B}{\sqrt{(\mathbf{v}_B-\mathbf{v}_e)^2 - \mathbf{v}_B{}^2\mathbf{v}_{e\perp}^2}}\, i\,\log\Big(\frac{iM_W}{\delta_W + i\Gamma}\Big). \qquad (8)$$

In this equation $\mathbf{v}_{e\perp}$ is the component of the vector $\mathbf{v}_e$ transverse to the vector $\mathbf{n}_B$. Note that the factor in front of the logarithm is nothing but the Lorentz boosted Coulomb factor $\alpha/|\mathbf{v}_1 - \mathbf{v}_2|$. This factor has the following limits: when velocities are small it turns to $\alpha/|\mathbf{v}_B - \mathbf{v}_e|$ hence reproducing usual expansion parameter for the Coulomb problem [13] while in the limit $|\mathbf{v}_e| \to 1$ or $|\mathbf{v}_B| \to 1$ this factor equals to α and hence appears to be independent from the kinematic of the process.

Let us now discuss the photon exchange between W and B (see fig.1). On the first glance this graph does not seem to be non-factorizable correction we are interested in. However gauge invariance arguments do not allow us to exclude this graph from the consideration. We study this graph in the soft-photon approximation neglecting the contribution of the photon pole (see sect.3.2). Calculation is quite similar to the previous one and results in the following contribution to the amplitude:

$$M_b = \alpha\, M_0\,\frac{1-\mathbf{v}_W\mathbf{v}_B}{|\mathbf{v}_W - \mathbf{v}_B|}\, i\,\log\Big(\frac{iM_W}{\delta_W + i\Gamma}\Big). \qquad (9)$$

There are no other corrections of the non-factorizable origin which influence differential distributions. For example the interaction of the W with the electron is of the initial-final state interaction type [4] and hence has rather simple pole structure. The infrared contribution from this graph is completely cancelled by the corresponding real emission (see also discussion in the ref. [14] for the stable particle case). Hence all radiative corrections which are of the non-factorizable nature and are not cancelled by the emission of the soft photons are given by the sum of two amplitudes presented above. The sum of this amplitudes gives us the result for the non-factorizable corrections:

$$M_{n/fact} = \alpha\, M_0\, i\,\log\Big(\frac{iM_W}{\delta_W + i\Gamma}\Big)\Big(\frac{1-\mathbf{v}_W\mathbf{v}_B}{|\mathbf{v}_W - \mathbf{v}_B|} - \frac{1-\mathbf{v}_e\mathbf{v}_B}{\sqrt{(\mathbf{v}_B-\mathbf{v}_e)^2 - \mathbf{v}_B{}^2\mathbf{v}_{e\perp}^2}}\Big). \qquad (10)$$

Let us write corresponding contribution to the cross-section in the following form:

$$d\sigma = d\sigma_0\, K, \quad K = -2\,\eta\,\mathrm{arctg}\Big(\frac{\delta_W}{\Gamma_W}\Big), \qquad (11)$$

$$\eta = \alpha\,\Big(\frac{1-\mathbf{v}_W\mathbf{v}_B}{|\mathbf{v}_W - \mathbf{v}_B|} - \frac{1-\mathbf{v}_e\mathbf{v}_B}{\sqrt{(\mathbf{v}_B-\mathbf{v}_e)^2 - \mathbf{v}_B{}^2\mathbf{v}_{e\perp}^2}}\Big).$$

Here $d\sigma_0$ is the lowest order cross section. The important point to be noted here is that in the relativistic limit for this equation *the cancellation* between contributions from WB and Be interaction occurs. This result recovers the "non–observability" of the Coulomb phase. We could expect this compensation because in the ultra–relativistic limit the spectator ("B"-particle) does not distinguish transverse movement of the electron and hence does not notice that the charge movement has been changed. As the result, the Coulomb phases of the resonance and its decay products add coherently to a pure imaginary quantity reproducing the non-observability of the Coulomb phase.

Following above discussion we can exponentiate our results given in the eqs. (12)-(13) (cf. ref. [14]) to get:

$$M_{n/fact} = M_0\Gamma(1+i\eta)e^{i\phi_0}\Big(\frac{M_W}{\delta_W + i\Gamma_W}\Big)^{i\eta} \tag{12}$$

where $i\phi_0$ is an imaginary phase which is not relevant. It is straightforward to find the contribution of this amplitude to the differential cross-section:

$$d\sigma = d\sigma_0\ \frac{\pi\eta}{sh(\pi\eta)}\ \exp\{-2\ \eta\ \mathrm{arctg}(\frac{\delta_W}{\Gamma_W})\}. \tag{13}$$

To see how this correction disappears when we proceed to the total cross section [4-6] we integrate previous equation over the range of the W masses (of course with the usual approximations in mind) and get

$$\sigma = \sigma_0\ (1 + O\Big(\alpha\ \frac{\Gamma_W}{M_W}\Big)). \tag{14}$$

On the one-loop level this cancellation is clearly visible since the $\mathrm{arctg}(\delta_W/\Gamma_W)$ is the odd function of the resonance off-shellness, while the usual Breit-Wigner is the even one. Therefore their convolution is zero.

It is clear however that this correction influences differential distributions in the invariant mass of the produced resonance. To get an idea of what one gets in the realistic situation let us imagine that we deal with the production of two equal mass resonances and the integration over invariant mass of one of them has been already performed. We treat the resonance which is "integrated out" as a stable particle. The velocity of the "electron" is taken equal to unity. Then as it has been noted above the factor η turns out to be independent from the scattering angles of the final particles. In this case this factor reads:

$$\eta = \frac{(1-\beta)^2}{2\beta}\alpha \tag{15}$$

where

$$\beta = \sqrt{1 - \frac{4{M_W}^2}{s}}$$

is the on-shell velocity of the produced resonance (as far as we are not too close to the threshold we can use this on-shell value for the velocity). We note here that the η–factor in the eq.(15) very quickly goes to zero if the total energy increases. For example, taking the mass of the resonance equal to 80

GeV one sees that for $\sqrt{s} = 170$ GeV we have $\eta = 0.65\ \alpha$ and for $\sqrt{s} = 200$ GeV this factor decreases up to $0.13\ \alpha$ suppressing this correction roughly to one order in magnitude.

However, this correction influences differential distributions in the invariant mass of the W-particle decay products moving the peak to the lower values of the resonance masses. The result for the corrected distribution is shown in the fig.2 for the usual values of the width and the mass of the W–boson and for different energies of the process. The position of the maximum of the distribution differs from the same quantity defined by the Breit-Wigner propagator. The position of the new maximum is:

$$\Big(\sqrt{p_W^2}\Big)_{max} - M_W \approx -\eta\Gamma. \tag{16}$$

Though the pole position is not affected too much for the realistic values of the particle width and the coupling constant it is still comparable with the planned accuracy of the W mass determination in the intermediate energy region $\sqrt{s} \sim 170 - 190$ GeV. For higher energies this corrections are strongly suppressed hence having no importance from the experimental point of view.

Let us make some comments now.

There exists the S–matrix approach for the description of the gauge boson pole which was originally proposed for the description of the s–channel production of the Z–boson [1]. The basic idea of this method is to start from the analytical properties of a given amplitude. Our analysis shows that in order to apply this method to the *charge* boson production one must claim that there is a *branching point* in the complex plane of the invariant mass of the resonance but not a pole. Corresponding intercept of the branching point appears to be non-trivial function of the kinematic of the process. From this we conclude that a theoretical analysis of this situation will be more complicated and there is no straightforward extension of the S–matrix pole scheme to the processes involving charge unstable gauge boson(s) production[5].

Our next remark concerns the top decay width. As it is clear from the exact expression for the η factor the non-factorizable corrections to the differential distributions over invariant mass of the W–boson decay products are negligible due to the small mass of the b-quark.

Let us also outline how the calculation of the radiative corrections to the single resonance production must be performed. We stress once more that the radiative correction which is presented in the eq.(13) is the only one which is usually referred as non-factorizable. As we have also traced

[5] This fact and a possible modification of the pole scheme are discussed in the ref. [2]

the cancellation of the real emission against virtual photon poles we can formulate the *practical recipe* for the calculation of radiative corrections to the differential cross-section of the reactions with one unstable particle:

1) The leading order differential distribution is given by the known formula:

$$\frac{d\sigma(p_W^2)}{dp_W^2} = \frac{\sigma_0(M_W^2)}{\pi} \frac{M_W\ d\Gamma(W \to e\nu)}{(p_W^2 - M_W^2)^2 + \Gamma_W^2 M_W^2} \tag{17}$$

where σ_0 is the on-shell cross section for the production of the particles W and B, and $d\Gamma(W \to e\nu)$ is the on-shell differential partial width of the W-particle.

2) In order to compute $O(\alpha)$ corrections to this formula one needs:

- to substitute one-loop results for all quantities in the previous formula;
- to add our result for the "Coulomb phase" contribution (eq.13) to the above formula, since this is the only contribution of the order $O(\alpha)$ which comes from the non–factorizable interaction. This prescription already takes into account partial cancellation of the non–factorizable real corrections against corresponding real ones.

There is one subtle point in the preceeding discussion. One can get an idea that we make a double counting, i.e. we include the soft region of the triangle (bW interaction in the terms of the model) to the "narrow width approximation", while this region is also accounted in the calculation of the non-factorizable radiative corrections, which according to the recipe are added later by hands. We note in this respect that the soft region for the "on-shell" triangle is completely cancelled by the soft emission. In this case we do not get any contribution from this region because the Coulomb phase for the stable particles is pure imaginary and hence disappears from the observables. Consequently there is no double counting and our recipe is simple and reasonable.

3 Production of two resonances

Now we are in position to discuss similar problem for the case when two unstable particles are produced in an appropriate reaction. We note that non-factorizable radiative corrections to the processes involving production of two resonances have been discussed in the literature. Namely total cross-sections [4-6] and various distributions in the non-relativistic (threshold)

limit [11] have been analysed. As we have mentioned in the introduction, we do not discuss threshold region in what follows.

Below we calculate *double resonance* $O(\alpha)$ non-factorizable radiative corrections to the lowest order matrix element. Through out the paper we consistently neglect single-resonance and background contributions.

Next important remark concerns logarithmic and polylogarithmic functions which appear in the result of this calculation. Generally we need to evaluate this functions in the complex plane. Hence it is important to fix conventions for the cuts of this functions. All logarithmic and polylogarithmic functions which appear in our final formulae have the *usual* cuts i.e for the logarithms it goes from 0 to $-\infty$ along the real axis and for the polylogarithms from 1 to $+\infty$ along the real axis.

For concreteness we consider the process $\gamma^* \to t\bar{t} \to bW^+\bar{b}W^-$ as the basis for further discussion. However, for the energy region sufficiently far from the threshold the results of our calculation appear to be general and are not restricted to a concrete process.

It is worth to note from the very beginning that the results which one obtains for two resonances appear to be more complicated and are not so transparent from the physical point of view as compared to the case of a single resonance production. If it is possible we try to appeal to the physical picture rather than huge formulae. The generic graphs for the non-factorizable radiative corrections are presented in the fig.3.

3.1 Three-point function

We start our consideration with the usual three-point function. As it is clear from the previous section the contribution of this graph is unavoidable due to the gauge invariance arguments. Corresponding amplitude diverges logarithmically in the soft-gluon approximation. To avoid this divergence we introduce the cut-off Λ and restrict the integration region to the values of the loop momenta $\mathbf{k}^2 \leq \Lambda^2$. This regularization is not Lorentz-invariant, so we use the center of mass frame everywhere. The result of the calculation is Lorentz invariant anyhow.

As we work in the soft gluon approximation, we are interested in the contribution from the region $k \sim \Gamma$. The natural requirement for the cut-off Λ is then

$$\Gamma << \Lambda << \sqrt{s}, M_W.$$

When the momentum of the virtual or real gluon is much large than Γ we can neglect the width and the off-shellness in our formulae and work with

Starting from the expression for the amplitude presented in the eq.(18) we introduce Schwinger-Fock proper time for each of the resonances. The integration over loop momentum reduces to the evaluation of the Fourier transform of the gluon propagator to the coordinate space. The result is well-known:

$$\int \frac{d^4k}{(2\pi)^4}\frac{e^{ikx}}{k^2+i\epsilon} = \frac{i}{4\pi^2}\frac{1}{x^2-i\epsilon}. \qquad (24)$$

In our case the four-coordinate of the gluon propagator is the difference in Lorentz coordinates of the resonances:

$$x^\mu = p_1{}^\mu\tau_1 - p_2{}^\mu\tau_2.$$

In the center of mass frame we rewrite the result in the following way:

$$M = -M_0\frac{\alpha_s C_F}{\pi}(1+\beta^2)\int d\tau_1 d\tau_2 \frac{\exp i(D_1\tau_1 + D_2\tau_2)}{(\tau_1-\tau_2)^2 - \beta^2(\tau_1+\tau_2)^2 - i\epsilon}. \qquad (25)$$

This expression exhibits poles on the integration path. The position of this poles corresponds to the movement of one particle in the field produced by the other when retardation effects are taken into account. The residues in these poles provide us with (we again drop all pure imaginary quantities):

$$M_{pole} = M_0\,\frac{\alpha_s C_F}{\pi}\frac{(1+\beta^2)}{4\beta}\Big(i\pi\log(\xi(-1)) + i\pi\log(\xi(1)) + \pi^2\Big). \qquad (26)$$

This expression has all desirable symmetry properties and corresponds to the Coulomb phases of two resonances which they acquire in the field of their partners.

As the space-time picture shows that our understanding of the Coulomb effects is still valid we proceed further and extract the residual Coulomb phase contribution from the gluon pole. We will not use the proper time representation systematically and continue evaluation of the three point function in the momentum space.

3.1.2 Gluon poles— So far we have studied the "particle" pole contribution to the amplitude. Now we are in position to discuss the contribution of the gluon pole. The separation of particle and gluon poles in our calculation is useful due to the fact that the contribution from the gluon pole of the virtual graph is in very close analogy to the corresponding real emission. Hence if we get the contribution of the virtual gluon pole it is a matter of machinery substitutions to obtain the amplitude for the soft real emission.

Taking the residue of the gluon propagator, which is located in the lower half-plane of the k_0 variable, we find that the integration over transverse

component of the loop momentum is again logarithmic and hence straightforward. We perform one integration by parts and arrive finally to the following representation for the gluon pole contribution:

$$M_g = M_0\frac{\alpha_s C_F}{\pi}\frac{(1+\beta^2)}{4\beta}(A_1+A_2+A_3+A_4) \qquad (27)$$

where

$$A_1 = \int_{-\Lambda}^{\Lambda} dk_z \log\Big(\frac{D_1+D_2}{Em_t} + \frac{2\beta k_z}{m_t}\Big)\times$$
$$\times\Big(\frac{\beta}{D_1/E - \Lambda + \beta k_z} - \frac{\beta}{D_2/E + \Lambda + \beta k_z}\Big),$$

$$A_2 = \int_{-\Lambda}^{\Lambda} dk_z \log\Big(\frac{D_1+D_2}{Em_t} + \frac{2\beta k_z}{m_t}\Big)\times$$
$$\times\Big(\frac{1-\beta}{D_1/E - (1-\beta)k_z} - \frac{1+\beta}{D_1/E + (1+\beta)k_z}\Big),$$

$$A_3 = \int_{-\Lambda}^{\Lambda} dk_z\theta(k_z) \log\Big(\frac{D_1+D_2}{Em_t} + \frac{2\beta k_z}{m_t}\Big)\times$$
$$\times\Big(\frac{1+\beta}{D_1/E + (1+\beta)k_z} + \frac{1+\beta}{D_2/E + (1+\beta)k_z}\Big),$$

$$A_4 = \int_{-\Lambda}^{\Lambda} dk_z\theta(-k_z) \log\Big(\frac{D_1+D_2}{Em_t} + \frac{2\beta k_z}{m_t}\Big)\times$$
$$\times\Big(\frac{\beta-1}{D_1/E - (1-\beta)k_z} + \frac{\beta-1}{D_2/E - (1-\beta)k_z}\Big).$$

Let us discuss the advantages of this representation: evaluation of the integral A_1 can be immediately reduced to the integration over the semi-circle of the radius Λ, as a consequence it will not depend on the off-shellness and the width of the resonances. Therefore it will be completely cancelled by the real emission. The A_2 term is the extracted contribution of the particle pole (hidden Coulomb phase, as it has been called above), the last two terms are specific for the gluon pole. The calculation of this integrals is straightforward due to the fact that all of them are of a polylogarithmic type. It is clear that

we need to evaluate polylogarithms and logarithms as the functions of the complex argument. We note in this respect that all important points for performing logarithmic and polylogarithmic integrals in the complex plane have been discussed long ago in the ref. [16].

As has already been mentioned, the contribution from the A_1 term is completely canceled by the real emission hence we do not present it here. The result for the A_2 reads:

$$A_2 = -\pi^2 - 2\pi i \ \log(\xi(1)). \tag{28}$$

The most involved is the evaluation of both A_3 and A_4 contributions. The result which one obtains after direct integration is:

$$\begin{aligned} A_3 + A_4 &= \log(\zeta)\log\Big(\frac{(1-\beta^2)E^2}{|z_1||z_2|m_t^2}\Big) - \frac{1}{2}\log^2(|d_{12}|) - \log(|d_{12}|)\log\Big(\frac{|z_1|}{|z_2|}\Big) \\ &+ 2\mathrm{Li}_2(\zeta) - \mathrm{Li}_2(-\zeta d_{12}) - \mathrm{Li}_2\Big(-\frac{\zeta}{d_{12}}\Big) + \frac{\pi^2}{2} \\ &+ \frac{1}{2}(\phi_1-\phi_2)^2 - (\nu_2-\nu_1)(\phi_2-\phi_1) - \pi(\nu_1+\nu_2). \end{aligned} \tag{29}$$

Here the following notations are used:

$$\begin{aligned} z_1 = \xi(1), \quad z_2 = \xi(-1), \\ \nu_i = \arg(z_i), \quad \phi_i = \arg(D_i), \\ d_{12} = \frac{D_1}{D_2}, \quad \zeta = \frac{1-\beta}{1+\beta}. \end{aligned}$$

In the presentation of this result we split the answer into the modulus and the phase parts, and write each of them in a way which allows straightforward investigation of the $\beta \to 1$ limit.

The result for the three-point function is then:

$$M_{t\bar{t}} = M_{part} + A_2 + A_3 + A_4. \tag{30}$$

Here M_{part} is defined in the eq.(22).

Now let us discuss how corresponding real emission can be obtained from these quantities. In particular we mean the interference of the gluons emitted by different resonances. It is straightforward to write the contribution of this interference term to the differential cross-section in the soft-gluon approximation. Direct examination of the momentum integral shows that it is sufficient to perform the following modifications in the result obtained for the virtual gluon pole to get a contribution due to the real emission:

- $D_1 \to -D_1{}^*$;
- change the sign of the result.

It is important to note here that this transformation does not influence analytical properties of the amplitude, hence we can perform it in the final result. It is seen from the eqs.(30-31) that the virtual pole contribution is not invariant under this transformation, hence the real emission will not cancel the contribution of the virtual gluon pole for the three-point function.

To demonstrate this point we study the limit $\beta \to 1$. It is straightforward to obtain the following from the eqs. (24), (30–32):

- Particle pole: $2\pi\phi_2$;
- Gluon pole : $2\pi\phi_1 - \pi(\phi_1+\phi_2) - \frac{1}{2}(\phi_1-\phi_2)^2 + const$;
- Virtual correction = Particle pole + Gluon pole.

Transition to the real emission discussed above reduces to the transformation $\phi_1 \to \pi - \phi_1$. As a result the sum of the real emission and the virtual correction in the limit $\beta \to 1$ equals to:

$$\pi(\phi_1+\phi_2) + 2\phi_1\phi_2 + const. \tag{31}$$

The constant term is independent from widths and off-shellnesses of the resonances and we do not present it here. The first term is the Coulomb phase of two resonances in the limit $\beta \to 1$ and the second is a correlation between the phases of two resonances.

3.2 Four point function

As a next step we consider the graphs with the gluon exchange between $t\bar{b}$ or $\bar{t}b$. Evidently there is a symmetry between these two and having the result for one of them it is straightforward to reconstruct it for the other. For concreteness we study the interaction between t and $\bar{b}$.

3.2.1 Particle poles— We start with the discussion of the particle poles. In this case it is not so easy to apply direct integration discussed in respect with the evaluation of the three point function and we use the following trick to reduce the necessary amount of work: in the soft approximation the product of two propagators of the unstable particles (cf. eq.(3)) can be decomposed as:

$$D(p_1-k)\ D(p_2+k) = \frac{1}{(D_1+D_2)+4\mathbf{pk}}\Big(\frac{1}{D_1-2p_1k} + \frac{1}{D_2+2p_2k}\Big). \tag{32}$$

Here $\mathbf{p}$ is the three momentum of the produced unstable particles ($\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$). Examining the poles in the complex plane one finds that by appropriate choice of the integration contour the second term in this decomposition gives no particle pole contribution while for the first one it is sufficient to take the pole corresponding to the $\bar{b}$ propagator. In fact this decomposition leads to a mixture of the poles of the original expression. Hence strictly speaking the particle poles which are discussed below are some combinations of the original particle and gluon poles. However as we have seen during discussing the triangle graph particle and gluon poles are hardly separated when we deal with the amplitude involving two unstable particles. So the "names" here are just a matter of taste.

Taking the residue of the $\bar{b}$ propagator we get:

$$M_{part} = \overline{M}_0 \frac{4\pi\alpha_s C_F}{D_1} \frac{1 - \mathbf{v}_1\mathbf{v}_4}{E}\, I,$$

$$I = \int \frac{d^3k}{(2\pi)^3} \frac{1}{((\mathbf{v}_4\mathbf{k})^2 - \mathbf{k}^2)((D_1 + D_2)/E + 2\mathbf{v}_1\mathbf{k})(D_1/E - (\mathbf{v}_4 - \mathbf{v}_1)\mathbf{k})}.$$

Here $\overline{M}_0$ is the Born amplitude with the extracted unstable propagators, $\mathbf{v}_1$ and $\mathbf{v}_4$ stand for the top and the $\bar{b}$-quark three velocities respectively. As it is clear from this equation, the integration can be performed in a way similar to the case of the single resonance production. We introduce a proper time for each of the resonance propagators and exponentiate them. The integration over $\mathbf{k}$ is then the same as in the single resonance case (section 2). Finally we get:

$$M_{part} = \overline{M}_0 \frac{\alpha_s C_F}{D_1} \frac{1 - \mathbf{v}_1\mathbf{v}_4}{E} \int \frac{d\tau d\tau_1}{\sqrt{{r_1}^2 + {r_2}^2}} \exp\left\{i\left(\frac{D_1 + D_2}{E}\tau + \frac{D_1}{E}\tau_1\right)\right\}. \tag{33}$$

In what follows we denote the angle between velocity $\mathbf{v}_i$ of the particle labelled i and the top quark velocity $\mathbf{v}_1$ by θ_i. Then r_1 and r_2 are:

$$r_1 = 2\beta\cos\theta_4\tau - (1 - \beta\cos\theta_4)\tau_1$$

and

$$r_2^2 = \frac{{m_4}^2}{{E_4}^2}\beta^2\sin^2\theta_4(2\tau + \tau_1)^2.$$

Next we make the change of the variables: $\tau = \lambda x$ and $\tau_1 = \lambda(1 - x)$ with $0 \le x \le 1$ and $0 \le \lambda \le \infty$. Integrating further over λ we obtain:

$$M_{part} = \overline{M}_0 \frac{i\alpha_s C_F}{D_1}(1 - \mathbf{v}_1\mathbf{v}_4)\times$$

$$\times \int_0^1 \frac{dx}{D_1 + D_2 x} \frac{1}{\sqrt{\left[(1 - \beta x_4) - x(1 + \beta x_4)\right]^2 + \mu^2(1 + x)^2}}.$$

We denote $\cos\theta_4$ as x_4 and :

$$\mu^2 = \frac{{m_4}^2}{{E_4}^2}\beta^2(1 - x_4^2).$$

Examining previous equations we see that the leading term under the square route can go through zero within the integration region if $x_4 \ge 0$.In this case this "would be" divergence is regularized by keeping the mass of the light particle finite. This means that the divergence is collinear. Actually this divergence can appear only if the "mass" of the gluon is zero, i.e. when the gluon pole in the original expression is taken. Hence the appearance of this divergence in the particle pole means that decomposition of the unstable propagators (see eq.(34)) which we use for the evaluation of this graph has really mixed gluon and particle poles of the original expression in a nontrivial way.

We insert the identity $1 = \theta(x_4) + \theta(-x_4)$ inside the integral. After this we get:

$$M_{part} = \overline{M}_0 \frac{i\alpha_s C_F(1 - \mathbf{v}_1\mathbf{v}_4)}{D_1\xi(x_4)m_t^2}\Big(\theta(-x_4)\ A + \theta(x_4)\ B\Big), \tag{34}$$

$$A = \log(\frac{D_1 + D_2}{D_1}) + \log(1 - \beta x_4) - \log(-2\beta x_4),$$

$$B = \log\left(\frac{\xi(x_4)m_t^2}{D_1 + D_2}\right) - \log\left(\frac{\xi(x_4)m_t^2}{D_1}\right) + \log(1 - \beta x_4) + \log(2\beta x_4)$$

$$- \log\left[\beta^2(1 - {x_4}^2)\right] + \log\left(\frac{{E_4}^2}{{m_4}^2}\right).$$

As it has been mentioned before and is quite clear from the above equation there are collinear logarithms associated with the massless b-quark in the final state. We discuss below (see section 3) how collinear logarithms cancel in the final result.

2.2.2 Gluon pole— Using decomposition eq.(34) for the resonance propagators and performing the integration over the contours discussed above we are forced to take the lower pole of the gluon propagator for the first term

in the decomposition and the upper one for the second. Performing the integration over the modulus of the three-momentum we obtain the following representation for the amplitude:

$$M_g = \overline{M}_0 \frac{\alpha_s C_F}{\pi D_1 m_t^2} \frac{1 - \mathbf{v}_1\mathbf{v}_4}{2}(J_1 + J_2). \tag{35}$$

Where J_1 and J_2 are:

$$J_1 = \int \frac{d^3\mathbf{n}_k}{2\pi\xi(x)(1-\mathbf{n}_k\mathbf{n}_3)}\Big(\log\Big(\frac{2\beta x + i\epsilon}{1-\beta x}\Big) + \log\Big(\frac{D_1}{D_1+D_2}\Big) - i\pi\Big), \tag{36}$$

$$J_2 = -\int \frac{d^3\mathbf{n}_k}{2\pi\xi(-x)(1-\mathbf{n}_k\mathbf{n}_3)}\Big(\log\Big(\frac{2\beta x + i\epsilon}{1-\beta x}\Big) + \log\Big(\frac{D_2}{D_1+D_2}\Big) - i\pi\Big). \tag{37}$$

Here $\mathbf{n}_k$ is the unit vector, by x we denote $\cos\theta_k = \mathbf{n}_k\mathbf{n}_1$ and $\mathbf{n}_i$ is the unit vector parallel to the velocity of the particle i.

The integration over azimuthal angle is easily performed using the following equation:

$$\int_0^{2\pi} \frac{d\varphi}{1-\mathbf{n}_k\mathbf{n}_i} = \frac{2\pi}{\sqrt{(\cos\theta_k - \cos\theta_i)^2}}. \tag{38}$$

This equation exhibits collinear singularities which appear when the momentum of the gluon is parallel to the momentum of the (anti)quark. We regularize them keeping the mass of the light particle in the singular terms. The exact formula reads:

$$|x - x_i| \to \sqrt{(x-x_i)^2 + \frac{{m_i}^2}{{E_i}^2}(1-{x_i}^2)}. \tag{39}$$

Finally changing the sign of the integration variable in the eq.(39) $\cos\theta_k \to -\cos\theta_k$ we get:

$$J_1 + J_2 = \int_{-1}^{1} \frac{dx}{\xi(x)\sqrt{(x-x_4)^2}}\Big(\log\Big(\frac{1+\beta x}{1-\beta x}\Big) + \log\Big(\frac{D_1}{D_2}\Big) + i\pi\theta(-x) - i\pi\theta(x)\Big). \tag{40}$$

It is rather straightforward to calculate the integral in the last equation. We split the integration region into two parts to rewrite the square root correctly and use a partial fractioning to obtain Spence–like integrals.

It is quite useful here to examine a part of the previous expression which contains θ-functions. The evaluation is straightforward. The result is:

$$M_{g\theta} = \overline{M}_0 \frac{\alpha_s C_F}{\pi} \frac{(1 - \mathbf{v}_1\mathbf{v}_4)}{2D_1\xi(x_4)m_t^2}\, A, \tag{41}$$

$$A = (i\pi\theta(-x_4) - i\pi\theta(x_4))\, A_1 + 2i\pi\theta(x_4)\, A_2 + 2i\pi\theta(-x_4)\, A_3.$$

Where $M_{g\theta}$ is the piece of the gluon pole part proportional to the θ-functions and

$$A_1 = \log\Big(\frac{\xi(x_4)}{\xi(-1)}\Big) + \log\Big(\frac{\xi(x_4)}{\xi(1)}\Big) + \log\Big(\frac{4{E_4}^2}{{m_4}^2}\Big),$$

$$A_2 = \log\Big(\frac{D_1+D_2}{\xi(-1)m_t^2}\Big) + \log\Big(\frac{1+x_4}{x_4}\Big),$$

$$A_3 = -\log\Big(\frac{D_1+D_2}{\xi(1)}\Big) - \log\Big(\frac{1-x_4}{-x_4}\Big).$$

If we sum the particle pole contribution elaborated above and the "θ"-part of the contribution due to the gluon pole the result appears to be simple and all θ-functions drop out.

$$M_{part} + M_{g\theta} = \overline{M}_0 \frac{\alpha_s C_F}{\pi} \frac{(1-\beta x_4)}{2} \frac{i\pi}{D_1\xi(x_4)}\, K, \tag{42}$$

$$K = 2\log\Big(\frac{\xi(x_4)}{D_1}\Big) + \log\Big(\frac{\xi(1)}{\xi(-1)}\Big) + 2\log\Big(\frac{1-\beta x_4}{\beta(1-x_4)}\Big) + L_4.$$

In this equation L_4 stands for

$$L_4 = \log\Big(\frac{{E_4}^2}{{m_4}^2}\Big).$$

This notation will be used further.

To evaluate remaining contributions due to gluon pole it is convenient to use additional functions introduced in the Appendix. Finally we get the following result for the four-point function:

$$M_{t\bar{b}} = \overline{M}_0 \frac{\alpha_s C_F}{2\pi} \frac{1-\beta x_4}{D_1\xi(x_4)m_t^2}\, A_{t\bar{b}}, \tag{43}$$

$$A_{t\bar{b}} = A_1 + A_2 + A_3,$$

$$A_1 = -F_1(-D_0, \beta|x_4) + F_1(-D_0, -\beta|x_4) + F_1(x_4, \beta|x_4) - F_1(x_4, -\beta|x_4),$$

$$A_2 = \log\left(\frac{D_1}{D_2}\right)\Big[F_2(x_4|x_4) - F_2(-D_0|x_4)\Big],$$

$$A_3 = i\pi\Big\{2\log\left(\frac{\xi(x_4)m_t^2}{D_1}\right) + \log\left(\frac{\xi(1)}{\xi(-1)}\right) + 2\log\left(\frac{1-\beta x_4}{\beta(1-x_4)}\right) + L_4\Big\}.$$

In this equation we denote:

$$D_0 = \frac{D_1 + D_2}{\beta(D_1 - D_2)}.$$

It is also straightforward to consider gluon pole of the original matrix element (without decomposing resonance propagators eq.(34)). We need it due to the study of the bremsstrahlung integral, namely the interference of the gluon radiation from t and $\bar{b}$ quarks. The calculation is similar to the one described above. Finally we get:

$$M_g = \overline{M}_0 \frac{\alpha_s C_F}{2\pi} \frac{1-\beta x_4}{D_1 \xi(x_4) m_t^2}\ A_g, \tag{44}$$

$$A_g = A_1 + A_2,$$

$$A_1 = -F_1(-D_0, \beta|x_4) + F_1(-D_0, -\beta|x_4) + F_1(x_4, \beta|x_4) - F_1(x_4, -\beta|x_4),$$

$$A_2 = \left(\log\left(\frac{D_1}{D_2}\right) + i\pi\right)\Big(F_2(x_4|x_4) - F_2(-D_0|x_4)\Big).$$

Corresponding bremsstrahlung integral can be obtained from the previous equation by the standard change (cf. discussion after eq.(32)) in the relevant piece of the differential cross-section.

3.3 Five-point function

We are finally left with the last non-factorizable graph which corresponds to the interaction between b and $\bar{b}$. We again calculate a contribution due to particle and gluon poles separately.

3.3.1. Particle poles— Evaluation of the particle pole proceeds in a way similar to the one which has been used for the four-point function. We use decomposition for the propagators (eq.(34)) and then choose appropriate contour for the integration. The integral naturally splits into two pieces which represent the movement of a system of particles in the Coulomb field produced by a quark or an antiquark. These two pieces are symmetric and complementary to each other.

Let us examine one of them. We exponentiate the propagators to obtain the Coulomb-like three momentum integral. As we have one more propagator

here in comparison with the four-point function we need to introduce three "times" instead of two:

$$M_{part,4} = \overline{M}_0\ 4\pi i\alpha_s C_F \frac{(1-\mathbf{v}_3\mathbf{v}_4)}{E^2}\ I, \tag{45}$$

$$I = \int d\tau d\tau_1 dt \exp\Big\{i\Big(\frac{D_1+D_2}{E}\tau + \frac{D_1}{E}\tau_1\Big)\Big\}\int \frac{d^3k}{(2\pi)^3}\frac{\exp\Big\{i\mathbf{kr}\Big\}}{(\mathbf{v}_4\mathbf{k})^2 - \mathbf{k}^2}$$

where $\mathbf{r}$ stands for the following vector:

$$\mathbf{r} = 2\mathbf{v}\tau + (\mathbf{v}-\mathbf{v}_4)\tau_1 + (\mathbf{v}_3-\mathbf{v}_4)t. \tag{46}$$

Here the quantity $\mathbf{v}$ is the on-shell velocity of the top quark. Integrating this equation over $\mathbf{k}$ we get:

$$M_{part,4} = i\alpha_s C_F \frac{(1-\mathbf{v}_3\mathbf{v}_4)}{E^2}\overline{M}_0 \int \frac{d\tau d\tau_1 dt}{\sqrt{(\mathbf{n}_4\mathbf{r})^2 + \mathbf{r}_\perp^2}}\times$$

$$\times \exp\Big\{i\Big(\frac{D_1+D_2}{E}\tau + \frac{D_1}{E}\tau_1\Big)\Big\}. \tag{47}$$

Here $\mathbf{n}_4$ is the unit vector parallel to the velocity of the particle 4 and $\mathbf{r}_\perp$ is the component of the vector $\mathbf{r}$ perpendicular to the vector $\mathbf{n}_4$.

The second term ($M_{part,3}$) can be obtained from the eq.(52) by the following set of substitutions:

$$\mathbf{v}_3 \to -\mathbf{v}_4, \quad \mathbf{v}_4 \to -\mathbf{v}_3, \quad D_1 \to D_2, \quad D_2 \to D_1. \tag{48}$$

The evident intention then is to perform the integration over t. The integral appears to be logarithmically divergent on the upper limit. This reflects the fact that infrared singularities of the five-point function can not be completely regularized by the virtualities and widths of the unstable particles. However, we anticipate that the above divergence corresponds to the Coulomb phase of the b quark in the field of the antiquark $\bar{b}$. Hence we expect that this divergence is pure imaginary and drops from the observable quantities (as it occurs in the infrared limit of the "stable" theory [12]). This expectation is verified by direct calculation. Below we omit this infinite piece from all expressions.

The integration in the eq.(49) is then straightforward. We do not present its results because it is much more reasonable to present the sum of the particle pole and the "θ"-terms from the gluon pole.

3.3.2 Gluon poles— Let us discuss the contribution due to gluon poles. As we have chosen appropriate contour to evaluate particle poles we are forced to take the lower and the upper poles in the gluon propagator for the first and the second term in the eq.(32) respectively. As in the case of the four-point function we perform the integration over the modulus of the three momentum and get:

$$M_g = -\overline{M}_0 \frac{\alpha_s C_F}{2\pi}(1-\mathbf{n}_3\mathbf{n}_4)\int \frac{d^2\mathbf{n}_k}{(1-\mathbf{n}_k\mathbf{n}_3)(1-\mathbf{n}_k\mathbf{n}_4)}\Psi(D_1,D_2,\cos\theta_k). \tag{49}$$

The function Ψ can be written in the following way:

$$\Psi(D_1,D_2,x) = \Psi_0(D_1,D_2,x)+\Psi_1(D_1,D_2,\beta,x)+\Psi_1(D_2,D_1,-\beta,x)+\Psi_\theta \tag{50}$$

where:

$$\Psi_0(D_1,D_2,x) = \frac{1}{D_1D_2}\Big(\log\frac{m_i^2}{\epsilon E}-i\pi\Big),$$

$$\Psi_1(D_1,D_2,\beta,x) = \frac{1-\beta x}{D_1\xi(x)m_i^2}\log\Big(\frac{D_1}{(1-\beta x)m_i^2}\Big),$$

$$\Psi_\theta = \frac{2i\pi\beta x(\theta(x)-\theta(-x))}{m_i^2\xi(x)(D_1+D_2)}.$$

The parameter ϵ in this equation is the infrared cut-off. The first term in the previous equation does not contribute to the observable quantities. Indeed, the infrared log is canceled by the real emission while the $i\pi$ term is pure imaginary and hence does not interfere with the Born amplitude.

Next we evaluate the integral in the eq.(51). As the function Ψ does not depend on the azimuthal angle, we calculate the following integral:

$$I_{34} = \int_0^{2\pi}\frac{d\varphi}{2\pi(1-\mathbf{n}_k\mathbf{n}_3)(1-\mathbf{n}_k\mathbf{n}_4)}. \tag{51}$$

Direct integration gives:

$$I_{34} = \frac{A_{34}}{(1-\mathbf{n}_3\mathbf{n}_4)(1+x_{34})(x-x_a)(x-x_b)}, \tag{52}$$

$$A_{34} = \frac{N_3-xK_3}{|x-x_3|}+\frac{N_4-xK_4}{|x-x_4|},$$

$$x_{a(b)} = \frac{\cos\theta_3+\cos\theta_4 \pm i\ \sin\theta_3\sin\theta_4\sin\varphi_{34}}{1+\cos\theta_{34}},$$

$$N_3 = 1-\cos\theta_{34}-\cos\theta_3(\cos\theta_3-\cos\theta_4),$$

$$N_4 = 1-\cos\theta_{34}-\cos\theta_4(\cos\theta_4-\cos\theta_3),$$

$$K_3 = \cos\theta_4-\cos\theta_3\cos\theta_{34},$$

$$K_4 = \cos\theta_3-\cos\theta_4\cos\theta_{34}.$$

Here we denote by $x=\cos\theta$, by $x_i=\cos\theta_i$ and by $x_{34}=\cos\theta_{34}$. Here the angle θ_{34} is the angle between the vectors $\mathbf{n}_3$ and $\mathbf{n}_4$.

As it is seen from this equation, the result of the azimuthal integration is a rational function of the $\cos(\theta)$. The remainder of the integrand consists of logs and constants, hence it is quite clear that the integration can be performed in terms of the Spence functions and logarithms.

The other point is that divergence which occurs for $x=x_{3,4}$ is the collinear one and hence its regularization is clear. Explicit formula reads:

$$|x-x_i| \to \sqrt{(x-x_i)^2+\frac{{m_i}^2}{{E_i}^2}(1-{x_i}^2)}.$$

We write eq.(54) in the following way:

$$I_{34} = \frac{2\pi}{1-\mathbf{n}_3\mathbf{n}_4}(I_3(x)+I_4(x)) \tag{53}$$

where

$$I_3 = \frac{N_3-xK_3}{(1+x_{34})(x-x_a)(x-x_b)|x-x_3|}. \tag{54}$$

Let us study the θ-terms of the Ψ function and show how they cancel against corresponding parts of the particle pole. As both particle and gluon poles are naturally splitted into two terms (3 and 4) we present them separately. Further evaluation is straightforward. The sum of the particle pole contribution and the θ-terms of the gluon pole reads:

$$M_{41} = M_{part,4}+M_{g,\theta,4} = -\overline{M}_0\frac{i\alpha_s C_F}{2}\ K, \tag{55}$$

$$K = \frac{2}{D_2D}\log\Big(\frac{D}{im_i^2}\Big)-\frac{2}{D_2D_1}\log\Big(\frac{D_1}{im_i^2}\Big)+$$

$$\frac{2(1-\beta x_4)}{D_1\xi(x_4)m_i^2}\log\Big(1-\beta x_4\Big)-\frac{2(1+\beta x_4)}{D_2\xi(x_4)m_i^2}\log\Big(\frac{D}{D_1}\Big)+$$

$$\frac{1}{D}\Big\{R_{4\xi}\Big(2\log\Big(\frac{\xi(x_4)}{D}\Big)+\log\Big(\frac{\xi(1)}{\xi(-1)}\Big)\Big)+$$

$$\sum_{i=\pm}R_i\Big(\log(1-x_i)-\log(-1-x_i)-2\log(-x_i)+2\log(x_4-x_i)\Big)+$$

$$R_4\Big(2\ \log(\beta)+2\log(1-x_4)-L_4\Big)\Big\}.$$

and

$$M_{31}=M_{part,3}+M_{g,\theta,3}=-\overline{M}_0\frac{i\alpha_s C_F}{2}\ K, \tag{56}$$

$$K=\frac{2}{D_1 D}\log\Big(\frac{D}{im_t^2}\Big)-\frac{2}{D_2 D_1}\log\Big(\frac{D_2}{im_t^2}\Big)+$$

$$\frac{2(1+\beta x_3)}{D_2\xi(x_3)m_t^2}\log\Big(1+\beta x_3\Big)-\frac{2(1-\beta x_3)}{D_1\xi(x_3)m_t^2}\log\Big(\frac{D}{D_2}\Big)-$$

$$\frac{1}{D}\Big\{R_{3\xi}\Big(2\log\Big(\frac{\xi(x_3)m_t^2}{D}\Big)+\log\Big(\frac{\xi(-1)}{\xi(1)}\Big)\Big)-$$

$$\sum_{i=\pm}R_i\Big(\log(1-x_i)-\log(-1-x_i)+2\log(-x_i)-2\log(x_3-x_i)\Big)+$$

$$R_3\Big(2\log(\beta)+2\log(1+x_3)-L_3\Big)\Big\}.$$

We denote $D=D_1+D_2$ in the above expression . The exact expressions for the quantities R_i can be found in the Appendix.

Then we are left with the integration of the Ψ_1 function. The result of the integration is:

$$M_{42}=-\overline{M}_0\frac{\alpha_s C_F}{2\pi}\Big[\ \frac{(1-\beta x_4)}{D_1\xi(x_4)m_t^2}\Big(\log\Big(\frac{4{E_4}^2}{{m_4}^2}\Big)\log\Big(\frac{D_1}{m_t^2}\Big)-F_1(x_4,-\beta|x_4)\Big)-$$

$$\sum_{i=\pm}\frac{(1-\beta x_i)}{2D_1\xi(x_i)m_t^2}\Big(\log\Big(\frac{D_1}{m_t^2}\Big)F_2(x_i|x_4)-F_1(x_i,-\beta|x_4)\Big)-$$

$$\frac{R_{4\xi}}{D_1+D_2}(\log\Big(\frac{D_1}{m_t^2}\Big)F_2(-D_0|x_4)-F_1(-D_0,-\beta|x_4)\Big]+$$

$$(D_1\to D_2,\beta\to-\beta). \tag{57}$$

Similar term (M_{32}) which corresponds to the particle 3 can be then obtained by the direct substitution $3\to 4$ in the eq.(59). Our final result for the

radiative correction due to the $b\bar{b}$ interaction can be constructed from the above quantities:

$$M_{b\bar{b}}=M_{41}+M_{42}+M_{31}+M_{32}. \tag{58}$$

Finally we present the contribution of the "true" gluon pole (i.e. without decomposition of the resonance propagators eq.(34)) of the virtual five point function:

$$M_g=M_{42}+M_{32}+M_{34} \tag{59}$$

$$M_{34}=-\overline{M}_0\frac{i\alpha_s C_F}{2}\Big[\ \frac{(1-\beta x_4)}{D_1\xi(x_4)m_t^2}\log\Big(\frac{4{E_4}^2}{{m_4}^2}\Big)-\sum_{i=\pm}\frac{(1-\beta x_i)}{2D_1\xi(x_i)m_t^2}F_2(x_i|x_4)$$

$$+\frac{R_{4\xi}}{D}F_2(-D_0|x_4)\Big]+(4\to 3).$$

This concludes our evaluation of the five-point function.

4 Analyses of the general formulae

So far we have derived general formulae for the double resonance radiative corrections to the matrix element of the production of two resonances. Here we discuss some general properties of the obtained formulae.

4.1 Collinear singularities

As it is clearly seen from the above formulae each of the separate contributions to the non-factorizable radiative corrections exhibits collinear logarithms. Normally these logarithms are cancelled against the real emission. Let us note that T. D. Lee and M. Nauenberg [17] have used quite general approach to prove the absence of the similar divergencies in any quantum mechanical system. The basis for the proof is the existence of the unitary S-matrix. As is well-known from the work by M. Veltman [18], it is indeed possible to construct the unitary S-matrix in the field theory with the unstable particle. Hence the arguments of the ref. [17] must apply also here. However it is necessary to clarify the level of the inclusiveness which is necessary for this cancellation to occur when unstable particles are considered.

For this aim we extract all the terms which are singular in the limit $m_i\to 0$, $i=3,4$ from the above formulae and calculate their contribution to the total cross-section. In spite the fact that these terms are quite complicated

in the individual graphs the sum of all these contributions appear to be very simple. We first write its contribution to the differential cross section:

$$\frac{d\sigma_{col}}{d\sigma_0} = 2\,\frac{\alpha_s C_F}{2\pi}\;\mathrm{Re}\Big\{i\pi\Big(\frac{D_2}{D}L_4 + \frac{D_1}{D}L_3\Big)\Big\}. \qquad (60)$$

We remind that the quantities L_3, L_4 are defined by the eq.(42).

Let us discuss now the properties of this equation. First we note that the source of this large logarithms are the virtual contribution due to the five–point function. Of course there are collinear logarimths also in the real interference but these are cancelled against similar pieces in the virtual corrections.

We can also reexpress the terms in the eq. (60) to indicate exactly the mass singularities which we find in this case:

$$\frac{d\sigma_{mass}}{d\sigma_0} = \frac{\alpha_s\, C_F}{2\pi}\;\mathrm{Re}\Big\{i\pi\Big(\frac{D_2}{D} - \frac{D_1}{D}\Big)\Big\}\log\frac{m_3^2}{m_4^2}. \qquad (61)$$

All other terms which have been dropped in the transition from the eq. (60) to the eq. (61) are smooth in the limit when the masses $m_i,\ \ i = 3, 4$ go to zero.

We see therefore that if the masses of light particles in the final state are equal then we do not get any mass singularities. This is the case for instance for the reaction $e^+e^- \to t\bar{t} \to W^+W^-b\bar{b}$. However such mass singularities will appear in the reactions like $e^-e^+ \to W^+W^- \to e^-\bar{\nu}_e\mu^+\nu_\mu$. They drop out if the sum of charge conjugate channels is considered simultaneously – for instance $e^-e^+ \to W^+W^- \to e^-\bar{\nu}_e\mu^+\nu_\mu$ and $e^-e^+ \to W^+W^- \to \mu^-\bar{\nu}_\mu e^+\nu_e$. In any case if the integration over invariant masses of the produced resonances is performed, these singularities drop out from the observable cross section.

4.2 Real emission and virtual gluon poles

We discuss here how the cancellation of the real emission and the virtual corrections occurs when unstable particles are produced. We begin with the single resonance production (cf. section 1).

First we examine *Be* interaction (in terms of the section 1). It is straightforward to write the cross-section for the real emission and the contribution of the virtual photon pole to the cross-section in the soft photon approximation:

$$d\sigma_\gamma{}^{virt} + d\sigma^{emiss} = 4\pi\alpha|M_{Born}|^2 4p_2p_3\times$$

$$\times\int\frac{d^3k}{2\pi^3 2|\mathbf{k}|}2\mathrm{Re}\Big\{\frac{D(p_1+k)+D(p_1-k)}{(2p_3k)(2p_2k)D(p_1)}\Big\}.$$

It is seen from this expression that gluon momentum enters the propagator of the unstable particle with different signs in virtual and real corrections. This is the illustration of the statement in the ref. [4] where the authors claim that the cancellation is not local in the momentum space in contrast to the usual situation. However the above expression is well-defined and we can evaluate it explicitly. The result of this calculation appears to be pure imaginary and hence does not contribute to the cross-section. The same situation also occurs for the usual triangle graph with one unstable particle. However the case with two resonances appears to be much more unusual.

As is well known the usual thing in dealing with the soft limit of the Feynman graphs is the cancellation between real and virtual corrections. The essence of this cancellation is the fact that the particle movement is not affected by emission and absorbtion of soft massless quanta. Therefore the probability of a process remains the same. The piece of the virtual corrections that cancels real emission is the residue of the massless gauge boson propagator (photon or gluon).

This simple remark verifies similar cancellation in the case when the integration over invariant masses of the unstable particles has been performed. In this case, as it is clear from our consideration, we effectively recover the situation with the stable particles. However the differential distributions represent a different case.

Explicit investigation of the contribution due to the gluon pole from the virtual correction and the real emission shows that they cancel each other in a non-trivial way. Let us fix the off-shellness eq.(1) of one of the resonances δ_1. Then the virtual gluon pole contribution calculated for the off-shellness of the other resonance δ_2 cancels the real emission for the off-shellness $-\delta_2$. The reason is that for negative values of δ the particle is more likely to absorb gluons (the particle prefers to make its invariant mass larger) while for positive δ's the situation is opposite; exactly on the mass shell $\delta = 0$ there is no difference. This shows that in the case of the unstable particle we have one more degree of freedom – the invariant mass which is sensitive to the soft ($k \sim \Gamma$) emission and absorbtion. Averaging over invariant masses we "lose" this degree of freedom (technically non-local cancellation in the space of the invariant masses occurs), but when the distribution in the invariant mass is studied we meet some unusual properties.

5 Conclusions

We have derived general formulae for the non-factorizable radiative corrections to the invariant mass distributions for both single and double resonance production. We find these corrections to be important for the accurate description of this distribution in the vicinity of the resonance peak.

Our approach is motivated by the observation that non-factorizable corrections to the Born amplitude are governed by the soft limit in order to give resonant contributions. This fact justifies the use of the soft photon (gluon) approximation for this problem. As usual the soft photon approximation provides universal results in the sense that they are not restricted to a concrete process.

The gauge-invariant current can only be constructed if one takes into account both the current of the resonance and the current of its decay products. Gauge invariance is responsible for the cancellation of the whole effect for high energies and the most probable kinematical configuration, i.e. when the charge decay products follow the direction of motion of the resonance.

We hope that our study provides better understanding of the structure of the infrared limit of the theories with the unstable particles. We note that the usual cancellation between soft real and virtual corrections is not complete even in the well known theories like QED: in fact the "photon" poles from the virtual corrections cancel the real emission, while the "particle" poles (which also give infrared divergencies) appear to be pure imaginary and physically correspond to the Coulomb phase [12], [14].

In the case when we deal with the unstable particles the "particle" poles provide non-vanishing corrections to the observable quantities. The origin of this correction is very simple: the decay of the resonance accidentally changes the movement of the charge and hence destroys a coherence necessary to acquire "proper" Coulomb phase. Dealing with the Born amplitudes describing resonance production we can recognize that the integration over invariant masses of the resonance restores the "stable particle scenario". As for the non-factorizable radiative corrections we know that they disappear if the integration over invariant masses is performed [4-6]. As the integration over invariant masses restores the stable particle scenario, the absence of the contribution due to non-factorizable corrections in the integrated quantities is in accordance with the non-observability of the Coulomb phase in the familiar theories with the stable particles.

As for the cancellation of the virtual photon poles against the real emission we argue that this cancellation occurs only if the integration over invariant mass of at least one of the resonances is performed.

Let us give a summary of the formulae presented in the text:

- Non-factorizable radiative correction to the differential cross-section for a single resonance production is given by eq.(11).
- Non-factorizable corrections for the matrix elements describing production of two resonances are given by:
  1. three-point function – eq.(30);
  2. four-point function – eq.(43);
  3. five-point function – eq.(58).

From the phenomenological side our study is motivated by a future investigation of heavy unstable particles. The vivid example is provided by the study of the reaction $e^-e^+ \to W^-W^+$ at LEP 2. It seems that the planning accuracy of the measurement and the proposed technique of measuring the line shape of the invariant mass distribution requires taking into account QED non-factorizable corrections as well.

As the dominant contribution to $e^-e^+ \to W^-W^+$ comes from the t-channel neutrino exchange, on the first glance it seems that the six-point function is actually needed for this case. However, simple estimates show that practically for the whole phase-space of the final particles factorization of the Born amplitude is still valid. Hence it is sufficient to use the five-point function for the description of the production of two W bosons at LEP2.

We note that the energy region for the LEP 2 ($\sqrt{s} = 170-200$ GeV) is the intermediate but not really threshold energy region. Consequently one has to consider the effects of the final state interaction between decay products of the resonances as well: it is likely that the Coulomb correction alone (which is definitely the leading one in the threshold region) is not sufficient for the LEP2 energy region.

As it has been indicated above our results beeing obtained in the soft approximation are universal. For the illustrative purposes we apply them to the process $e^+e^- \to W^+W^- \to e^+\nu_e e^-\bar{\nu}_e$. The values of different sources of virtual contributions as well as corresponding pieces in real interference are presented in the figs.5–6.

We stress however that these *numerical* consequences of our results for the observable quantities seem to depend strongly on the experimental procedure which will be used in the real life experiments. It must be clear from the above discussion that the cancellation of the soft real emission against the virtual corrections is quite delicate in the case of the production of the unstable particles. Therefore a more realistic treatment of the soft ($\omega \sim \Gamma$)radiation

is necessary. This is basically the main reason why we do not see much sense in an exhaustive numerical analyses of our formulae.

Another phenomenological issue which we mention here is the possibility to measure the invariant mass distribution of the top quark at e^+e^- and $\gamma\gamma$ colliders. Our formulae can be also applied for the $O(\alpha_s)$ non-factorizable corrections in this case in the spirit of [6], [11]. Let us note however that in this case the influence of the hadronization on the precise determination of the top mass should be considered. The discussion of this important issue can be found in the ref. [19].

To conclude, we want to emphasize once more that the non-factorizable corrections change the shape of the invariant-mass distribution while preserve the total probability [4-6]. As the study of the properties of the unstable fundamental particles requires the measurement of the invariant mass distributions our results must be taken into account while preparing for the analysis of the forthcoming high-precision experiments on the unstable particle production. Not only high statistics will be important but also our possibilities to make the correct correspondence between the results of the perturbative calculations of the masses and widths of the unstable particles in the framework of the Standard Model with the experimentally measured quantities.

6 Acknowledgments

We are indebted to Prof. J. G. Körner for support and encouragement in the course of this work. We would like to thank Prof. I. F. Ginzburg for a number of useful discussions. We are grateful to referees of journal of *Nucl. Phys. B*, whose criticism has been decisive in making this manuscript intelligible. K.M. is grateful to the Graduiertenkolleg "Elementarteilchenphysik" of the Mainz University for the support. O.I.Y. acknowledges support of the Deutsche Forschung Gemeinschaft.

Appendix

Let us introduce the following integral:

$$F_1(a,\beta|x_i)=\int\limits_{-1}^{1}\frac{dx}{x-a}\log(1+\beta x)(\theta(x-x_i)-\theta(x_i-x)). \qquad (62)$$

Here a is a complex number with the non-zero imaginary part. Also x_i is an arbitrary number satisfying $-1\le x_i\le 1$. The result of the integration then reads:

$$F_1(a,\beta|x_i)=-2\log\Big(\frac{(a-x_i)\beta}{1+\beta a}\Big)\log(1+\beta x_i)$$
$$+\log\Big(\frac{(1+a)\beta}{1+\beta a}\Big)\log(1-\beta)+$$
$$\log\Big(\frac{(a-1)\beta}{1+\beta a}\Big)\log(1+\beta)-2\mathrm{Li}_2\Big(\frac{1+\beta x_i}{1+a\beta}\Big)$$
$$+\mathrm{Li}_2\Big(\frac{1-\beta}{1+a\beta}\Big)+\mathrm{Li}_2\Big(\frac{1+\beta}{1+a\beta}\Big).$$

In our formulas we also need this function in the case when a is real but in some restricted cases, namely $a=x_i$. In this special case the divergence is of the collinear origin and we regularize it keeping the mass of the light particle finite. Hence the result for this function with $a=x_i$ reads:

$$F_1(x_i,\beta|x_i)=\log(1+\beta x_i)\log\Big(\frac{4{E_i}^2}{{m_i}^2}\Big)-\mathrm{Li}_2\Big(\frac{(1+x_i)\beta}{1+x_i\beta}\Big)-\mathrm{Li}_2\Big(\frac{(x_i-1)\beta}{1+x_i\beta}\Big).$$

Our next function is defined as following:

$$F_2(a|x_i)=\int\limits_{-1}^{1}\frac{dx}{x-a}(\theta(x-x_i)-\theta(x_i-x)). \qquad (63)$$

The result of the integration is:

$$F_2(a|x_i)=-2\log(x_i-a)+\log(-1-a)+\log(1-a). \qquad (64)$$

When $a=x_i$ this function equals:

$$F_2(x_i|x_i)=\log\Big(\frac{4{E_i}^2}{{m_i}^2}\Big). \qquad (65)$$

Next we present the quantities necessary for the eqs.(57-59):

$$R_4=\frac{2\beta x_4}{m_i^2\xi(x_4)},\quad R_{i=+,-}=\frac{-\beta x_i}{m_i^2\xi(x_i)}, \qquad (66)$$

$$R_{\xi,4}=\frac{2\beta(D_1+D_2)(K_4(D_1+D_2)+\beta(D_1-D_2)N_4)}{m_i^6(1+x_{34})\xi(x_4)\xi(x_+)\xi(x_-)}$$

where all notations are the same as in the main text of the paper. The quantities for the index 3 can be obtained from the previous ones by direct substitution $4\to 3$.

References

[1] R. G. Stuart, University of Michigan preprint UM-TH-95-13(1995) (hep-ph **9504308**) and references there in.

[2] I. Aeppli, G. Oldenborgh, D. Wyler, Nucl. Phys. **B428** (1994), 126.

[3] V.A. Khoze, T. Stjöstrand, Phys. Rev. Lett. **72** (1994), 28; Z. Physik **C62** (1994), 281.

[4] V. S. Fadin, V. A. Khoze and A. D. Martin, Phys. Rev. **D49** (1994), 2247.

[5] V. S. Fadin, V. A. Khoze and A. D. Martin, Phys. Lett. **D320** (1994), 141.

[6] K. Melnikov and O. Yakovlev, Phys. Lett. **B324** (1994), 217.

[7] V. S. Fadin, V. A. Khoze, Soviet JETP Lett. **46** (1987), 525.

[8] L. H. Orr, W. J. Stirling, V. A. Khoze, Nucl. Phys. **B378** (1992), 413.

[9] W. Beenakker and A. Denner, Int. Journ. Mod. Phys. **A9** (1994), 4837.

[10] W. Bernreuther et al., DESY report **92-123A**, e^+e^- Collisions at 500 GeV: The Physics potential, p.327.

[11] K. Fujii, T. Matsui, Y. Sumino, Phys. Rev. **D50** (1994), 4341.

[12] R. Yennie, S. C. Frautschi, H. Suura, Annals of Physics **13** (1961), 379.

[13] A. Sommerfeld, Atombau und Sperktrallinie, Braunschweig Vieweg, 1939

[14] S. Weinberg, Phys. Rev. **140** , 2B (1968), 516.

[15] D. Bardin, A. Denner and W. Beenakker, Phys. Lett. **B317** (1993), 213.

[16] G. t'Hooft, M. Veltman Nucl. Phys. **B153** (1979), 365.

[17] T. D. Lee, M. Nauenberg, Phys. Rev. **B133** (1964), 1549.

[18] M. Veltman, Physica **25** (1963), 186.

[19] V. A. Khoze, T. Sjöstrand, Phys. Lett. **B328** (1994), 466.

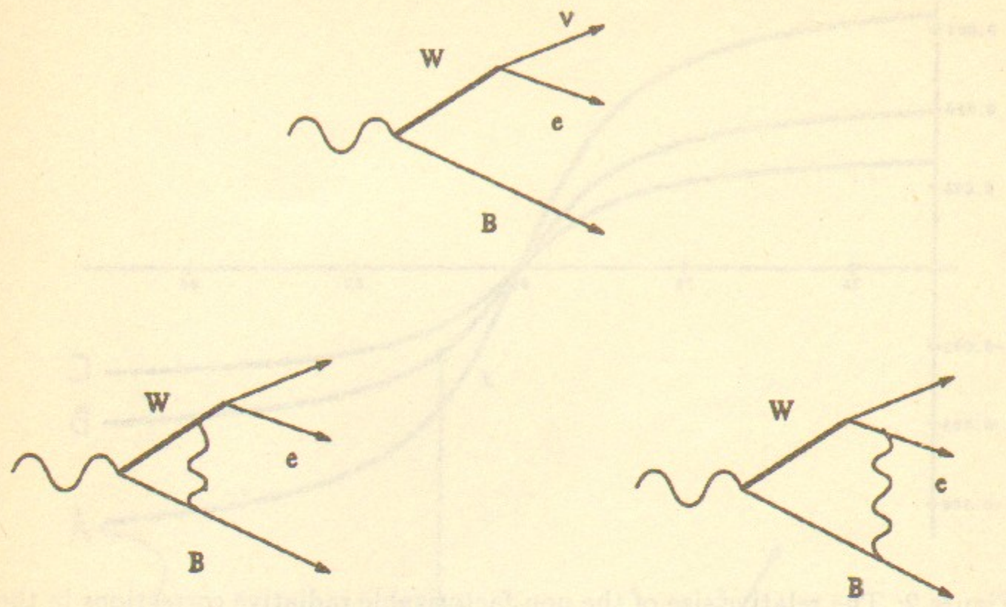

Figure 1: Born graph and graphs responsible for the non-factorizable corrections for the simple model (see sect.2).

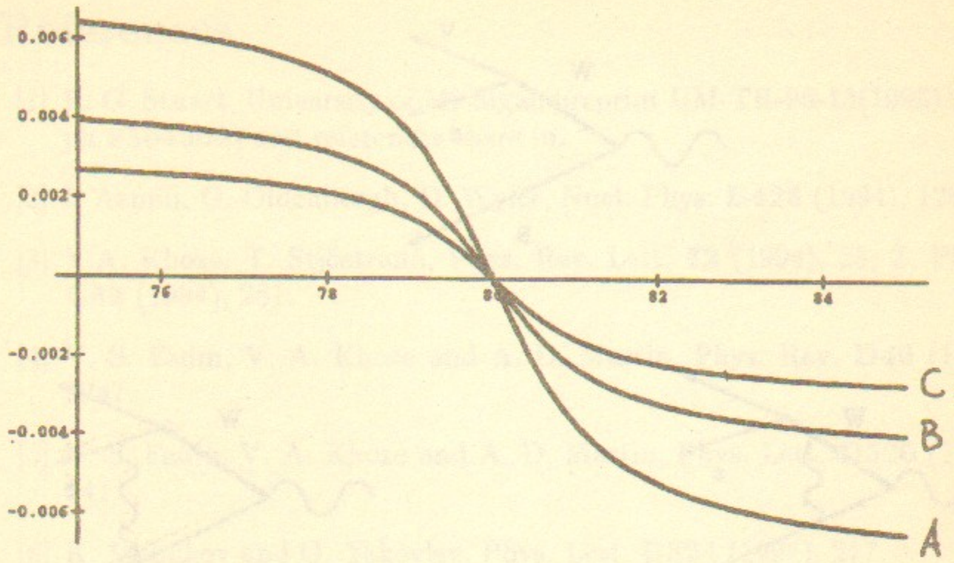

Figure 2: The relative size of the non-factorizable radiative corrections in the simple model (see eq.(11) with η from eq.(15)). Curves A, B, C correspond to the total energies $\sqrt{s} = 180$, 190, 200 GeV respectively. We use $m_W = 80$ GeV and $\alpha = 1/137$.

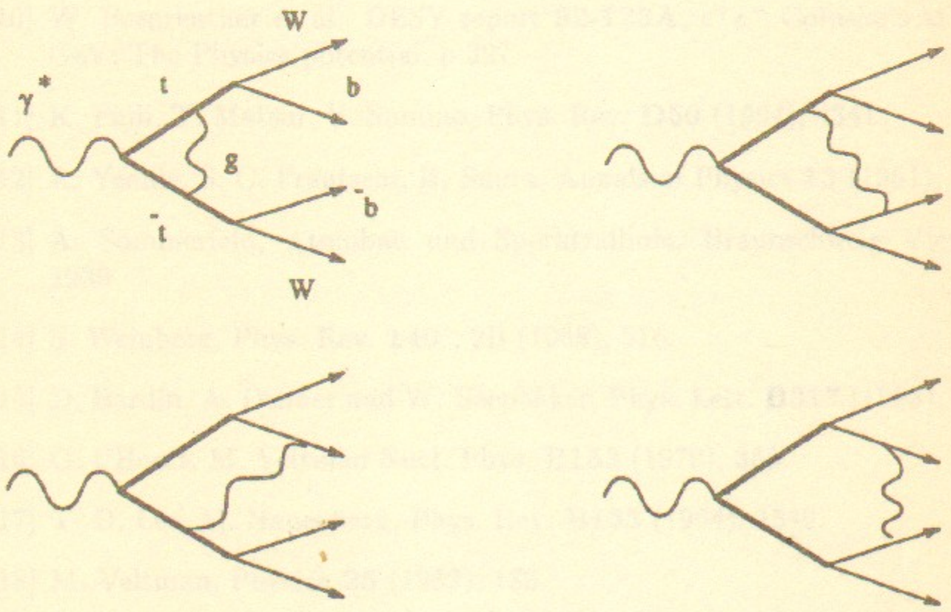

Figure 3: Non-factorizable graphs for the process $\gamma^* \to t\bar{t} \to bW^+\bar{b}W^-$

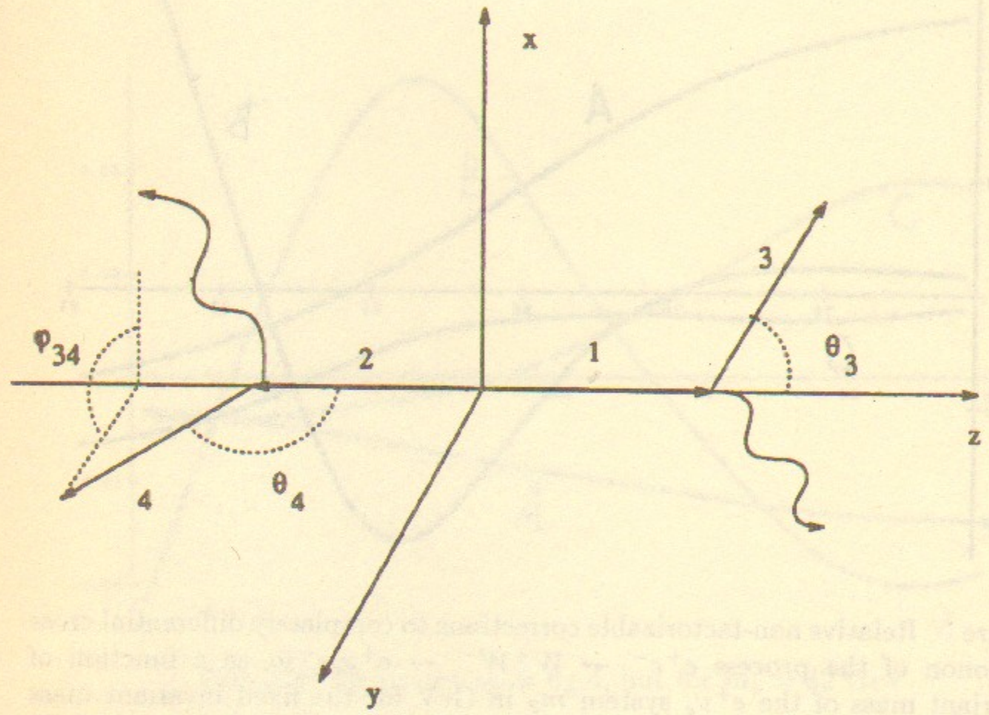

Figure 4: Geometry of the discussed reactions.

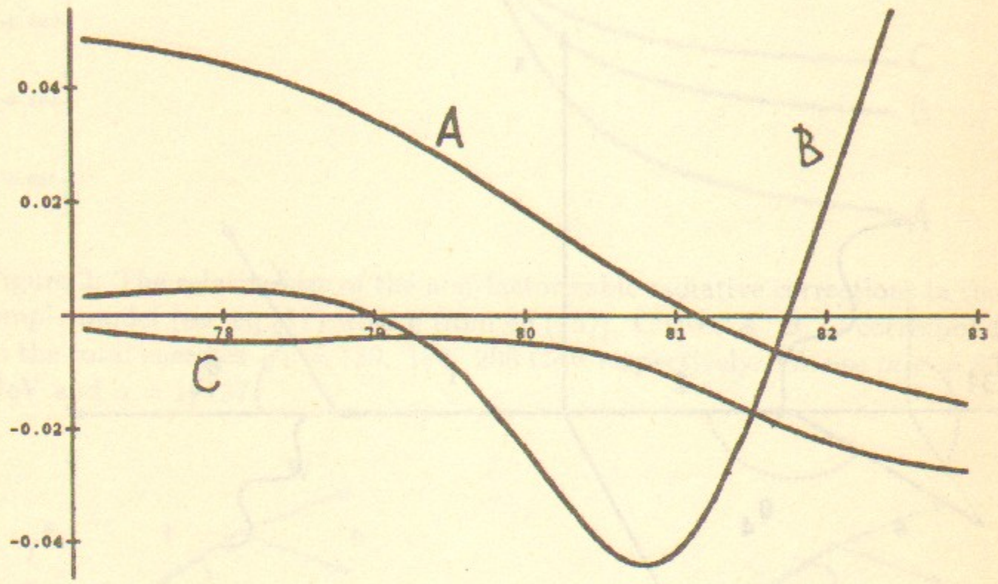

Figure 5: Relative non-factorizable corrections to completely differential cross sectionon of the process $e^+e^- \to W^+W^- \to e^+\nu_e e^-\bar{\nu}_e$ as a function of invariant mass of the $e^+\nu_e$ system m_2 in GeV for the fixed invariant mass of $e^-\bar{\nu}_e$ $m_1 = 78$ GeV. We use $\sqrt{s} = 180$ GeV, $m_W = 80$ GeV, $\alpha = 1/137$, $\theta_{W-e^-} = 30°$, $\theta_{W-e^+} = 150°$, $\varphi_{e^+e^-} = 0$. Curves A, B, C correspond to the conrtibutions due to three–, four–, and five–point functions respectively.

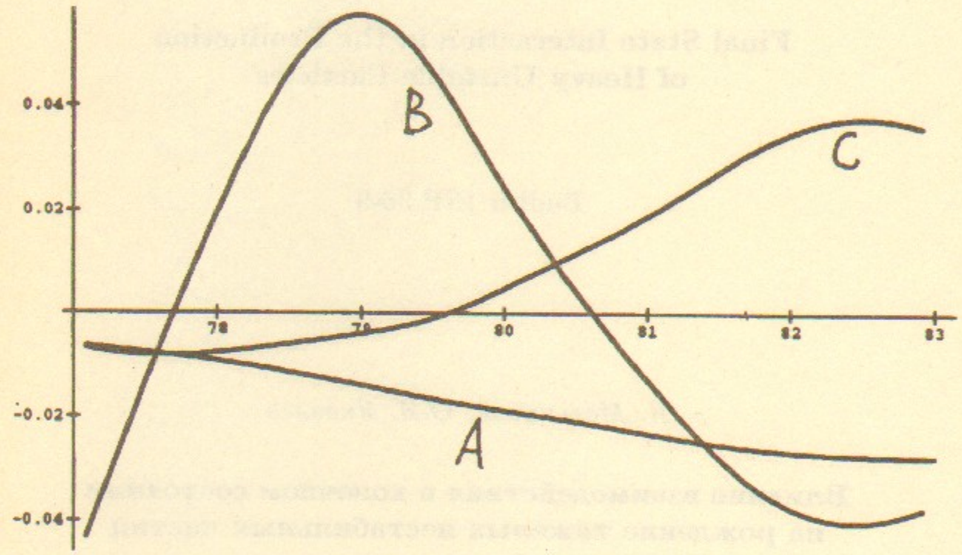

Figure 6: The same as in fig.4, but for $m_1 = 82$ GeV.

K. Melnikov, O.I. Yakovlev

Final State Interaction in the Production of Heavy Unstable Particles

Budker INP 96-9

К. Мельников, О.И. Яковлев

Влияние взаимодействия в конечном состоянии на рождение тяжелых нестабильных частиц

Ответственный за выпуск С.Г. Попов
Работа поступила 04.03.1996 г.

Сдано в набор 06.03.1996 г.
Подписано в печать 06.03.1996 г.
Формат бумаги 60×90 1/16 Объем 2.2 печ.л., 1.8 уч.-изд.л.
Тираж 150 экз. Бесплатно. Заказ N° 9

Обработано на IBM PC и отпечатано на
ротапринте ГНЦ РФ "ИЯФ им. Г.И. Будкера СО РАН",
Новосибирск, 630090, пр. академика Лаврентьева, 11.