

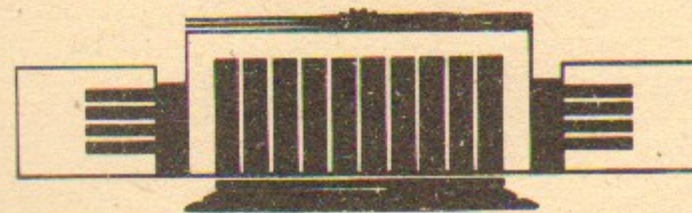


ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ  
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COUPLINGS OF HEAVY HADRONS  
WITH SOFT PIONS  
FROM QCD SUM RULES

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## Couplings of heavy hadrons with soft pions from QCD sum rules

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### ABSTRACT

We estimate the couplings in the Heavy Hadron Chiral Theory (HHCT) lagrangian from the QCD sum rules in an external axial field. Stability of the sum rules at moderate values of the Borel parameter is poor that probably signals a slow convergence of the OPE series. At large values of the Borel parameter they stabilize, and yield the couplings much lower than the constituent quark model expectations. This region is not trustworthy for baryons, but in the meson case only unexpectedly large contributions of a few lowest excitations could invalidate the prediction  $g_1 \approx 0.2$ .

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## 1 Introduction

It is well known that the QCD lagrangian with  $n_l$  massless flavours has the  $SU(n_l)_L \times SU(n_l)_R$  symmetry spontaneously broken to  $SU(n_l)_V$  giving the  $(n_l^2 - 1)$ -plet of pseudoscalar massless Goldstone mesons (pions)  $\pi_j^i$  ( $\pi_i^i = 0$ ). Their interactions at low momenta are described by the chiral lagrangian (see e. g. [1])

$$L_\pi = \frac{f^2}{8} \text{Tr} \partial_\mu \Sigma^+ \partial_\mu \Sigma + \dots, \quad \Sigma = \exp \frac{2i\pi}{f}, \quad (1)$$

where the pion constant  $f \approx 132\text{MeV}$  is defined by

$$\langle 0 | j_{j\mu}^i | \pi \rangle = i f e_j^i p_\mu, \quad j_{j\mu}^i = \bar{q}_j \gamma_\mu \gamma_5 q^i \rightarrow i \frac{f^2}{2} (\Sigma^+ \partial_\mu \Sigma - \Sigma \partial_\mu \Sigma^+)_j^i$$

( $e_j^i$  is the pion flavour wave function), and dots mean terms with more derivatives. Light quark masses can be included perturbatively, and lead to extra terms in (1).  $SU(n_l)_L \times SU(n_l)_R$  transformations act as  $\Sigma \rightarrow L \Sigma R^+$ . Let's define  $\xi = \exp i\pi/f$ ,  $\Sigma = \xi^2$ ; it transforms as  $\xi \rightarrow L \xi U^+ = U \xi R^+$  where  $U$  is a  $SU(n_l)$  matrix depending on  $\pi(x)$ . The vector  $v_\mu = \frac{1}{2}(\xi^+ \partial_\mu \xi + \xi \partial_\mu \xi^+)$  and the axial vector  $a_\mu = \frac{i}{2}(\xi^+ \partial_\mu \xi - \xi \partial_\mu \xi^+)$  transform as  $v_\mu \rightarrow U(v_\mu + \partial_\mu)U^+$ ,  $a_\mu \rightarrow U a_\mu U^+$ . There is a freedom in transformation laws of matter fields such as  $\psi^i$  because it is always possible to multiply them by a matrix depending on  $\pi$ . The only requirement is the correct transformation with respect to  $SU(n_l)_V$  ( $L = R$ ). It is convenient to choose  $\psi \rightarrow U \psi$ . Then the covariant derivative  $D_\mu = \partial_\mu + v_\mu$  transforms as  $D_\mu \psi \rightarrow U D_\mu \psi$ . Covariant derivatives of tensors with more flavour indices are defined similarly.



Hadrons with a heavy quark are now successively investigated in the framework of the Heavy Quark Effective Theory (HQET) [2] (for review and references see [3]). To the leading order in  $1/m$ , the heavy quark spin does not interact and can be rotated or switched off at all (spin-flavour and superflavour symmetry). The  $\bar{Q}q$  mesons with a spinless heavy quark form the  $j^P = \frac{1}{2}^+$   $n_l$ -plet  $\psi^i$ . The  $Qqq$  baryons can have  $j^P = 0^+$  or  $1^+$  giving the scalar flavour-antisymmetric  $n_l(n_l - 1)/2$ -plet  $\Lambda^{ik}$  and the vector flavour-symmetric  $n_l(n_l + 1)/2$ -plet  $\bar{\Sigma}^{ik}$ . Switching the heavy quark spin on gives degenerate  $0^-$  and  $1^-$   $B$  and  $B^*$  mesons,  $\frac{1}{2}^+$   $\Lambda$  baryons, and degenerate  $\frac{1}{2}^+$  and  $\frac{3}{2}^+$   $\Sigma$  and  $\Sigma^*$  baryons.

Interaction of these ground-state heavy hadrons with soft pions is described by the Heavy Hadron Chiral Theory [4, 5]. Excited mesons were incorporated in [6], and electromagnetic interactions—in [7]; chiral loop effects were considered in [8, 5]. We start from the HHCT lagrangian with the heavy quark spin switched off:

$$L = L_\pi + \bar{\psi}_i iD_0 \psi^i + \Lambda_{ij}^* iD_0 \Lambda^{ij} + \bar{\Sigma}_{ij}^* \cdot (iD_0 - \Delta) \bar{\Sigma}^{ij} \quad (2)$$

$$+ g_1 \bar{\psi}_i \hat{a}_j^i \gamma_5 \psi^j + 2ig_2 \bar{\Sigma}_{ik}^* \cdot \bar{a}_j^i \times \bar{\Sigma}^{jk} + 2g_3 \left( \Lambda_{ik}^* \bar{a}_j^i \cdot \bar{\Sigma}^{jk} + \bar{\Sigma}_{ik}^* \cdot \bar{a}_j^i \Lambda^{jk} \right),$$

where  $\Delta$  is the  $\Sigma$ - $\Lambda$  mass difference. The possibility of consideration of the  $\Sigma\Lambda\pi$  interaction in HHCT relies on the fact that this difference is small compared to the chiral symmetry breaking scale though formally both of them are of the order of the characteristic hadron mass scale. The matrix elements of the axial current between heavy hadrons are easily obtained using PCAC:

$$\langle M' | \bar{j}_j^i | M \rangle = g_1 \bar{u}_j^i \bar{\gamma} \gamma_5 u^i, \quad (3)$$

$$\langle \Sigma' | \bar{j}_j^i | \Sigma \rangle = 2ig_2 \bar{e}_{jk}^i \times \bar{e}^{ik}, \quad \langle \Lambda | \bar{j}_j^i | \Sigma \rangle = 2g_3 e_{jk}^i \bar{e}^{ik},$$

where  $u^i$ ,  $e^{ij}$ ,  $\bar{e}^{ij}$  are the  $M$ ,  $\Lambda$ ,  $\Sigma$  wave functions, and the nonrelativistic normalization of the states and wave functions is assumed. If we switch the heavy quark spin on, we obtain the usual HHCT lagrangian [4, 5].

The HHCT couplings  $g_i$  should be in principle calculable in the underlying theory—HQET, but this is a difficult nonperturbative problem. Some experimental information is available only on  $g_1$ . If we neglect  $1/m_c$  corrections, then  $\Gamma(D^{*+} \rightarrow D^0 \pi^+) = \frac{g_1^2 P_\pi^3}{6\pi f^2}$ , and similarly for  $D^+ \pi^0$  (with the extra  $1/2$ ). The experimental upper limit [9] on  $\Gamma(D^{*+})$  combined with the branching ratios [10] give  $g_1 < 0.68$ . A combined analysis of  $D^*$  pionic and radiative decays was performed in [7]; it gives  $g_1 \sim 0.4$ – $0.7$ .

In the constituent quark model,  $g_1$  is the axial charge  $g$  of the constituent light quark in the heavy meson. Moreover, following the folklore definition “constituent quark is  $B$  meson minus  $b$  quark”, this is the most clear way to define  $g$  of the constituent quark. The baryonic couplings  $g_{2,3}$  are also equal to  $g$  in this model. The most naive estimate is  $g \approx 1$ ; the nucleon axial charge is  $g_A = \frac{5}{3}g$  in the constituent model, and in order to obtain  $g_A = 1.25$  we should assume  $g = 0.75$ .

Sum rules [11] were successfully used to solve many nonperturbative problems in QCD and HQET. The currents with the quantum numbers of the ground state mesons and baryons with the heavy quark spin switched off are

$$j_M^i = Q^* \frac{1 + \gamma_0}{2} q^i, \quad j_{\Lambda 1}^{ij} = (q^{T[i} C \gamma_5 q^{j]}) Q, \quad \bar{j}_{\Sigma 1}^{ij} = (q^{T(i} C \bar{\gamma} q^{j)}) Q, \quad (4)$$

where  $Q$  is the spinless static quark field,  $C$  is the charge conjugation matrix,  $q^T$  means  $q$  transposed,  $(ij)$  and  $[ij]$  mean symmetrization and antisymmetrization. There are also currents  $j_{\Lambda 2}^{ij}$  and  $\bar{j}_{\Sigma 2}^{ij}$  with the additional  $\gamma_0$ . Correlator of the mesonic currents was investigated in [12, 13], and of the baryonic ones—in [14]. The sum rules results are in a qualitative agreement with the constituent quark model: the massless quark propagator plus the quark condensate contribution simulate the constituent quark propagation well enough.

The sum rules method was generalized to the case of a constant external field for calculation of such static characteristics of hadrons as the magnetic moments [15]. Sum rules in an external axial field were used [16] for calculation of  $g_A$  of light baryons. In the present work we use HQET sum rules in an axial field to calculate  $g_{1,2,3}$ . Sum rules for the  $D^* D \pi$  coupling have been studied earlier [17]; their result (in the terms used here) is  $g_1 = 0.2$ . We disagree with some results of this paper.

## 2 Mesons

We introduce the external axial field  $A_{j\mu}^i$  ( $A_{i\mu}^i = 0$ ) by adding the term

$$\Delta L = j_{j\mu}^i A_{i\mu}^j \quad (5)$$

to the lagrangian. We are going to calculate correlators of the currents (4) up to the terms linear in  $A$  (these terms are denoted by the subscript  $A$ ). The light quark propagator in the gauge  $x_\mu A_\mu(x) = 0$  gets the contribution

$$S_{jA}^i(x, 0) = -\frac{iA_j^i \cdot x}{2\pi^2} \left( \frac{\hat{x} \gamma_5}{x^4} - \frac{x_\mu \tilde{G}_{\mu\nu} \gamma_\nu}{4x^2} \right) + \dots \quad (6)$$



The  $G^2$  term in  $S_A$  vanishes after the vacuum averaging; we are not going to calculate gluonic contributions beyond  $G^2$  and hence may omit this term.<sup>4</sup> The axial field induces the quark condensates<sup>5</sup>

$$\langle q^{ia\alpha}(x)\bar{q}_{jb\beta}(0) \rangle_A = \frac{\delta_b^a}{4N} \left\{ \left[ f^2 \hat{A}_j^i + \frac{i}{6} \langle \bar{q}q \rangle \left( [\hat{A}_j^i, \hat{x}] + 6A_j^i \cdot x \right) - \frac{m_1^2 f^2}{36} \left( 5x^2 \hat{A}_j^i - 2A_j^i \cdot x \hat{x} \right) \right] \gamma_5 \right\}_\beta^\alpha, \quad (7)$$

$$\langle q^{ia\alpha} g \tilde{G}_{\mu\nu}^A \bar{q}_{jb\beta} \rangle_A = \frac{(t^A)_b^a}{12C_F N} m_1^2 f^2 (A_{j\mu}^i \gamma_\nu - A_{j\nu}^i \gamma_\mu)_\beta^\alpha,$$

where  $m_1^2 \approx 0.2 \text{ GeV}^2$  is defined by [18]<sup>6</sup>  $\langle 0 | \bar{d} g \tilde{G}_{\mu\nu} \gamma_\nu \gamma_5 u | \pi^+ \rangle = i m_1^2 f p_\mu$ ,  $N = 3$  is the number of colours,  $C_F = \frac{N^2 - 1}{2N}$ . We assumed  $p \cdot A = 0$  where  $p \rightarrow 0$  is the momentum of the field  $A$ ; in general these formulae should contain  $A_\perp = A - \frac{A \cdot p}{p^2} p$ .

The correlator of the meson currents (4) has the  $A$ -term

$$i \langle T j^i(x) \bar{j}_j(0) \rangle_A = \frac{1 + \gamma_0}{2} \hat{A}_j^i \gamma_5 \frac{1 + \gamma_0}{2} \delta(\vec{x}) \Pi_A(x_0) \quad (8)$$

that depends only on  $\vec{A}_j^i$  and not  $A_{j0}^i$ . The correlator possesses the usual dispersion representation at any  $A_{j\mu}^i$ . The meson contribution at  $A_{j\mu}^i = 0$  is  $\rho(\omega) = F^2 \delta(\omega - \varepsilon)$  where  $\varepsilon$  is the meson energy and  $\langle 0 | j^i | M \rangle = i F u^i$  (the usual meson constant is  $f_M = \frac{2F}{\sqrt{m}}$ ). Switching on the external field produces the energy shift  $\varepsilon \rightarrow \varepsilon - g_1 \gamma_5 \vec{\gamma} \cdot \vec{A}$  ( $s_n = \frac{1}{2} \gamma_5 \vec{\gamma} \cdot \vec{n} = \pm \frac{1}{2}$  is the meson spin projection). Therefore the meson contribution is  $\rho_A(\omega) = g_1 F^2 \delta'(\omega - \varepsilon) + c \delta(\omega - \varepsilon)$  where the second term originates from the change of  $F^2$ . Besides that there is a smooth continuum contribution  $\rho_A^{\text{cont}}(\omega)$ . Thus we obtain

$$\Pi_A(\omega) = \frac{g_1 F^2}{(\omega - \varepsilon)^2} + \frac{c}{\omega - \varepsilon} + \Pi_A^{\text{cont}}(\omega). \quad (9)$$

In other words, the lowest meson's contribution in both channels (Fig. 1a) gives the double pole at  $\omega = \varepsilon$ ; mixed lowest-higher state contributions

<sup>4</sup>The first term in (6) was presented in [16], but we don't understand how the authors calculated  $G^2$  corrections without the second term and the above statement about the third one.

<sup>5</sup>These formulae were presented in [16] but with a different second term in the first equation; their term does not satisfy the relation  $\langle (\hat{D}q)\bar{q} \rangle = 0$  that follows from the equation of motion.

<sup>6</sup>the sum rule considered in the second paper of [18] yields the relation  $m_1^2 \approx m_0^2/4$ .

(Fig. 1b) give a single pole at  $\omega = \varepsilon$  plus a term with a spectral density in the continuum region after the partial fraction decomposition; higher states' contributions (Fig. 1c) have a spectral density in the continuum region only.

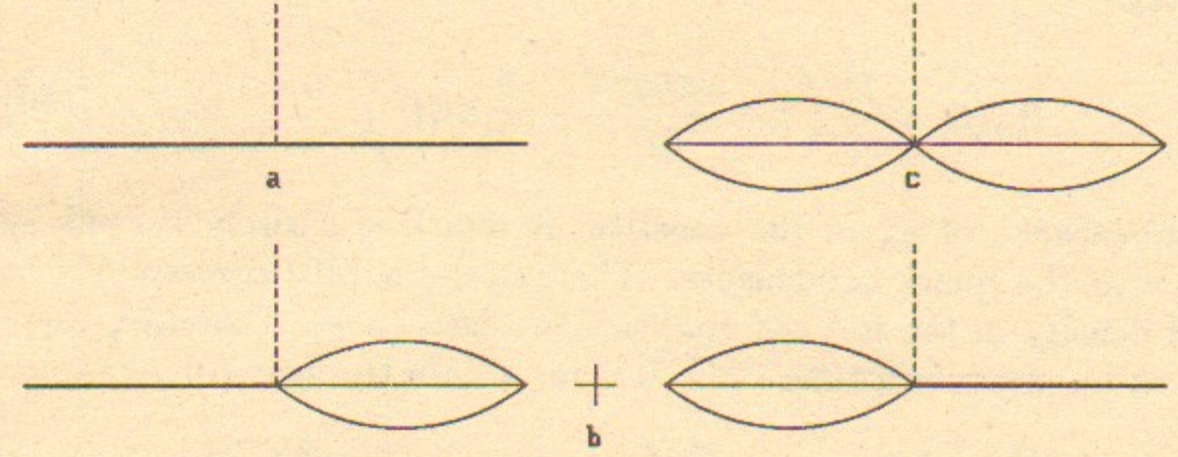


Figure 1: Physical states' contributions to the correlator

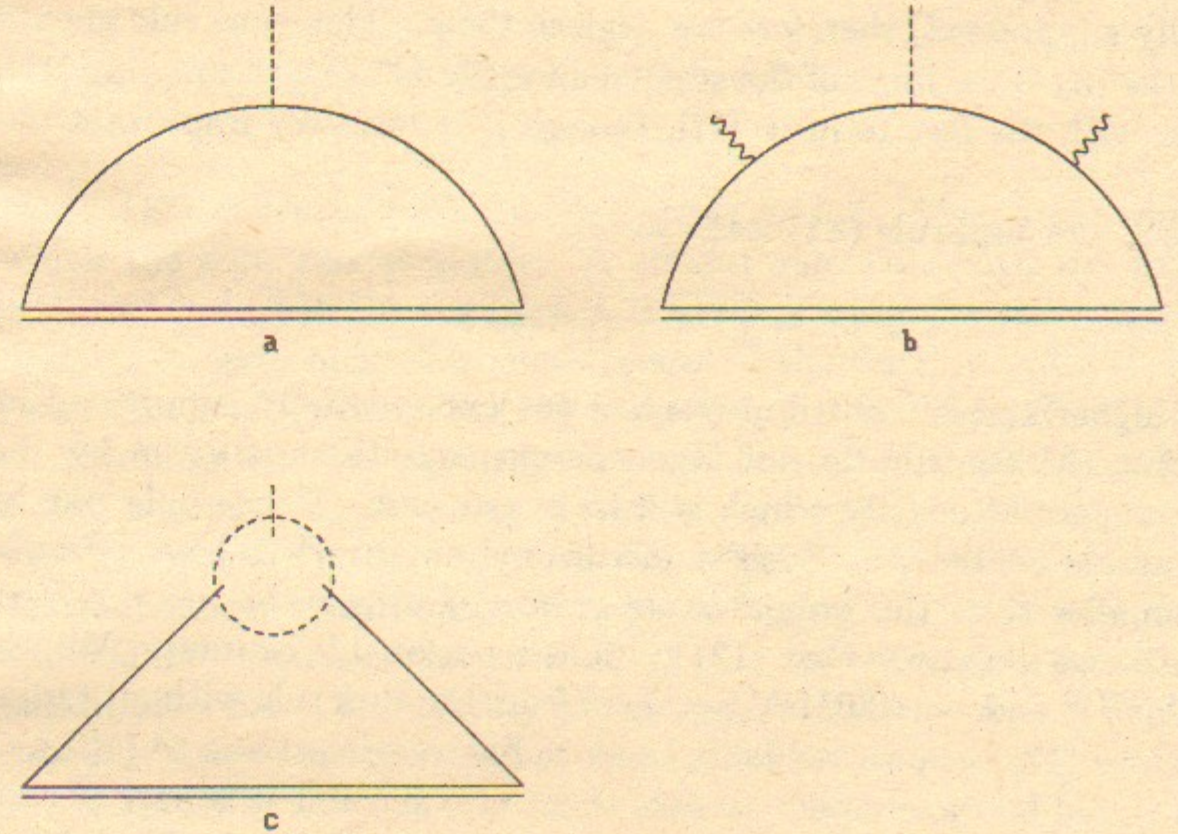


Figure 2: Correlator of meson currents



We can also calculate the correlator using OPE. Gluons don't interact with the heavy quark in the fixed point gauge  $x_\mu A_\mu(x) = 0$ . The diagrams with non-cut light quark line (Fig. 2a, b) vanish. The diagram with cut line (Fig. 2c) gives

$$\Pi_A(t) = \frac{f^2}{4} \left( 1 - \frac{i\langle\bar{q}q\rangle t}{3f^2} - \frac{5}{36} m_1^2 t^2 \right). \quad (10)$$

Thus the appearance of  $g_A$  of the constituent quark is entirely caused by interaction with the quark condensate. The correlator (10) corresponds to the spectral density in the form of  $\delta(\omega)$  and its derivatives. Equating these expressions at an imaginary time  $t = -i/E$  we obtain the sum rule

$$\left( \frac{g_1 F^2}{E} + c \right) e^{-\varepsilon/E} = \frac{f^2}{4} \left( 1 - \frac{\langle\bar{q}q\rangle}{3f^2 E} + \frac{5}{36} \frac{m_1^2}{E^2} \right). \quad (11)$$

At sufficiently high energies the hadronic spectral density is equal to the theoretical one, and it vanishes. Moreover, higher states' contributions are exponentially suppressed, therefore we neglect them. This sum rule should agree with the  $m_c \rightarrow \infty$  limit of the sum rule for the  $D^* \rightarrow D\pi$  coupling [17]. We disagree with the last term in [17], though it is not very important numerically.

At large  $E$  the sum rule (11) leads to

$$g_1 F^2 = \frac{1}{4} f^2 \varepsilon - \frac{1}{12} \langle\bar{q}q\rangle. \quad (12)$$

Of course, higher states' contributions are not exponentially suppressed at large  $E$ . But the contribution of high energy excitations is given by the perturbative spectral density which is zero in our case. Hence only one or few lowest states contribute. Excited mesons are supposed to have  $F^2$  substantially smaller than the ground state meson, and may be are not very important. So we may hope that (12) gives us a reasonable estimate. We use  $F^2 = (240\text{MeV})^3$  and  $\varepsilon = 430\text{MeV}$  obtained from the sum rule without radiative corrections [12] because we don't know radiative corrections to (10) and feel that it would be inconsistent to take them into account in a part of sum rules (this would lead to large errors e. g. in the sum rule for the Isgur-Wise function). Of course, the existence of large radiative corrections [13] leads to an additional uncertainty in the result. If higher states' contributions to (12) are not important, we obtain  $g_1 \approx 0.2$ .

We can multiply (11) by  $\exp(\varepsilon/E)$  and differentiate in  $E$  in order to exclude  $c$ . The result is shown in Fig. 3. Of course, at large enough  $E \gtrsim$

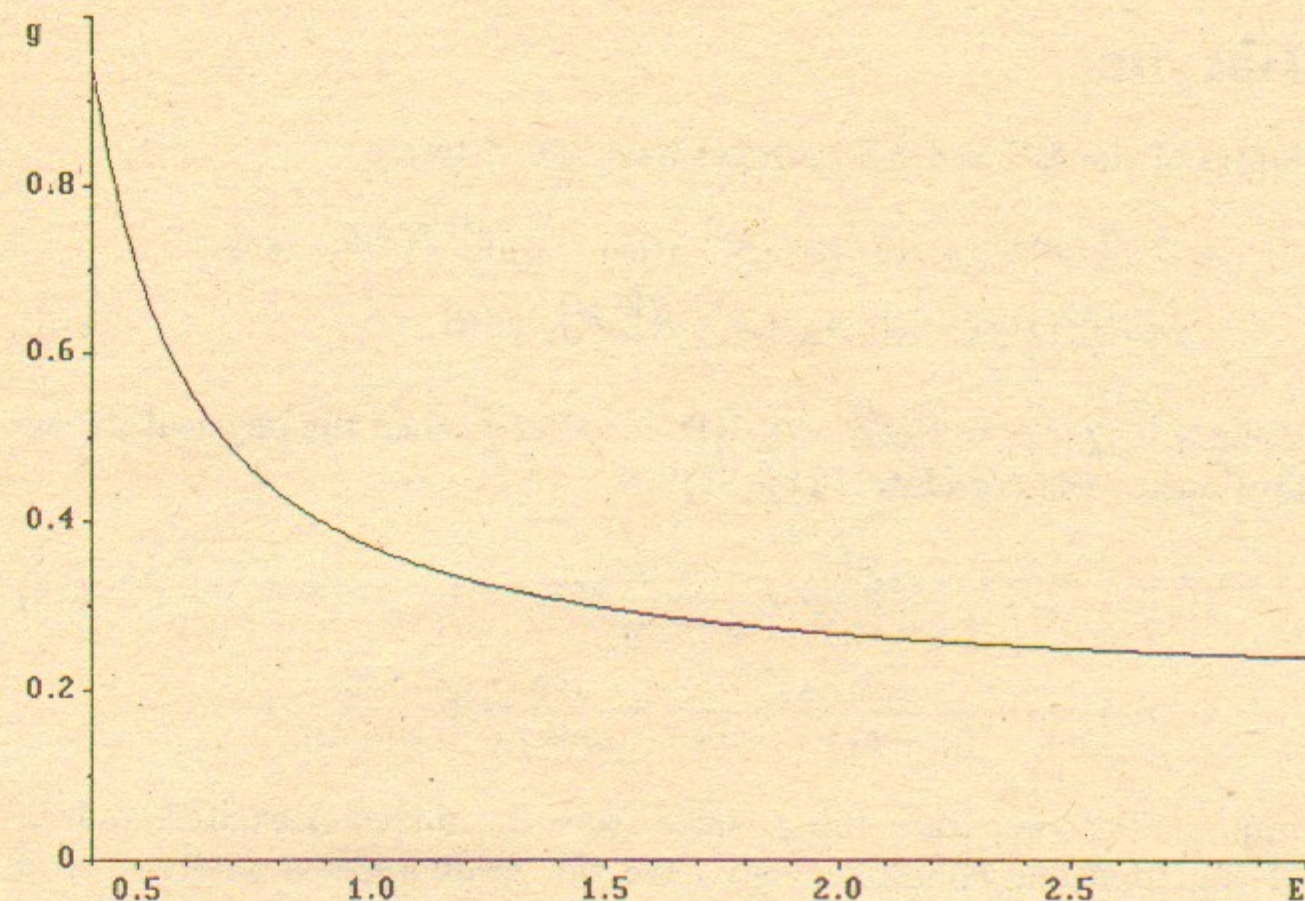


Figure 3: Sum rule for  $g_1$

2GeV (12) is reproduced, and we get  $g_1 \approx 0.2$ . This is the only region in which the sum rule is stable. A similar sum rule with the finite  $m_c$  was analyzed in [17] in the region  $E = 1.5-3\text{GeV}$  with the same result. But this requires an extremely slow convergence of the OPE in order to forbid using lower  $E$ . The OPE series (11) looks like  $1 + 200\text{MeV}/E + (170\text{MeV}/E)^2$ , so it seems that the expansion is applicable at  $E > 500\text{MeV}$ . This is close to the lower bound of the applicability region of the ordinary sum rule [12]. The upper bound of its applicability region is about 800MeV because the continuum contribution is too large at higher  $E$ . If we suppose that the applicability region of the sum rule (11) is approximately the same, then we obtain  $g_1 \sim 0.4-0.7$ . Stability of the sum rule in this region is poor. In this case we have to assume that the higher states' contribution is significant at  $E > 800\text{MeV}$ .



### 3 Baryons

Correlators of the  $\Sigma\Sigma$  and  $\Lambda\Sigma$  currents have the  $A$ -terms

$$\begin{aligned} i\langle T j_{\Sigma l}^{ij}(x) j_{\Sigma i' j' m}^*(0) \rangle_A &= i\epsilon_{lmn} A_{n(i' \delta_{j'})}^{(i \delta_j)} \delta(\vec{x}) \Pi_A(x_0), \\ i\langle T j_{\Lambda}^{ij}(x) j_{\Sigma i' j'}^*(0) \rangle_A &= \bar{A}_{(i' \delta_{j')} }^{[i \delta_j]} \delta(\vec{x}) \Pi_A(x_0). \end{aligned} \quad (13)$$

If we define  $\langle 0 | j_{\Lambda}^{ij} | \Lambda \rangle = F_{\Lambda} e^{ij}$ ,  $\langle 0 | j_{\Sigma}^{ij} | \Sigma \rangle = F_{\Sigma} \bar{e}^{ij}$ , then the physical states' contributions to the correlators (Fig. 1) are

$$\begin{aligned} \Pi_A(\omega) &= \frac{2g_2 F_{\Sigma}^2}{(\omega - \epsilon_{\Sigma})^2} + \frac{c}{\omega - \epsilon_{\Sigma}} + \dots \\ \Pi_A(\omega) &= \frac{2g_3 F_{\Lambda} F_{\Sigma}}{(\omega - \epsilon_{\Lambda})(\omega - \epsilon_{\Sigma})} + \frac{c_{\Lambda}}{\omega - \epsilon_{\Lambda}} + \frac{c_{\Sigma}}{\omega - \epsilon_{\Sigma}} + \dots \end{aligned} \quad (14)$$

It is impossible to separate the  $g_3$  term from the mixed  $\Lambda$ -excited and  $\Sigma$ -excited contributions unambiguously. We can do it approximately if  $\Delta = \epsilon_{\Sigma} - \epsilon_{\Lambda} \ll \epsilon_c - \epsilon_{\Lambda, \Sigma}$  because in such a case partial fraction decomposition of the first term would give large contributions to  $c_{\Lambda, \Sigma} \sim 1/\Delta$  with the opposite signs while the natural scale of  $c_{\Lambda, \Sigma}$  in (14) is  $c_{\Lambda, \Sigma} \sim 1/(\epsilon_c - \epsilon_{\Lambda, \Sigma})$ . This is not a defect of the sum rule but the uncertainty inherent to  $g_3$  which can be defined only when  $\Delta$  is small compared to the chiral symmetry breaking scale. We choose to require  $c_{\Lambda} = c_{\Sigma}$ ; the choices  $c_{\Lambda} = 0$  or  $c_{\Sigma} = 0$  would be equally good.

The baryonic correlators in the OPE framework are described by the diagrams in Fig. 4. The diagrams Fig. 4a-c with the non-cut quark line interacting with the axial field vanish due to (6). We use the factorization approximation for the four-quark condensate in Fig. 4e. In this approximation two diagonal correlators  $\langle j_1 j_1 \rangle$  and  $\langle j_2 j_2 \rangle$  coincide in both  $\Sigma\Sigma$  and  $\Lambda\Sigma$  cases, as well as  $\langle j_{\Lambda 1} j_{\Sigma 2} \rangle$  and  $\langle j_{\Lambda 2} j_{\Sigma 1} \rangle$ . This is similar to the usual heavy baryon sum rules [14], and confirms the observation that  $F_{\Lambda 1} = F_{\Lambda 2}$  and  $F_{\Sigma 1} = F_{\Sigma 2}$  within the factorization approximation to the sum rules. Only even-dimensional condensates contribute to the diagonal  $\Sigma\Sigma$  and the nondiagonal  $\Lambda\Sigma$  correlators:

$$\Pi_A(t) = \frac{2N! f^2}{N\pi^2 t^3} \left[ 1 - \left( \frac{5}{3} \pm \frac{C_B}{C_F} \right) \frac{m_1^2 t^2}{12} + \frac{\pi^2 \langle \bar{q} q \rangle^2 t^4}{6N f^2} \right], \quad (15)$$

where  $C_B/C_F = 1/(N-1)$  (this term comes from the diagram Fig. 4f). Only odd-dimensional condensates contribute to the nondiagonal  $\Sigma\Sigma$  and

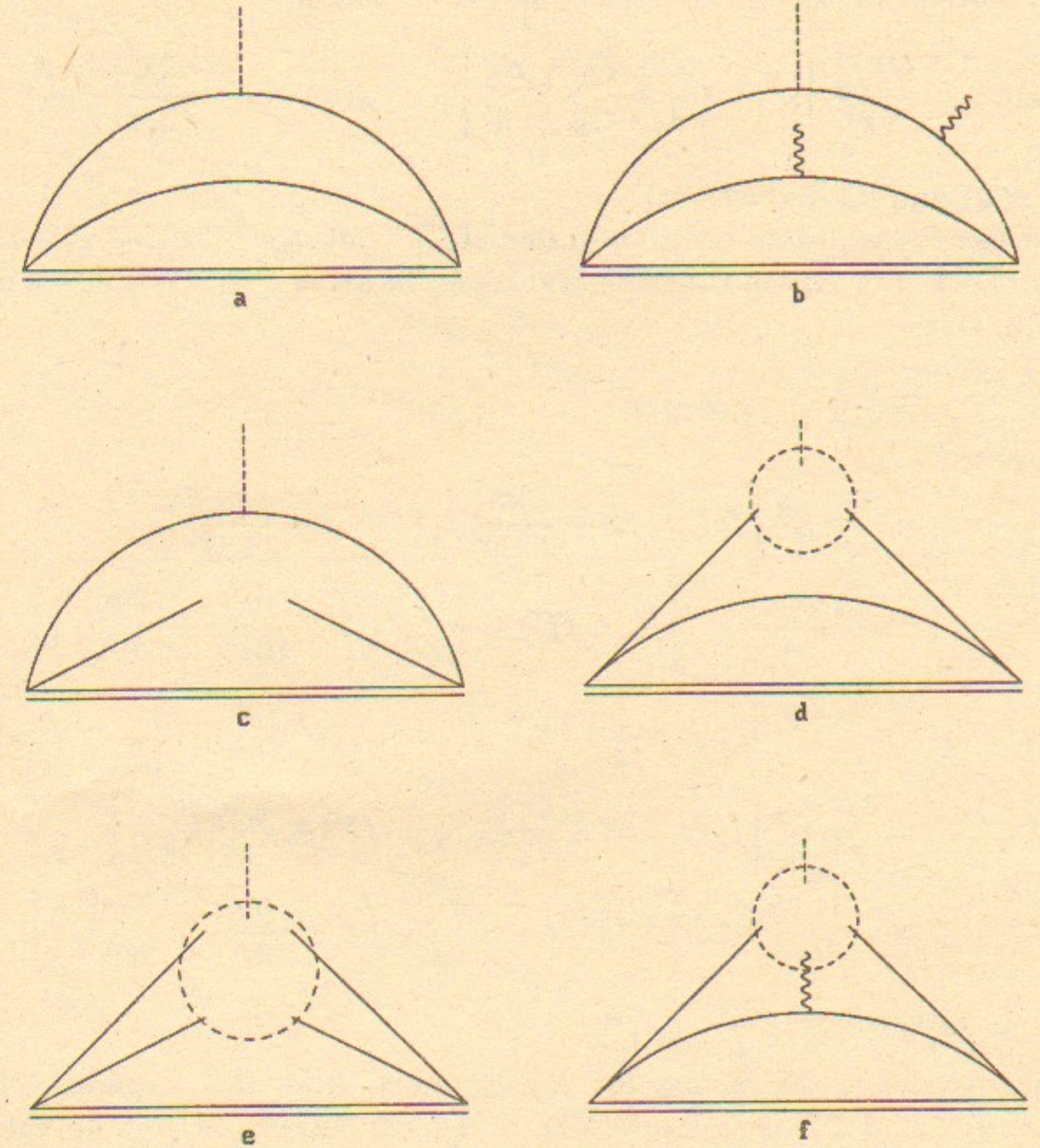


Figure 4: Correlator of baryon currents



the diagonal  $\Lambda\Sigma$  correlators:

$$\Pi_A(t) = \frac{2N! \langle \bar{q}q \rangle}{3N\pi^2 t^2} \left[ 1 - \frac{3\pi^2 f^2 t^2}{2N} \left( 1 + \frac{1}{16} m_0^2 t^2 - \frac{5}{36} m_1^2 t^2 \right) \right]. \quad (16)$$

These correlators correspond to the spectral densities

$$\rho_A(\omega) = \frac{N! f^2}{N\pi^2} \left[ \omega^2 + \left( \frac{5}{3} \pm \frac{C_B}{C_F} \right) \frac{m_1^2}{6} \right], \quad \rho_A(\omega) = -\frac{2N! \langle \bar{q}q \rangle \omega}{3N\pi^2} \quad (17)$$

(plus  $\delta(\omega)$  and its derivatives).

We use the standard continuum model  $\rho_A^{\text{cont}}(\omega) = \rho_A^{\text{theor}} \vartheta(\omega - \varepsilon_c)$ . Equating the OPE (15, 16) and the spectral representation at  $t = -i/E$ , we obtain the sum rules

$$\begin{aligned} & \left( \frac{2g_2 F_\Sigma^2}{E} + c \right) e^{-\varepsilon_\Sigma/E} \\ &= \frac{4f^2}{\pi^2} E^3 \left[ f_2(\varepsilon_c/E) + \frac{13m_1^2}{72E^2} f_0(\varepsilon_c/E) + \frac{\pi^2 \langle \bar{q}q \rangle^2}{18f^2 E^4} \right] \\ &= -\frac{4\langle \bar{q}q \rangle}{\pi^2} E^2 \left[ f_1(\varepsilon_c/E) + \frac{\pi^2 f^2}{2E^2} \left( 1 - \frac{m_0^2}{16E^2} + \frac{5m_1^2}{36E^2} \right) \right], \\ & \left( \frac{2g_3 F_\Lambda F_\Sigma}{\Delta} \text{th} \frac{\Delta}{2E} + \frac{c}{2} \right) (e^{-\varepsilon_\Lambda/E} + e^{-\varepsilon_\Sigma/E}) \\ &= \frac{4f^2}{\pi^2} E^3 \left[ f_2(\varepsilon_c/E) + \frac{7m_1^2}{72E^2} f_0(\varepsilon_c/E) + \frac{\pi^2 \langle \bar{q}q \rangle^2}{18f^2 E^4} \right] \\ &= -\frac{4\langle \bar{q}q \rangle}{\pi^2} E^2 \left[ f_1(\varepsilon_c/E) + \frac{\pi^2 f^2}{2E^2} \left( 1 - \frac{m_0^2}{16E^2} + \frac{5m_1^2}{36E^2} \right) \right], \end{aligned} \quad (18)$$

where  $f_n(x) = 1 - e^{-x} \sum_{m=0}^n \frac{x^m}{m!}$ .

Fig. 5a shows the results for  $g_{2,3}$  obtained from the diagonal  $\Sigma\Sigma$  and the nondiagonal  $\Lambda\Sigma$  correlators (the first formulae in (18)). The values of  $\varepsilon_{\Lambda,\Sigma}$  and  $F_{\Lambda,\Sigma}$  are taken from the diagonal sum rules [14] in the middle of the stability plateau  $E \approx 300\text{MeV}$ . We use the same values of the continuum threshold as in [14]:  $(5.6 \pm 0.5)k$  for  $\Sigma$  and  $(4.6 \pm 0.5)k$  for  $\Lambda$  where  $k^3 = -\frac{\pi^2}{6} \langle \bar{q}q \rangle$ ,  $k = 260\text{MeV}$ . We put the continuum threshold in the sum rule for  $g_2$  ( $\Sigma\Sigma$  correlator) equal to that in the  $\Sigma$  sum rule; in the case of  $g_3$  ( $\Lambda\Sigma$  correlator) we put it equal to the average of the thresholds in the  $\Lambda$  and  $\Sigma$  channels. Of course, the effective continuum threshold for the  $A$ -term in the correlators does not necessary coincide with that in the absence of the

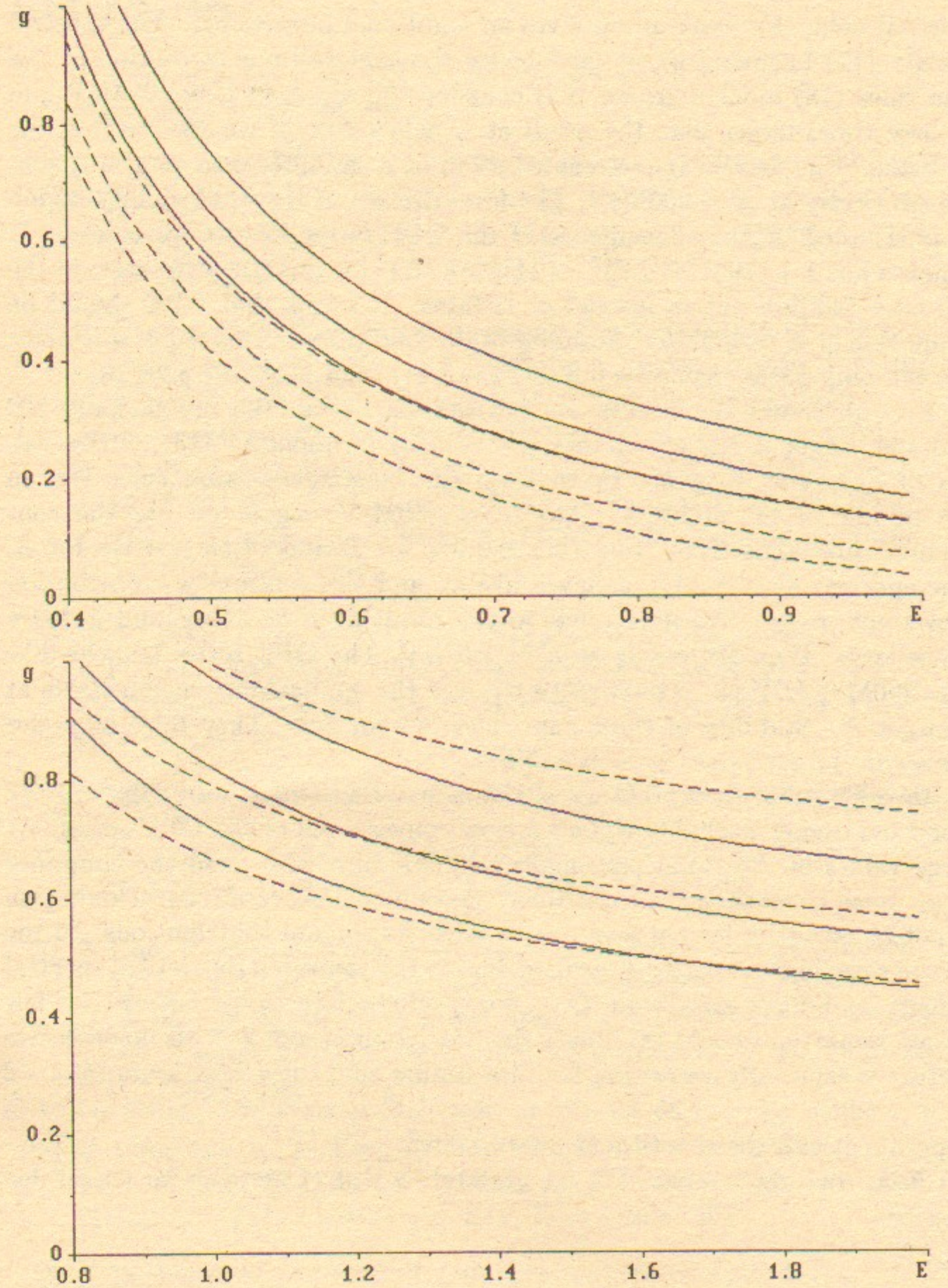


Figure 5: Sum rules for  $g_{2,3}$  (solid and dashed lines): a—diagonal  $\Sigma\Sigma$  and nondiagonal  $\Lambda\Sigma$  sum rules; b—nondiagonal  $\Sigma\Sigma$  and diagonal  $\Lambda\Sigma$  sum rules.



external field; this assumption gives an additional uncertainty. The spectral density (17) behaves like  $\omega^2$ , and hence the continuum contribution to the sum rules (18) quickly grows: it is equal to the result at  $E \approx 600\text{MeV}$  and is three times larger than the result at  $E \approx 900\text{MeV}$ . If we assume that the uncertainty in the continuum contribution is, e. g., 30%, then we can't trust the sum rules at  $E > 900\text{MeV}$ . The lower bound of the applicability region is determined by the convergence of the OPE series for the correlators. It behaves like  $1 + (190\text{MeV}/E)^2 + (245\text{MeV}/E)^4$  for the  $\Sigma\Sigma$  correlator; in the  $\Lambda\Sigma$  case  $140\text{MeV}$  enters instead of  $190\text{MeV}$ . It seems that OPE should be applicable at  $E > 400\text{MeV}$ . Stability of the sum rules in this window is poor. We can only guess that  $g_2 \sim 0.2-1$ ,  $g_3 \sim 0.1-1$ , and probably  $g_3 < g_2$ .

Fig. 5b shows the results for  $g_{2,3}$  obtained from the nondiagonal  $\Sigma\Sigma$  and the diagonal  $\Lambda\Sigma$  correlators (the second formulae in (18)). The values of  $\varepsilon_{\Lambda,\Sigma}$  and  $F_{\Lambda,\Sigma}$  are taken from the nondiagonal sum rules [14] in the middle of the stability plateau  $E \approx 650\text{MeV}$ . Again we use the continuum thresholds from [14]:  $(5.6 \pm 0.5)k$  for  $\Sigma$  and  $(3.65 \pm 0.5)k$  for  $\Lambda$ . The spectral density (17) behaves like  $\omega$ , and the continuum contribution grows not so quickly: it is equal to the result at  $E \approx 1\text{GeV}$  and is three times larger than the result at  $E \approx 1.6\text{GeV}$ . The OPE series behaves like  $1 + (290\text{MeV}/E)^2 [1 - (150\text{MeV}/E)^2]$ , and the applicability region starts at a larger  $E$ . Stability of these sum rules is poor too. They tend to prefer somewhat larger values  $g_2 \sim g_3 \sim 0.5-1$ .

In conclusion, poor stability of the sum rules at moderate values of the Borel parameter probably signals a slow convergence of the OPE series. At large values of the Borel parameter they stabilize, and yield the couplings  $g_{1,2,3}$  much lower than the constituent quark model expectations. This region is not trustworthy for baryons due to large continuum contributions. In the meson case, higher excited states are dual to the vanishing theoretical spectral density, and don't contribute. Only unexpectedly large contributions of a few lowest excitations could invalidate the prediction  $g_1 \approx 0.2$ . This inconclusive picture is especially surprising because similar sum rules in an external axial field produce  $g_A = 1.25$  for the nucleon [16] in excellent agreement with experiment and the constituent quark model.

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from QCD sum rules**

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