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COHERENT UNDULATOR RADIATION OF ELECTRON BEAM, MICROBUNCHED FOR THE FEL POWER OUTCOUPLING

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НОВОСИБИРСК

Coherent undulator radiation of electron beam, microbunched for the FEL power outcoupling

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Abstract

The intensity of the coherent undulator radiation of electron beam, preliminarily microbunched by the FEL master oscillator for the FEL power outcoupling, is approximately calculated by simple analytic considerations, taking into account the transverse emittance and the energy spread of the electron beam.

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In our previous paper [1] we discussed the conservation of spatial-temporal correlations of longitudinal density of the electron beam in two undulators, which are separated by achromatic magnetic system. This property causes the mutual coherency of radiation from these undulators, which we had really observed earlier [2]. If the first undulator is a part of magnetic system of the oscillator FEL (see Fig.1), i.e. is situated on the axis of the FEL optical resonator, then the electron beam will be longitudinally modulated on energy

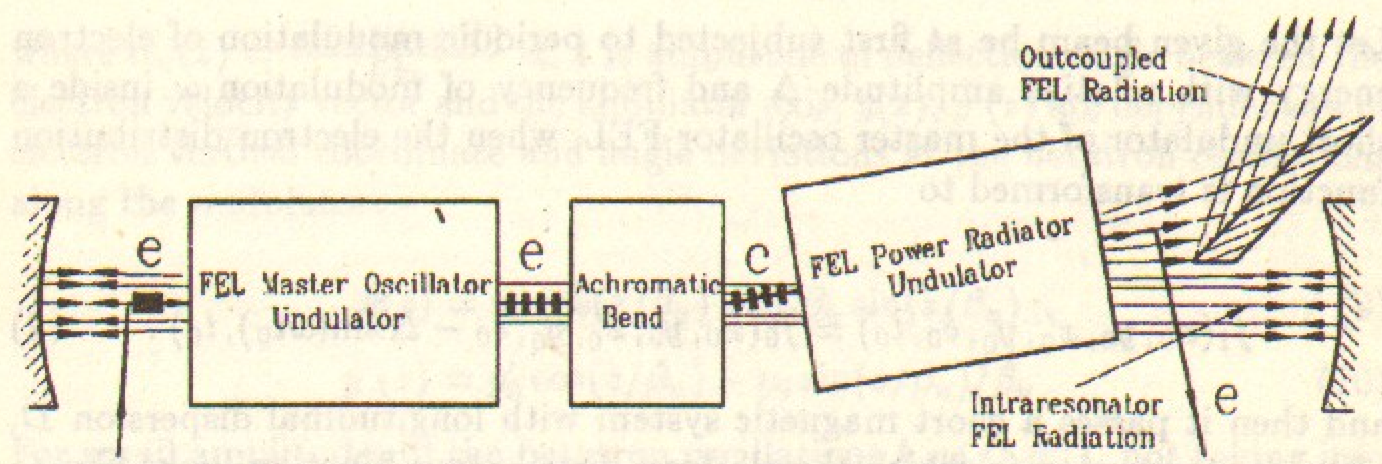

Fig. 1. The schematic diagram of the electron radiation outcoupling from the oscillator FEL.

and density. Then, going through the second undulator (radiator), this beam will coherently radiate on the wavelength of its longitudinal modulation (the wavelength of the master oscillator FEL). We can deflect this coherent radiation from the axis and take it out of the optical resonator, having placed the magnetic system of achromatic bend on a small angle between the first and the second undulator. Let's note, that in this case the transverse mode composition of intraresonator FEL radiation doesn't influence on the angular distribution of the outcoupled FEL radiation, that's why the use of confocal optical resonator in the master oscillator FEL seems attractive here [3,4].

Now we'll discuss the quantity characteristics of the coherent undulator radiation of the electron beam, which was preliminary modulated in the master oscillator FEL. Such radiation has been already discussed in a number of works [5,6] without taking into account the influence of emittance and energy spread of electron beam in undulator. Let's study the stationary continuous beam of relativistic electrons, which spreads with velocity v, close to the light velocity c ($v \approx c$) along the axis Z, with the electron distribution function $f_0(x_0, y_0, x'_0, y'_0, \delta_0, t_0)$ in plane $z = const$ on their deviations of transverse coordinates x_0, y_0, of angles x'_0, y'_0 and relative deviation of energy δ_0 (t_0 is the time of particle passing through this plane). Let's note, that f_0 is the density of particle flow in a phase space with the normalization

$$\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} f_0(x_0, y_0, x'_0, y'_0, \delta_0, t_0)\,dx_0\,dy_0\,dx'_0\,dy'\,d\delta_0 = 1\,. \tag{1}$$

Let the given beam be at first subjected to periodic modulation of electron energy with relative amplitude Δ and frequency of modulation ω inside a short undulator of the master oscillator FEL, when the electron distribution function is transformed to

$$f_1(x_0, y_0, x'_0, y'_0, \delta_0, t_0) = f_0(x_0, y_0, x'_0, y'_0, \delta_0 - \Delta\sin(\omega t_0), t_0)\,. \tag{2}$$

and then it passes a short magnetic system with longitudinal dispersion D, after the conversion in which the electron distribution function looks like

$$f_2(x_0, y_0, x'_0, y'_0, \delta_0, t_0) = f_1(x_0, y_0, x'_0, y'_0, \delta_0, t_0 - \delta_0 D/c) = \tag{3}$$

$$= f_0(x_0, y_0, x'_0, y'_0, \delta_0 - \Delta\sin(\omega t_0 - kD\delta_0), t_0 - \delta_0 D/c)\,, \tag{4}$$

where $k = \omega/c$.

Then, let such modulated electron beam pass through a planar undulator, in median plane of which the magnetic field H is parallel to axis Y and

oscillates along the axis Z with amplitude H_0. Its dependence on coordinates y and z near the undulator axis is described as

$$H_y(y, z) = H_0 ch(k_u y)\cos(k_u z)\,, \tag{5}$$

where $k_u = 2\pi/\lambda_u$, λ_u is a period of undulator. Such distribution of magnetic field provides focussing of electron vertical betatron oscillations with longitudinal period $2\pi\beta_u \gg \lambda_u$, where β_u is matched beta-function of electron beam in undulator:

$$\beta_u = \frac{pc\sqrt{2}}{eH_0} \approx \frac{E\sqrt{2}}{eH_0}\,.$$

p and E are momentum and energy of electrons, respectively. Inside the undulator, in components of electron velocity vector oscillating parts appear with the longitudinal frequency k_u:

$$v_x(z) = v[x'_0 - \alpha_u(z)\sin(k_u z)]\,, \tag{6}$$

(6)

$$v_y(z) = vy'(z)\,, \tag{7}$$

$$v_z(z) = \sqrt{v^2 - v_x^2 - v^2} \approx v\{1 - [(x'_0\alpha_u(z)\sin(k_u z))^2 + y'^2(z)]/2\}, \tag{8}$$

where $\alpha_u(z) = \frac{eH_0 ch[k_u y(z)]}{Ek_u} \ll 1$ is amplitude of deflection angle between the electron velocity vector and the undulator axis; $y(z)$, $y'(z)$ are dependencies of electron vertical coordinate and angle deviations at the betatron oscillations along the undulator:

$$y(z) = y_0\cos(z/\beta_u) + y'_0\beta_u\sin(z/\beta_u)\,; \tag{9}$$

$$y'(z) = y'_0\cos(z/\beta_u) - y_0\sin(z/\beta_u)/\beta_u\,. \tag{10}$$

For small amplitudes of the betatron oscillations $k_u y(z) \ll 1$, not taking into consideration small quickly oscillating parts, supposing

$$\beta_u \approx \frac{E_0\sqrt{2}}{eH_0}\,, \tag{11}$$

we'll get from (8)

$$v_z(z) \approx v\left\{1 - \frac{[(1-2\delta)\alpha_{u0}^2/2 + +x_0'^2 + y_0^2/\beta_u^2 + y_0'^2]}{2} + \right.$$

$$+\frac{\alpha_{u0}^2}{4}\cos[2k_u z]\Bigg\}, \tag{12}$$

where $\alpha_{u0} = \frac{eH_0}{E_0 k_u}$ and E_0 are mean deflection angle amplitude and mean energy of the electrons in the undulator, respectively. From (12), in particular, one can see that the electron longitudinal velocity v_z, averaged over the undulator period, keeps its initial value at passing along the undulator. In more general case, when the undulator construction provides both the vertical and horizontal focussing, the vertical magnetic field near the undulator axis is described by the dependence

$$H_y(x,y,z) = H_0 ch(k_{ux}x) ch(k_{uy}y)\cos(k_u z), \tag{13}$$

where $k_{ux}^2 + k_{uy}^2 = k_u^2$. Then the dependence of the longitudinal electron velocity along the undulator gets a form, analogical to (12):

$$v_z(z) \approx v\Bigg\{1 - \frac{[(1-2\delta)\alpha_{u0}^2/2 + x_0^2/\beta_{ux}^2 + x_0'^2 + y_0^2/\beta_{uy}^2 + y_0'^2]}{2} +$$

$$+\frac{\alpha_{u0}^2}{4}\cos[2k_u]\Bigg\}, \tag{14}$$

where

$$\beta_{ux} = \beta_u k_u/k_{ux}, \quad \beta_{uy} = \beta_u k_u/k_{uy}, \quad 1/\beta_{ux}^2 + 1/\beta_{uy}^2 = 1/\beta_u^2, \tag{15}$$

and the transverse components of the velocity vector are expressed as

$$v_x(z) \approx v[x'(z) - \alpha_{u0}\sin(k_u z)], \tag{16}$$

$$v_y(z) = v y'(z), \tag{17}$$

$$x'(z) = x_0'\cos(z/\beta_{ux}) - x_0\sin(z/\beta_{ux})/\beta_{ux}, \tag{18}$$

$$y'(z) = y_0'\cos(z/\beta_{uy}) - y_0\sin(z/\beta_{uy})/\beta_{uy}. \tag{19}$$

Let's describe the radiation field of moving electron by Fourier-harmonic of its vector-potential [7]

$$\vec{A}_\omega = \frac{e}{c}\int \frac{\vec{v}(t)}{R(t)}\exp\{i\omega[t + R(t)/c]\}dt, \tag{20}$$

where $R(t)$ is distance from the electron to the observation point at the time t. In far zone of radiation field

$$\vec{R}(t) = \vec{R}_0 - \vec{r}(t), \quad |\vec{R}_0| \gg |\vec{r}(t)|, \tag{21}$$

where $\vec{R}_0$ is radius-vector from the undulator to the observation point, $\vec{r}(t)$ is radius-vector of electron moving along the undulator; making replacement of the integration variable

$$t = t_0' + \int_0^z \frac{dz_1}{v_z(z_1)}, \tag{22}$$

where t_0' is the time of electron coming to the undulator input, we'll get

$$\vec{A}_\omega = \frac{e}{cR_0}\exp(ikR_0)\int_0^L \frac{\vec{v}(z)}{v_z(z)}\exp\left\{i\left[\omega t_0' + \omega\int_0^z \frac{dz_1}{v_z(z_1)} - \vec{k}\,\vec{r}(z)\right]\right\}dz, \tag{23}$$

where L is length of the undulator, $\vec{k}$ – wave vector of radiation ($|\vec{k}| = \omega/c$), directed to the observation point,

$$\vec{r}(z) = [x(z) + \alpha_{u0}\cos(k_u z)/k_u,\ y(z),\ z],$$

$$x(z) = x_0\cos(z/\beta_{ux}) + x_0'\beta_{ux}\sin(z/\beta_{ux}), \tag{24}$$

$$y(z) = y_0\cos(z/\beta_{uy}) + y_0'\beta_{uy}\sin(z/\beta_{uy}). \tag{25}$$

The total vector-potential of radiation field of electron beam with the distribution function at the undulator input $f_2(x_0, y_0, x_0', y_0', \delta_0', t_0')$, taking into account the normalization (1), is expressed as

$$\vec{A}(t) = 2\mathrm{Re}\Bigg\{\frac{I\omega\exp(ikR_0 - i\omega t)}{2\pi cR_0}\int_0^L\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\pi/\omega}^{+\pi/\omega}\frac{\vec{v}(z)}{v_z(z)}\times$$

$$\times\exp\left\{i\left[\omega t_0' + \omega\int_0^z \frac{dz_1}{v_z(z_1)} - \vec{k}\,\vec{r}(z)\right]\right\}\times$$

$$f_2(x_0, y_0, x_0', y_0', \delta_0', t_0')\,dt_0' dx_0 dy_0 dx_0' dy_0' d\delta_0' dz\Bigg\}, \tag{26}$$

where I is total current of the electron beam.

Knowing, that the total power direction diagram of the first harmonic of electron undulator radiation is concentrated in the angular cone with the opening angle about $1/\gamma_*$ [8] (where $\gamma_* = \gamma/\sqrt{1+K^2/2}$, γ is electron relativistic factor, $K = \gamma\alpha_{u0}$ is undulator parameter), let's define the area of the subsequent approximate consideration of the coherent undulator radiation, having limited r.m.s. dispersion of the electron angular spread $\sigma_{x',y'}$ in the beam

$$\sigma_{x',y'} \ll 1/\gamma_* \tag{27}$$

and diffractive divergence of the coherent radiation of electron beam with r.m.s. dispersion of its transverse size $\sigma_{x,y}$

$$\frac{1}{k\sigma_{x,y}} \ll 1/\gamma_*\,, \quad \text{i.e.}$$

$$\sigma_{x,y} \gg \gamma_*/k\,. \tag{28}$$

With such approximation, having omitted small parts and rewritten in (26) subintegral expressions as

$$\frac{v_x(z)}{v_z(z)} \approx -\alpha_{u0}\sin(k_u z)\,, \tag{29}$$

$$\omega\int_0^z \frac{dz_1}{v_z(z_1)} - \vec{k}\,\vec{r}(z) \approx$$

$$\approx k\{[(1-2\delta_0')/\gamma_*^2 + \theta_x^2 + \theta_y^2 + x0^2/\beta_{ux}^2 + x_0'^2 + y_0^2/\beta_{uy}^2 + y_0'^2]\,z/2-$$

$$-\frac{\alpha_{u0}^2}{8k_u}\sin[2k_u z] - \theta_x x[z] - \theta_y y[z]\}, \tag{30}$$

we'll get, that the vector-potential of considered undulator radiation field of the first harmonic at observation small angles, relative to the undulator axis ($\theta_{x,y} \ll 1/\gamma_*$), mainly contains only one component:

$$A_x(t) \approx \mathrm{Re}\left\{\frac{-iI}{cR_0}\exp(ikR_0 - i\omega t)\alpha_{u0}\left[J_0(X) - J_1(X)\right]\cdot I_z\right\}, \tag{31}$$

where

$$I_z = \frac{\omega}{2\pi}\int_0^L\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-pi/\omega}^{+\pi/\omega} f_2(x_0, y_0, x_0', y_0', \delta_0', t_0')\times$$

$$\times\exp\{i\,[\omega t_0' + +[k((1-2\delta_0')/\gamma_*^2 + \theta_x^2 + \theta_y^2 + x_0^2/\beta_{ux}^2 + x_0'^2 + y_0^2/\beta_{uy}^2 + y_0'^2)/$$

$$/2 - k_u]\,z - \theta_x x(z) - \theta_y y(z)\Big]\Big\}\, dt_0' dx_0 dy_0 dx_0' dy_0' d\delta_0' dz\,. \tag{32}$$

$X = \frac{k\alpha_{u0}^2}{8k_u}$ is argument of Bessel functions of zero $J_0(X)$ and first $J_1(X)$ orders. Taking into account (4) and having made replacements of variables

$$t_0' = t_0 + \delta_0' D/c = t_0 + [\delta_0 + \Delta\sin(\omega t_0)]D/c, \;\; \delta_0' = \delta_0 + \Delta\sin(\omega t_0), \tag{33}$$

from (32) we get

$$I_z = \frac{\omega}{2\pi}\int_0^L\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-pi/\omega}^{+\pi/\omega} f_0(x_0, y_0, x_0', y_0', \delta_0, t_0)\times$$

$$\times\exp\{i\,[\omega t_0 - \sin(\omega t_0)(X_0 + kz\Delta/\gamma_*^2) - \delta_0 X_0/\Delta + [k((1-2\delta_0)\,/$$

$$/\,\gamma_*^2 + \theta_x^2 + \theta_y^2 + +x_0^2/\beta ux^2 + x_0'^2 + y_0'^2/\beta_{uy}^2 + y'^2)/2 - k_u]\,z-$$

$$-\theta_x x(z) - \theta_y, y(z)\Big]\Big\}\, dt_0 dx_0 dy_0 dx_0' dy_0' d\delta_0 dz\,, \tag{34}$$

where $X_0 = -kD\Delta$ is the parameter of beam bunching at the undulator input. For the initial beam with the distribution function, independent on time, after integrating over t_0 we'll get

$$I_z = \frac{\omega}{2\pi}\int_0^L\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_0(x_0, y_0, x_0', y_0', \delta_0)\times$$

$$\times J_1(X_0 + kz\Delta/\gamma_*^2)\exp\Big\{i\,[-\delta_0 X_0/\Delta +$$

$$+[((1-2\delta_0)/\gamma_*^2 + \theta_x^2 + \theta_y^2 + x_0^2/\beta_{ux}^2 + x_0'^2 + y_0^2/\beta_{uy}^2 + y_0'^2)/2 - k_u]\,z-$$

$$-\theta_x x(z) - \theta_y y(z)\Big]\Big\}\, dx_0 dy_0 dx_0' dy_0' d\delta_0 dz \tag{35}$$

Then, expressing the radiation magnetic field as

$$H_{\omega y} \approx ikA_{\omega x}\,, \tag{36}$$

we write an expression for the angular distribution of coherent radiation power:

$$\frac{dP}{d\Omega} \approx \frac{\bar{H}_y^2}{4\pi} cR_0^2 \approx \frac{\bar{A}_x^2}{4\pi} ck^2 R_0^2 = \frac{\{Ik\alpha_{u0}[J_0(X) - J_1(X)]\}^2}{8\pi c} |I_z|^2. \tag{37}$$

The total power of coherent radiation can be expressed as

$$P = I^2 \cdot Z_u\,, \tag{38}$$

where

$$Z_u = \frac{\{k\alpha_{u0}[J_0(X) - J_1(X)]\}^2}{8\pi c} \int |I_z|^2 d\Omega \tag{39}$$

is effective impedance of the undulator. For small values of the transverse emittance, the energy spread and the energy modulation of the electron beam, when

$$kL\Delta/\gamma_*^2 \ll 1$$

and for all electrons of the beam along the undulator with $0 < z < L$

$$kz(-2\delta_0/\gamma_*^2 + x_0^2/\beta_{ux}^2 + x_0'^2 + y_0^2/\beta_{uy}^2 + y_0'^2)/2 - \delta_0 X_0/\Delta - \theta_x x(z) - \theta_y y(z) \ll 1,$$

the expression (34) takes the form

$$|I_{z0}| = J_1(X_0) \left| \int_0^L \exp\{i[k(1/\gamma_*^2 + \theta^2)/2 - k_u]z\} dz \right| =$$

$$= J_1(X_0) L \left| \frac{\sin(kL\theta^2/4 - \kappa)}{kL\theta^2/4 - \kappa} \right|, \tag{40}$$

where $\theta^2 = \theta_x^2 + \theta_y^2$, $\kappa \doteq \left[k_u - \frac{k}{2\gamma_*^2}\right] L/2 = kL\theta_0^2/4$ is resonance detuning of the undulator, and θ_0 is observation angle, which corresponds to the wavelength λ in angular dependence of the spontaneous undulator radiation spectrum

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + K^2/2 + \gamma^2\theta_0^2)\,,$$

and after subsequent integration over solid angle we get (with $\kappa \gg 1$)

$$\int |I_{z0}|^2 d\Omega = 4\pi J_1^2(X_0) L/k \int_{-\kappa}^{+\infty} \frac{\sin^2(\xi)}{\xi^2} d\xi \approx 4\pi^2 J_1^2(X_0) L/k. \tag{41}$$

In this case with

$$k \approx \frac{4\pi\gamma^2}{\lambda_u(1 + K^2/2)} \tag{42}$$

the maximum effective impedance of the undulator is expressed as

$$Z_{u0} \approx q \frac{2\{\pi K[J_0(X) - J_1(X)] J_1(X_0)\}^2}{c(1 + K^2/2)}\,, \tag{43}$$

where $q = L/\lambda_u$ is a number of undulator periods. For $K \gg 1$ and $\max[J_1(X_0)] \approx J_1(1.84) \approx 0.582$

$$Z_{u0} \approx q \cdot 196\ \text{Ohm}\,. \tag{44}$$

The expression (39) can be rewritten in the form

$$Z_u \approx Z_{u0} \cdot 0.47 \cdot \frac{\int |I_z|^2 d\Omega}{\lambda L}\,. \tag{45}$$

Having made replacements of the variables $\zeta = z/L$ and $\xi = kL\theta^2/4$ we'll rewrite the expression (40) in the form

$$|I_{z0}| = J_1(X_0) L \cdot \left| \int_0^1 \exp\{i[\xi - \kappa] \cdot 2\zeta\} d\zeta \right|. \tag{46}$$

Let's note, that at slow changing of the undulator detuning $\kappa = \kappa(\zeta)$ along its axis

$$\left| \frac{d\kappa}{d\zeta} \right| \frac{\lambda_u}{L} \ll 1\,, \tag{47}$$

with $\kappa \gg 1$ the Z_{u0} value is constant, since the integration over solid angle (41) is approximately reduced to a certain integral of the form

$$\int_{-\infty}^{+\infty}\left|\int_0^1 \exp(i\xi\zeta)\cdot\exp[iF(\zeta)]\,d\zeta\right|^2 d\xi \equiv 2\pi\,, \qquad (48)$$

for arbitrary real function $F(\zeta)$. Such slow changing of the detuning κ with the motion along the undulator axis can be caused by the decrease of electron beam energy, owing to the radiation losses, which are not limited in this case by maximum possible quantity of the relative radiation losses of electron energy ($\sim \frac{1}{4q}$) for the conventional oscillator FEL with homogeneous undulator.

Instead of using the well-known tapered undulator for the removal of limitation of radiation losses in the oscillator FEL, another variant of the electron FEL radiation outcoupling can also be useful. For instance (see Fig.2a), an achromatic bend can be removed from the electron outcoupling scheme at Fig.1, but instead of it, we can essentially detune the additional undulator-radiator (at high detuning $\kappa \gg 1$ the electron beam inside the radiator practically doesn't interact with intraresonator radiation from the master FEL undulator) and an additional intraresonator mirror with the opening, that let pass the main mode of intraresonator FEL radiation. The

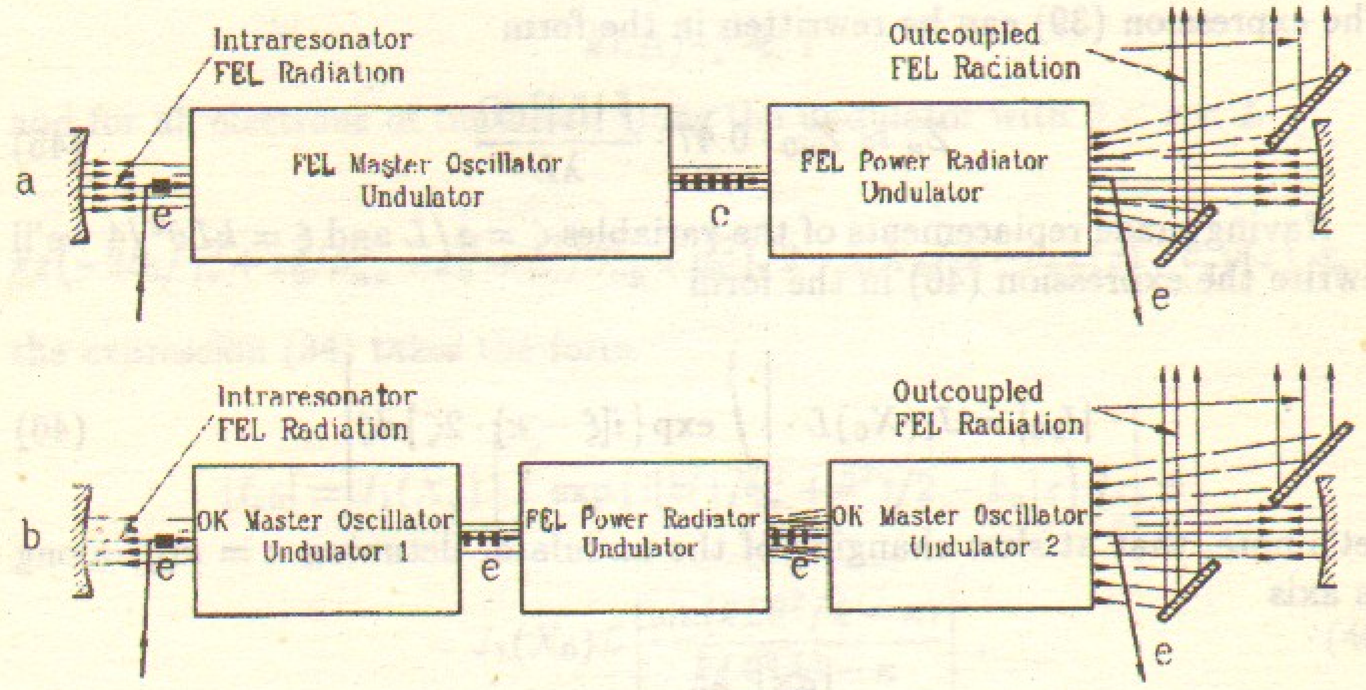

Fig. 2. The schematic diagram of the "cone" electron radiation outcoupling from the oscillator FEL (a) and the oscillator OK (b).

own gain coefficient of the detuned undulator should not exceed the value of radiation losses in optical resonator, which are regulated by change of the opening aperture of the intraresonator mirror. It will be necessarily to optimize the intraresonator radiation losses in the master FEL saturation regime in order to get maximum power of coherent radiation, which was emitted in a form of empty angular cone from the undulator-radiator and outcoupled by the intraresonator mirror. Such "cone" outcoupling seems also attractive, when the undulator-radiator is used as dispersive section in master oscillator optical klystron (OK), i.e. between its two main undulators (see Fig.2b). In this case an electron beam in the oscillator OK saturation regime comes to the OK undulator 2 input, having the beam bunching parameter X_0 (see expression 34), which essentially exceeds its optimum at $X_{0\,opt} \approx 1.84$ (see expression 44). Thus, the optimal beam bunching takes place inside of the undulator-radiator (the OK dispersive section), its effective impedance will therefore be the highest and where the greatest part of the OK coherent radiation will be outcoupled from the OK optical resonator.

The quantity of the effective impedance Z_u of the undulator-radiator can be much lower than Z_{u0}, due to the emittance and the energy spread of electron beam, the influence of whose we'll study further. Let's study the beam with the gaussian function of the electron distribution

$$f_0(x_0, y_0, x_0', y_0', \delta_0) = \frac{1}{(2\pi)^{5/2}\epsilon_x\epsilon_y\sigma_e}\times$$
$$\times \exp\left(-\frac{\delta_0^2}{2\sigma_e^2} - \frac{\beta_x x_0'^2 + 2\alpha_x x_0' x_0 + \gamma_x x_0^2}{2\epsilon_x} - \frac{\beta_y y_0'^2 + 2\alpha_y y_0' y_0 + \gamma_y y_0^2}{2\epsilon_y}\right), \qquad (49)$$

where σ_e is the relative dispersion of electron energy spread; $\epsilon_{x,y}$ are transverse emittances of the electron beam; $\alpha_{x,y}$, $\beta_{x,y}$, $\gamma_{x,y}$ are Twiss parameters for transverse phase ellipses of the electron beam at the undulator input. Having omitted details of integration (35) over the energy spread, the transverse coordinates and angles in the electron beam, we'll write its final result:

$$I_z = \int_0^L F_x(z)F_y(z)F_e(z)\exp\{iz[k(1/\gamma_*^2 + \theta_x^2 + \theta_y^2)/2 - k_u]\}dz\,, \qquad (50)$$

where

$$F_x(z) = F(\theta_x, \varepsilon_x, \alpha_x, \beta_x, \gamma_x, z) =$$

$$= \exp\left\{ -\frac{k^2\theta_x^2\varepsilon_x}{2[\gamma_x - ikz\varepsilon_x/\beta_{ux}^2]} \left[\cos^2(z/\beta_{ux}) + \frac{[(\gamma\beta_{ux} - ikz\varepsilon_x/\beta_{ux})\sin(z/\beta_{ux}) - \alpha_x\cos(z/\beta_{ux})]^2}{1 - ikz\varepsilon_x(\beta/\beta_{ux}^2 + \gamma_x) - (kz\varepsilon_x/\beta_{ux})^2}\right]\right\} / \sqrt{1 - ikz\varepsilon_x(\beta/\beta_{ux}^2 + \gamma_x) - (kz\varepsilon_x/\beta_{ux})^2}\,; \qquad (51)$$

$$F_y(z) = F(\theta_y, \varepsilon_y, \alpha_y, \beta_y, \gamma_y, z);$$

$$F_e(z) = J_1(X_0 + kz\Delta/\gamma_*^2)\exp\{-[(X_0 + kz\Delta/\gamma_*^2)\sigma_e/\Delta]^2/2\}\,. \qquad (52)$$

Let's study the case of small transverse emittances, when $F_{x,y}(z) \approx 1$ is carried out along the whole undulator ($0 < z < L$). Then we can approximately write the integral (50) in the form

$$I_{ze} \approx \int_0^L F_e(z)\exp\{ikz(\theta^2 - \theta_0^2)/2\}\,dz, \qquad (53)$$

and the following integration over solid angle in the form

$$\int |I_{ze}|^2 d\Omega \approx \pi \int_0^L\int_0^L\int_0^\infty F_e(z)F_e(z_1)\exp\{ik(z-z_1)(\theta^2-\theta_0^2)/2\}dz dz_1 d\theta^2. \quad (54)$$

For high undulator detuning ($\kappa \gg 1$), using that

$$\int_{-\infty}^{+\infty} \exp(ikx)dx = 2\pi\delta(k),$$

we'll get the expression

$$\int |I_{ze}|^2 d\Omega \approx 4\pi^2/k \int_0^L F_e^2(z)\,dz,$$

which maximum value for infinite long undulator is reached at $X_0 = 0$ and forms

$$\int |I_{ze}|^2 d\Omega \approx (2\pi\gamma_*/k)^2/\sigma \int_0^\infty J_1^2(\xi\Delta/\sigma)\exp(-\xi^2)\,d\xi. \qquad (55)$$

Taking into account, that

$$\max\left[\int_0^\infty J_1^2(\xi\Delta/\sigma)\exp(-\xi^2)\,d\xi\right] \approx 0.148 \quad \text{with} \quad \Delta/\sigma \approx 2.54, \qquad (56)$$

from (45) we'll get the expression for the undulator maximal possible impedance at the energy spread of the electron beam:

$$Z_{ue} \approx 6.83\,\text{Ohm}/\sigma_e\,. \qquad (57)$$

Accordingly to (44), at $Z_{ue} \gg 196$ Ohm the effective number of the undulator radiating periods can be defined as

$$q_e = Z_{ue}/196\,\text{Ohm} = 0.035/\sigma_e.$$

Let's note, that when using the magnetic system with positive value of longitudinal dispersion (signopposite to the undulator dispersion value) for the electron beam bunching at the undulator input, the maximum impedance of the infinite undulator is reached at $-X_0 \gg 1$ and is twice as much the quantity (57).

Let's study the case of small electron energy spread, when

$$F_e(z) \approx \max[J_1(X_0)] \approx J_1(1.84) \approx 0.582$$

is carried out along the whole undulator ($0 < z < L$). Then the integral (50) can be approximately rewritten as

$$I_{z\varepsilon} \approx 0.582 \cdot \int_0^L F_x(z)F_y(z)\exp\{ikz(\theta^2 - \theta_0^2)/2\}\,dz. \qquad (58)$$

Let's consider the long undulator with equal horizontal and vertical focussing and the electron beam with matched beta-functions at the undulator input:

$$\alpha_{x,y} = 0; \quad \beta_{x,y} = \beta_u; \quad \gamma_{x,y} = 1/\beta_u\,. \qquad (59)$$

From (51), supposing $\varepsilon_x = \varepsilon_y = \varepsilon$, we'll get

$$F_x(z)\cdot F_y(z) = \exp\left[-\frac{k^2\theta^2\beta_u\varepsilon}{2(1-ikz\varepsilon/\beta_u)}\right] / (1-ikz\varepsilon/\beta_u)^2. \qquad (60)$$

Passing on to the dimensionless variables $I_{z\varepsilon}$, z, ε, θ, θ_0:

$$I_{z\varepsilon}k\varepsilon/\beta \to I_{z\varepsilon};\; kz\varepsilon/\beta \to z;\; k\varepsilon \to \epsilon;\; \frac{\beta}{\varepsilon}\cdot\theta^2/2 \to \theta^2;\; \frac{\beta}{\varepsilon}\cdot\theta_0^2/2 \to \theta_0^2, \qquad (61)$$

from (58) we'll get for the infinite long undulator

$$I_{z\varepsilon}(\varepsilon,\theta_0^2,\theta^2) = 0.582\int_0^\infty \exp\left[-\frac{\varepsilon^2\theta^2}{(1-iz)} + iz(\theta^2-\theta_0^2)\right] / (1-iz)^2 dz. \qquad (62)$$

Having defined the function

$$F_\epsilon(\varepsilon) = \max[F_\varepsilon(\epsilon,\theta_0^2)] \quad \text{over} \quad \theta_0^2\,, \qquad (63)$$

where

$$F_\varepsilon(\epsilon,\theta_0^2) = \frac{2}{\pi^2}\int_0^\infty |I_{z\varepsilon}(\varepsilon,\theta_0^2,\theta^2)|^2 d\theta^2, \qquad (64)$$

and returning to dimensional variables we'll get from (45) an expression for the undulator maximal possible impedance at the transverse emittance of the electron beam:

$$Z_{u\varepsilon} \approx \frac{\beta_u}{\lambda_u}\cdot\frac{F_\varepsilon(k\varepsilon)}{k\varepsilon}\cdot 76.9\ \text{Ohm}. \qquad (65)$$

Accordingly to (44), at $Z_{u\varepsilon} \gg 196$ Ohm the effective number of the undulator radiating periods can be defined as

$$q_\varepsilon = Z_{u\varepsilon}/196\ \text{Ohm} = 0.39\cdot\frac{\beta_u}{\lambda_u}\cdot\frac{F_\varepsilon(k\varepsilon)}{k\varepsilon}.$$

The calculated curve of the function $F_\varepsilon(k\varepsilon)$ is given at Fig.3.

Now we'll study the influence of the values of the transverse emittance and the energy spread of the electron beam on the quantity of the undulator effective impedance (45) in particular, for the finite length undulator without horizontal focussing with high vertical focussing ($L/\beta_u \gg 1$). We'll consider the beam with the matched vertical beta-function at the undulator input:

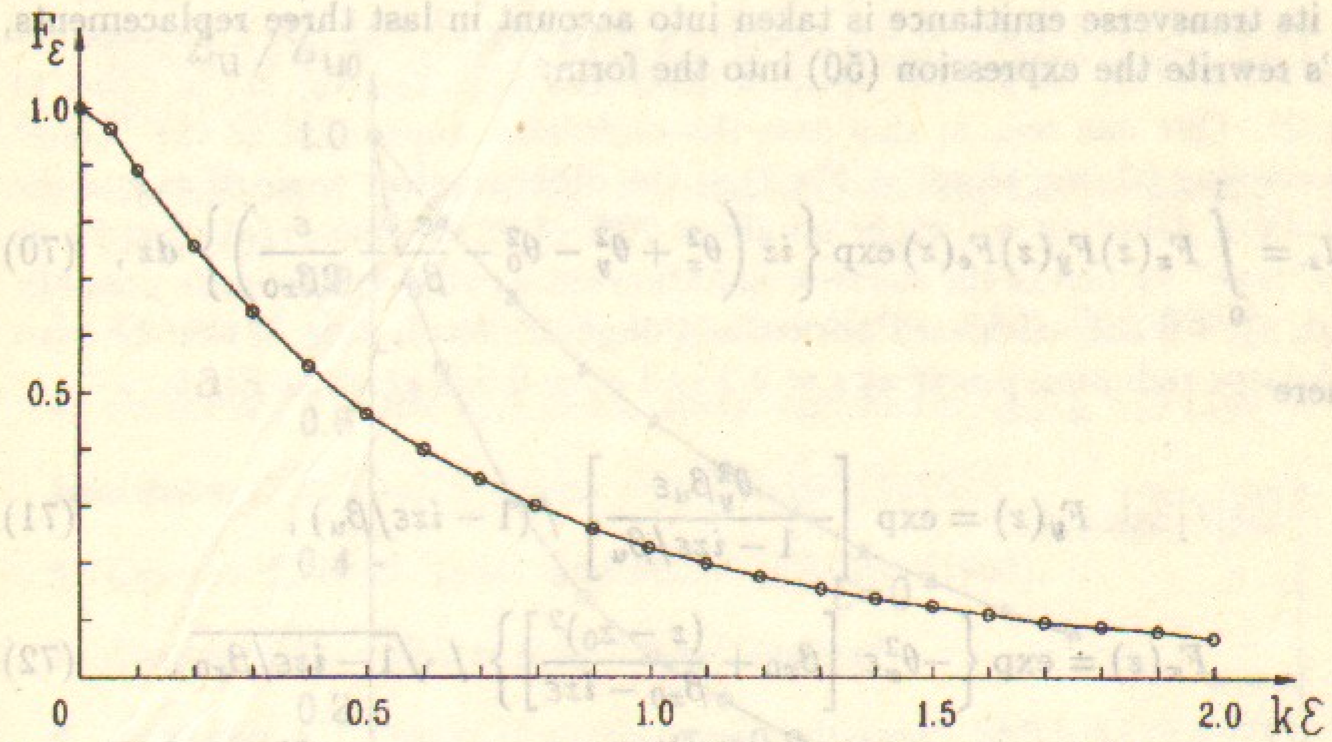

Fig. 3. The calculated curve of the function $F_\varepsilon(k\varepsilon)$.

$$\alpha_y = 0; \quad \beta_y = \beta_u; \quad \gamma_y = 1/\beta_u. \qquad (66)$$

In this case from (51), supposing $\varepsilon_x = \varepsilon_y = \varepsilon$, we'll get

$$F_y(z) = \exp\left[-\frac{k^2\theta_y^2\beta_u\varepsilon}{2(1-ikz\varepsilon/\beta_u)}\right] / (1-ikz\varepsilon/\beta_u), \qquad (67)$$

$$F_x(z) = \exp\left\{-\frac{k^2\theta_x^2\varepsilon}{2}\left[\beta_{x0} + \frac{(z-z_0)^2}{\beta_{x0}-ikz\varepsilon}\right]\right\} / \sqrt{1-ikz\varepsilon/\beta_{x0}}, \qquad (68)$$

where z_0, β_{x0} are position and quantity of the minimum of the beam horizontal beta-function along the undulator, respectively. Passing to the dimensionless variables I_z, z, z_0, β_u, β_{x0}, ε, $\theta_{x,y}$, θ_0, Δ, σ_e:

$$I_z/L \to I_z;\quad \frac{z}{L}\to z;\quad \frac{z_0}{L}\to z_0;\quad \frac{\beta_u}{L}\to\beta_u;\quad \frac{\beta_{x0}}{L}\to\beta_{x0};\quad k\varepsilon\to\varepsilon;$$

$$kL\theta_{x,y}^2/2 \to \theta_{x,y}^2;\quad 2\kappa = \left[k_u - k\left(\frac{1}{2\gamma_*^2} + \frac{\varepsilon}{\beta_u} + \frac{\varepsilon}{2\beta_{xo}}\right)\right]L = kL\theta_0^2/2 \to \theta_0^2;$$

$$\frac{4\pi q\Delta}{1+\gamma_*^2(\theta_0^2+2\varepsilon/\beta_u+\varepsilon/\beta_{x0})}\to\Delta;\quad \frac{4\pi q\sigma_e}{1+\gamma_*^2(\theta_0^2+2\varepsilon/\beta_u+\varepsilon/\beta_{x0})}/\sqrt{2}\to\sigma_e\,, \qquad (69)$$

where the dependence of the mean longitudinal velocity of the electron beam on its transverse emittance is taken into account in last three replacements, let's rewrite the expression (50) into the form:

$$I_z = \int_0^1 F_x(z)F_y(z)F_e(z)\exp\left\{iz\left(\theta_x^2+\theta_y^2-\theta_0^2-\frac{\varepsilon}{\beta_u}-\frac{\varepsilon}{2\beta_{x0}}\right)\right\}dz\,, \quad (70)$$

where

$$F_y(z) = \exp\left[-\frac{\theta_y^2\beta_u\varepsilon}{1-iz\varepsilon/\beta_u}\right] / (1-iz\varepsilon/\beta_u)\,, \quad (71)$$

$$F_x(z) = \exp\left\{-\theta_x^2\varepsilon\left[\beta_{x0}+\frac{(z-z_0)^2}{\beta_{x0}-iz\varepsilon}\right]\right\} / \sqrt{1-iz\varepsilon/\beta_{x0}}\,, \quad (72)$$

$$F_e(z) = J_1(X_0+\Delta z)\exp\{-[(X_0+\Delta z)\sigma_e/\Delta]^2\}\,, \quad (73)$$

and we'll write down the expression (45) for the undulator effective impedance in the normalized form:

$$Z_u/Z_{u0} \approx 0.6\cdot\int_0^{\infty}\int_0^{+\infty}|I_z|^2 d\theta_x d\theta_y\,. \quad (74)$$

For the undulator of powerful IR FEL on the base of CW race-track microtron-recuperator [9], which is being constructed in Budker Institute of Nuclear Physics, with β_u/L = 129cm/360cm=0.358 we give at Fig.4a two calculated dependencies (74) of the undulator normalized impedance Z_u/Z_{u0} on the dimensionless emittance ϵ of the beam with $\sigma_e = 0$ (at optimal quantities θ_0, β_{x0}, z_0) and on the dimensionless energy spread dispersion σ_e of the beam with $\varepsilon = 0$ (at optimal quantities X_0, Δ). Here the decrease of the undulator impedance by half of its maximum is observed at

$$\varepsilon \approx 0.2\,\frac{\lambda}{2\pi} \approx 0.32\,\mu\text{m for } \lambda \approx 10\,\mu\text{m}$$

or at

$$\sigma_e \approx 0.5\,\frac{\sqrt{2}}{4\pi q} \approx 0.14\% \text{ for } q = 40.$$

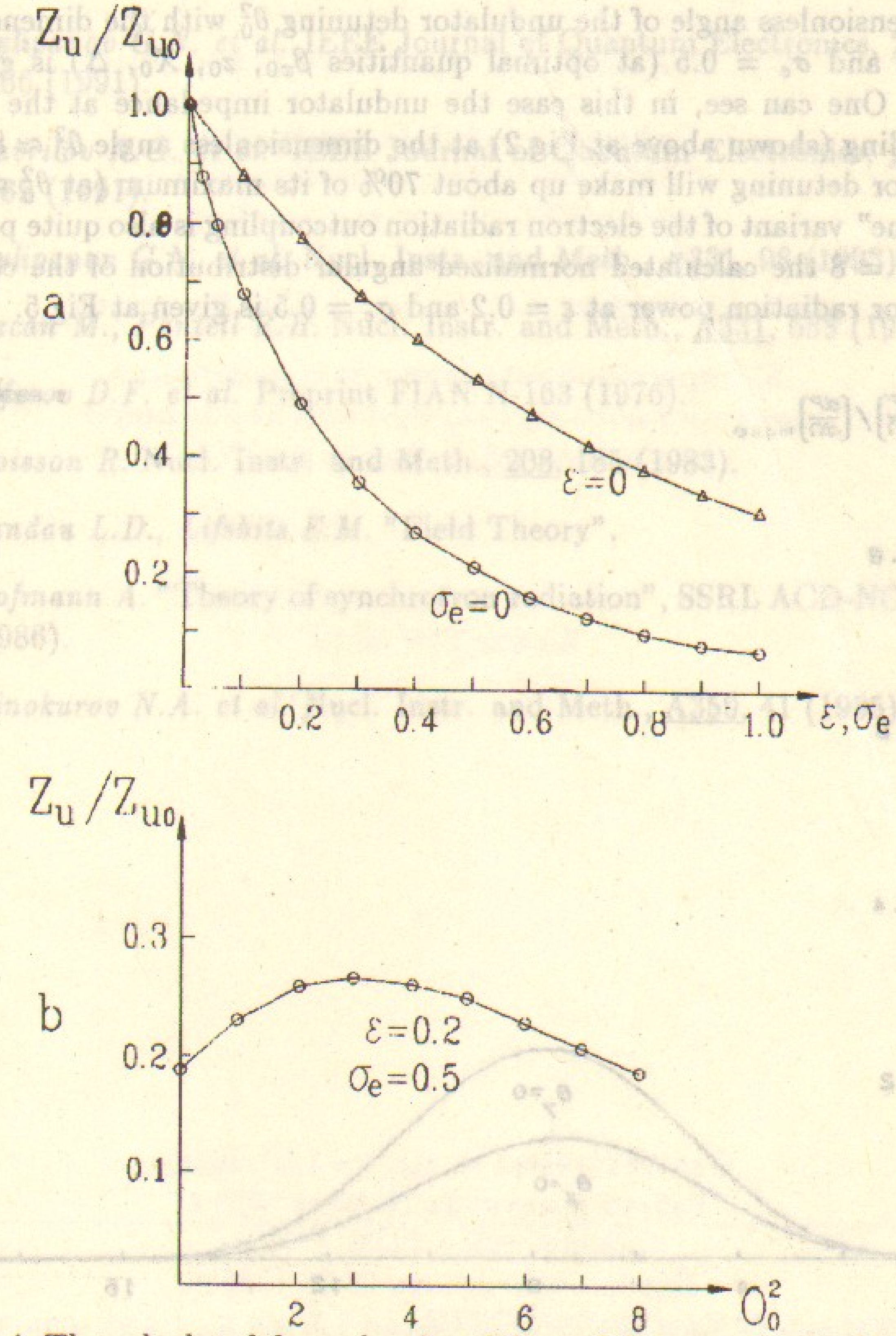

Fig. 4. The calculated dependencies of the undulator normalized impedance Z_u/Z_{u0}: (a) on the dimensionless emittance ε of the beam with $\sigma_e = 0$ (at optimal quantities θ_0, β_{x0}, z_0) and on the dimensionless energy spread dispersion σ_e of the beam with $\varepsilon = 0$ (at optimal quantities X_0, Δ); (b) on the dimensionless angle of the undulator detuning θ_0^2 with the dimensionless $\varepsilon = 0.2$ and $\sigma_e = 0.5$ (at optimal quantities β_{x0}, z_0, X_0, Δ).

The calculated dependence (74) of the normalized impedance Z_u/Z_{u0} on the dimensionless angle of the undulator detuning θ_0^2 with the dimensionless $\varepsilon = 0.2$ and $\sigma_e = 0.5$ (at optimal quantities β_{x0}, z_0, X_0, Δ) is given at Fig.4b. One can see, in this case the undulator impedance at the "cone" outcoupling (shown above at Fig.2) at the dimensionless angle $\theta_0^2 \approx 8$ of the undulator detuning will make up about 70% of its maximum (at $\theta_0^2 \approx 3$), so this "cone" variant of the electron radiation outcoupling is also quite possible. With $\theta_0^2 = 8$ the calculated normalized angular distribution of the coherent undulator radiation power at $\varepsilon = 0.2$ and $\sigma_e = 0.5$ is given at Fig.5.

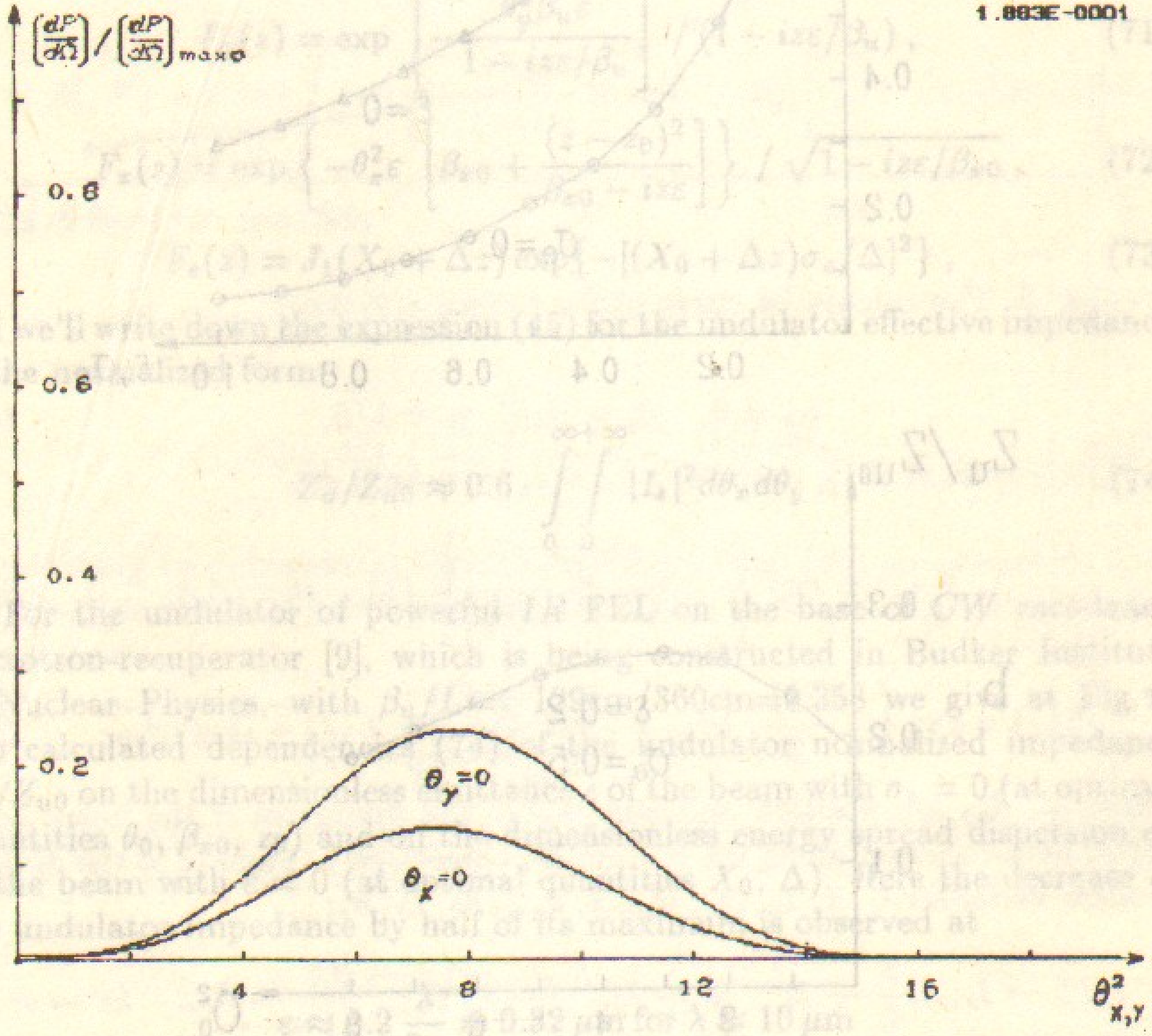

Fig. 5. The calculated normalized angular distribution of the coherent undulator radiation power at $\varepsilon = 0.2$ and $\sigma_e = 0.5$ with the undulator detuning $\theta_0^2 = 8$.

References

1. *Kulipanov G.N. et al.* IEEE Journal of Quantum Electronics, 27, (12) 2566 (1991).
2. *Gavrilov N.G. et al.* IEEE Journal of Quantum Electronics, 27, (12) 2569 (1991).
3. *Kulipanov G.N. et al.* Nucl. Instr. and Meth., A331, 98 (1993).
4. *Özcan M., Pantell R.H.* Nucl. Instr. and Meth., A331, 658 (1993).
5. *Alferov D.F. et al.* Preprint FIAN N-163 (1976).
6. *Coisson R.* Nucl. Instr. and Meth., 208, 185 (1983).
7. *Landau L.D., Lifshits E.M.* "Field Theory".
8. *Hofmann A.* "Theory of synchrotron radiation", SSRL ACD-NOTE, 38 (1986).
9. *Vinokurov N.A. et al.* Nucl. Instr. and Meth., A359, 41 (1995).

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