

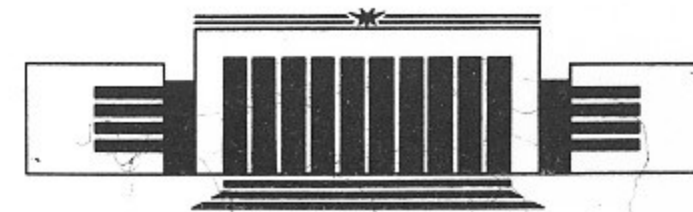


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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
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A METHOD OF ELECTROMAGNETIC
SHOWER IDENTIFICATION
AND MEASURING OF ITS POSITION
IN SEGMENTED CALORIMETERS

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НОВОСИБИРСК

**A Method of Electromagnetic
Shower Identification and Measuring
of its Position in Segmented Calorimeters**

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ABSTRACT

The Monte Carlo investigation of spatial resolution for electromagnetic showers of the calorimeter of the Spherical Neutral Detector (SND) has been performed. For description of the transverse distribution of energy in an electromagnetic shower the function $\exp(-\beta \cdot \sqrt{R})$ is introduced. On its base the method of estimation of shower angles and the criterion of close photon separation have been worked out. It is shown that for the photons with the energy less than 700 MeV this method provides the best results compared to the other methods.

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properties of the event with the calorimeter. The distribution of the energy in the shower is represented in section 2. The energy profile of the shower is represented in section 3. The energy profile of the shower is represented in section 3. The energy profile of the shower is represented in section 3.

2 The simulation

The electromagnetic calorimeter of SND consists of three spherical layers of lead crystals (Fig. 1). The crystals are represented by cylindrical pyramids. The number of the crystal layers is 3. The number of the crystal layers is 3. The number of the crystal layers is 3. The number of the crystal layers is 3.

To simulate electromagnetic showers the universal Monte Carlo code UNIMOD [2] was used.

1 Introduction

In high energy physics experiments electromagnetic calorimeters based on heavy crystals NaI, CsI, BGO, and etc. are widely used. Such calorimeters provide high energy resolution up to 1% for 1 GeV showers, and also they have good spatial resolution. This spatial resolution is determined by cross size of a shower as well as by the granularity of a calorimeter. Many authors studied in detail transverse profile of energy in the showers of high energy and used the centre of gravity method and other methods of shower coordinate determination [1-6]. From the results of these works it is clear that there is no unambiguous recipe to determine the shower coordinates which is applicable for all calorimeters and for all electromagnetic shower energies.

SND detector is intended for the experiments at e^+e^- collider VEPP-2M and at the future ϕ -factory in Novosibirsk [7]. The range of shower energies in these experiments is (10-700) MeV. In this work the possibilities of maximum spatial resolution obtaining for the SND are considered. The comparison of various methods of shower coordinate estimation which were used in the previous works [1-6] was performed, and a new method was suggested. The results obtained may be of interest also for other detectors, for instance, which are developed for the experiments at B-factories.

In section 2 the structure of the SND calorimeter and Monte Carlo simulation are briefly discussed. In section 3 the results of studying of the transverse energy profile are represented. Section 4 is devoted to the description of various methods of estimation of isolated shower coordinates. In section 5 the

processing of the events with close photons is described. The discussion is represented in section 6.

2 The simulation

The electromagnetic calorimeter of SND consists of three spherical layer of the NaI(Tl) crystals (Fig.1). The crystals represent by themselves truncated tetrahedral pyramids. The number of the crystal types is 8. Angle size for 6 central types of the crystals is 9° as in azimuthal direction as in polar one. Two types of the crystals, close to the axis of beams, have azimuthal angle 18° . Each crystal is wrapped into mylar reflector and disposed into aluminum container 150μ thick. The total thickness of calorimeter is 34.7 cm or about $13X_0$.

To simulate electromagnetic showers the universal Monte Carlo code UNIMOD2 [9] was used which was developed for experiments at VEPP-2M and VEPP-4M colliders in Novosibirsk. The previous version of the program worked out for simulation of ND [10] and MD-1 [11] experiments. In [12] it has been shown that the results of electromagnetic shower simulation obtained with UNIMOD2 coincide with a high accuracy with those ones according to the GEANT [13]. It is not surprising since for electromagnetic shower simulation practically the same formulas describing the physical processes in a shower are used in the both codes.

For the investigation of the transverse energy profile in a shower two variants of the simulation were performed. In the first variant the simplest model of calorimeter, i.e., three continuous spherical layers of NaI(Tl) dividing by spherical layers of effective absorber was considered. The initial photons were directed along axis Z from the sphere centre. Into the output file all trajectory steps of all charged particles in the calorimeter were recorded together with the energy loss of each particle. While processing recorded information there was a possibility to divide these energy losses between any given in a space volumes. It allowed to use comparably small statistics of the simulation, but, with this, to study the character of transverse distribution of the energy in detail. Such simulation worked out only at the initial stage of the research.

The main results were obtained with the use of complete simulation of showers in SND. At that all absorbers in the detector, including containers, phototriodes, pre-amplifiers, and elements of the calorimeter construction were taken into account. While simulating the threshold for the energy of Bremsstrahlung photon was assumed to be 0.2 MeV, the threshold for the

energy of delta-electron was also 0.2 MeV. The small non-homogeneity of the light collection along the crystals was taken into account. In cluster reconstruction algorithm the energy thresholds were used for the energy loss in one crystal—1 MeV and for cluster energy deposition—10 MeV. The noise of the electronics channel (average equivalent value is about 0.1 MeV) and the fluctuations of the number of photoelectrons were not taken into account.

The simulation of single photons emitted from the detector centre with uniform angular distribution was performed. Besides, the systems of two photons of fixed energies and with fixed spatial angle between them were simulated. In such events one of photons was distributed uniformly according to the angle. The another one was distributed symmetrically along the cone round the direction of the first photon. The simulation of electron showers was also performed.

3 Transverse energy distribution

The transverse energy deposition distribution, according to the quantity of energy which is deposited outside the cylinder with the radius R , surrounding the direction of the photon, is one of the important functions usually used for shower description. It is well known [1-6] that this function may be expressed in terms of the sum of two exponents, i.e., "slow" and "quick" ones. Because the spherical form of the SND calorimeter in the present work the shower coordinates were measured in degrees and the distribution function of energy deposition outside the cone with the angle θ around the shower direction was investigated (Fig.2). Even the preliminary studies using the simple model of the calorimeter have shown that the distribution function of energy may be also expressed in terms of two exponents. However, the effort to find more simple expression was performed. As a result, the following function depending only on two free parameters was found:

$$E(\theta) = \alpha \cdot \exp(-\beta \cdot \sqrt{\theta}). \quad (1)$$

With this, over the interval of photon energy (50-700) MeV the parameters α and β turned out to be practically independent of the photon energy.

Using the simple model of calorimeter the shower fluctuations in various directions from its axis and the correlations of energy depositions have been studied. It was shown that the fluctuations of energy with good accuracy were independent. It resulted in the understanding that energy distributions on polar and azimuthal angle in the calorimeter may be considered independently. It considerably simplifies the statement of the problem.

In particular, in the paper all results are given, mainly, for the distributions on the polar angle.

In order to obtain these distributions it is necessary to introduce some additional values and designations. In Fig.1 the section of the calorimeter in the plane passing through beam axis and the crystal centres is shown. Three crystals from three different layers having the same polar and azimuthal angles form the calorimeter tower. The tower in which the given photon directs will be called "central tower" of the given shower. Spherical form of the calorimeter allows to bring distributions for the photons emitted under various central towers into coincidence.

For the purpose, the polar angle of a photon θ will be considered from the axis of the central tower (Fig. 1). It will vary in the limits $\pm 4.5^\circ$. Let E_+ be total energy deposition in the towers disposed from the right from the central one on the polar angle, divided into the total energy deposition of the shower; let E_- be the normalized energy deposition in the towers disposed from the left.

The analysis of mean values of E_+ and E_- has shown that with the photon energy higher than 50 MeV the mean value distribution $E(\theta)$ does not depend on the energy, and it may be good approximated by formula (1) also (Fig.2). Note that the using of the simple model of the calorimeter allowed to observe the distribution in the wide range of θ angles. The complete simulation was convenient to use for the obtaining distributions on θ E_+ and E_- in the range (0-9) degrees. The Analogous distribution was made also for electron showers (Fig.3). The fitted parameters of the both distributions for the SND calorimeter are shown in Table 1.

Since function (1) had not been used earlier it was necessary to perform independent additional tests of applicability of such distribution function for other calorimeters. For this purpose the data [4] on the transverse distribution of energy in the forward electromagnetic calorimeter of the DELPHI detector were fitted. It turned out that if in function (1) instead of the angle θ one used the radius R then such the function good described these data. The parameters obtained as the result of approximation are represented in Table 1.

Another verification of the result obtained follows from [14], where the algorithm of fast simulation of electromagnetic showers in the calorimeter of the ZEUS detector is described. On the basis of the data on the complete simulation of the calorimeter the distribution of energy deposition in the thin layers of this calorimeter is approximated by the function represented by itself the product of the factor $\exp(-\beta \cdot \sqrt{R})$ and some function of radius and the depth of the layer. The value β is given in Table 1. Using the estimation of

the average depth of the shower maximum for the SND one may recalculate the angle in expression (1) into the Molier units. Then the coefficients β from Table 1 turn to be close to 0.5 with an accuracy about 30% for different detectors, i.e., β weakly depends on the matter of calorimeter.

Thus, it may be concluded that function (1) is the function of distribution of cross energy deposition in the SND calorimeter no accidentally. Most likely, in common case this dependence adequately describes the processes of multiple scattering and transport of energy by soft photons in a shower. Note that big transverse fluctuations of the shower do not contribute to this distribution since they result in the appearance of additional "false" clusters, whose energy are not added to the energy of the main cluster. It means that practically it is necessary to use function (1) only in the limited interval of angles. In the case of the SND detector this interval is approximately (0-30) degrees. Note that the simple exponent (Fig.2) may describe this dependence in considerably less interval (0-3) degrees.

Table 1

The fitted parameters of the distribution of transverse energy deposition in electromagnetic showers in the SND calorimeter, those ones in the forward calorimeter of the DELPHI detector (see, Fig.5,6 in [4]) and in the calorimeter of ZEUS [14].

Detector	Shower	α	β
SND	50-700 MeV γ	0.72	1.30°
SND	300 MeV e^-	0.90	1.36°
DELPHI	20 GeV e^-	9.06	1.42 mm
ZEUS	1-100 GeV e^-	—	1. mm

4 Spatial resolution for isolated showers

From the literature [1-6] a series of the methods of estimation of shower coordinates according to transverse distribution of the energy in a calorimeter is considered. All of them were used in the given work. For the comparison of the results obtained by various methods we denote each of them by the following symbol:

- G — The method of centre of gravity;
- F — The maximum likelihood method;
- L — The logarithmic method;
- S — The method of hyperbolic sine;
- W — The method of non-linear weighting.

In particular, in the paper all results are given, mainly for the distributions on the polar angle.

In order to obtain these distributions it is necessary to introduce some corrections. Below all these methods will be discussed in detail. The polar angle related to the beam axis obtained as a result of the using of some method we denote as θ' . It may vary in the limits (18–162) degrees. If one consider this very angle related to the axis of the central tower in the given event, then its region of changing become considerably less. Denote such relative angle θ^* . The most part of the distributions in the work are obtained for this angle.

Because of the symmetry of the detector and the uniform angle distribution of showers the value $(\theta^* - \theta)$ constructed according to the whole statistics of simulation with the use of any method of the processing is distributed symmetrically, i.e., with zero mean value. The mean square deviation of this value σ_θ characterizes the mean spatial resolution of calorimeter.

If one considers the sampling of a half of showers with positive value of emitted angle θ then the distribution of the value $(\theta^* - \theta)$ turns to be shifted (Fig.4). The mean value of this distribution will be called "systematic shift" and denoted by θ_S . Another half of the statistics gives the reflecting related to the Y axis in Fig.4 distribution. If one divides the statistics into parts on the θ angle with the step 0.5° , then it is possible to make this calculation for each sample the separate distribution and to find the dependence θ_S on θ (Fig.5).

4.1 The method of centre of gravity

The most simple and natural method of determination of shower angle in hodoscopic calorimeter (G-method) is the method of centre of gravity [1–6]

$$\theta' = \frac{\sum_i E_i \theta_i}{\sum_i E_i} \quad (2)$$

here E_i is the energy deposed in i-th tower, θ_i is the angle of the centre of i-th tower. The summing is performed on the all towers which were included in the cluster by the program of event reconstruction.

It is well known from the literature that G-method provides not bad mean resolution but large shift. It is confirmed for the SND calorimeter (Fig.4–6). The size of the crystal in SND is more than the Molier radius. So, even in the case when a photon is sufficiently near the crystal side, the largest part of the shower energy is deposited in one crystal. Then calculated according to (2) the value θ' , as a rule, will be close to this tower centre. It corresponds to the negative value θ_S . For the SND calorimeter the absolute value of systematic shift in G-method turned out to be close to the resolution.

4.2 The method of hyperbolic sine

In order to remove such big systematic uncertainty in the determination of photon angle, i.e., to succeed in the setting to zero the mean value θ_S one may to try to transform the obtained value θ^* taking into account the distributions in Fig.5. The form of this curve may be found analytically for instance, using the distribution of shower energy only at considerably small distances from the shower axis (Fig.2) when for its approximation it is sufficiently to consider one simple exponent.

$$E(\theta) = \gamma \exp(-\delta\theta). \quad (3)$$

Using relation (3) we may transform θ^* obtained by G-method and get the value of the angle θ^{**} (S-method [2,5]):

$$\theta^{**} = \frac{1}{\delta} \operatorname{arcsinh} h \left(\frac{\theta^* \sinh(\delta\theta_0)}{\theta_0} \right), \quad (4)$$

Here θ_0 is the angular half-size of a tower. The results of the simulated events processing (Fig.5,6b) show that the average systematic shift of θ_S , when using S-method, is close to zero nearly within the entire range of the photon energies considered, but the average resolution σ_θ thus obtained is the worse among all methods considered in the paper (Fig.6a). Note that similar results were obtained in [5] for high energy showers.

4.3 Logarithmic method

Since a new function (1) is derived in this paper for describing the transverse energy distribution in the shower, it is natural to use this function to improve the accuracy of the shower angle estimation (L-method). This method, as well as the previous one, is realized in two steps. First the value of the angle θ' is determined with G-method, which helps in finding of the central tower. Then the normalized energy depositions outside the central tower E_+ and E_- are found. Rather rough segmentation of the SND calorimeter and large energy fluctuations in the shower cause one of these values to be small, which makes impossible the direct usage of (1). Therefore, two intermediate values are calculated being the estimates of the shower angle with respect to the central tower sides:

$$\theta_+ = \left(\frac{1}{\beta} \ln \left(\frac{E_+}{\alpha} \right) \right)^2,$$

$$\theta_- = \left(\frac{1}{\beta} \ln \left(\frac{E_-}{\alpha} \right) \right)^2, \quad (5)$$

If E_+ or E_- are very small, then the values θ_+ or θ_- are outside the central tower limits. To suppress these fluctuations, an additional limiting parameter θ_M is introduced, and finally the shower angle with respect to the central tower axis is determined as

$$\begin{aligned} \theta^* &= \frac{(\theta_0 + \theta_+)E_+ + (\theta_- - \theta_0)E_-}{E_+ + E_-}, & \theta_+ \leq \theta_M, & \theta_- \leq \theta_M, \\ \theta^* &= \theta_0 - \theta_+, & \theta_+ \leq \theta_M, & \theta_- > \theta_M; \\ \theta^* &= \theta_- - \theta_0, & \theta_+ > \theta_M, & \theta_- \leq \theta_M; \\ \theta^* &= \theta^0, & \theta_+ > \theta_M, & \theta_- > \theta_M. \end{aligned} \quad (6)$$

The resolution and systematic shift as a function of parameter θ_M for SND calorimeter are shown in Fig.7. Using these data, we chose the value of parameter $\theta_M = 4.5^\circ$.

The results obtained with L-method are presented in Figs.4-6. Fig.4 illustrates the systematic shift decrease as compared with G-method. The dependence of resolution and systematic shift on photon energy for L-method are shown in Figs.5,6. Within the entire range of energies this method gives systematic shift smaller by 1.5-2 times than G-method. For photon energies over 200 MeV the resolution obtained by L-method is the best one, while for smaller energies it is slightly worse than the resolution obtained by G-method.

Additional analysis of resolution depending on the reconstructed angle θ^* has proved that there is some non-uniformity of resolution - it improves with θ^* increasing. Clearly, the resolution increases with the shower energy increasing. We found an analytical dependence of resolution on these parameters for L-method for the SND calorimeter:

$$\begin{aligned} \sigma_\theta &= 0.7^\circ + \frac{0.6^\circ}{\sqrt{E(\text{GeV})}}, & |Q^*| \leq 3^\circ, & E \geq 0.1 \text{ GeV}; \\ \sigma_\theta &= 2.7^\circ - \frac{2}{3}|\theta^*| + \frac{0.6^\circ}{\sqrt{E(\text{GeV})}}, & |Q^*| > 3^\circ, & E \geq \left(\frac{0.6}{\frac{2}{3}|\theta^*| - 0.1} \right)^2 \text{ GeV}; \\ \sigma_\theta &= 2.6^\circ. \end{aligned} \quad (7)$$

Note that allowances for decrease in the linear tower size during the polar angle changing from the calorimeter equator to its pole were made for corresponding calculations for the azimuthal shower angle. All results for the azimuthal angle are in good agreement with the data for the polar angle.

4.4 Maximum likelihood method

It is well known that the maximum likelihood method (F-method) can provide the best estimate of the model parameters. Unfortunately, this method is sometimes difficult to use because it is not easy to formulate the maximum likelihood function or to find its extremum. In this case, examining the results of the simplest variant of the shower simulation in a calorimeter, it was established that the root-mean-square deviation of the energy distribution coincides well with the average value of this distribution. Besides, the distribution has a "falling" character, which suggests a simple form of the distribution function

$$F(E_+, \theta) = \frac{1}{E(\theta)} \exp \left(-\frac{E_+}{E(\theta)} \right), \quad (8)$$

where $E(\theta)$ is the average energy deposition outside the plane constituting the angle θ with the shower axis, this value is given by (1). More detailed consideration shows that for $\theta > 0$ the most exact description is provided by gamma-distribution, but this angle region is rather narrow whereas the function is too cumbersome to use.

Using (8), one may write a logarithmic likelihood function $f(\theta^*)$ for the case when the energy E_+ was registered to the right of the tower, and the energy E_- - to the left.

$$f(\theta^*) = \frac{E_+}{E(\theta_0 - \theta^*)} + \frac{E_-}{E(\theta_0 + \theta^*)} + \ln(E(\theta_0 - \theta^*) \cdot E(\theta_0 + \theta^*)). \quad (9)$$

The function f minimum corresponds to the required value of the angle θ^* . Even in this form F-method is most inconvenient to use, since the function f minimum is to be found by successive approximation method. The results are presented in Fig.6. They show that F-method offers no actual advantages over L-method. Within the entire range of photon energies under consideration L-method has slightly better resolution, but F-method has smaller systematic shift.

4.5 Nonlinear weighting method

The authors of [6] suggest a method for coordinate determination using the weighting method (W-method) nonlinear with respect to the calorimeter tower energies:

$$\theta' = \frac{\sum_i W_i \cdot \theta_i}{\sum_i W_i}, \quad (10)$$

$$W_i = \max(0, W + \ln \left(\frac{E_i}{\sum_i E_i} \right)),$$

where θ_i is the i -th tower angle, E_i is the energy deposition in the i -th tower, W is the method parameter.

This method is as simple as G-method but it is rather efficient for determining the high energy shower directions. As is shown in [6], a good choice of W may provide nearly zero systematic shift, optimum resolution and e/π separation for a wide range of shower energies.

The resolution and systematic shift are shown in Fig.8 as a function of parameter W for the SND calorimeter. We used the value $W=3.5$ for photon energy 300 MeV. The systematic shift depending on the shower angle θ is somewhat smaller for W-method (Fig.5) than for L-method, but the resolution for the optimum values of parameter W (Fig.8) is worse than the resolution obtained by L-method (Fig.6a).

It is seen from Fig.8 that the optimum value of parameter W is reduced with the photon energy decreasing within the energy region of interest. Such a dependence of the optimum value of W on the shower energies is confirmed by the data presented in [6]. This means actually that the advantages of W-method vanish for low energy showers.

5 Close photons separation

The choice of the method for determining of the shower coordinates for a definite calorimeter cannot be done only on the basis of consideration of the isolated shower parameters. An important criterion is the resolution for close photons and also the possibility of separation of particles by their type. The problem of close showers separation is immediately connected with the problem of isolated shower splitting due to transverse fluctuations of energy. The probabilities of the shower mergence and "false" photons appearance are dependent both on the calorimeter segmentation and on the algorithm of the shower reconstruction. To process the SND data, the algorithm to search the central tower of the shower is developed with the further addition of the neighboring towers. It allows the separation of close photons if the energy depositions are sufficiently large in both clusters.

To examine the characteristics of the reconstruction methods, we used simulation of isolated showers and simulation of two photons with fixed energy emitted with fixed spatial angles $\Delta\omega$ in the range (9-30) degrees. The separation criteria of two hypotheses (one photon, two photons) described below were applied to the simulation events of these two classes. Independently of the method of determining the shower angles, the energy deposition in the towers was divided between the reconstructed photons using the distribution function (1).

5.1 The separation parameters

Mathematical statistics has a basic theorem of statistic solutions theory, which is also known as Neumann-Pierson lemma [15]. It may be formulated as follows: the optimum test allowing the discrimination of two hypotheses yields the relation

$$\ln(F_1/F_2) > C, \quad (11)$$

where F_1, F_2 are the likelihood functions of these hypotheses, C is a constant which is a separation parameter determined empirically, proceeding from the required confidence level for selecting one of the hypotheses. In this case, hypothesis 1 confirms to the assumption that one photon is registered, hypothesis 2 – two photons. The function F_1 depends on the energy distribution around the central tower. The function F_2 depends on the location of two central towers, on the estimation of the energy of both photons and on the energy distribution around the two central towers.

If the algorithm for cluster search in the calorimeter has found only one isolated cluster, the search for two central towers is rather problematic, but even more uncertain magnitudes are the energy estimates of these two photons. Thus, the function F_2 has a large number of parameters, while the accuracy and credibility of their determination is small. Therefore, theoretically powerful criterion can be hardly realized in practice, which has been confirmed by the attempts of applying criterion (11) to SND events. A simplified criterion

$$\ln(F_1) > C, \quad (12)$$

turned out to be more efficient. It may be applied to the events where only one photon is found, and to events with two photons assuming for calculating the function F_1 that all the towers of these two photons belong really to one photon.

5.2 Methods of separation of close photons

The likelihood function F_1 or its logarithm f_1 used in (12) are dependent on the method for coordinate reconstruction. For G - or W - methods, when a photon direction is determined by weighting, the logarithmic likelihood function was found through the sum of the variances of distribution by the polar and azimuthal angles. The root of this sum was used as a separation criterion.

$$f_1 = \sqrt{D(\theta) + D(\phi)},$$

$$D(\theta) = \frac{\sum_i W_i \theta_i^2}{\sum_i W_i} - \left(\frac{\sum_i W_i \theta_i}{\sum_i W_i} \right)^2, \quad (13)$$

here W_i is a weight factor coinciding with the energy deposition in the tower for G -method and determined by relation (10) for W -method, $D(\phi)$ is calculated similarly to $D(\theta)$. Fig.9 represents the distributions by this separation parameter for isolated showers with the energies 300 Mev and for two fairly close showers of the same total energy. The data in Fig.9 show that W -method provides much better separation of the peaks. However, imposing the requirement of 99% efficiency of the correct discrimination of showers from isolated photons, then W -method loses its advantages because of the distribution tails, and the efficiency of two photons reconstruction is higher in G -method (Fig.10).

For L -method the logarithmic likelihood function results from summation of the likelihood functions with respect to the polar and azimuthal angles given by (9). Figure 11 shows the distribution by this parameter for events with one or two photons. The distribution character in Fig.11 is sufficiently different from Fig.9. In this case the probability density function for isolated photons is very narrow and lies in the negative region. The probability density function for two photon events is wide, it lies mostly in the positive region. In this case it is easy to formulate the separation criterion suppressing the shower splitting, which stipulates high efficiency of identification of two photon events at the efficiency level of 99% to one photon events (Fig.10).

5.3 Determination of the close photon parameters

Thus, L -method seems preferable for the SND calorimeter from the viewpoint of the possibility of close showers separation. Below we present some results of processing the simulation events using L -method. Table 2 shows

the efficiency of one photon reconstruction depending on its energy. Its difference from 100% is related to the shower splitting, with the probability of late shower conversion and with the probability of 2-photon conversion in the coordinate system. In the range of the photon energies (50-700) MeV the average efficiency of isolated photon reconstruction and the probability of "false" photon appearance are nearly constant and equal $(98.24 \pm 0.12)\%$ and $(0.51 \pm 0.06)\%$, respectively.

Table 2.

Efficiency of an isolated photon reconstruction in SND calorimeter depending on its energy.

$E(\text{MeV})$	12	15	20	25	50	75	100	200	300	500	700
$\varepsilon(\%)$	70.6	86.8	94.5	97.0	98.6	98.4	98.1	98.6	98.4	97.9	97.6

Using the simulated two photons events with a fixed angle $\Delta\omega$ between them, two photon calorimeter efficiency was investigated (Fig.12). The results show the efficiency to be dependent on the photon energies ratio. Note that the efficiency shown in Fig.12 is slightly lower than that in Fig.10, since the algorithm of events reconstruction cannot always provide the separation of close showers. In this case only one summational shower is found, but its parameter f_1 , equal to the logarithmic likelihood function of one photon hypothesis, shows that this shower actually consists of two showers. The average energy release in each reconstructed close photon (Fig.13a,b) is practically independent on the angle between them. The average spatial resolution for close photons (Fig.13c,d) becomes worse because of the distribution tails containing a small number of events with a large error of the angle determination. Their appearance is explained by the incorrect determination of the central towers in the reconstructed shower. In spite of the fact, expression (7) for the angular accuracy of the calorimeter obtained for isolated showers, is applicable for the majority of close photons.

An important parameter of the calorimeter is the efficiency of identification of π^0 mesons. Table 3 shows this efficiency as a function of π^0 energy for those events where both photons resulting from π^0 decay arrived at the calorimeter. π^0 meson losses are connected with splitting of one shower, with photon conversion in the coordinate system, with the loss of soft shower and with the shower mergence.

Table 3.

Efficiency of π^0 meson reconstruction in the SND calorimeter depending on its energy.

E(MeV)	200	300	400	500	600	700
$\epsilon(\%)$	97.9	97.8	98.0	96.2	95.6	93.7

6 Discussion

Analyzing various methods for the estimation of isolated shower angles in the SND calorimeter (Figs.4-8), we come to the conclusion that if systematic shift is considerably reduced or even eliminated (S, W-methods), this inevitably deteriorates the resolution. Evidently, there exists a restriction on the accuracy of the shower angles determination caused by the nature of transverse energy distribution. It may be interpreted by the known relation of mathematical statistics called the Kramer-Rao information inequality [15]

$$\sigma_{\theta}^2(\theta) \geq \theta_s^2(\theta) + \left(1 + \frac{d\theta_s(\theta)}{d\theta}\right)^2 / \left(\frac{\partial f(\theta)}{\partial \theta}\right)^2 \quad (14)$$

Here f is the logarithmic likelihood function. As it follows from this relation, the possibility of suppressing systematic shift without losses in resolution is limited by the form of this function. With the deposited energy fluctuations in the calorimeter, the equating of the first additive in (14) to zero gives rise to the second one, deteriorating the total resolution.

To confirm additionally that the limitations are connected with the distributions rather than with the definite processing methods, another method for the angle reconstruction has been found. This method gives practically zero systematic shift. This method is a modification of W - method when $\sqrt{W_i}$ is substituted for W_i in (10). As a result of this method, the resolution in the required energy range was close to the resolution obtained by S-method. This proves the assumption that the restrictions imposed by the information inequality are related to large fluctuations of transverse energy.

Thus, for the SND calorimeter there is no unique choice of the processing method providing simultaneously the maximum resolution and the minimum systematic shift for isolated photons. For very low energies resolution is weakly dependent on the processing method (Fig.6a). This is connected with the fact that the entire energy of the soft shower is often released in one tower. L-method looks more preferable, though W- and F-methods give

similar results. G-method yields the best results for resolution for the energies lower than 100 MeV but it has the maximum systematic shift.

One of the additional criteria for selecting the processing method is the form of the distribution function for θ^* (Fig.14). G-method gives the distribution close to rectangular while L-method provides practically normal distribution, which is convenient for the further use in kinematic fits.

Resolution of any of the methods described depends on the reconstructed angle θ^* of the shower. In L-method the resolution improves for the values of the angle θ^* approaching 4.5 degrees, i.e. in those cases when the shower passes very close to the border between the towers. As follows from Fig.5, systematic shift decreases as well. The use of dependence (7) and similar dependence for the azimuthal angle allowed the estimation of the RMS of the angle measurement with good accuracy when L-method is used. The distribution of $(\theta^* - \theta)/\sigma(\theta^*)$ and similar distributions for the angle ϕ turned out to be close to normal with RMS equal to 1 with the accuracy as good as 10% in the entire range of photon energies under study.

The analysis of the results of processing of events with close photons shows that L-method has essential advantages as compared with G and W methods. This seems to be connected with the fact that the separation criterion in L-method employs the likelihood function (9) possessing all the available information about the shower. Energies E_+ and E_- appearing in this function are obtained by the summation of the energy deposited in the tower group, fluctuations being thus partially compensated. On the other hand, when the variance is calculated using (13), the energy deposition in every tower is used independently, therefore, the fluctuations of these energies deteriorate the power of the separation criteria in G and W -methods.

7 Conclusion

In the present paper possibilities of obtaining the maximum spatial resolution for electromagnetic showers in the SND calorimeter were considered. For this purpose, the distribution functions of the transverse energy profile in the shower were studied. Six methods for determining the shower angles were examined. The following basic results were obtained:

1. A simple two-parameter function $\exp(-\beta \cdot \sqrt{R})$ for describing the transverse energy distribution in electromagnetic shower is suggested. The comparison of the SND simulation data and the data from [4,14] shows that the profile of transverse distribution of the energy in various calorimeters may be described by similar function.

2. Using the suggested distribution function, a logarithmic method for determination of the shower angles was developed (L-method). This method is shown to give the best resolution for the SND calorimeter in the region of shower energies less than 700 MeV. The centre of gravity method (G-method) provides nearly the same resolution, but the systematic shift here is by 1.5–2 times larger than for L-method.

3. The method of the maximum likelihood (F-method) and the method of nonlinear weighting (W-method) give approximately the same results for resolution and systematic shift as L-method, but their use is hampered by technical difficulties. The hyperbolic sine method (S-method) and modified W-method yield nearly zero systematic shift, but their accuracy of the shower angle determination is by 1.5–2 times worse than in L-method. The deterioration of resolution when the systematic shift is compensated may be qualitatively explained by the information inequality.

4. The L-method allows the construction of a most efficient criterion for close photons separation. Using this method, one has practically the same accuracy of determining the parameters of close and isolated photons.

The logarithmic method suggested in this paper is likely to be useful for other electromagnetic calorimeters. It may be used to determine transverse coordinates of photon, to create the separation parameters for close showers, and for classification of particles.

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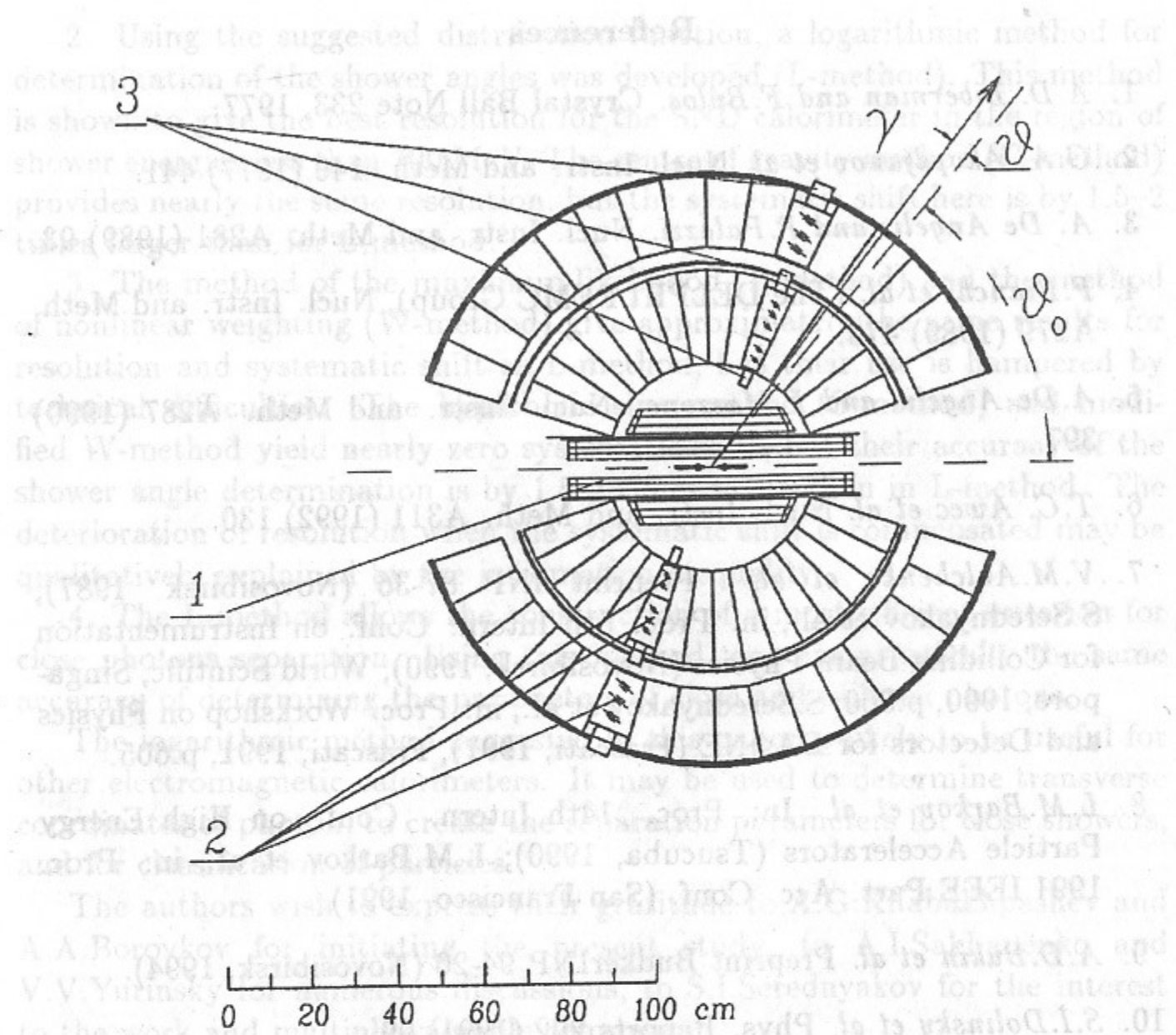


Fig.1. View of the SND calorimeter. 1 - coordinate system; 2 - NaI(Tl) crystals; 3 - phototriodes. The dashed line denotes the central tower axis, i.e. the tower where the $\gamma(e)$ landed. θ_0 is the central tower polar angle. θ is the angle of the $\gamma(e)$ with respect to the central tower axis.

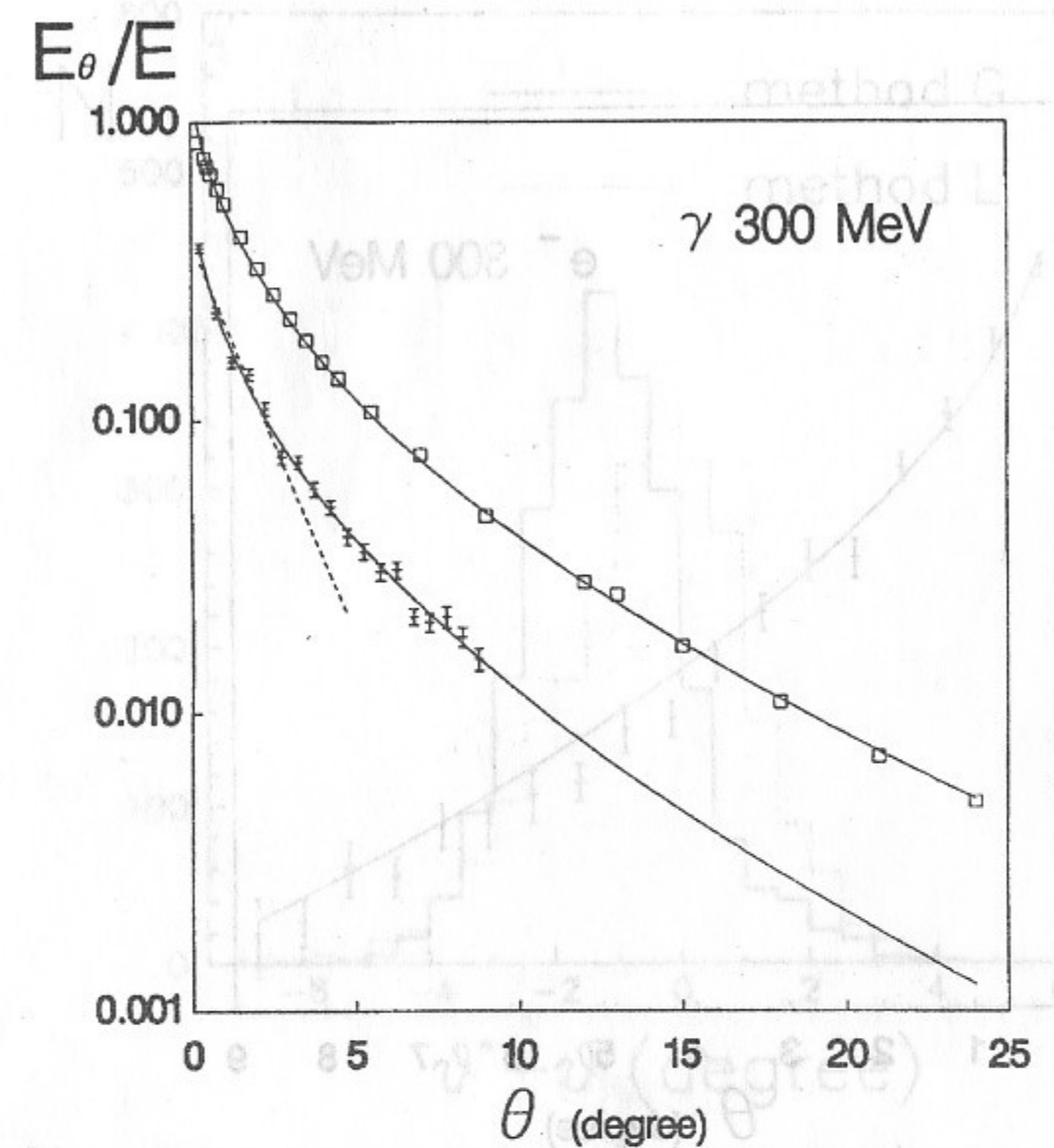


Fig.2. Angular distribution functions of the photon shower energy deposition normalized for the total energy deposited in the shower. \square - average fraction of γ energy deposited outside the cone with the angle θ whose axis coincide with γ direction (simple simulation of spherical NaI layers). \times - the average fraction of the energy deposited outside the plane be situated under the angle θ to the direction of the incident γ (complete simulation of the SND calorimeter). The solid lines show the approximation of data by the function (1), the dashed line is a simple exponent (3).

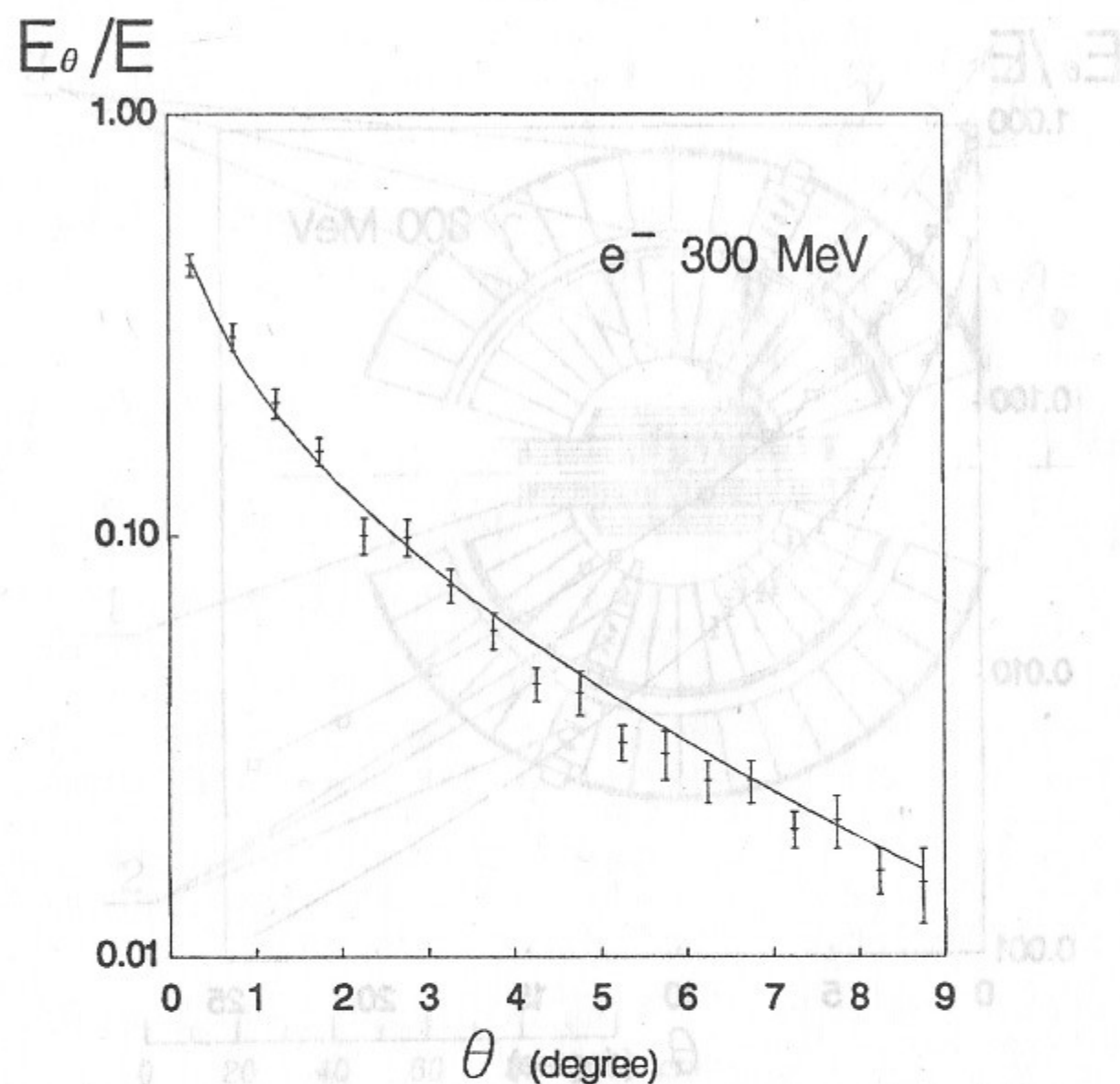


Fig.3. Angular distribution function of the electron shower energy deposition normalized for the total energy deposited in the shower, i.e. the average fraction of the energy deposited outside the plane be situated under the angle θ to the direction of the incident electron. The points denote simulation of SND calorimeter. The solid line is the approximation by dependence (1).

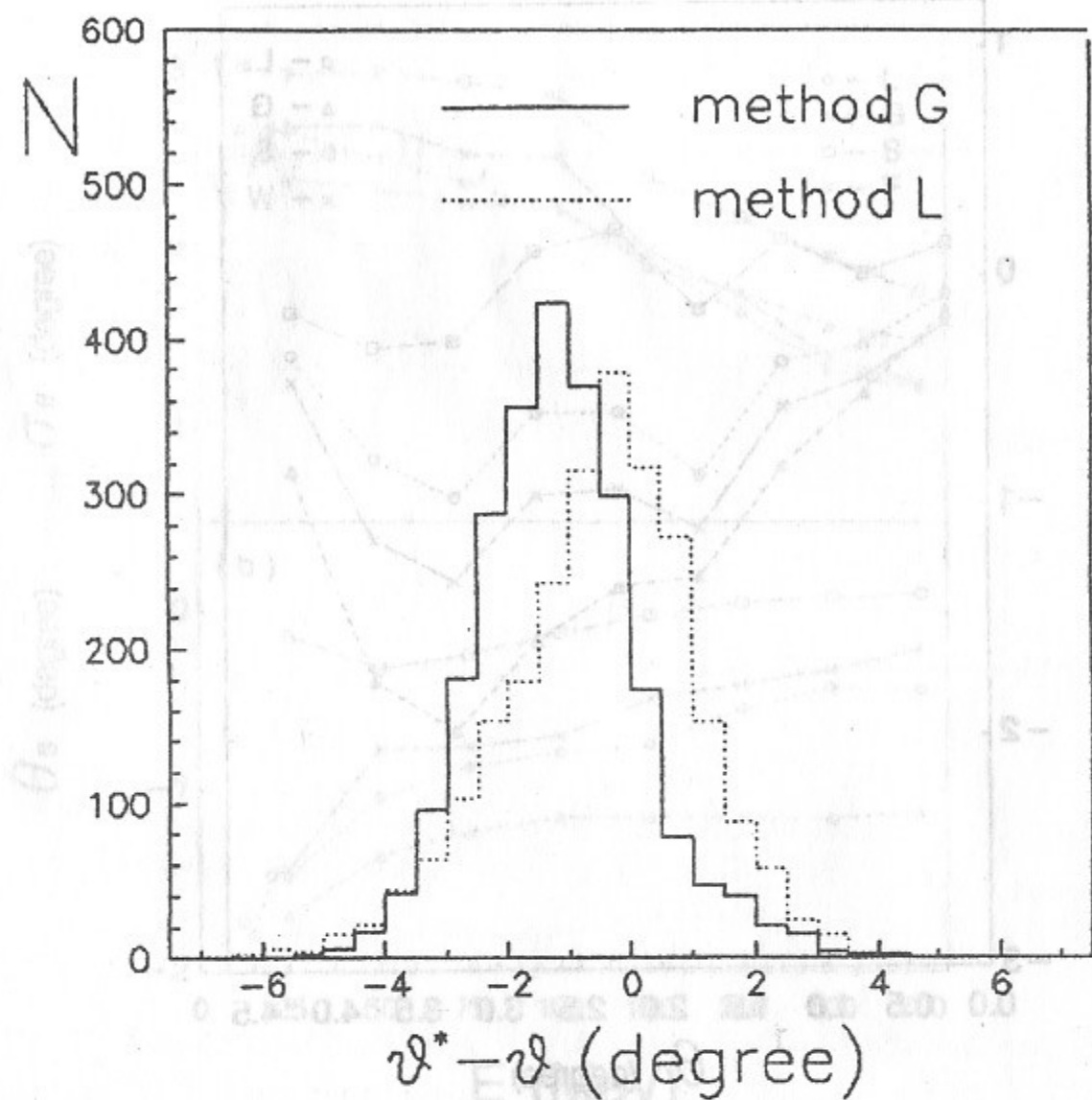


Fig.4. Calorimeter resolution obtained by the centre of gravity method (G-method) and logarithmic method (L-method) for the photon showers with the angles θ in the interval (0-4.5) degrees. The photon energy is 300 MeV. The average value of each distribution is the systematic shift.

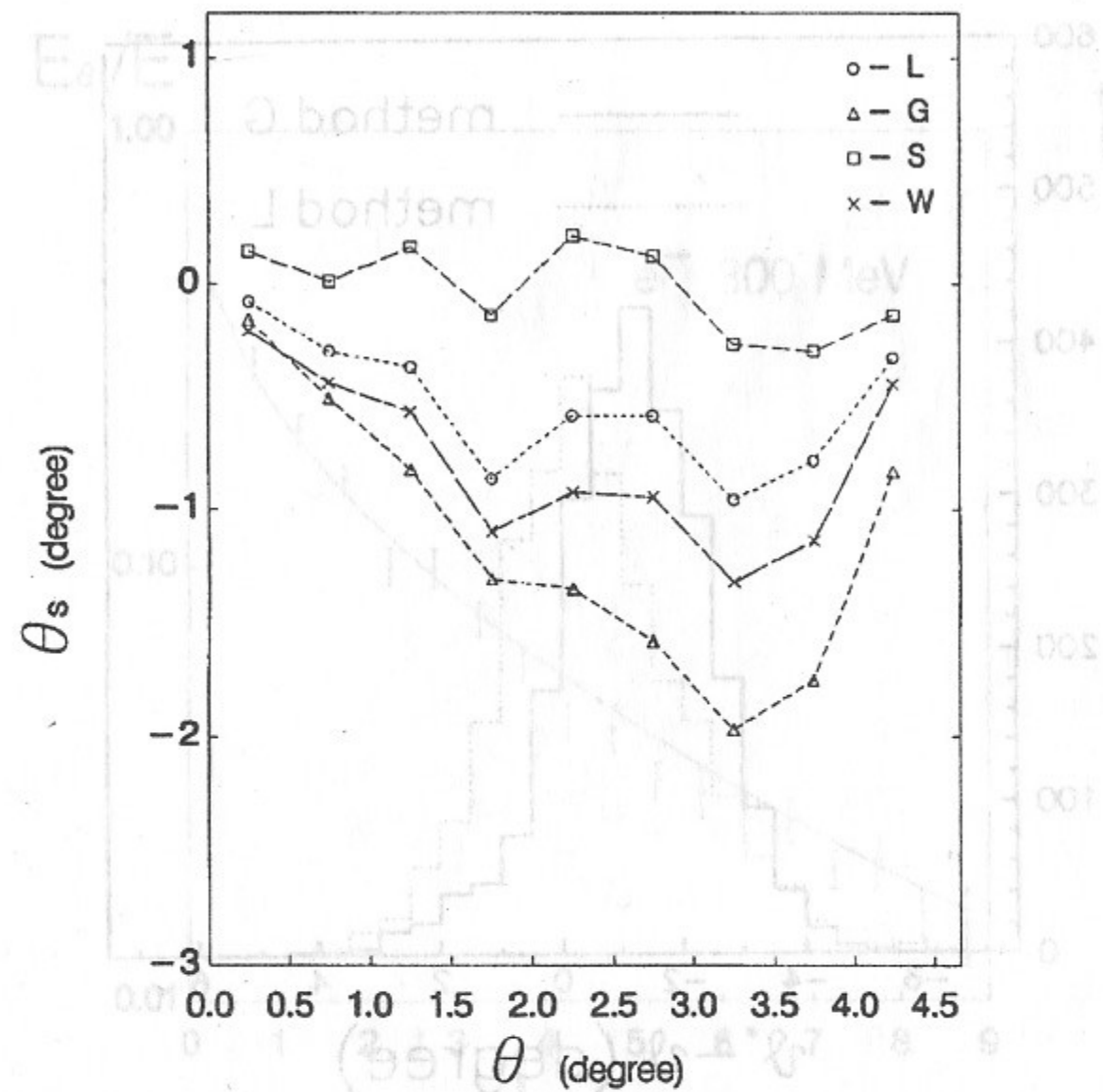


Fig. 5. Systematic shift dependences on the photon angle θ for various processing methods (photon energy is 300 MeV).

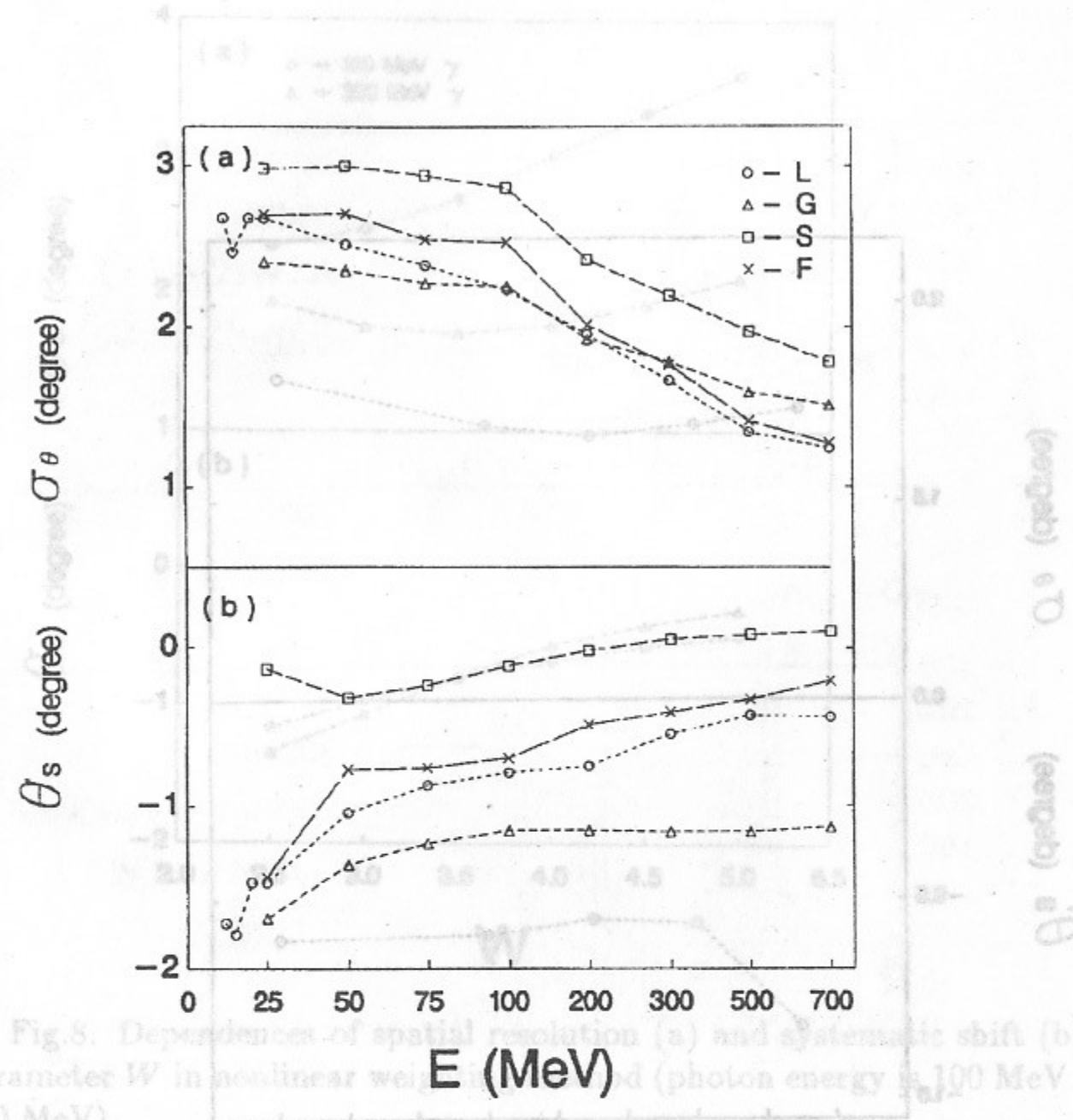


Fig. 6. Dependences of spatial resolution (a) and systematic shift (b) on the photon energy for various processing methods.

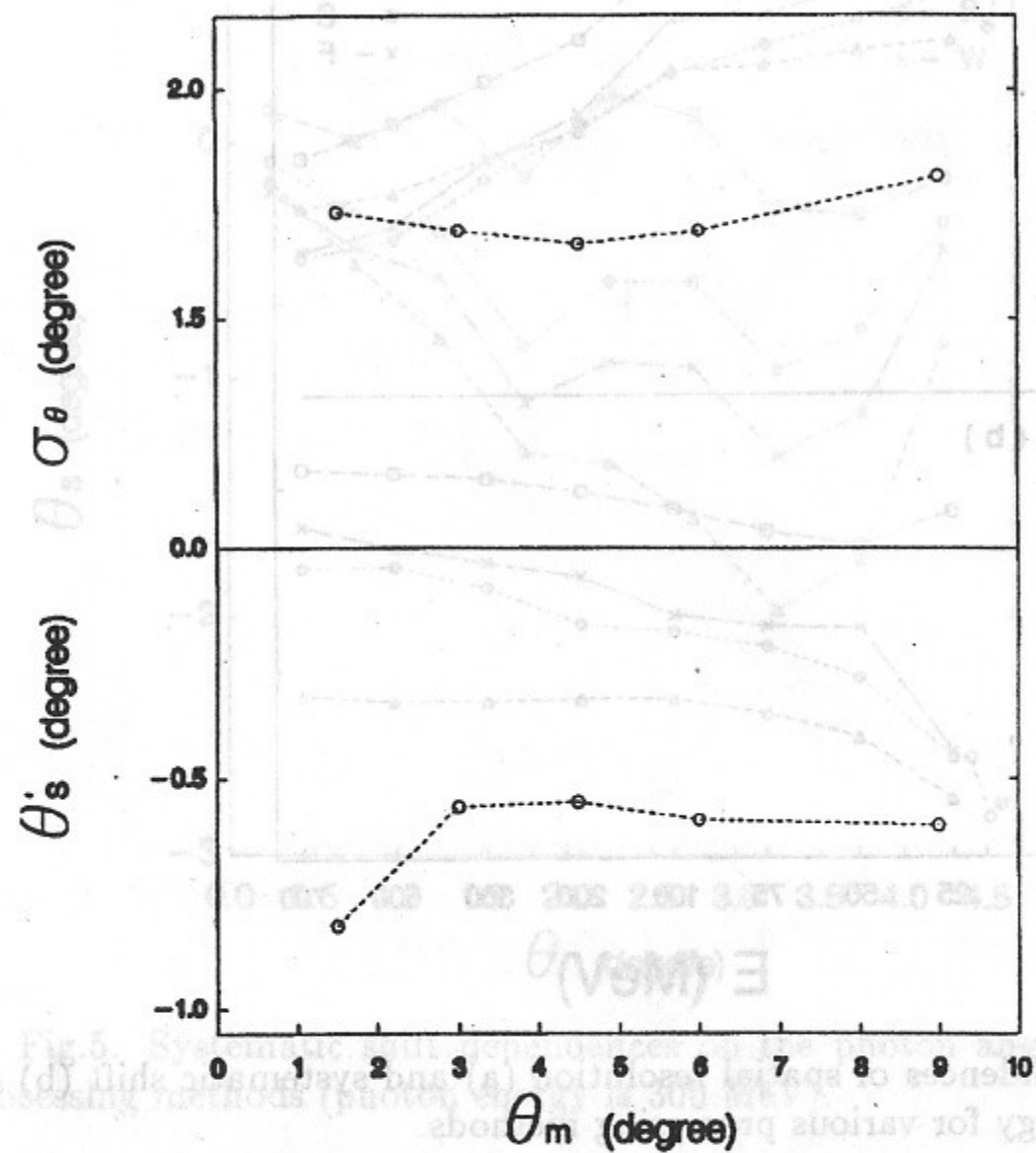


Fig.7. Dependences of spatial resolution (a) and systematic shift (b) on parameter θ_M in logarithmic method (photon energy is 300 MeV).

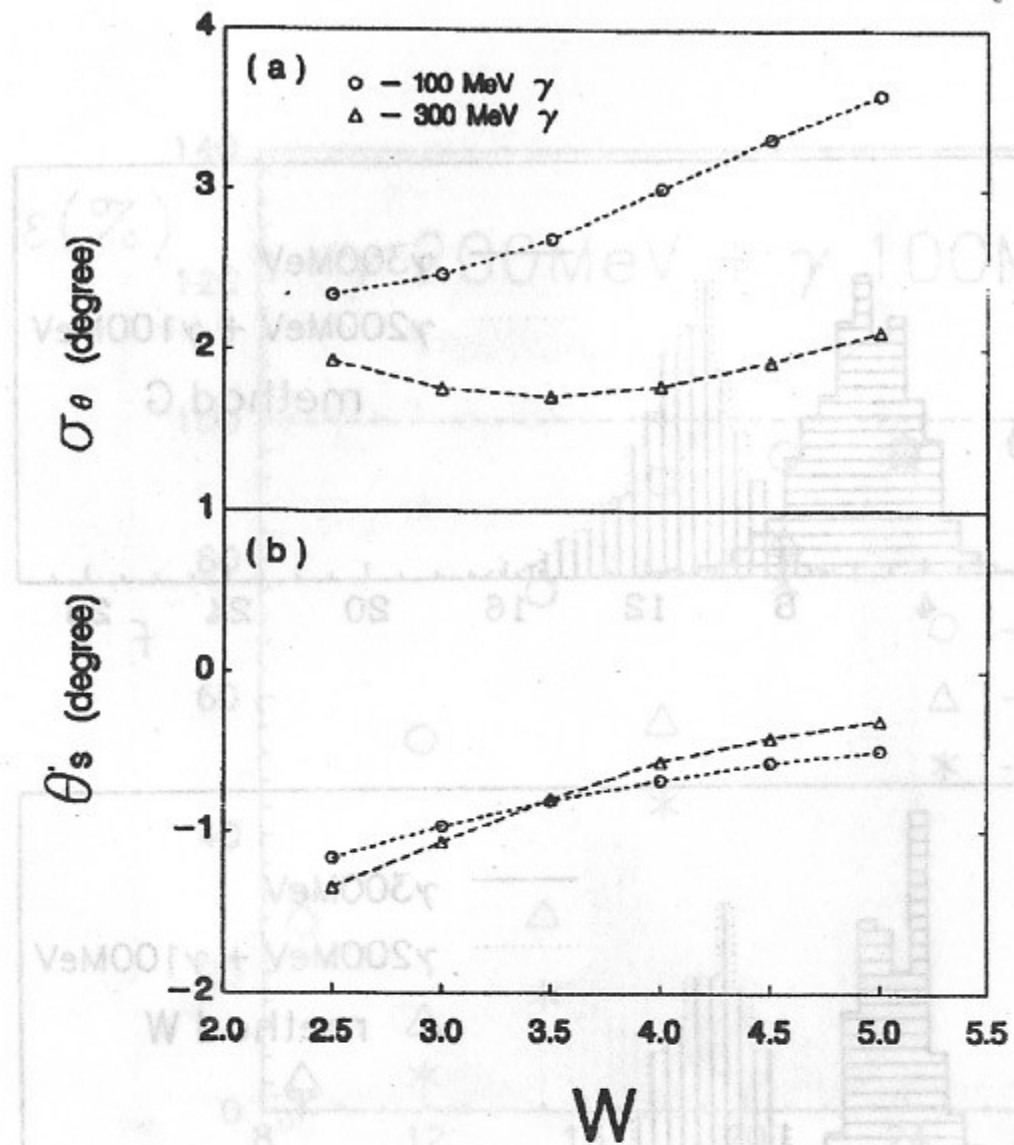


Fig.8. Dependences of spatial resolution (a) and systematic shift (b) on parameter W in nonlinear weighting method (photon energy is 100 MeV and 300 MeV).

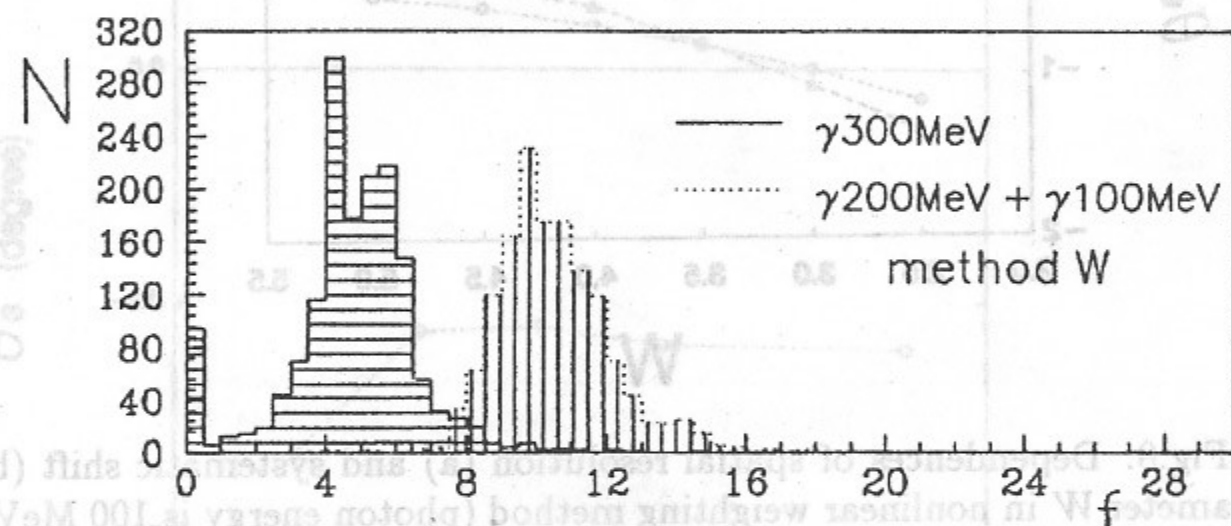
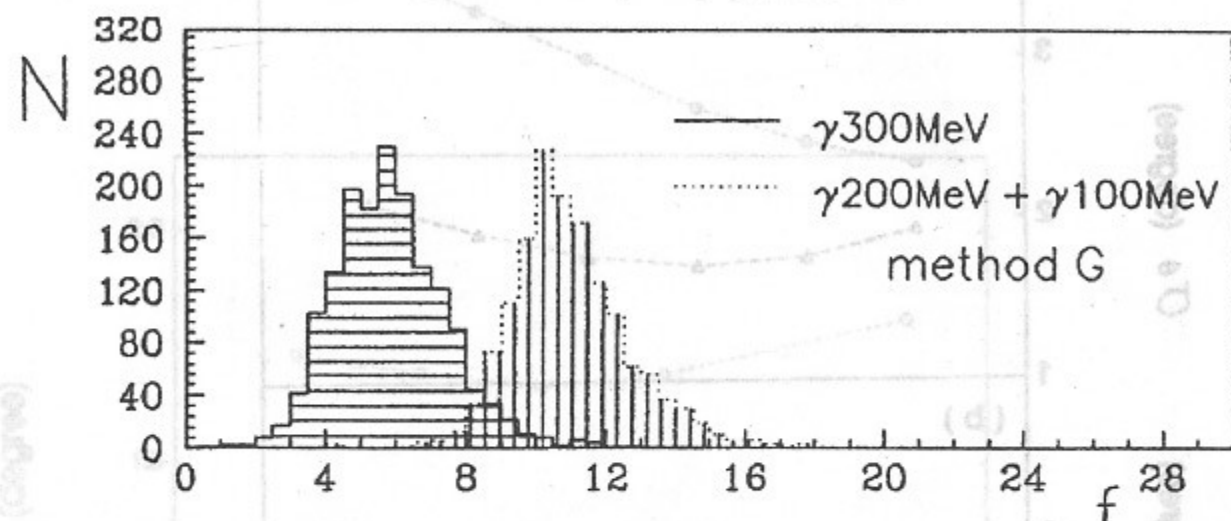


Fig.9. Distributions on separation parameter of two- and one-photon events by the centre of gravity method (G) and by nonlinear weighting method (W). Events with an isolated photon are shown as well as events with two photons with a spatial angle 18 degrees between them. The total photon energy is 300 MeV.

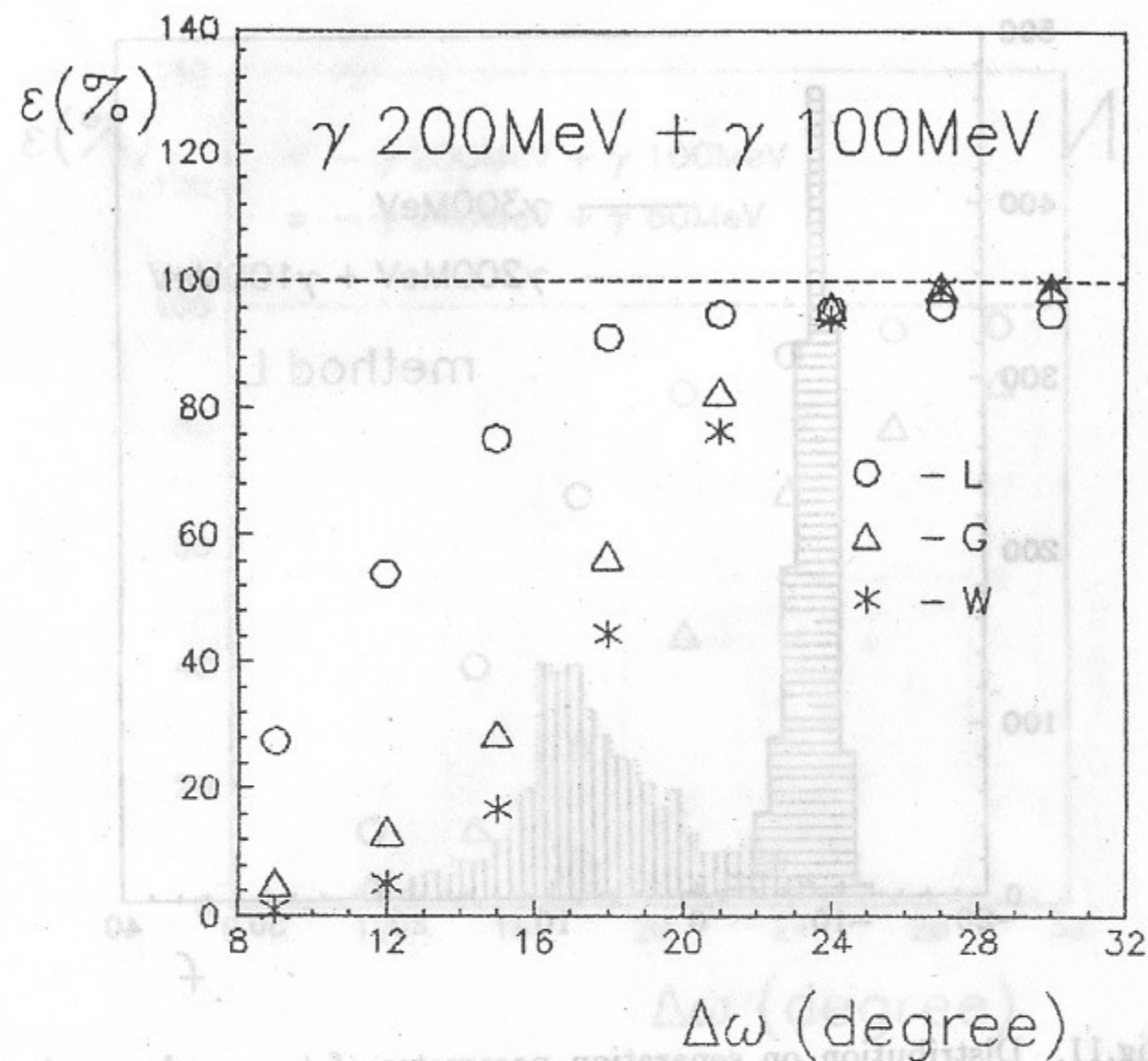


Fig.10. Efficiency of identification of two-photon events depending on the spatial angle between the photons for 99% efficiency of identification of one-photon events. The results are shown for centre of gravity method (G), logarithmic method (L) and nonlinear weighting method (W). The total photon energy is 300 MeV.

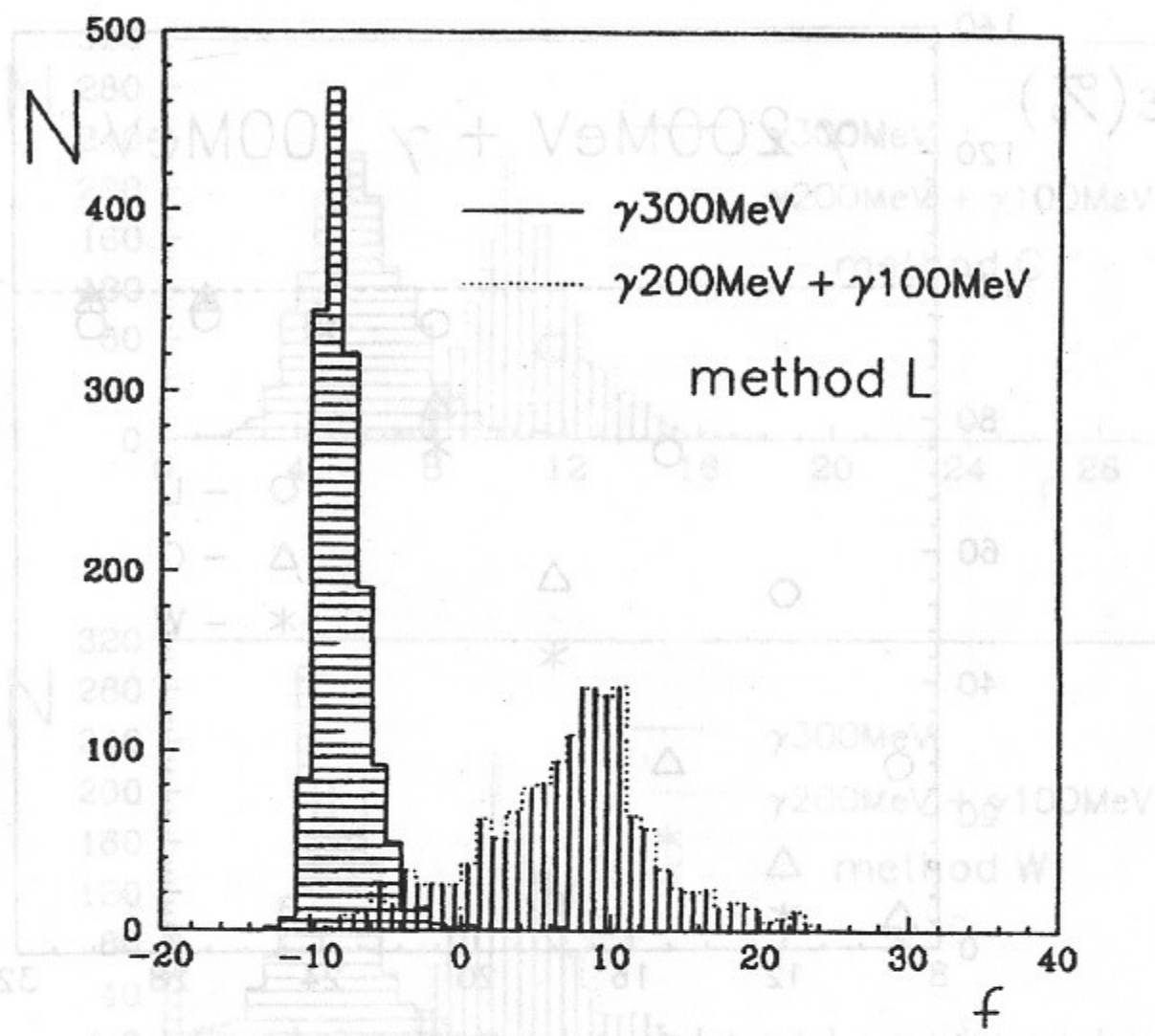


Fig.11. Distribution on separation parameter of two- and one-photon events by the logarithmic method. Events with an isolated photon are shown as well as events with two photons with a spatial angle 18° between them. The total photon energy is 300 MeV.

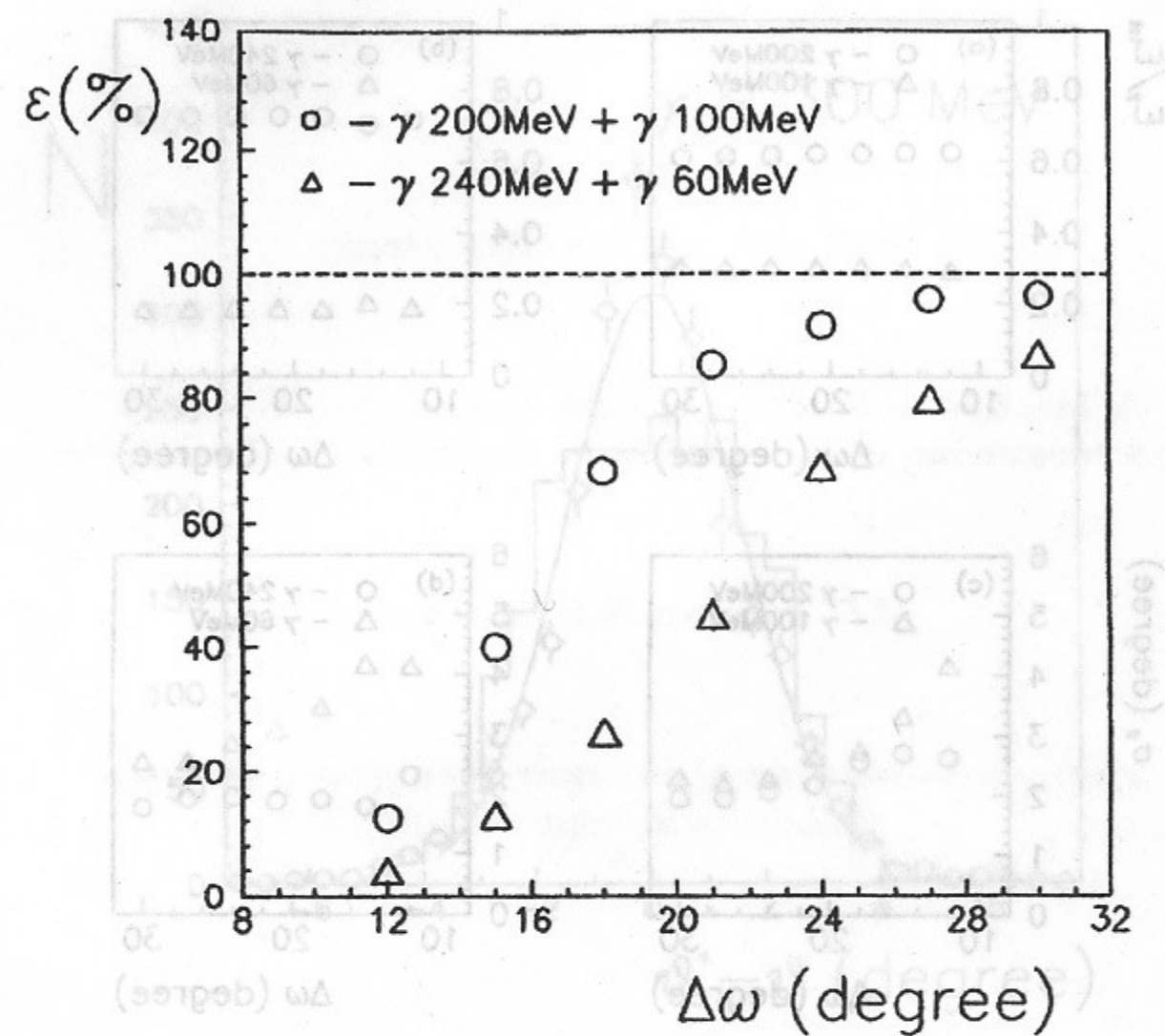


Fig.12. Efficiency of complete reconstruction of two-photon events in the SND calorimeter depending on the spatial angle between the photons. The total photon energy is 300 MeV.

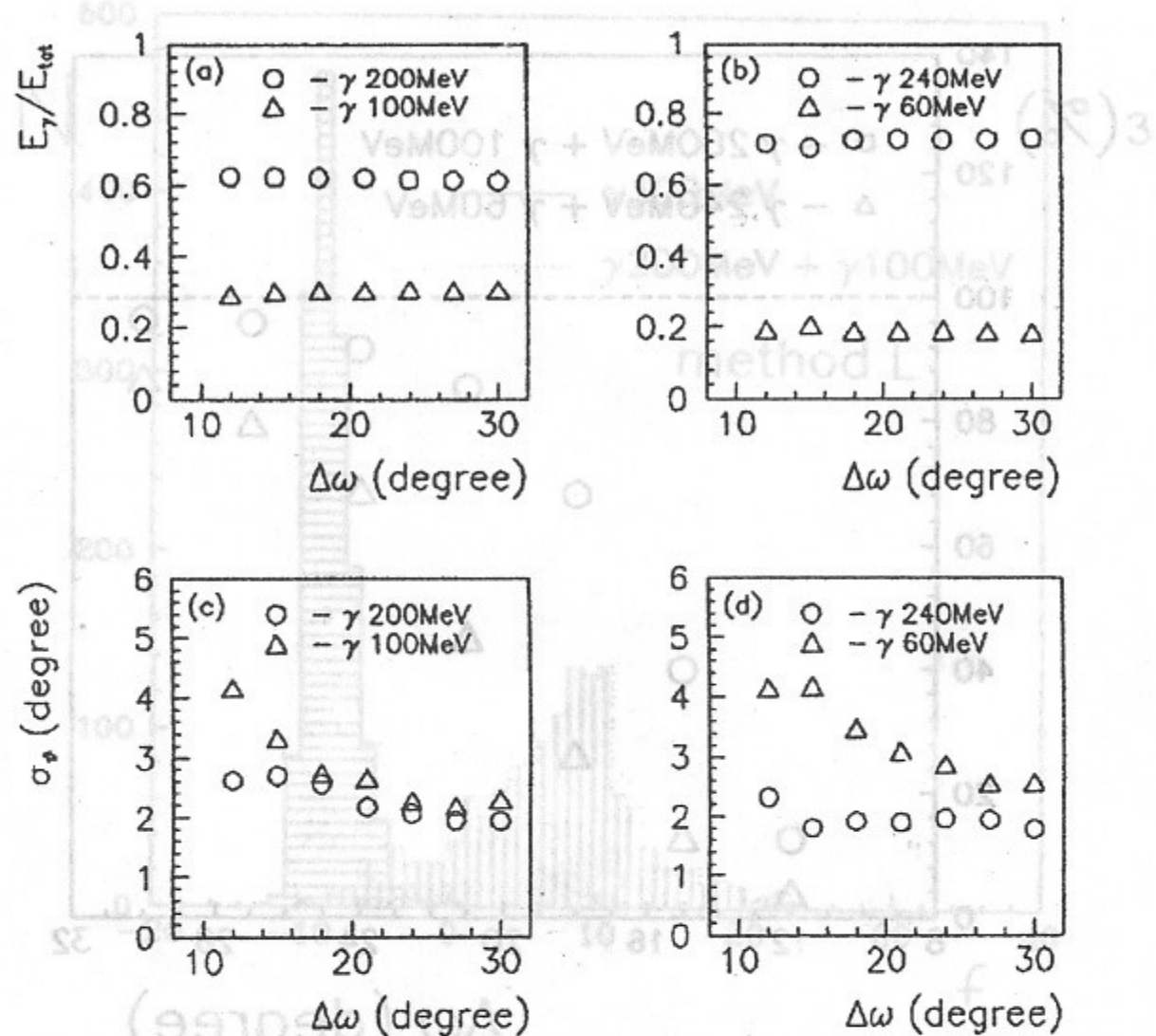


Fig.13. Parameters of reconstructed two-photon events depending on the spatial angle between the photons. (a,b) is the average energy deposition, (c,d) is the spatial resolution.

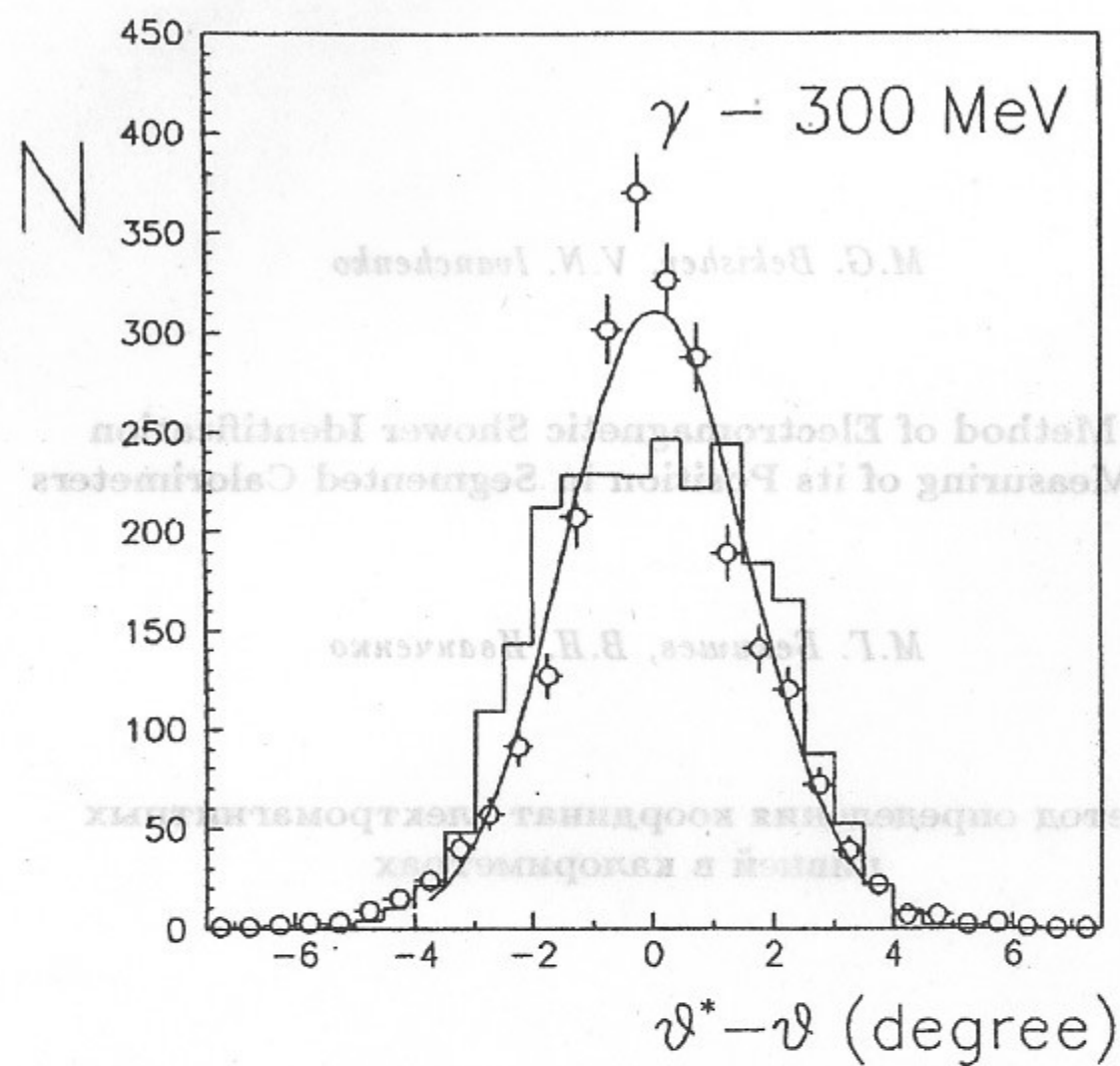


Fig.14. The SND calorimeter resolution for photons with the energy 300 MeV. The histogram illustrates the centre of gravity method. The points show logarithmic method, the solid line - fit by the normal distribution.

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