ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
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TOP NEAR THRESHOLD: ALL α_S CORRECTIONS ARE TRIVIAL

BUDKERINP 93-18

НОВОСИБИРСК

Top Near Threshold: all α_S Corrections are Trivial

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[3] Supported in part by grant from Soros fund "Culture initiative"

Abstract

We have calculated part of $O(\alpha_S)$ QCD radiative correction to the total cross-section of top's threshold production in $e^+e^- \to t\bar{t} \to W^- bW^+\bar{b}$ reaction. We found out a curious fact: there is no $O(\alpha_S)$ correction to the total cross-section, originated from $b\bar{b}$, $t\bar{b}$, $\bar{t}b$ gluon exchange and corresponding gluon emission. Thus, all $O(\alpha_S)$ corrections which are non-zero, can be classified as: 1) corrections to $t\bar{t}$ nonrelativistic potential 2) corrections to the top's width 3) hard gluon correction to $\gamma t\bar{t}$-vertex. These corrections are well-known and were described in the previous papers.

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1. Introduction

The problem of top production near threshold has been posed and solved by Fadin and Khoze [1].This and following papers [2–6] led people to conclusion that it would be possible to measure both top's mass and width and strong coupling constant α_S using top's threshold production. This belief is based on some important grounds:

- non-perturbative corrections are small [1, 2, 3]
- runnig of QCD coupling constant can be consistently taking into account [3]
- effects originated from top's width dependence from virtuality are in principle calculacable [4]

On this way an important problem has not been solved yet — it is the problem of $O(\alpha_S)$ QCD radiative corrections calculation. In this letter we are going to study this problem in details.

Let us firstly remind the reader about results obtained in the leading order approximation in ref. [1]. Summing "Coulomb-like" ladder Fadin and Khoze obtained the following cross-section for top's threshold production

$$\sigma(e^+e^- \to t\bar{t} \to W^- bW^+\bar{b}) = N_c {Q_t}^2 \sigma(e^+e^- \to \mu\bar{\mu}) \frac{4\pi}{s} \mathrm{Im} P_t(s)$$
$$\sigma(e^+e^- \to \mu\bar{\mu}) = \frac{4\pi\alpha_{QED}^2}{3s} \qquad (1)$$

Here $Q_t$is the top's charge and $P_t(s)$ stands for vacuum polarization by top's vector current. $s = q^2$, $P_t^{\mu\nu}(s) = P_t(s)(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu})$. $P_t(s)$ for $| s - 4m_t^2 | \ll 4m_t^2$ is connected with the non-relativistic Green function of Coulomb-like problem:

$$P_t(s) = \frac{3}{2{m_t}^2} G_{E+i\Gamma}(0,0) \qquad (2)$$

$$G_{E+i\Gamma}(r,0) = \langle \vec{r} \mid \frac{1}{\hat{H} - E - i\Gamma} \mid \vec{r} = 0 \rangle = \int \frac{d\vec{p}}{(2\pi)^3} \frac{V(E,\vec{p}) e^{i\vec{p}\vec{r}}}{E - \frac{\vec{p}^2}{m} + i\Gamma} \qquad (3)$$

$$\hat{H} = \frac{\vec{p}^2}{m} - \frac{4}{3}\frac{\alpha_S}{r} \qquad (4)$$

Here $E = \sqrt{s} - 2m_t$ and $V(E,\vec{p})$ is the $\gamma t\bar{t}$ nonrelativistic vertex function (Fig.1). We'll use all this notations throughout the paper.

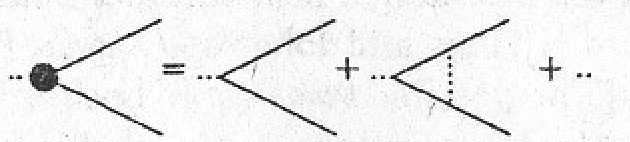

Fig.1

What we are going to do is to study $O(\alpha_S)$ QCD radiative corrections to the Eq.(1). On this way, one must recognize that there is no additional "smallness" due to electroweak coupling constants in the leading order cross-section because they are compensated by top's resonanse propagators. Hence, we are interested in a correction, that does not shift quark's propogator out of the pole. Dealing with "jet-jet" interaction, we see that this requirement results in the fact that we must consider "soft" gluon contribution; the required "softness", however, depends greatly from the diagram in question.

2. Method of calculation

As we intend to study the correction to the total cross-section, it is convinient to choose the the following way of work (for concreteness let us consider the typical graph, depictured in Fig. 2): firstly, integrate over the phase-space of bW^+ ("cut" block A) and $\bar{b}W^-$ ("cut" block B). On this way one'll obtain two quanaties: I_μ for the upper block and J_μ for the lower one (Fig. 2,a). Following ideas of ref. [1], the non-relativistic approximation can be used to simplify this quantaties. Secondly, write down the amplitude

of Fig. 2,a in term of I_μ and J_μ and integrate over loop momentum and top's virtuality.

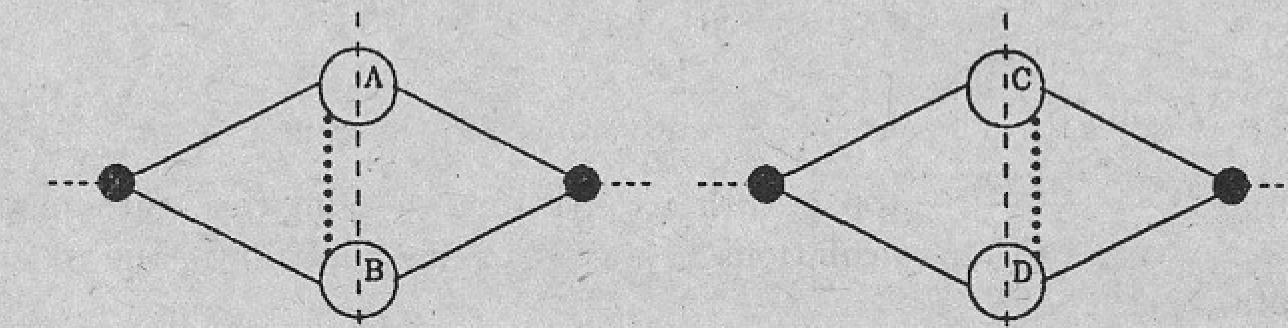

Fig.2a Fig.2b

As we are not far from threshold, it is sufficient to use nonrelativistic approximation [1] for t and $\bar{t}$ propogators:

$$S_t(p_t) = \frac{1+\gamma_0}{2} \frac{1}{\varepsilon_t - \frac{\vec{p}^2}{2m_t} + i\frac{\Gamma_t}{2}} \qquad (5)$$

$$S_{\bar{t}}(-p_{\bar{t}}) = \frac{1-\gamma_0}{2} \frac{-1}{\varepsilon_{\bar{t}} - \frac{\vec{p}^2}{2m_t} + i\frac{\Gamma_t}{2}} \qquad (6)$$

Here $p_t = (m_t + \varepsilon_t, \vec{p})$, $p_{\bar{t}} = (m_t + \varepsilon_{\bar{t}}, -\vec{p})$ is top's four momentum. We ignore in (5), (6) the momentum-dependence of the top quark width.

Let us now turn to the discussion of different diagrams contribution to the radiative correction. Studing various types of "jet-jet" interaction we found it convinient to use the Coulomb gauge. We also found the same result in the Feynman gauge.

3. Contribution of different diagrams to the total cross-section

3.1 Subgraphs calculation

First of all, let us calculate I_μ and J_μ — quantaties, introduced earlier. Applying Feynman rules, we get in nonrelativistic approximation:

1) for time-like gluon:

$$I_0 = -\sqrt{4\pi\alpha_S}\hat{w}_+ \frac{\Gamma_t}{k} \times$$
$$\times \left\{ \frac{1}{2} \ln\left(\frac{w-k+i\varepsilon}{w+k+i\varepsilon} \right) \gamma_0 - \frac{2x-1}{2x+1} \left(1 + \frac{w}{2k} \ln\left(\frac{w-k+i\varepsilon}{w+k+i\varepsilon} \right) \frac{\vec{\gamma}\vec{k}}{k} \right\} \qquad (7)$$

2) for space-like gluon :

$$I_i = \sqrt{4\pi\alpha_S}\,\hat{w}_+ \frac{\Gamma_t}{2k}\gamma_i \frac{(2x-1)}{(2x+1)}\left\{\frac{w^2-k^2}{2k^2}\ln\left(\frac{w-k+i\varepsilon}{w+k+i\varepsilon}\right)+\frac{w}{k}\right\} \qquad (8)$$

Here $k = |\vec{k}|$; w, $\vec{k}$ — energy and momentum of gluon, $x = \frac{m_W^2}{m_t^2}$, $\Gamma_t = m_t g_W{}^2 \frac{(1-x)^2(1+2x)}{8\pi x}$ — top's width, α_S — strong coupling constant and $\hat{w}_+ = 1+\gamma_5$. For further calculations it is convinient to introduce the following notations:

$$I_0 = w_+\gamma_0 A + w_+\gamma_i B^i, \qquad I_i = w_+\gamma_i C \qquad (9)$$

Straightforward calculation give

$$J_0 = w_+\gamma_0 A^* + w_+\gamma_i B^{i*}, \qquad J_i = w_+\gamma_i C^* \qquad (10)$$

There is a set of "cut" subgraph $\bar{I}(p_t,k)$ and $\bar{J}(p_t,k)$ of similar nature, presented at Fig. 2,b (Block D, C). They are connected with $I(p_t,k)$ and $J(p_t,k)$ in following way:

$$\bar{J}_\mu = I_\mu \qquad \bar{I}_\mu = J_\mu \qquad (11)$$

3.2 Time-like gluon exchange between $b\bar{b}$ - quarks

In this case, we deal with the graphs in Figs. 2,a,b. Their contribution to the function $\mathrm{Im}P_t$ reads: $\mathrm{Im}P_t^{b\bar{b}} = \frac{2}{3}\mathrm{Re}M^{b\bar{b}}(E)$. (Here $M^{b\bar{b}} = T^{*(one-loop)}T^{(Born)}$ and T stands for the amplitude of the process $\gamma^* \to t\bar{t} \to W^- bW^+\bar{b}$). Here $M^{b\bar{b}}$ reads:

$$M^{b\bar{b}} = -i\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}C_F\mathrm{Sp}\Big(\gamma_i V^*(\vec{p})S^*{}_t(p_t)I_0 \times$$
$$\times S_t(p_t+k)\gamma^i V(\vec{p}+\vec{k})S_{\bar{t}}(-p_{\bar{t}}+k)J_0 S^*{}_{\bar{t}}(-p_{\bar{t}})\Big)D(\vec{k}) \qquad (12)$$

where $C_F = \frac{N_c^2-1}{2N_c}$, $D(\vec{k}) = \frac{1}{|\vec{k}|^2}$ stands for photon propogator. Integration over $d\varepsilon$ can be performed explicitly by taking two residues in the points:

$$\varepsilon = \frac{\vec{p}^{\,2}}{2m_t} + i\frac{\Gamma_t}{2}, \qquad \varepsilon = E - w - \frac{(\vec{p}+\vec{k})^2}{2m_t} + i\frac{\Gamma_t}{2} \qquad (13)$$

After that $M^{b\bar{b}}$ takes the form:

$$M^{b\bar{b}} = -\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\int\frac{dw}{(2\pi)}\left(3AA^* + \vec{B}\vec{B}^*\right)\frac{C_F}{|\vec{k}|^2}\frac{4\Delta}{w^2-\Delta^2}\times$$
$$\times\frac{V^*(\vec{p})}{E-\frac{\vec{p}^{\,2}}{m_t}-i\Gamma_t}\frac{V(\vec{p}+\vec{k})}{E-\frac{(\vec{p}+\vec{k})^2}{m_t}+i\Gamma_t} \qquad (14)$$

Here $\Delta = -\frac{(\vec{p}+\vec{k})^2}{2m_t}+\frac{(\vec{p})^2}{2m_t}+i\Gamma_t$ and A, $\vec{B}$, C are introduced through Eqs. (7–9). Simple dimensional analyses leads us to conclusion, that the only integration region, wich can give correction of $O(\alpha_S)$ order is the region $k, w < \Gamma$. For example, the region $w \sim \Gamma$, $k \sim p_t \sim \alpha_S m_t$ will give $\alpha_S\frac{\Gamma_t}{p_t} \sim \frac{\Gamma_t}{m_t} \sim \alpha_W$, where α_S is the electroweak coupling constant ($\alpha_W \sim \alpha_S^2$). Hence, we are only interested in the integration over region $k, w < \Gamma$, but it is not difficult to see from Eq. [14] that in this case $M^{b\bar{b}}$ is purely imaginary ($\Delta = i\Gamma_t$). Hence $\mathrm{Im}P_t^{b\bar{b}} = \frac{2}{3}\mathrm{Re}M^{b\bar{b}}$ appears to be zero up to $O(\alpha_S)$ order.

3.3. Time-like gluon exchange between t and $\bar{b}$ ($\bar{t}$ and b) -quarks.

Different diagrams corresponding to this case are presented in Figs. 3,a,b. Their contribution to the total cross-section appears to be identical. This fact can be checked by straightforward calculation or by applying charge

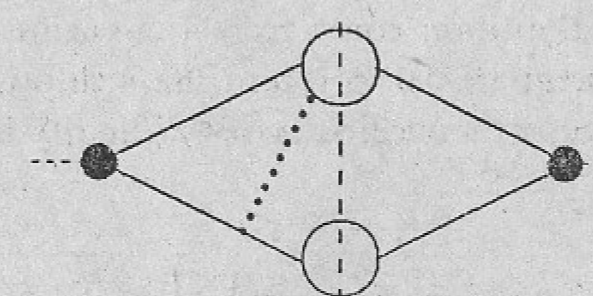

Fig.3a

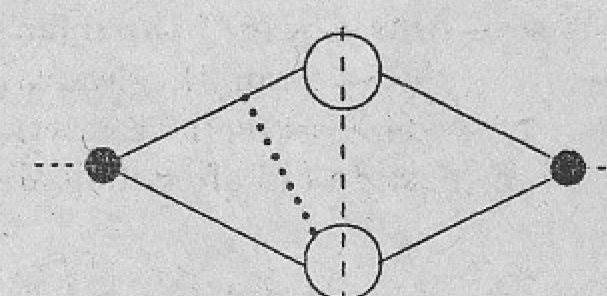

Fig.3b

conjugation to one of these diagrams. The contribution to $\mathrm{Im}P_t(E)$ reads:

$$\mathrm{Im}P_t^{t\bar{b}} = \frac{4}{3}\mathrm{Re}M^{t\bar{b}}(E)$$

$$M^{t\bar{b}} = -i\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}C_F\mathrm{Sp}\Big(\cdot\gamma_i V^*(\vec{p})S^*{}_t(p_t)\hat{\Gamma}S_t(p_t)\times$$

$$\times(\sqrt{4\pi\alpha_S}\gamma_0)S_t(p_t+k)\gamma^i V(\vec{p}+\vec{k})S_{\bar{t}}(-p_{\bar{t}}+k)J_0S^*{}_{\bar{t}}(-p_{\bar{t}})\Big)D(\vec{k}) \qquad (15)$$

Here $\hat{\Gamma} = w^+\gamma_0\Gamma_t$. Performing the trace and integrating over $d\varepsilon$, we get

$$M^{t\bar{b}} = -\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\int\frac{dw}{(2\pi)}\frac{(\sqrt{4\pi\alpha_S}A^*)}{k^2}\frac{12\Delta}{w^2-\Delta^2}\times$$

$$\times\frac{V^*(\vec{p_t})}{E-\frac{\vec{p}^2}{m_t}-i\Gamma_t}\frac{V(\vec{p_t}+\vec{k})}{E-\frac{(\vec{p}+\vec{k})^2}{m_t}+i\Gamma_t} \qquad (16)$$

In Eq. (16), we omit the terms which after integration over w are zero. We can perform integration over w taking into account analytical properties of the logarithm $\ln\left(\frac{w-k+i\varepsilon}{w+k+i\varepsilon}\right)$. After that one obtain:

$$M^{t\bar{b}} = i\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\Phi(|\,\vec{k}\,|)G^*(\vec{r}=0,\vec{p}\mid E+i\Gamma_t)G(\vec{r}=0,\vec{p}+\vec{k}\mid E+i\Gamma_t) \qquad (17)$$

The function $\Phi(k)$ reads here:

$$\Phi(|\,\vec{k}\,|) = 12\pi\alpha_S\frac{\Gamma_t}{k^3}\left(\pi-2\mathrm{arctg}\left(\frac{\Gamma_t}{k}\right)\right) \qquad (18)$$

As is seen from Eq. (14) the main contribution come from the region $k\sim p\sim m_t\alpha_S$; thus we can't neglect k in comparison with p in the argument of Green function. However, we can overcome this difficulty inserting full set of state $(|\,\vec{r}\rangle\langle\vec{r}\,|=1)$ and after that:

$$M^{t\bar{b}} = i\int d^3r|\,G(\vec{r}=0,\vec{r}\mid E+i\Gamma)\,|^2\int\frac{d^3k}{(2\pi)^3}\Phi(|\,\vec{k}\,|)e^{i\vec{k}\vec{r}} \qquad (19)$$

So, we see, that $M^{t\bar{b}}$ appears to be pure imaginary, hence these diagrams give no contribution to $O(\alpha_S)$ correction.

3.4 Space-like gluon exchange between $b\bar{b}$ and corresponding emission of real gluon.

Let us now study the space-like gluon exchange. The first note is that we can attach space-like gluon only to b-quark, because, as is well-known, space-like gluon interection to quark current is proportional to quarks velocity which (for the top) is of $O(\alpha_S)$ order, while for b quark it is of order 1. That's why we consider the graphs with space-like gluon exchange between $b\bar{b}$, and corresponding gluon emission Figs. $2a, b$, $4,a,b$. In latter case there are two possibilities: 1) the interference between radiation from b and $\bar{b}$

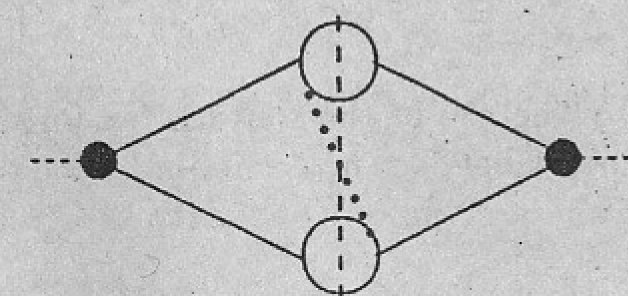
Fig.4a

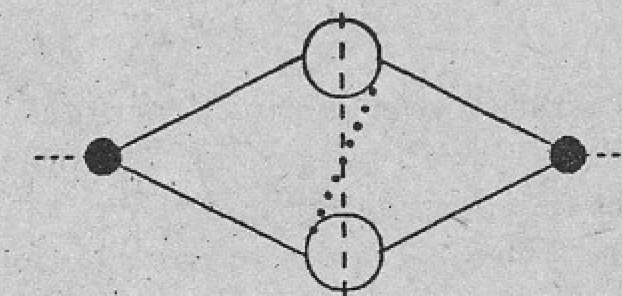
Fig.4b

2) b $(\bar{b})$ radiation. Our interest is concentrated on the first possibility, because the second one is included to the correction to $t\to Wb$ $(\bar{t}\to W\bar{b})$ width. The contribution to $\mathrm{Im}P_t(E)$ reads:

$$\mathrm{Im}P^{b\bar{b}}_{sp} = \frac{2}{3}\mathrm{Re}M^{b\bar{b}}_{sp} + \frac{2}{3}\mathrm{Re}M^{b\bar{b}}_{real}$$

$$M^{b\bar{b}}_{sp} = -\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\frac{1}{3}\mathrm{Sp}\Big(\gamma_l V^*(\vec{p_t})S^*{}_t(p_t)I_i\times$$

$$\times S_t(p_t+k)\gamma^l V(\vec{p_t}+\vec{k})S_{\bar{t}}(-p_{\bar{t}}+k)J_jS^*{}_{\bar{t}}(-p_{\bar{t}})\Big)D^{ij}(\vec{k},w) \qquad (20)$$

$$M^{b\bar{b}}_{real} = -\int\frac{d^4p}{(2\pi)^4}\int\frac{d^3k}{(2\pi)^3 2w}C_F\mathrm{Sp}\Big(\gamma_l V^*(\vec{p_t})S^*{}_t(p_t)I_i\times$$

$$\times S_t(p_t+k)\gamma^l V(\vec{p_t}+\vec{k})S_{\bar{t}}(-p_{\bar{t}}+k)\bar{J}_jS^*{}_{\bar{t}}(-p_{\bar{t}})\Big)e^{*i}e^j \qquad (21)$$

$M_{sp}^{b\bar{b}}$ is the virtual gluon contribution, $M_{real}^{b\bar{b}}$ is the radiation interference contribution. Also $D^{ij}(\vec{k},w) = \frac{i}{w^2-|\vec{k}|^2+i0}(\delta^{ij} - n^i n^j)$, $\vec{n} = \frac{\vec{k}}{|\vec{k}|}$, $e^{\mu} = (0,\vec{e})$ — wave function of gluon, for summation over polarization of gluon, we use formula $\Sigma e_i e_j^* = (\delta_{ij} - n_i n_j)$. Simple dimensional analyses leads us to conclusion, that the only integration region, wich can give correction of O(α_S) order is the region $k, w < \Gamma$. Performing the trace, and integrating over the energy of the top, we get:

$$\mathrm{Im}P_{sp}^{b\bar{b}} = -\mathrm{Re}\int \frac{d^3p}{(2\pi)^3}\int \frac{d^3k}{(2\pi)^3}\frac{16\Gamma_t}{3}\frac{V^*(\vec{p}_t)}{E-\frac{\vec{p}^2}{m_t}-i\Gamma_t}\frac{V(\vec{p}_t+\vec{k})}{E-\frac{(\vec{p}+\vec{k})^2}{m_t}+i\Gamma_t}\times$$

$$\left\{\int\frac{dw}{(2\pi)}\frac{C(w,\vec{k})C^*(w,\vec{k})}{(w^2+\Gamma_t{}^2)}\frac{i}{w^2-|\vec{k}|^2+i0} - \frac{C(w=|\vec{k}|)C^*(w=|\vec{k}|)}{2k(k^2+\Gamma_t{}^2)}\right\} \quad (22)$$

Integration over w can be perfomed by the use of the well-known formulae: $\int dx\frac{1}{x+i0} = P(\frac{1}{x}) - i\pi\delta(x)$. The principal value does not contribute due to the fact that it is purely imaginary, while the pole part of the virtual correction cancel the real emission contribution.

3.5 Other corrections.

There are other corrections, originated from the space-like gluon exchange between b-quarks and Coulomb gluon, and real gluon emission from Coulomb gluon. Using technique discussed above, one can conclude that they don't give $O(\alpha_S)$ correction to the total cross-section.

4.Conclusion.

So we have considered the O(α_S) QCD radiative correction to top threshold production cross-section $e^+e^- \to t\bar{t} \to W^-bW^+\bar{b}$, originated from "jet-jet" interection in the final state. We found out that thous corrections give nothing in O(α_S) order. Hence, O(α_S) radiative correction appears only from:

1. Corrections to $t\bar{t}$ nonrelativistic potential;
2. Corrections to the top's width;
3. Hard gluon correction to $\gamma t\bar{t}$-vertex.

Thus all they are trivial in the sence, that we can use the general formula for the total cross-section obtained in the leading order and then substituting $V(r)^{coulomb} \to V(r)^{one-loop}$, $\Gamma^{Born} \to \Gamma^{one-loop}$, $\sigma^{Born}(e^+e^- \to \mu\bar{\mu}) \to \sigma^{Born}(e^+e^- \to \mu\bar{\mu})(1+\frac{8\alpha_S}{3\pi})$, we obtain the top's threshold production cross-section up to O(α_S) order.

$$\sigma(e^+e^- \to t\bar{t} \to W^-bW^+\bar{b}) = \sigma^{Born}(e^+e^- \to \mu\bar{\mu})\left(1+\frac{8\alpha_S}{3\pi}\right)\frac{6\pi Q_t^2}{m_t^2}\times$$

$$\times\mathrm{Im}\langle r=0 \mid \frac{1}{\hat{H}-E-i\Gamma^{one-loop}} \mid r=0\rangle \quad (23)$$

$$\hat{H} = \frac{\vec{p}^2}{m} + V(r)^{one-loop}$$

This is the general answer for the problem, outlined in the introduction. From above discussion, we can conclude that similar situation takes place for all unstable particles threshold production. This result can be expressed in the form of a general statment:

There is no O(α_{QED} or α_{QCD}) corrections for the total cross-section, originated from the "jet-jet" interection.

We want to add in conclusion, that, for our mind, the absence of O(α_S) correction to the total cross section, originated from "jet-jet" interection, must have simple physical meaning and now we are looking for it.

After this work was complited, we received the paper [6] in which W^+W^- production near threshold is discussed.

Acknowledgement:

The authors are very gratefull to Profs. V.S. Fadin, I.F. Ginzburg and Dr. A.G. Grozin for discussions.

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Рождение t-кварков около порога:
КХД радиационная поправка

BUDKERINP 93-18

Ответственный за выпуск С.Г. Попов

Работа поступила 24 февраля 1993 г.
Подписано в печать 24.02 1993 г.
Формат бумаги 60×90 1/16 Объем 0,7 печ.л., 0,6 уч.-изд.л.
Тираж 220 экз. Бесплатно. Заказ N° 18

Обработано на IBM PC и отпечатано на
ротапринте ИЯФ им. Г.И. Будкера СО РАН,
Новосибирск, 630090, пр. академика Лаврентьева, 11.