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AZIMUTHAL ASYMMETRY IN PROCESSES OF NONLINEAR QED FOR LINEARLY POLARIZED PHOTON

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НОВОСИБИРСК

Azimuthal Asymmetry in Processes of Nonlinear QED for Linearly Polarized Photon

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Abstract

Cross sections of nonlinear QED processes(photon-photon scattering, photon splitting in a Coulomb field, and Delbrück scattering) are considered for linearly polarized initial photon. The cross sections have sizeable azimuthal asymmetry.

The nonlinear effects of QED are due to the interaction of a photon with electron-positron field. These processes are the photon-photon scattering, Delbrück scattering (coherent photon scattering in the Coulomb field), the coalescence of photons, and the photon splitting into two photons, see Fig. 1. Among these processes only Delbrück scattering has been observed and investigated experimentally (see experimental papers [1] and [2] and reviews [3] and [4]), whilst observation of others may be considered still as a challenge.

Modern theory of photon-photon scattering and the review of previous papers concerning this problem can be found in [5, 6]. The total cross section of the photon splitting was estimated very roughly in [7, 8]. Then the differential cross section of the process $\frac{d^5\sigma}{d^3k_3d\Omega_4}$ was found in [9, 6] in the lowest order of the perturbation theory and in the Born approximation over the interaction with the Coulomb field (we use the photon momentum notation of [6]: k_2 is the initial photon, k_3 and k_4 are the final photons). The last cross section is extremely cumbersome one. The partially integrated differential cross section in the form which is suitable for the comparison with experiment as well as the integral cross section was calculated in [10] in the frame of equivalent photon method. Later the same cross section was found numerically after laborious calculations in [11] on the basis of formulae of [9, 6]. The results obtained in [11] agree with [10] within twenty per cent.

Recently the possibility to observe the photon splitting in the high-energy region ($\omega_2 \gg m$, ω is the photon energy, m is the electron mass) has been discussed in [12]. The history of experimental observation of the photon splitting is quite dramatic. In experiment [1] designed to measure the Delbrück scattering some surplus of photons was observed and was attributed to the

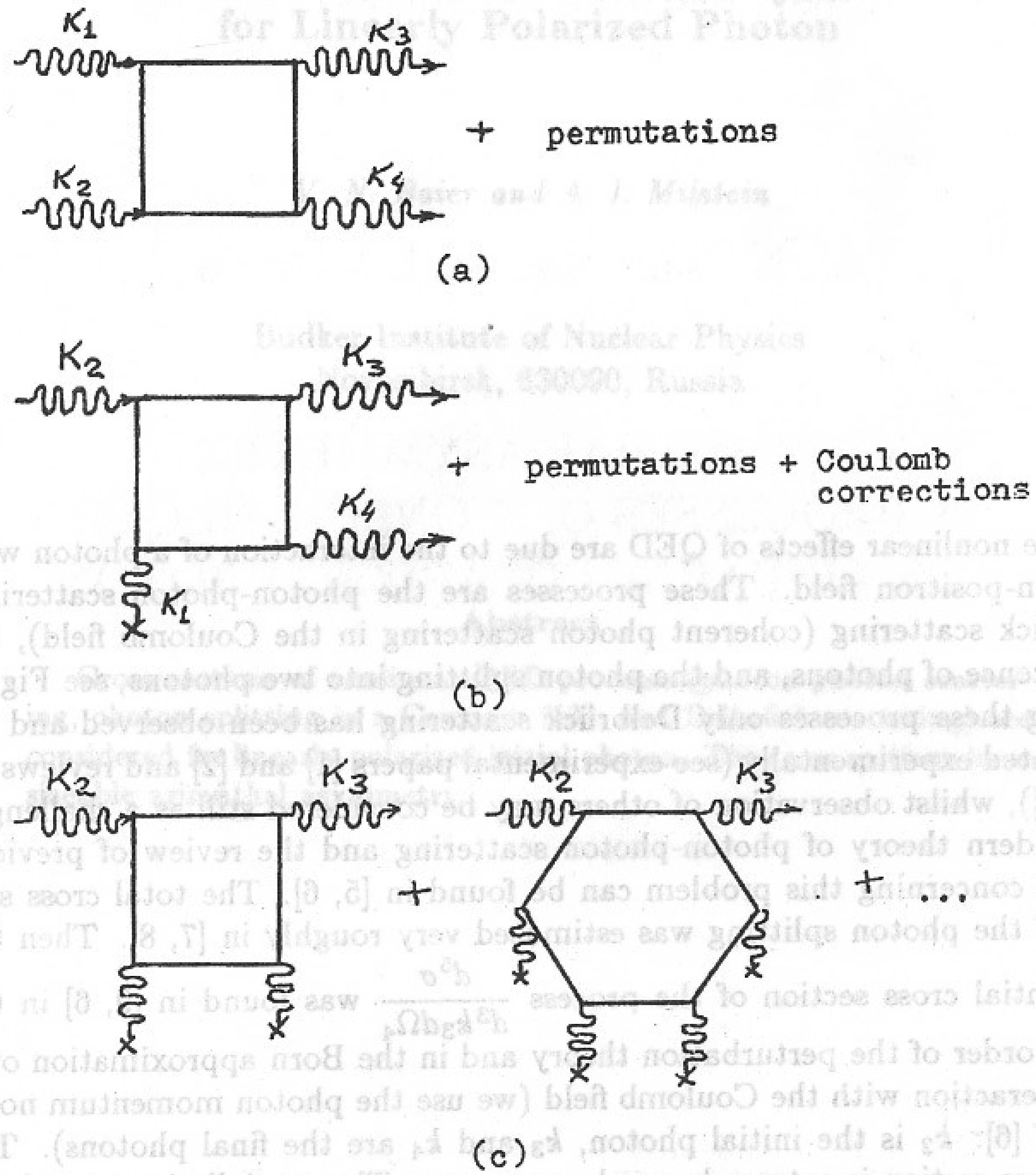

Fig. 1. Nonlinear processes of QED: (a) photon-photon scattering; (b) photon splitting; (c) Delbruck scattering.

photon splitting. Analysis made in [10] showed that the cross section of photon splitting is two order of magnitude smaller than the cross section observed in [1]. It was concluded in [10] that observed extra photons have nothing to do with the photon splitting. In [13] it was shown that these photons are due to e^+e^- pair creation with hard photon bremsstrahlung from each particle. So, the process of photon splitting is still unobserved.

Since the processes of photon-photon scattering and photon splitting are still escape observation it is important to look for additional signature for these processes. In this paper we show that for linearly polarized initial photon at definite conditions the differential cross sections of the processes under discussion possess substantial azimuthal asymmetry. This is also true for Delbrück scattering and may be useful in the further investigation of the process.

Let us consider some QED process initiated by a polarized photon. We represent the amplitude of the process as $M = \mathbf{eJ}$, where $\mathbf{e}$ is the polarization vector of initial photon and $\mathbf{J}$ is the current for the sum of corresponding Feynman diagrams. We introduce the definite coordinate frame and the helicity polarization vectors for the initial photon

$$\mathbf{e}_\lambda = \frac{1}{\sqrt{2}}(\mathbf{e}_2 - i\lambda \mathbf{e}_1), \qquad \lambda = \pm , \tag{1}$$

where $\mathbf{e}_{1,2}$ are unit vectors directed along the coordinate axes. This definition coincides with used in the book [14]. The corresponding helicity amplitudes are $M_\lambda = \mathbf{e}_\lambda \mathbf{J}$. Let us consider the linear polarized initial photon with the angle between the polarization vector $\mathbf{e}$ and $\mathbf{e}_1$ equal to φ. Then

$$\mathbf{e} = \frac{i}{\sqrt{2}}\left(e^{i\varphi}\mathbf{e}_+ - e^{-i\varphi}\mathbf{e}_-\right) . \tag{2}$$

Therefore, for the amplitude M we have

$$M = \frac{i}{\sqrt{2}}\left(e^{i\varphi}M_+ - e^{-i\varphi}M_-\right) , \tag{3}$$

For the square of amplitude modulus which defines the differential cross section of the process we get:

$$|M|^2 = |\overline{M}|^2 - \mathrm{Re}\left[M_+ M_-^* e^{2i\varphi}\right] , \tag{4}$$

where

$$|\overline{M}|^2 = \frac{1}{2}\left(|M_+|^2 + |M_-|^2\right). \qquad (5)$$

In the processes of photon-photon scattering and Delbrück scattering the initial and final photons lie at the same plane. This is also valid with the logarithmic accuracy (see below) for the process of photon splitting of high energy photon. Let us chose the vector $\mathbf{e}_1$ belonging to this plane and the vector $\mathbf{e}_2$ perpendicular to one. Then, by virtue of parity conservation, for the cross section of photon-photon scattering averaged with respect to the polarization of the second initial photon and summed over the final photon polarizations, as far as for the cross sections of photon splitting and Delbrück scattering in a Coulomb field summed over the final photon polarizations, we have:

$$|M|^2 = |\overline{M}|^2\,(1 + A\cos 2\varphi)\,, \qquad (6)$$

where

$$A = -\frac{\mathrm{Re}\,[M_+ M_-^*]}{|\overline{M}|^2}. \qquad (7)$$

This is the basic formula for our further consideration.

Let us start with the case of photon-photon scattering. Due to the P and T invariance and the identity of two initial (final) photons, only five amplitudes are independent among sixteen helicity amplitudes $M_{\lambda_1\lambda_2\lambda_3\lambda_4}$ ($\lambda_n = \pm$). They may be chosen as (see [14])

$$M_{++++}, \quad M_{++--}, \quad M_{+-+-}, \quad M_{+--+}, \quad M_{+++-}$$

The crossing invariance gives two additional equality between these amplitudes, so that it is sufficient to calculate only three of sixteen amplitudes, for instance M_{++++}, M_{++--} and M_{+++-} (they were obtained in [5, 6], we use the amplitudes given in [14], where the cross section is given in terms of these amplitudes as well). Using the relations between amplitudes we get from (7) the parameter A of azimuthal asymmetry for the photon-photon scattering:

$$A = -\frac{1}{|\overline{M}|^2}\mathrm{Re}\left[\left(M_{++++} + M_{+--+} + M_{+-+-} + M_{++--}\right)M_{+++-}^*\right], \qquad (8)$$

where

$$|\overline{M}|^2 = \frac{1}{2}\left[|M_{++++}|^2 + |M_{++--}|^2 + |M_{+-+-}|^2 + |M_{+--+}|^2 + 4\,|M_{+++-}|^2\right]. \qquad (9)$$

Substituting the explicit expression for the amplitudes we obtain that $A > 0$. So the cross section has maximum at $\varphi = 0$ and minimum at $\varphi = \pi/2$. As it is known, the cross section of photon-photon scattering depends on kinematic invariants s, t and u $(s+t+u=0)$ which in c.m.s. are of the form

$$s = 2k_3k_4 = 4\omega^2,$$

$$t = -2k_2k_3 = -2\omega^2(1-\cos\vartheta), \quad u = -2k_2k_4 = -2\omega^2(1+\cos\vartheta)\,, \qquad (10)$$

where k_1 and k_2 are momenta of initial photons and k_3 and k_4 are momenta of final photons, ω is the photon energy, ϑ is the angle between vectors $\mathbf{k}_1$ and $\mathbf{k}_3$. In Fig. 2 the asymmetry parameter A is given as a function of ratio $-t/s = (1-cos\vartheta)/2$ for different value of $s/4m^2$. The parameter A is a monotone increasing function of s. It mounts up to 25% at $s/4m^2 \gg 1$. So, the discussed azimuthal asymmetry is quite sizable in this limit.

The case of both linearly polarized initial photons is of interest too. If the angles between the vectors of polarization and vector $\mathbf{e}_1$ are equal to φ_1 and φ_2 respectively, then the square of the modulus amplitude of photon-photon scattering summed only over the final photon polarizations is

$$|M|^2 = |\overline{M}|^2\left[1 + A(\cos 2\varphi_1 + \cos 2\varphi_2) + B\cos 2(\varphi_1+\varphi_2) + C\cos 2(\varphi_1-\varphi_2)\right], \qquad (11)$$

where A is given in (8),

$$B = \frac{1}{|\overline{M}|^2}\left[\mathrm{Re}\,(M_{++++}M_{++--}^*) + |M_{+++-}|^2\right],$$
$$C = \frac{1}{|\overline{M}|^2}\left[\mathrm{Re}\,(M_{+-+-}M_{+--+}^*) + |M_{+++-}|^2\right]. \qquad (12)$$

Averaging over one of the initial polarization we return to (6). Formula (11) contains the parameters of linear polarizations of photons in the same way as the cross section of conversion of two linearly polarized photons into electron-positron pair as it should be on the ground of general consideration.

Now we consider the photon splitting into two photons in the Coulomb field. In the high energy limit ($\omega_2 \gg m$) the method of equivalent photons may be used to calculate the cross section of the process. This method allows one to express the cross section of this process through the photon-photon scattering cross section (see [10])

$$d\sigma_{\gamma\to\gamma\gamma} = \frac{Z^2\alpha}{\pi}\frac{ds}{s} L d\sigma_{\gamma\gamma\to\gamma\gamma}, \qquad (13)$$

where $d\sigma_{\gamma\gamma\to\gamma\gamma}$ is the cross section of photon-photon scattering, and

$$L = \int\limits_{\Delta^2_{min}}^{\Delta^2_{ef}} \frac{d\Delta^2}{\Delta^2}\left[1 - F(\Delta^2)\right]^2, \quad \Delta^2 = -k_1^2, \qquad (14)$$

$F(\Delta^2)$ is the atomic electron form-factor. It is convenient to express the kinematic invariants (10) in terms of real photon momenta. In lab frame and in small angle region

$$s = \frac{\omega_2^2\theta_3^2 x}{1-x}, \quad t = -\omega_2^2\theta_3^2 x, \quad u = -\frac{\omega_2^2\theta_3^2 x^2}{1-x}, \qquad (15)$$

where $\omega_3 = \omega_2 x$, $\omega_4 = \omega_2(1-x)$, θ_3 is the angle between vectors $\mathbf{k}_2$ and $\mathbf{k}_3$. For Δ^2_{min} and Δ^2_{ef} one has

$$\Delta^2_{min} = \left(\frac{\omega_2\theta_3^2 x}{2(1-x)}\right)^2, \quad \Delta^2_{ef} = (\omega_2\theta_3 x)^2.$$

Substituting eqs.(7)—(9) into (13) we obtain the photon splitting cross section for linearly polarized initial photon. In Fig. 3 the asymmetry parameter A is given as a function of x for fixed $\omega_2\theta_3$. Here also the cross section is maximum at $\varphi = 0$ and is minimum at $\varphi = \pi/2$. The curves in Fig. 3 are not symmetric with respect to $x = 1/2$ since variable x enters in kinematic variables (15) in non-symmetric form. The cross section of photon splitting integrated over the polar angle θ_3 is of interest also:

$$d\sigma = \frac{d\varphi}{2\pi}\left(\sigma_0 + \sigma_1 \cos 2\varphi\right). \qquad (16)$$

This is the averaged characteristic of azimuthal asymmetry. For photon energy 300 MeV in c.m.s. the ratio $\sigma_1/\sigma_0 \approx 7\%$.

Let us note that the same property has the cross section of electron-positron pair creation by a photon in the Coulomb field (particles of pair

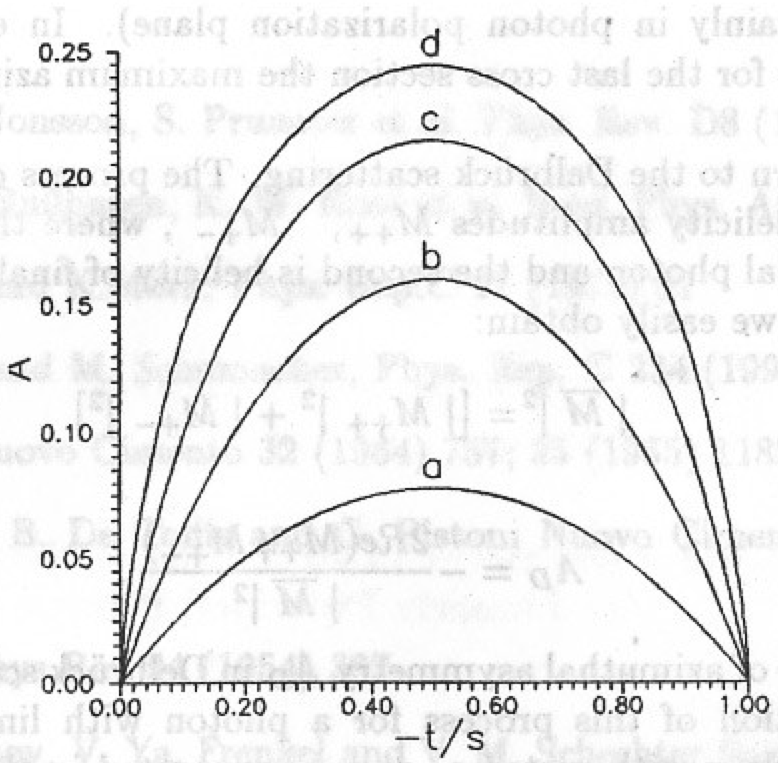

Fig. 2. Azimuthal asymmetry A in the photon-photon scattering cross section for different values of parameter $\delta = s/4m^2$: (a) for $\delta = 3$; (b) for $\delta = 10$; (c) for $\delta = 30$; (d) for $\delta = 100$.

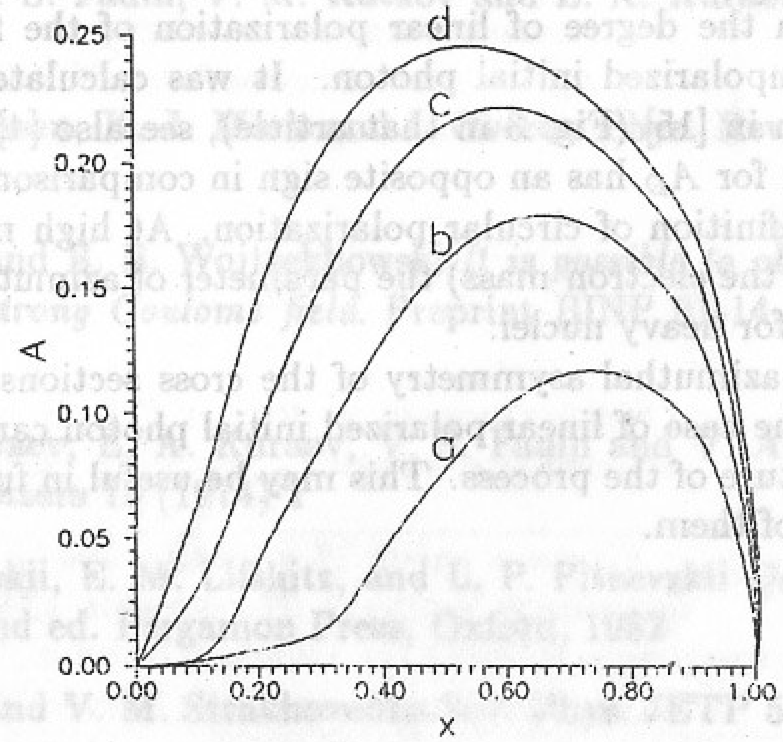

Fig. 3. Azimuthal asymmetry A in the photon splitting cross section for different values of parameter $\delta_s = (\omega\theta_3/2m)^2$: (a) for $\delta_s = 3$; (b) for $\delta_s = 10$; (c) for $\delta_s = 30$; (d) for $\delta_s = 100$.

are moving mainly in photon polarization plane). In equivalent photons approximation for the last cross section the maximum azimuthal asymmetry is $\approx 33\%$.

Now we turn to the Delbrück scattering. The process can be described in terms of two helicity amplitudes M_{++}, M_{+-}, where the first subscript is helicity of initial photon and the second is helicity of final photon. As in the previous case, we easily obtain:

$$|\overline{M}|^2 = [|M_{++}|^2 + |M_{+-}|^2] \tag{17}$$

and

$$A_D = -\frac{2Re(M_{++}M_{+-}^*)}{|\overline{M}|^2}. \tag{18}$$

The parameter of azimuthal asymmetry A_D in Delbrück scattering is positive. The cross section of this process for a photon with linear polarization **e** making an angle φ with vector $\mathbf{e}_1$ may be presented in the form

$$d\sigma_D = d\sigma_D^0(1 + A_D \cos 2\varphi), \tag{19}$$

where $d\sigma_D^0$ is the cross section for unpolarized photons. The function A_D coincides with the degree of linear polarization of the final photon ξ_3 for the case of unpolarized initial photon. It was calculated in quasiclassical approximation in [15] (Fig. 3 in that article), see also [4]. Note that in [15] the expression for A_D has an opposite sign in comparison with (18) because of different definition of circular polarization. At high momentum transfer $\Delta \gg m$ (m is the electron mass) the parameter of azimuthal asymmetry A_D achieves 80% for heavy nuclei.

Thus, the azimuthal asymmetry of the cross sections of nonlinear QED processes in the case of linear polarized initial photon can be used as an additional signature of the process. This may be useful in further experimental investigation of them.

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