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INTRODUCTION TO THE HEAVY QUARK EFFECTIVE THEORY
PART 1

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НОВОСИБИРСК

Introduction to the Heavy Quark Effective Theory
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ABSTRACT

Heavy Quark Effective Theory (HQET) is a new approach to QCD problems involving a heavy quark. In the leading approximation, the heavy quark is considered as a static source of the gluon field; $1/m$ corrections can be systematically included in the perturbation theory. New symmetry properties not apparent in QCD appear in HQET. They are used, in particular, to obtain relations among heavy hadron form factors. HQET also simplifies lattice simulation and sum rules analysis of heavy hadrons.

Part 1 contains discussion of the effective lagrangian, mesons, baryons, and renormalization. Part 2 will contain $1/m$ corrections, nonleptonic decays, and interaction with soft pions.

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1 Effective Lagrangian

Recently an interesting new approach to QCD problems involving a heavy quark was proposed, namely the Heavy Quark Effective Theory (HQET). In the leading approximation, the heavy quark is considered as a static source of the gluon field; $1/m$ corrections can be systematically included in the perturbation theory. This simplification is similar to considering a hydrogen atom instead of a positronium. New symmetry properties not apparent in QCD appear in HQET. They are used, in particular, to obtain relations among heavy hadron form factors. However, in QCD even such a simplified problem is unsolvable. Approximate methods such as lattice simulation or sum rules are necessary to obtain quantitative results. Here again HQET allows to proceed much further than QCD.

There are several good reviews of HQET [1, 2, 3] to which we address the reader for an additional information. Here we widely use the properties of currents' correlators to obtain general results. This approach is inspired by sum rules, though we shall not consider details of sum rules calculations. We shall start from a very simple though approximate picture in the Sections 1–3; some complications are discussed later.

Let's start from the QCD Lagrangian

$$L = \overline{Q}(i\widehat{D} - m)Q + \overline{q}i\widehat{D}q - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \cdots \tag{1.1}$$

where Q is the heavy quark field, q are light quark fields (their masses are not written down for simplicity), $G^a_{\mu\nu}$ is the gluon field strength, and dots mean gauge fixing and ghost terms. It is well known that the free heavy quark Lagrangian $\overline{Q}(i\hat{\partial} - m)Q$ gives the dependence of the energy on the momentum $\varepsilon = \sqrt{m^2 + \vec{p}^2}$. We shall consider problems with a single heavy quark approximately at rest, and all characteristic momenta $|\vec{p}| \ll m$. Then we can simplify the dispersion law to $\varepsilon = m$. It corresponds to the Lagrangian $\overline{Q}(i\gamma_0\partial_0 - m)Q$. In such problems it is convenient to measure all energies relative to the level m. This means that instead of the true energy ε we shall use the effective energy $\widetilde{\varepsilon} = \varepsilon - m$. Then the heavy quark energy $\widetilde{\varepsilon} = 0$ independently on the momentum. The free Lagrangian giving such a dispersion law is $\overline{Q}i\gamma_0\partial_0 Q$. The spin of the heavy quark at rest can be described by a 2-component spinor $\widetilde{Q}$ (we can also consider it as a 4-component spinor with the vanishing lower components: $\gamma_0\widetilde{Q} = \widetilde{Q}$). Reintroducing the interaction with the gluon field by requirement of the gauge invariance, we arrive at the HQET Lagrangian [4]

$$L = \widetilde{Q}^+ iD_0\widetilde{Q} + \overline{q}i\hat{D}q - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \cdots \qquad (1.2)$$

The static quark field $\widetilde{Q}$ contains only annihilation operators. There are no heavy antiquarks in the theory, because processes of heavy quark-antiquark pair production are suppressed by $1/m$. The heavy antiquark (if present) is described by a separate field. The field theory (1.2) is not Lorentz-invariant, because the heavy quark defines a selected frame—its rest frame.

The Lagrangian (1.2) gives the static quark propagator

$$\widetilde{S}(\widetilde{p}) = \frac{1}{\widetilde{p}_0 + i0}, \quad \widetilde{S}(x) = \widetilde{S}(x_0)\delta(\vec{x}), \quad \widetilde{S}(t) = -i\vartheta(t). \qquad (1.3)$$

In the momentum space it depends only on $\widetilde{p}_0$ but not on $\vec{p}$ because we have neglected the kinetic energy. Therefore in the coordinate space the static quark does not move. The unit 2×2 matrix is assumed in the propagator (1.3). It is often convenient to use it as a 4×4 matrix; in such a case the projector $\frac{1+\gamma_0}{2}$ excluding the lower components is assumed. The static quark interacts only with A_0; the vertex is $ig\delta_{0\mu}t^a$.

One can watch how expressions for QCD diagrams tend to the corresponding HQET expressions in the limit $m \to \infty$ [5]. The QCD heavy quark propagator is

$$S(p) = \frac{\hat{p} + m}{p^2 - m^2} = \frac{m(1+\gamma_0) + \hat{\widetilde{p}}}{2m\widetilde{p}_0 + \widetilde{p}^2} = \frac{1+\gamma_0}{2\widetilde{p}_0} + O\left(\frac{\widetilde{p}}{m}\right). \qquad (1.4)$$

A vertex $ig\gamma_\mu t^a$ sandwiched between two projectors $\frac{1+\gamma_0}{2}$ may be replaced by $ig\delta_{0\mu}t^a$ (one may insert the projectors at external heavy quark legs too).

Therefore any tree QCD diagram equals the corresponding HQET one up to $O(\widetilde{p}/m)$ terms. In loops, momenta can be arbitrarily large, and the relation (1.4) can break. But regions of large loop momenta are excluded by the renormalization in both theories, and for convergent integrals one may use (1.4) (see Sec. 4).

The Lagrangian (1.2) can be rewritten in covariant notations:

$$L = \widetilde{Q}iv_\mu D_\mu\widetilde{Q} + \cdots \qquad (1.5)$$

where the static quark field $\widetilde{Q}$ is a 4-component spinor obeying the relation $\hat{v}\widetilde{Q} = \widetilde{Q}$ and v_μ is the quark velocity. The true total momentum p_μ is related to the effective one $\widetilde{p}_\mu$ by

$$p_\mu = mv_\mu + \widetilde{p}_\mu, \quad |\widetilde{p}_\mu| \ll m. \qquad (1.6)$$

The static quark propagator is

$$\widetilde{S}(\widetilde{p}) = \frac{1+\hat{v}}{2}\frac{1}{v_\mu\widetilde{p}_\mu + i0}, \qquad (1.7)$$

and the vertex is $igv_\mu t^a$. In the limit $m \to \infty$ the heavy quark can't change its velocity v_μ in any processes with bounded momenta $\widetilde{p}_\mu$. Therefore there exists the velocity superselection rule [6]: heavy quarks with each velocity v_μ can be treated separately and described by a separate field $\widetilde{Q}_v$. If we are interested in a transition of a heavy hadron with the velocity v_1 into a heavy hadron with the velocity v_2, we can use the Lagrangian

$$L = \sum_i \overline{\widetilde{Q}}_i iv_{i\mu}D_\mu\widetilde{Q}_i + \cdots \qquad (1.8)$$

where $\widetilde{Q}_i$ is the static quark field with the velocity v_i (the quark Q_1 is present in the initial hadron and Q_2—in the final one). These quarks have different propagators (1.7) and vertices. They can be of the same or different flavour; it doesn't matter because they can't transform into each other except by an external current with an unbounded momentum transfer (of order m). It is even possible to write a Lorentz-invariant Lagrangian [6]

$$L = \int\frac{d^3\vec{v}}{2v_0}\overline{\widetilde{Q}}_v iv_\mu D_\mu\widetilde{Q}_v + \cdots \qquad (1.9)$$

describing static quarks with all possible velocities at ones. But in any specific problem only several heavy quarks with several velocities are involved; all fields $\widetilde{Q}_v$ except few ones are in the vacuum state and are irrelevant, and finite sums (1.8) are sufficient.

There is an ambiguity what quark mass m should be used in (1.6) [7]. In general the HQET Lagrangian is $\overline{\widetilde{Q}}(iv_\mu D_\mu - \delta m)\widetilde{Q}$; the residual mass δm is shifted when we change m. Physical quantities, of course, don't depend on this choice. The most convenient definition of the heavy quark mass m is one that gives $\delta m = 0$. It corresponds to the pole of the quark propagator at $v_\mu \widetilde{p}_\mu = 0$, or $p^2 = m^2$ in QCD. This pole mass is gauge-invariant. There is also an ambiguity in the exact choice of v_μ in (1.6) [8]. This reparametrization invariance relates coefficients of terms of different orders in $1/m$ expansion. Quantization of the theory (1.8) was discussed in [9].

The HQET Lagrangian (1.2) possesses the $SU(2)$ spin symmetry [10]. The heavy quark spin does not interact with gluon field in the limit $m \to \infty$ because its chromomagnetic moment vanishes. If there are n_h heavy quark flavours with the same velocity, there is the $SU(2n_h)$ spin-flavour symmetry. For example, in the problem of transition from a heavy hadron with the velocity v_1 to a different flavour heavy hadron with the velocity v_2, at equal velocities $v_1 = v_2$ the Lagrangian (1.8) has the $SU(4)$ spin-flavour symmetry which relates all form factors to the form factor of a single hadron at zero momentum transfer (equal 1). At non-equal velocities, it has only the $SU(2) \times SU(2)$ spin symmetry relating form factors to each other. The Lagrangian (1.9) has the symmetry $SU(2n_h)^\infty \times SO(3,1)$.

Not only the orientation but also the magnitude of the heavy quark spin is irrelevant in HQET. This leads to a supersymmetry group called the superflavour symmetry [11]. It allows one to predict properties of hadrons with a scalar or vector heavy quark appearing in supersymmetric extensions of the Standard Model, in technicolor models, and in some composite models. The scalar and vector static quark Lagrangians

$$L = \widetilde{\varphi}^+ iD_0 \widetilde{\varphi} + \cdots, \quad L = \vec{\widetilde{V}}^+ iD_0 \vec{\widetilde{V}} + \cdots \tag{1.10}$$

have the $SU(n_h)$ and $SU(3n_h)$ spin-flavour symmetry. This idea can also be applied to baryons with two heavy quarks [12] because they form a small size (of order $1/m\alpha_s$) spin 0 or 1 bound state antitriplet in color.

HQET has great advantages over QCD in lattice simulation of heavy quark problems. Indeed, the applicability conditions of the lattice approximation to problems with light hadrons are that the lattice spacing is much less than the characteristic hadron size, and the total lattice length is much larger than this size. For simulation of QCD with a heavy quark, the lattice spacing must be much less than the heavy quark Compton wavelength $1/m$.

For b quark it is impossible at present. The HQET Lagrangian does not involve the heavy quark mass m, and the applicability conditions of the lattice approximation are the same as for light hadrons [13]. Relation of the lattice HQET to the continuum one was investigated in [14, 15, 16]. Simulation results can be found in [17].

2 Mesons

Due to the heavy quark spin symmetry, hadrons may be classified according to the light fields' angular momentum and parity j^P which are conserved quantum numbers. In other words, we can switch off the heavy quark spin using the superflavour symmetry, and then the hadron's momentum and parity will be j^P. The $\overline{Q}q$ mesons are the QCD analog of the hydrogen atom. The ground-state (S-wave) meson has $j^P = \frac{1}{2}^+$; the excited P-wave mesons have $j^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$. When we switch the heavy quark spin on, each of these mesons becomes a degenerate doublet. Its components are transformed into each other by the heavy quark spin symmetry operations. We have the ground-state doublet 0^-, 1^-, and the excited P-wave doublets 0^+, 1^+, and 1^+, 2^+. Splittings in these doublets (hyperfine splittings) are due to the heavy quark chromomagnetic moment interaction violating the spin symmetry, and are proportional to $1/m$ (Sect. 5).

Form factors of the ground-state mesons in HQET were considered in [10, 18, 19]; applications to semileptonic B decays were discussed in the review [20] and papers [21], and to e^+e^- annihilation—in [22, 23]. Transition form factors to the P-wave mesons were considered in [24, 25]. A general method of counting independent form factors applicable both to mesons and baryons was proposed in [26], and an elegant explicit construction—in [27]. It was applied to ground state to arbitrary excited meson transitions in [28]. Two-point HQET sum rules were investigated in [29, 30], and three-point ones—in [31]. Mesons in two-dimensional QCD with the large number of colors were considered in [32]. Here we shall use correlators of currents with the quantum numbers of mesons in order to investigate properties of mesons in HQET.

When the heavy quark is scalar, there is one bilinear heavy-light current without derivatives $\widetilde{\jmath}_s = \widetilde{Q}_s^+ q$ ($\widetilde{Q}_s^+$ is the heavy antiquark field). It has no definite parity; the currents $\widetilde{\jmath}_\pm = \frac{1\pm\gamma_0}{2}\widetilde{\jmath}_s$ have the parity $P = \pm 1$ because the P-conjugation acts as $q \to \gamma_0 q$. The current $\widetilde{\jmath}_+$ has the quantum numbers of the ground-state $\frac{1}{2}^+$ meson, and $\widetilde{\jmath}_-$—of the P-wave $\frac{1}{2}^-$ meson. Currents with the quantum numbers of mesons with higher j necessarily involve derivatives.

In the case of real-world spin $\frac{1}{2}$ heavy quark, there are 4 bilinear currents without derivatives $\tilde{\jmath} = \bar{\tilde{Q}}\Gamma q$. Indeed, because of $\gamma_0\tilde{Q} = \tilde{Q}$, the current with $\Gamma = \gamma_0$ reduces to $\Gamma = 1$; $\gamma_0\gamma_5$—to γ_5; σ_{0i}—to γ_i; σ_{ij}—to $\varepsilon_{ijk}\gamma_k\gamma_5$. We are left with $\Gamma = \gamma_5$, $\tilde{\jmath}$ and 1, $\tilde{\jmath}\gamma_5$. The first pair with Γ anticommuting with γ_0 has the quantum numbers of the ground-state 0^-, 1^- doublet; the second pair with Γ commuting with γ_0—of the P-wave 0^+, 1^+ doublet.

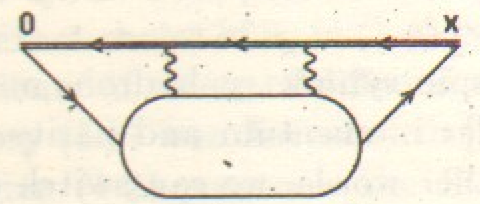

Figure 1: Correlator of two HQET heavy-light currents

A correlator of any two currents containing the static quark field has the form (Fig. 1)

$$i\langle T\tilde{\jmath}_2(x)\tilde{\jmath}_1^+(0)\rangle = \delta(\vec{x})\Pi(x_0), \qquad (2.1)$$
$$\Pi(\omega) = \int \Pi(t)e^{i\omega t}dt, \quad \Pi(t) = \int \Pi(\omega)e^{-i\omega t}\frac{d\omega}{2\pi}.$$

It obeys the dispersion representation

$$\Pi(\omega) = \int_0^\infty \frac{\rho(\varepsilon)d\varepsilon}{\varepsilon - \omega - i0} + \cdots, \quad \Pi(t) = -\tilde{S}(t)\int_0^\infty \rho(\omega)e^{-i\omega t}d\omega + \cdots \qquad (2.2)$$

A subtraction polynomial in $\Pi(\omega)$ (denoted by dots) gives $\delta(t)$ and its derivatives in $\Pi(t)$. We can analytically continue a correlator from the half-axis $t > 0$ to imaginary $t = -i\tau$. Then $\Pi(\tau)$ and $\rho(\omega)$ are related by the Laplace transform

$$\Pi(\tau) = i\int_0^\infty \rho(\omega)e^{-\omega\tau}d\omega, \quad \rho(\omega) = \frac{1}{2\pi}\int_{a-i\infty}^{a+i\infty} \Pi(\tau)e^{\omega\tau}d\tau, \qquad (2.3)$$

where a is to the right from all singularities of $\Pi(\tau)$.

The contribution of an intermediate state $|h\rangle$ with the energy $\tilde{\varepsilon}$ to $\Pi(t)$, $\Pi(\omega)$, $\rho(\omega)$ is

$$\begin{aligned}
\Pi_h(t) &= i\langle 0|\tilde{\jmath}_2|h\rangle i\tilde{S}(t)e^{-i\tilde{\varepsilon}t}\langle h|\tilde{\jmath}_1^+|0\rangle, \\
\Pi_h(\omega) &= i\langle 0|\tilde{\jmath}_2|h\rangle\frac{i}{\omega - \tilde{\varepsilon} + i0}\langle h|\tilde{\jmath}_1^+|0\rangle, \\
\rho_h(\omega) &= \langle 0|\tilde{\jmath}_2|h\rangle\langle h|\tilde{\jmath}_1^+|0\rangle\delta(\omega - \tilde{\varepsilon}).
\end{aligned} \qquad (2.4)$$

We remind the reader that the HQET energy $\tilde{\varepsilon}$ means the true energy minus the heavy quark mass.

The correlator of two meson currents with the scalar heavy quark has the γ-matrix structure (Fig. 1)

$$i\langle T\tilde{\jmath}_s(x)\bar{\tilde{\jmath}}_s(0)\rangle = \delta(\vec{x})\Pi_s(x_0), \quad \Pi_s = A + B\gamma_0. \qquad (2.5)$$

For the currents with the definite parity P we have

$$i\langle T\tilde{\jmath}_P(x)\bar{\tilde{\jmath}}_P(0)\rangle = \delta(\vec{x})\Pi_P(x_0)\frac{1+P\gamma_0}{2}, \Pi_P = A + PB = \frac{1}{4}\operatorname{Tr}(1+P\gamma_0)\Pi_s. \qquad (2.6)$$

Due to the linear relations (2.1–2.3), the same γ-matrix structures and relations between Π_s and Π_P hold in both the coordinate space and the momentum one, and also for spectral densities. When calculating the correlator using the Operator Product Expansion (OPE), even-dimensional terms contain an odd number of γ-matrices along the light quark line and after all integrations contribute to B; odd-dimensional terms contain even number of γ-matrices and contribute to A. If we denote $\langle 0|\tilde{\jmath}_+|M, \frac{1}{2}^+\rangle = \tilde{f}_{M,\frac{1}{2}^+}u$ where u is the meson M wave function, then the meson's contribution to $\rho_s(\omega)$ summed over polarizations is $\tilde{f}^2_{M,\frac{1}{2}^+}\delta(\omega - \tilde{\varepsilon})\sum u\bar{u}$, or the contribution to $\rho_+(\omega)$ is $\tilde{f}^2_{M,\frac{1}{2}^+}\delta(\omega - \tilde{\varepsilon})$. Similar formulae hold for $\frac{1}{2}^-$ mesons.

Now let's switch the heavy quark spin on. The correlator is (Fig. 1)

$$i\langle T\tilde{\jmath}_2(x)\tilde{\jmath}_1^+(0)\rangle = \delta(\vec{x})\Pi(x_0), \quad \Pi = \operatorname{Tr}\Gamma_2\frac{1-\gamma_0}{2}\bar{\Gamma}_1\Pi_s. \qquad (2.7)$$

In Π_s, γ_0 may be replaced by $P = \pm 1$ for $\Gamma_{1,2}$ (anti-) commuting with γ_0, and Π_s becomes the scalar function Π_P:

$$\Pi = \Pi_P\operatorname{Tr}\Gamma_2\frac{1-\gamma_0}{2}\bar{\Gamma}_1. \qquad (2.8)$$

The correlators of the currents $\overline{\widetilde{Q}}\gamma_5 q$ and $\overline{\widetilde{Q}}\gamma_i q$ with the quantum numbers of the ground-state 0^-, 1^- mesons are equal to $2\Pi_+$ and $2\delta_{ij}\Pi_+$. If we denote $<0|\overline{\widetilde{Q}}\gamma_5 q|M,0^-> = \widetilde{f}_{M,0^-}$, $<0|\overline{\widetilde{Q}}\vec{\gamma} q|M,1^-> = \widetilde{f}_{M,1^-}\vec{e}$ (where $\vec{e}$ is the 1^- meson's polarization vector) then the meson's contributions to the spectral densities are $\widetilde{f}^2_{M,0^-}\delta(\omega-\widetilde{\varepsilon}_{0^-})$ and $\delta_{ij}\widetilde{f}^2_{M,1^-}\delta(\omega-\widetilde{\varepsilon}_{1^-})$. Therefore the spin symmetry requires that the mesons in 0^- and 1^- channels are degenerate ($\widetilde{\varepsilon}_{0^-}=\widetilde{\varepsilon}_{1^-}=\widetilde{\varepsilon}$), and $\widetilde{f}_{M,0^-}=\widetilde{f}_{M,1^-}=\sqrt{2}\widetilde{f}_{M,\frac{1}{2}^+}$. Similar formulae hold for P-wave 0^+, 1^+ mesons.

In QCD, the meson constants are usually defined as $<0|\overline{Q}\gamma_\mu\gamma_5 q|M,0^-> = f_{M,0^-}p_\mu$, $<0|\overline{Q}\gamma_\mu q|M,1^-> = mf_{M,1^-}e_\mu$, where the meson states are normalized in the relativistic way: $<M,\vec{p}'|M,\vec{p}> = 2p_0\delta(\vec{p}'-\vec{p})$. This normalization is senseless in HQET; we must use the non-relativistic normalization $<M,\vec{p}'|M,\vec{p}> = \delta(\vec{p}'-\vec{p})$ instead. Then the definitions read $\sqrt{2m}<0|\overline{\widetilde{Q}}\gamma_0\gamma_5 q|M,0^-> = mf_{M,0^-}$, $\sqrt{2m}<0|\overline{\widetilde{Q}}\vec{\gamma} q|M,1^-> = mf_{M,1^-}\vec{e}$. Finally we obtain the scaling law

$$f_{M,0^-} = f_{M,1^-} = \frac{2\widetilde{f}_{M,\frac{1}{2}^+}}{\sqrt{m}}. \tag{2.9}$$

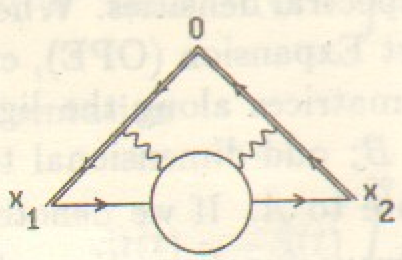

Figure 2: Correlator of two HQET heavy-light currents and a heavy-heavy current

To investigate hadron form factors in HQET, we consider correlators of two currents $\widetilde{\jmath}_{1,2}$ containing the static quark fields $\widetilde{Q}_{1,2}$ with the velocities $v_{1,2}$ and the heavy-heavy velocity-changing current $\widetilde{J}$ (Fig. 2):

$$i^2<T\widetilde{\jmath}_2(x_2)\widetilde{J}(0)\widetilde{\jmath}_1^+(x_1)> = \int_0^\infty dt_2\delta(x_2-v_2t_2)\int_0^\infty dt_1\delta(x_1+v_1t_1)K(t_2,t_1),$$

$$K(\omega_2,\omega_1) = \int K(t_2,t_1)e^{i\omega_2t_2+i\omega_1t_1}dt_2dt_1, \tag{2.10}$$

$$K(t_2,t_1) = \int K(\omega_2,\omega_1)e^{-i\omega_2t_2-i\omega_1t_1}\frac{d\omega_2}{2\pi}\frac{d\omega_1}{2\pi}.$$

They obey the double dispersion representation

$$K(\omega_2,\omega_1) = \int\frac{\rho(\varepsilon_2,\varepsilon_1)d\varepsilon_2d\varepsilon_1}{(\varepsilon_2-\omega_2-i0)(\varepsilon_1-\omega_1-i0)}+\cdots \tag{2.11}$$

$$K(t_2,t_1) = \widetilde{S}(t_2)\widetilde{S}(t_1)\int\rho(\omega_2,\omega_1)e^{-i\omega_2t_2-i\omega_1t_1}d\omega_2d\omega_1+\cdots$$

Subtraction terms in $K(\omega_2,\omega_1)$ (denoted by dots) are polynomial in ω_1 with coefficients that are arbitrary functions of ω_2 (given by single dispersion integrals) plus vice versa. These terms give $\delta(t_1)$ and its derivatives times arbitrary functions of t_2 plus vice versa in $K(t_2,t_1)$. We can analytically continue $K(t_2,t_1)$ from $t_{1,2}>0$ to $t_{1,2}=-i\tau_{1,2}$. Then $K(\tau_2,\tau_1)$ and $\rho(\omega_2,\omega_1)$ are related by the double Laplace transform

$$K(\tau_2,\tau_1) = -\int\rho(\omega_2,\omega_1)e^{-\omega_2\tau_2-\omega_1\tau_1}d\omega_2d\omega_1, \tag{2.12}$$

$$\rho(\omega_2,\omega_1) = \frac{1}{(2\pi)^2}\int_{a-i\infty}^{a+i\infty}d\tau_2\int_{a-i\infty}^{a+i\infty}d\tau_1K(\tau_2,\tau_1)e^{\omega_2\tau_2+\omega_1\tau_1}.$$

The contribution of intermediate states $|h_{1,2}>$ with the energies $\widetilde{\varepsilon}_{1,2}$ to $K(\tau_2,\tau_1)$, $K(\omega_2,\omega_1)$, and $\rho(\omega_2,\omega_1)$ is

$$K_{h_2h_1}(t_2,t_1) = i^2<0|\widetilde{\jmath}_2|h_2>i\widetilde{S}(t_2)e^{-i\widetilde{\varepsilon}_2t_2}<h_2|\widetilde{J}|h_1>i\widetilde{S}(t_1)e^{-i\widetilde{\varepsilon}_1t_1}<h_1|\widetilde{\jmath}_1^+|0>,$$

$$K_{h_2h_1}(\omega_2,\omega_1) = i^2<0|\widetilde{\jmath}_2|h_2>\frac{i}{\omega_2-\widetilde{\varepsilon}_2+i0}<h_2|\widetilde{J}|h_1>\frac{i}{\omega_1-\widetilde{\varepsilon}_1+i0}<h_1|\widetilde{\jmath}_1^+|0>,$$

$$\rho_{h_2h_1}(\omega_2,\omega_1) = <0|\widetilde{\jmath}_2|h_2><h_2|\widetilde{J}|h_1><h_1|\widetilde{\jmath}_1^+|0>\delta(\omega_2-\widetilde{\varepsilon}_2)\delta(\omega_1-\widetilde{\varepsilon}_1), \tag{2.13}$$

where the sum over $h_{1,2}$ polarizations is assumed. Let's introduce the spin wave functions $\psi_{1,2}$ of $h_{1,2}$. Then $<0|\widetilde{\jmath}_{1,2}|h_{1,2}> = \widetilde{f}_{1,2}\psi_{1,2}$, $<h_2|\widetilde{J}|h_1> = \psi_2^+\widetilde{f}_{21}\psi_1$, where $\widetilde{f}_{21}$ is a form factor matrix in the spin space. The spectral density of the correlator of the currents $\widetilde{\jmath}^+_{1,2}\psi_{1,2}$ with some specific polarizations $\psi_{1,2}$ is $\psi_2^+\rho(\omega_2,\omega_1)\psi_1$. The contribution of $|h_{1,2}>$ to it is $\widetilde{f}_2\widetilde{f}_1\psi_2^+\widetilde{f}_{21}\psi_1\delta(\omega_2-\widetilde{\varepsilon}_2)\delta(\omega_1-\widetilde{\varepsilon}_1)$.

The correlator of two meson currents with the scalar heavy quark $\tilde{\jmath}_{s2}$, $\bar{\tilde{\jmath}}_{s1}$ and the scalar heavy-heavy current $\tilde{J}_s = \tilde{Q}^+_{s1}\tilde{Q}_{s2}$ has the γ-matrix structure (Fig. 2)

$$K_s = A + B_1\hat{v}_1 + B_2\hat{v}_2 + C\hat{v}_2\hat{v}_1. \qquad (2.14)$$

For the currents $\tilde{\jmath}_{P_2}$, $\bar{\tilde{\jmath}}_{P_1}$ with the definite parities we have

$$K_{P_2P_1} = A + P_1B_1 + P_2B_2 + P_1P_2C = \frac{\frac{1}{4}\,\mathrm{Tr}(1+P_1\hat{v}_1)(1+P_2\hat{v}_2)K_s}{1+P_1P_2v_1\cdot v_2}. \qquad (2.15)$$

Due to the linear relations (2.10–2.12), the same γ-matrix structures and relations between K_s and $K_{P_2P_1}$ hold in both the coordinate space and the momentum one, and also for spectral densities. When calculating the correlator using the OPE, even-dimensional terms contribute to B_1, B_2, and odd-dimensional ones—to A, C.

It is convenient to use the "brick wall" frame in which $\vec{v}_1 = -\vec{v}_2$ is directed along z for counting form factors. Angular momentum projection onto z is conserved: $j_{2z} = j_{1z}$. The reflection in any plane containing z transforms the state $|j, j_z>$ to $Pi^{2j}|j, -j_z>$. Therefore the amplitude for $-j_{1z}$, $-j_{2z}$ is equal to that for j_{1z}, j_{2z} up to a phase factor; the $0 \to 0$ transition is allowed only if the "naturalness" $P(-1)^j$ is conserved [26]. For example, $\frac{1}{2}^+ \to \frac{1}{2}^+$, $\frac{1}{2}^+ \to \frac{1}{2}^-$, and $\frac{1}{2}^+ \to \frac{3}{2}^-$ transitions are described by one form factor each:

$$\begin{aligned}
<M, \tfrac{1}{2}^+ |\tilde{J}_s| M, \tfrac{1}{2}^+> &= \xi(\mathrm{ch}\,\varphi)\bar{u}_2u_1, \\
<M, \tfrac{1}{2}^- |\tilde{J}_s| M, \tfrac{1}{2}^+> &= \tau_{1/2}(\mathrm{ch}\,\varphi)\bar{u}_2\gamma_5u_1, \qquad (2.16)\\
<M, \tfrac{3}{2}^- |\tilde{J}_s| M, \tfrac{1}{2}^+> &= \tau_{3/2}(\mathrm{ch}\,\varphi)v_{1\mu}\bar{u}_{2\mu}u_1,
\end{aligned}$$

where $\mathrm{ch}\,\varphi = v_1 \cdot v_2$ is the cosine of the Minkovskian angle between the world lines of the incoming and the outgoing heavy quark, and u_μ is the Rarita-Schwinger wave function of the spin $\frac{3}{2}$ meson. Here we have slightly changed the notations as compared to [24]: in the original notations right-hand sides of (2.16) should contain $2\tau_{1/2}$ and $\sqrt{3}\tau_{3/2}$. The contribution of $\frac{1}{2}^+$ mesons to the spectral density $\rho_s(\omega_2, \omega_1)$ is $\tilde{f}_2\tilde{f}_1\xi(\mathrm{ch}\,\varphi)\frac{1+\hat{v}_2}{2}\frac{1+\hat{v}_1}{2}\delta(\omega_2 - \tilde{\varepsilon}_2)\omega_1 - \tilde{\varepsilon}_1)$, i. e. the contribution to $\rho_{++}(\omega_2, \omega_1)$ is $\tilde{f}_2\tilde{f}_1\xi(\mathrm{ch}\,\varphi)\delta(\omega_2 - \tilde{\varepsilon}_2)\omega_1 - \tilde{\varepsilon}_1)$. The spectral density of the correlator of the currents $\tilde{\jmath}_{s1,2}u_{1,2}$ with some specific polarizations $u_{1,2}$ is $\bar{u}_2\rho_s(\omega_2, \omega_1)u_1$. The contribution of $\frac{1}{2}^+$ mesons to it is $\tilde{f}_2\tilde{f}_1\bar{u}_2\xi(\mathrm{ch}\,\varphi)u_1\delta(\omega_2 - \tilde{\varepsilon}_2)\delta(\omega_1 - \tilde{\varepsilon}_1)$. Similar formulae hold for $\frac{1}{2}^-$ mesons.

Now let's switch the heavy quark spin on. The correlator of $\tilde{\jmath}_2 = \bar{\tilde{Q}}_2\Gamma_2 q$, $\bar{\tilde{\jmath}}_1 = \bar{q}\bar{\Gamma}_1\tilde{Q}_1$, and $\tilde{J} = \bar{\tilde{Q}}_1\Gamma\tilde{Q}_2$ is (Fig. 2)

$$K = \mathrm{Tr}\,\Gamma_2\frac{1-\hat{v}_2}{2}\Gamma\frac{1-\hat{v}_1}{2}\bar{\Gamma}_1K_s. \qquad (2.17)$$

In K_s, $\hat{v}_{1,2}$ may be replaced by $P_{1,2} = \pm 1$ for $\Gamma_{1,2}$ (anti-) commuting with $\hat{v}_{1,2}$, and K_s becomes the scalar function $K_{P_2P_1}$:

$$K = K_{P_2P_1}\,\mathrm{Tr}\,\Gamma_2\frac{1-\hat{v}_2}{2}\Gamma\frac{1-\hat{v}_1}{2}\bar{\Gamma}_1. \qquad (2.18)$$

Let's introduce the currents $\bar{q}\mathcal{M}\tilde{Q}$, $\mathcal{M} = \bar{\Gamma}\psi$ where ψ is the spin wave function, i. e. $\mathcal{M}_{0^-} = \bar{\gamma}_5 = -\gamma_5$, $\mathcal{M}_{1^-} = \bar{\gamma}_\mu e_\mu = \hat{e}$. The spectral density of their correlator is $\rho_{P_2P_1}\,\mathrm{Tr}\,\overline{\mathcal{M}}_2\frac{1-\hat{v}_2}{2}\Gamma\mathcal{M}_1$; the mesons' contribution to it is $\tilde{f}_{M_2}\tilde{f}_{M_1}<M_2|\tilde{J}|M_1>\delta(\omega_2 - \tilde{\varepsilon}_2)\delta(\omega_1 - \tilde{\varepsilon}_1)$. Hence we obtain [10, 18]

$$<M_2|\tilde{J}|M_1> = \frac{1}{2}\xi(\mathrm{ch}\,\varphi)\,\mathrm{Tr}\,\overline{\mathcal{M}}_2\frac{1-\hat{v}_2}{2}\Gamma\frac{1-\hat{v}_1}{2}\mathcal{M}_1. \qquad (2.19)$$

This formula expresses all form factors of transitions of a ground-state 0^-, 1^- meson to a ground-state 0^-, 1^- meson under the action of any heavy-heavy current $\tilde{J} = \bar{\tilde{Q}}_2\Gamma\tilde{Q}_1$ via one universal Isgur-Wise form factor $\xi(\mathrm{ch}\,\varphi)$. A similar formula with $\tau_{1/2}(\mathrm{ch}\,\varphi)$ holds for $\frac{1}{2}^+ \to \frac{1}{2}^-$ transitions; $\mathcal{M} = 1$, $\hat{e}\gamma_5$ for 0^+, 1^+ mesons.

A three-point correlator at $\varphi = 0$ is expressed via the corresponding two-point one:

$$\begin{aligned}
K(t_2, t_1) &= \Pi(t_1 + t_2), \\
K(\omega_2, \omega_1) &= \frac{\Pi(\omega_1) - \Pi(\omega_2)}{\omega_1 - \omega_2}, \qquad (2.20)\\
\rho(\omega_2, \omega_1) &= \rho(\omega_1)\delta(\omega_2 - \omega_1).
\end{aligned}$$

The first two forms follow from each other by the Fourier transform. The third form is necessary and sufficient for the first one because of (2.2); it can be obtained by the double backward Laplace transform (2.12) of the single forward Laplace transform (2.3) of the first form at imaginary times. The third form can be also obtained from the second one by taking the double discontinuity: the discontinuity of the first term in ω_2 is $\pi i\delta(\omega_2 - \omega_1)\Pi(\omega_1)$, and the discontinuity of this expression in ω_1 is $\frac{1}{2}(2\pi i)^2\rho(\omega_1)\delta(\omega_2 - \omega_1)$; the second term contributes equally.

To prove the first form, let's consider any diagram for the two-point correlator in the coordinate space (for simplicity, with the scalar heavy quark). Vertices along the heavy quark line have the times $t_0 < t_1 < \cdots < t_{n-1} < t_n$, and the integration in $t_2, \ldots, t_{n-1}$ is performed. The integrand is an integral over coordinates of all vertices not belonging to the heavy quark line. Now consider all diagrams for the three-point correlator obtained by inserting the heavy-heavy vertex with time t (and $\varphi = 0$) to all the possible places along the heavy quark line. These diagrams have the same integrand and the integration regions $t_0 < t_1 < \cdots < t_{m-1} < t < t_m < \cdots < t_{n-1} < t_n$ ($m = 1, \ldots, n$). These regions span the whole integration region of the original diagram. Therefore the sum of this set of three-point diagrams is equal to the two-point diagram.

The second form can be easily proved in the momentum space in the exact analogy with the QED Ward identity using the relation $i\widetilde{S}(\omega_1 + \omega')ii\widetilde{S}(\omega_2 + \omega') = \frac{i\widetilde{S}(\omega_1) - i\widetilde{S}(\omega_2)}{\omega_1 - \omega_2}$ (Fig. 3). In particular, it implies $K(\omega, \omega) = \frac{d\Pi(\omega)}{d\omega}$.

Comparing the mesons' contributions to $\rho(\omega_2, \omega_1)$ and $\rho(\omega)$, we see

$$\xi(1) = 1 \tag{2.21}$$

for any $\frac{1}{2}^+$ meson. For non-diagonal $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions $\xi(1) = 0$ because $\rho(\omega_2, \omega_1) = 0$ off the diagonal. The physical meaning of this is simple: when the current $\widetilde{J}$ replaces the old heavy quark by the new one with the same velocity and color, light fields don't notice it. The formulae for the form factors at $\varphi = 0$ equivalent to (2.19, 2.21) were first proposed in the quark model framework [33].

The variable $\operatorname{ch}\varphi$ is related to the momentum transfer q^2 (for simplicity in the case of equal heavy quark masses) by the formula $q^2 = 2m^2(1 - \operatorname{ch}\varphi)$. Form factors are analytic functions of q^2 with the cut in the annihilation channel from $4m^2$ to $+\infty$. Therefore the Isgur-Wise function $\xi(\operatorname{ch}\varphi)$ is an analytic function in the $\operatorname{ch}\varphi$ plane with the cut from -1 to $-\infty$. Geometrically speaking, $ch\varphi > 1$ corresponds to Minkovskian angles between the incoming and outgoing heavy quark world lines (scattering or decay); $\operatorname{ch}\varphi = 1$ means the straight world line—no transition at all; nothing special happens at $\operatorname{ch}\varphi < 1$, only the angle becomes Euclidean; $\operatorname{ch}\varphi = -1$ is really a singular point where the quark returns along the same world line; $\operatorname{ch}\varphi < -1$ corresponds again to Minkovskian angles only one of the world lines is directed to the past (annihilation).

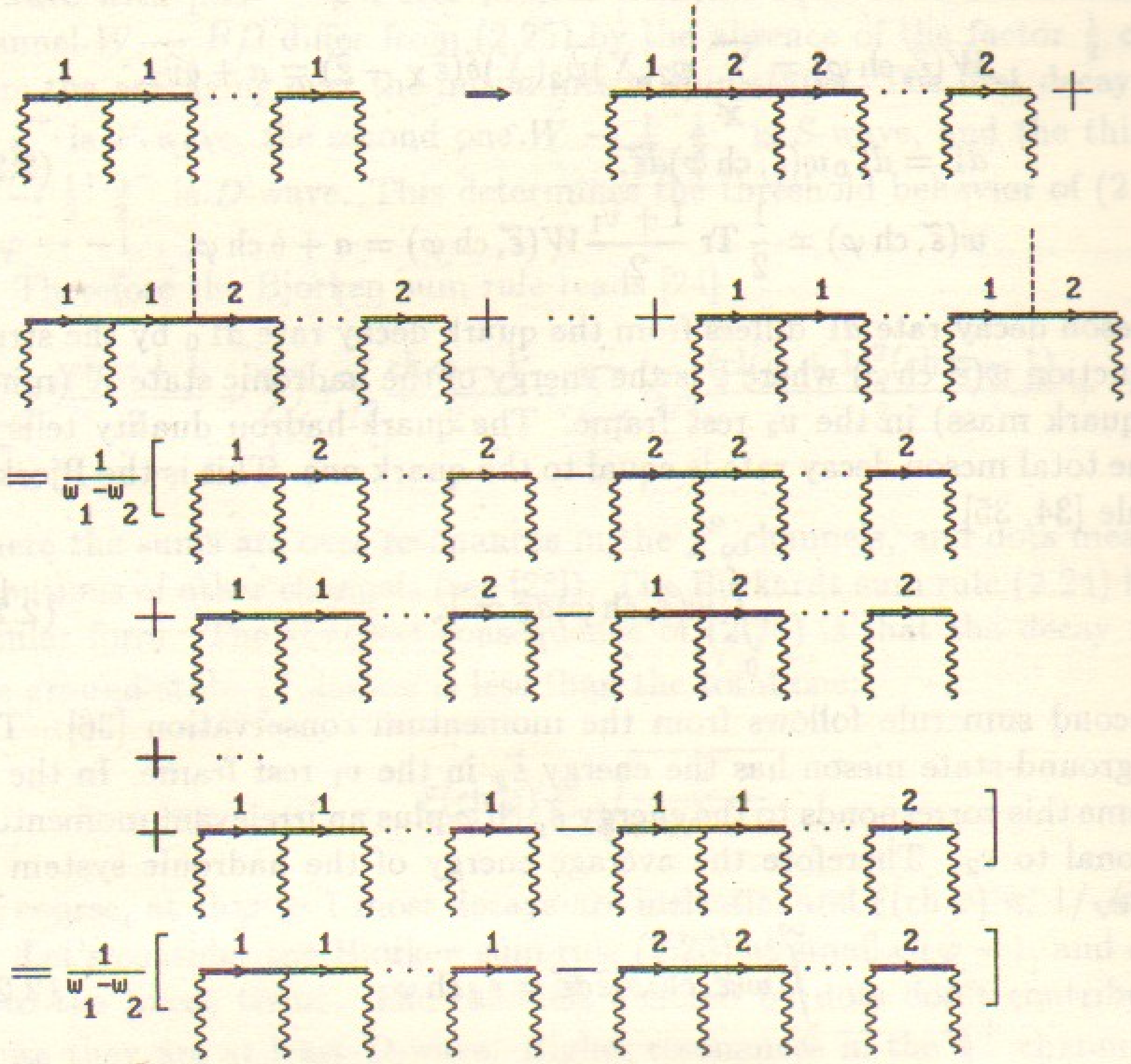

Figure 3: Proof of the Ward identity. A digit 1 (2) near a heavy quark line means that its energy includes ω_1 (ω_2)

In the rest of this Section, we shall for simplicity live in the world with the scalar b quark decaying into the scalar c quark plus the scalar W boson. Their masses can be adjusted in such a way as to give any desired $\operatorname{ch}\varphi$. The quark decay matrix element is simply $M = g$ where g is the coupling constant.

Until now we discussed exclusive decays of the $\frac{1}{2}^+$ B meson. Inclusive decays can be also treated in HQET [34, 35]. The matrix element of the decay $B \to X + W$ (where X is any hadronic state containing the c quark) has the structure $M = g\overline{\psi}_2(X)u_1$. Its spin-averaged square is $\overline{|M|^2} = \frac{g^2}{2} \operatorname{Tr} \frac{1+\hat{v}_1}{2} \psi_2(X)\overline{\psi}_2(X)$. Let's sum over hadronic states X with the energy $\tilde{\varepsilon}$:

$$
\begin{aligned}
W(\tilde{\varepsilon}, \operatorname{ch}\varphi) &= \sum_X \psi_2(X)\overline{\psi}_2(X)\delta(\tilde{\varepsilon}_X - \tilde{\varepsilon}) = a + b\hat{v}_2, \\
d\Gamma &= d\Gamma_0 w(\tilde{\varepsilon}, \operatorname{ch}\varphi)d\tilde{\varepsilon}, \qquad (2.22) \\
w(\tilde{\varepsilon}, \operatorname{ch}\varphi) &= \frac{1}{2}\operatorname{Tr}\frac{1+\hat{v}_1}{2}W(\tilde{\varepsilon}, \operatorname{ch}\varphi) = a + b\operatorname{ch}\varphi.
\end{aligned}
$$

The meson decay rate $d\Gamma$ differs from the quark decay rate $d\Gamma_0$ by the structure function $w(\tilde{\varepsilon}, \operatorname{ch}\varphi)$ where $\tilde{\varepsilon}$ is the energy of the hadronic state X (minus the c quark mass) in the v_2 rest frame. The quark-hadron duality tells us that the total meson decay rate is equal to the quark one. This is the Bjorken sum rule [34, 35]

$$
\int_0^\infty w(\tilde{\varepsilon}, \operatorname{ch}\varphi)d\tilde{\varepsilon} = 1. \qquad (2.23)
$$

The second sum rule follows from the momentum conservation [36]. The initial ground-state meson has the energy $\tilde{\varepsilon}_g$ in the v_1 rest frame. In the v_2 rest frame this corresponds to the energy $\tilde{\varepsilon}_g \operatorname{ch}\varphi$ plus an irrelevant momentum orthogonal to v_2. Therefore the average energy of the hadronic system X must be

$$
\int_0^\infty w(\tilde{\varepsilon}, \operatorname{ch}\varphi)\tilde{\varepsilon}d\tilde{\varepsilon} = \tilde{\varepsilon}_g \operatorname{ch}\varphi. \qquad (2.24)
$$

Inclusive semileptonic B decays in HQET were also discussed in [37].

Now we shall explicitly write down some contribution to this sum rule. The spin-averaged matrix elements squared for the decays $\frac{1}{2}^+ \to \frac{1}{2}^+$, $\frac{1}{2}^+ \to \frac{1}{2}^-$, and $\frac{1}{2}^+ \to \frac{3}{2}^-$ (2.16) are

$$
\begin{aligned}
\overline{|M|^2_{\frac{1}{2}^+}} &= \frac{g^2\xi^2}{2}\operatorname{Tr}\frac{1+\hat{v}_1}{2}\frac{1+\hat{v}_2}{2} = g^2\xi^2\frac{\operatorname{ch}\varphi+1}{2}, \\
\overline{|M|^2_{\frac{1}{2}^-}} &= \frac{g^2\tau^2_{1/2}}{2}\operatorname{Tr}\frac{1+\hat{v}_1}{2}\gamma_5\frac{1+\hat{v}_2}{2}\overline{\gamma}_5 = g^2\tau^2_{1/2}\frac{\operatorname{ch}\varphi-1}{2}, \qquad (2.25) \\
\overline{|M|^2_{\frac{3}{2}^-}} &= g^2\tau^2_{3/2}\frac{(\operatorname{ch}\varphi+1)^2(\operatorname{ch}\varphi-1)}{3}.
\end{aligned}
$$

Here we have used the Rarita-Schwinger density matrix. The decay $\frac{1}{2}^+ \to \frac{1}{2}^+$ is S-wave, hence $\overline{|M|^2}$ is constant at $\varphi \to 0$. The decays $\frac{1}{2}^+ \to \frac{1}{2}^-$, $\frac{1}{2}^+ \to \frac{3}{2}^-$ are P-wave, hence $\overline{|M|^2} \sim \varphi^2$. The decays to the D-wave mesons $\frac{3}{2}^+$, $\frac{5}{2}^+$ are

D-wave with $\overline{|M|^2} \sim \varphi^4$, etc. Matrix elements squared in the annihilation channel $W \to \overline{B}D$ differ from (2.25) by the absence of the factor $\frac{1}{2}$ coming from the averaging over the initial meson spin states. The first decay $W \to \frac{1}{2}^-\frac{1}{2}^+$ is P-wave, the second one $W \to \frac{1}{2}^-\frac{1}{2}^-$ is S-wave, and the third one $W \to \frac{1}{2}^-\frac{3}{2}^-$ is D-wave. This determines the threshold behavior of (2.25) at $\operatorname{ch}\varphi \to -1$.

Therefore the Bjorken sum rule reads [24]

$$
\sum_{\frac{1}{2}^+}\xi^2\frac{\operatorname{ch}\varphi+1}{2} + \sum_{\frac{1}{2}^-}\tau^2_{1/2}\frac{\operatorname{ch}\varphi-1}{2} + \sum_{\frac{3}{2}^-}\tau^2_{3/2}\frac{(\operatorname{ch}\varphi+1)^2(\operatorname{ch}\varphi-1)}{3} + \cdots = 1, \qquad (2.26)
$$

where the sums are over resonances in the j^P channels, and dots mean contributions of other channels (see [28]). The Burkardt sum rule (2.24) has the similar form. The simplest consequence of (2.26) is that the decay rate to the ground-state $\frac{1}{2}^+$ meson is less than the total one:

$$
\xi(\operatorname{ch}\varphi) \le \sqrt{\frac{2}{\operatorname{ch}\varphi+1}}. \qquad (2.27)
$$

Of course, at $\operatorname{ch}\varphi \gg 1$ most decays are inelastic, and $\xi(\operatorname{ch}\varphi) \ll 1/\sqrt{\operatorname{ch}\varphi}$.

Let's consider the Bjorken sum rule (2.26) at small $\operatorname{ch}\varphi - 1$, and expand it to the linear terms. The channels denoted by dots don't contribute because they are at least D-wave. Higher resonances in the $\frac{1}{2}^+$ channel don't contribute because they have $\xi(\operatorname{ch}\varphi) = O(\operatorname{ch}\varphi - 1)$. We are left with [24]

$$
\xi'(1) = -\frac{1}{4} - \frac{1}{4}\sum_{\frac{1}{2}^-}\tau^2_{1/2}(1) - \frac{2}{3}\sum_{\frac{3}{2}^-}\tau^2_{3/2}(1). \qquad (2.28)
$$

This gives us the Bjorken bound $\xi'(1) < -\frac{1}{4}$ (evident also from (2.27)). Similarly, the Burkardt sum rule (2.24) leads to the optical (Thomas-Reiche-Kuhn) sum rule [38]

$$
\frac{1}{4}\sum_{\frac{1}{2}^-}(\tilde{\varepsilon}_{1/2} - \tilde{\varepsilon}_g)\tau^2_{1/2}(1) + \frac{2}{3}\sum_{\frac{3}{2}^-}(\tilde{\varepsilon}_{3/2} - \tilde{\varepsilon}_g)\tau^2_{3/2}(1) = \frac{1}{2}\tilde{\varepsilon}_g. \qquad (2.29)
$$

It can be used for obtaining bounds on $\xi'(1)$ [38].

It is also possible to establish the bound on the Isgur-Wise form factor at the cut [39]: the decay rate $W \to \overline{B}D$ is less than the total decay rate $W \to \bar{b}c$. The meson decay rate for each flavour is given by (2.25) without the spin-averaging factor $\frac{1}{2}$; the quark decay rate is $\overline{|M|^2} = g^2 N_c$ where N_c is the number of colors. If there are n_l light flavours for which $\xi(\operatorname{ch}\varphi)$ is approximately the same, then

$$n_l|\xi(\operatorname{ch}\varphi)|^2|\operatorname{ch}\varphi + 1| \le N_c. \qquad (2.30)$$

In general the left-hand side is the sum over light flavours. The factor n_l was erroneously omitted in [39]; the phrase justifying this seems to have no sense. At $|\operatorname{ch}\varphi| \gg 1$, the $B\overline{D}$ channel constitutes a small fraction of the total $W \to b\overline{c}$ width, and $|\xi(\operatorname{ch}\varphi)| \ll 1/\sqrt{|\operatorname{ch}\varphi|}$. One could include also higher states' contributions in the left-hand side; this should be done with caution because of the possibility of double counting. The inequality (2.30) is applicable only sufficiently far from the threshold, at $|\operatorname{ch}\varphi + 1| \gg \pi^2\alpha_s^2$. Near the threshold the Coulomb interaction between the heavy quark and antiquark is essential. The total decay width on the right-hand side is not equal to its free-quark value N_c; it contains high narrow resonances at the quarkonium levels. Moreover, the very concept of the Isgur-Wise form factor is inapplicable in this region. The HQET picture is based on the fact that heavy quarks move along straight world lines, but at velocities $\sim \pi\alpha_s$ they really rotate around each other.

If the inequality (2.30) were true everywhere on the cut, we would immediately arrive at a paradox [40]. Consider the function $f(\operatorname{ch}\varphi)$ analytic in the $\operatorname{ch}\varphi$ plane with the cut from -1 to $-\infty$. On the cut $|f|^2 \le \frac{N_c}{2n_l}$, and $f(1) = 1$. This is consistent with the maximum modulus theorem only at

$$2n_l \le N_c \qquad (2.31)$$

what is not the case in our world. The more detailed analysis [40] shows that it is possible to obtain similar inequalities (with the constant smaller than 2) using weight functions that are rather insensitive to the threshold region, and the paradox remains.

HQET can also be used for description of heavy to light transitions [41] and rare B decays [42]. In these cases the heavy quark spin symmetry is not so restrictive, and more form factors are necessary. Relations between B and D decays can be established using the heavy quark spin-flavour symmetry and the isospin symmetry. Inclusive heavy to light decays are considered in [35]; they are described by several structure functions obeying sum rules in the deep inelastic region.

An interesting approach in which the parameter $\left(\frac{\varphi_{max}}{2}\right)^2 = \left(\frac{m_b - m_c}{m_b + m_c}\right)^2$ is considered small was proposed in [43]. To the leading order in this parameter,

the quark-hadron duality is perfect: the $b \to cW$ quark decay rate is equal to the $B \to DW$ meson decay rate (see (2.26)). This is true for all heavy quark polarizations; in particular, the hadronic tensor $\langle B|\tilde{\jmath}|X\rangle\langle X|\tilde{\jmath}^+|B\rangle$ summed over ground state mesons $X = D$, D^* is equal to the corresponding quark tensor summed over c polarizations.

3 Baryons

For ground-state baryons, the light quark spins can add giving $j^P = 0^+$ or 1^+. In the first case their spin wave function is antisymmetric, the Fermi statistics and the antisymmetry in color require an antisymmetric flavour wave function. Hence the light quarks must be different; if they are u, d then their isospin $I = 0$. With the heavy quark spin switched off, we have the 0^+ $I = 0$ baryon Λ_Q. If one of the light quarks is s, we obtain the isodoublet Ξ'_Q forming together with Λ_Q the $SU(3)$ antitriplet. In the 1^+ case the flavour wave function is symmetric; if the light quarks are u, d then their isospin $I = 1$. So we have the 1^+ isotriplet Σ_Q; with one s quark—the isodoublet Ξ_Q; with two s quarks—the isosinglet Ω_Q. Together they form the $SU(3)$ sextet. With the heavy quark spin switched on, the scalar baryons Λ_Q, Ξ'_Q become $\frac{1}{2}^+$; the vector baryons form degenerate $\frac{1}{2}^+$, $\frac{3}{2}^+$ doublets Σ_Q, Σ^*_Q; Ξ_Q, Ξ^*_Q; Ω_Q, Ω^*_Q.

Baryons and their form factors in HQET were considered in [44, 45, 46]. Two-point and three-point HQET sum rules were investigated in [47].

Baryon currents with the scalar heavy quark have the form $\tilde{\jmath}_s = \varepsilon^{abc}(q^{Ta}C\Gamma\tau q^b)\tilde{Q}^c_s$ where q^T means q transposed and C is the charge conjugation matrix (because $q^T C$ is transformed like $\overline{q}$ under the action of the Lorentz group). Here τ is a flavour matrix, symmetric for 0^+ baryons and antisymmetric for 1^+ ones. We shall abbreviate it to $\tilde{\jmath}_s = (q^T C\Gamma q)\tilde{Q}_s$. A light quark pair with $j^P = 0^+$ corresponds to the current $a = q^T C\gamma_5 q$, and with 1^+—to $\vec{a} = q^T C\vec{\gamma} q$ (one can easily check it using the P-conjugation $q \to \gamma_0 q$). It is also possible to insert γ_0 into these currents without changing their quantum numbers. So, the scalar heavy quark currents with the quantum numbers of Λ_Q, Σ_Q are $\tilde{\jmath}_{\Lambda s} = a\tilde{Q}_s$, $\vec{\tilde{\jmath}}_{\Sigma s} = \vec{a}\tilde{Q}_s$.

With the real-world spin $\frac{1}{2}$ heavy quark, the current $\tilde{\jmath} = a\tilde{Q}$ has the spin $\frac{1}{2}$; the current $\vec{\tilde{\jmath}} = \vec{a}\tilde{Q}$ contains spin $\frac{1}{2}$ and spin $\frac{3}{2}$ components. The part $\vec{\tilde{\jmath}}_{3/2} = \vec{\tilde{\jmath}} + \frac{1}{3}\vec{\gamma}\,\vec{\gamma}\cdot\vec{\tilde{\jmath}}$ satisfies the condition $\vec{\gamma}\cdot\vec{\tilde{\jmath}}_{3/2} = 0$ and hence has the spin $\frac{3}{2}$. The other part $\vec{\tilde{\jmath}}_{1/2} = -\frac{1}{3}\vec{\gamma}\,\vec{\gamma}\cdot\vec{\tilde{\jmath}} = \frac{1}{3}\vec{\gamma}\gamma_5\tilde{\jmath}_{1/2}$, $\tilde{\jmath}_{1/2} = \vec{a}\cdot\vec{\gamma}\gamma_5\tilde{Q}$ has the spin $\frac{1}{2}$.

Finally we obtain the currents $\widetilde{\jmath} = (q^T C\Gamma q)\Gamma'\widetilde{Q}$ with the quantum numbers of Λ_Q, Σ_Q, Σ_Q^* [47]

$$\widetilde{\jmath}_\Lambda = (q^T C\gamma_5 q)\widetilde{Q}, \quad \widetilde{\jmath}_\Sigma = (q^T C\vec{\gamma} q)\cdot\vec{\gamma}\,\gamma_5\widetilde{Q}, \quad \vec{\widetilde{\jmath}}_{\Sigma^*} = (q^T C\vec{\gamma} q)\widetilde{Q} + \frac{1}{3}\vec{\gamma}(q^T C\vec{\gamma} q)\cdot\vec{\gamma}\widetilde{Q}, \tag{3.1}$$

and similar currents with the extra γ_0 inside the brackets.

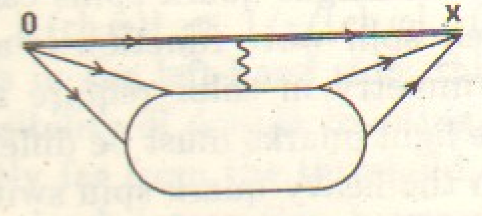

Figure 4: Correlator of two HQET baryonic currents

The correlators of two baryon currents with the scalar heavy quark have the structure (Fig. 4)

$$\begin{aligned} i<T\widetilde{\jmath}_{\Lambda s}(x)\widetilde{\jmath}^+_{\Lambda s}(0)> &= \delta(\vec{x})\,\mathrm{Tr}\,\tau^+\tau\Pi_\Lambda(x_0), \\ i<T\widetilde{\jmath}_{\Sigma si}(x)\widetilde{\jmath}^+_{\Sigma sj}(0)> &= \delta_{ij}\delta(\vec{x})\,\mathrm{Tr}\,\tau^+\tau\Pi_\Sigma(x_0). \end{aligned} \tag{3.2}$$

From now on we shall for simplicity assume the normalization $\mathrm{Tr}\,\tau^+\tau = 1$. If we denote $<0|\widetilde{\jmath}_\Lambda|\Lambda_Q, 0^+> = \widetilde{f}_{\Lambda,0^+}$, $<0|\vec{\widetilde{\jmath}}_\Sigma|\Sigma_Q, 1^+> = \widetilde{f}_{\Sigma,1^+}\vec{e}$, then the baryon contribution to $\rho_{\Lambda,\Sigma}(\omega)$ is $\widetilde{f}^2_{\Lambda,\Sigma}\delta(\omega - \widetilde{\varepsilon}_{\Lambda,\Sigma})$.

Now let's switch the heavy quark spin on. The correlators are (Fig. 4)

$$\Pi = \left(\Gamma_1' \frac{1+\gamma_0}{2}\overline{\Gamma}_2'\right)\Pi_s, \tag{3.3}$$

where tensor indices may be contracted between Π_s and $\Gamma'_{1,2}$. The same relation holds for $\Pi(t)$, $\Pi(\omega)$, and $\rho(\omega)$. For Λ_Q, Σ_Q, Σ_Q^* we obtain

$$\begin{aligned} \rho_\Lambda &= \frac{1+\gamma_0}{2}\rho_{\Lambda s}, \\ \rho_\Sigma &= \gamma_i\gamma_5\frac{1+\gamma_0}{2}\gamma_j\gamma_5\delta_{ij}\rho_{\Sigma s} = 3\frac{1+\gamma_0}{2}\rho_{\Sigma s}, \\ \rho_{\Sigma^*} &= \left(\delta_{ii'} - \frac{1}{3}\gamma_i\gamma_{i'}\right)\frac{1+\gamma_0}{2}\left(\delta_{jj'} - \frac{1}{3}\gamma_j\gamma_{j'}\right)\delta_{i'j'}\rho_{\Sigma s} \end{aligned} \tag{3.4}$$

$$= \frac{1+\gamma_0}{2}\left(\delta_{ij} + \frac{1}{3}\gamma_i\gamma_j\right)\rho_{\Sigma s}.$$

If we denote $<0|\widetilde{\jmath}_\Lambda|\Lambda_Q, \frac{1}{2}^+> = \widetilde{f}_{\Lambda,\frac{1}{2}^+}u$, $<0|\widetilde{\jmath}_\Sigma|\Sigma_Q, \frac{1}{2}^+> = \widetilde{f}_{\Sigma,\frac{1}{2}^+}u$, $<0|\vec{\widetilde{\jmath}}_{\Sigma^*}|\Sigma_Q^*, \frac{3}{2}^+> = \widetilde{f}_{\Sigma^*,\frac{3}{2}^+}\vec{u}$, then the baryon contributions to (3.4) are $\frac{1+\gamma_0}{2}\widetilde{f}^2_{\Lambda,\frac{1}{2}^+}\delta(\omega - \widetilde{\varepsilon}_\Lambda)$, $\frac{1+\gamma_0}{2}\widetilde{f}^2_{\Sigma,\frac{1}{2}^+}\delta(\omega - \widetilde{\varepsilon}_\Sigma)$, $\frac{1+\gamma_0}{2}\left(\delta_{ij} + \frac{1}{3}\gamma_i\gamma_j\right)\widetilde{f}^2_{\Sigma^*,\frac{3}{2}^+}\delta(\omega - \widetilde{\varepsilon}_\Sigma)$. Λ_Q is a spin symmetry singlet, therefore there are no interesting predictions of the spin symmetry in this channel. Baryons in Σ_Q and Σ_Q^* channels are degenerate: $\widetilde{\varepsilon}_{\Sigma,\frac{1}{2}^+} = \widetilde{\varepsilon}_{\Sigma^*,\frac{3}{2}^+} = \widetilde{\varepsilon}_{\Sigma,1^+}$, and $\frac{1}{\sqrt{3}}\widetilde{f}_{\Sigma,\frac{1}{2}^+} = \widetilde{f}_{\Sigma^*,\frac{3}{2}^+} = \widetilde{f}_{\Sigma,1^+}$. Note that both sides of the definitions $<0|j|B> = f_B u$ get the same factor $\sqrt{2m}$ when going from the relativistic normalization to the nonrelativistic one. Therefore the QCD quantities $f_B = \widetilde{f}_B$ don't depend on m (compare with (2.9)).

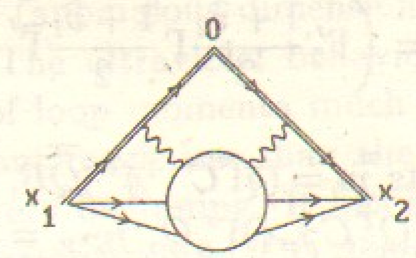

Figure 5: Correlator of two HQET baryonic currents and a heavy-heavy current

The correlators of two baryon currents with the scalar heavy quark and the scalar heavy-heavy current $\widetilde{J}_s$ have the structure (Fig. 5)

$$i^2<T\widetilde{\jmath}_{\Lambda s}(x_2)\widetilde{J}_s(0)\widetilde{\jmath}^+_{\Lambda s}(x_2)> = \mathrm{Tr}\,\tau^+\tau$$

$$\int_0^\infty dt_2\delta(x_2 - v_2t_2)\int_0^\infty dt_1\delta(x_1 + v_1t12)K_\Lambda(t_2, t_1), \tag{3.5}$$

$$i^2<T\widetilde{\jmath}_{\Sigma s\mu}(x_2)\widetilde{J}_s(0)\widetilde{\jmath}^+_{\Sigma s\nu}(x_2)> = \mathrm{Tr}\,\tau^+\tau$$

$$\int_0^\infty dt_2\delta(x_2 - v_2t_2)\int_0^\infty dt_1\delta(x_1 + v_1t12)K_{\Sigma\mu\nu}(t_2, t_1),$$

$$K_{\Sigma\mu\nu} = K_{\Sigma||}e_{2||\mu}e_{1||\nu} + K_{\Sigma\perp}\delta_{\perp\mu\nu},$$

where $e_{1||} = (v_2 - \operatorname{ch}\varphi v_1)/\operatorname{sh}\varphi$, $e_{2||} = -(v_1 - \operatorname{ch}\varphi v_2)/\operatorname{sh}\varphi$ are the Σ_Q polarization vectors in the scattering plane, $\delta_{\perp\mu\nu} = \sum e_{\perp\mu} e_{\perp\nu} = [\operatorname{ch}\varphi(v_{1\mu}v_{2\nu} + v_{2\mu}v_{1\nu}) - v_{1\mu}v_{1\nu} - v_{2\mu}v_{2\nu}]/\operatorname{sh}^2\varphi - g_{\mu\nu}$.

According to the rules [26], the transition $\Lambda_Q \to \Lambda_Q$ is described by one form factor ($j_{1z} = j_{2z} = 0$); $\Lambda_Q \to \Sigma_Q$ is forbidden by naturalness; $\Sigma_Q \to \Sigma_Q$ is described by two form factors ($j_{1z} = j_{2z} = 0$ and ± 1):

$$
\begin{aligned}
<\Lambda_Q|\widetilde{J}_s|\Lambda_Q> &= \xi_\Lambda(\operatorname{ch}\varphi), \\
<\Sigma_Q|\widetilde{J}_s|\Sigma_Q> &= \xi_{\Sigma\mu\nu} e^*_{2\mu} e_{1\nu}, \quad \xi_{\Sigma\mu\nu} = \xi_{\Sigma||}(\operatorname{ch}\varphi) e_{2||\mu} e_{1||\nu} + \xi_{\Sigma\perp}(\operatorname{ch}\varphi)\delta_{\perp\mu\nu}.
\end{aligned}
\tag{3.6}
$$

The contribution of Λ_Q, Σ_Q to $\rho_{\Lambda,\Sigma||,\Sigma\perp}(\omega_2,\omega_1)$ is $\widetilde{f}^2_{\Lambda,\Sigma}\xi_{\Lambda,\Sigma||,\Sigma\perp}\delta(\omega_2 - \widetilde{\varepsilon}_{\Lambda,\Sigma})\delta(\omega_1 - \widetilde{\varepsilon}_{\Lambda,\Sigma})$. The spectral density of the correlator of the currents $\widetilde{\jmath}^+_{\Sigma\mu} e_\mu$ with some specific polarizations $e_{1,2}$ is $\rho_{\Sigma\mu\nu} e^*_{2\mu} e_{1\nu}$; the Σ_Q contribution to it is $\widetilde{f}^2_\Sigma \xi_{\Sigma\mu\nu} e^*_{2\mu} e_{1\nu}$.

Now let's switch the heavy quark spin on. The correlators are (Fig. 5)

$$
K = \left(\Gamma'_2 \frac{1+\widehat{v}_2}{2} \Gamma \frac{1+\widehat{v}_1}{2} \overline{\Gamma}'_1\right) K_s \tag{3.7}
$$

Let's introduce the currents $\widetilde{\overline{\jmath}}u = (\overline{q}\overline{\Gamma}C^{-1}\overline{q}^T)\widetilde{\overline{Q}}\mathcal{B}$, $\mathcal{B} = \overline{\Gamma}'u$. Rewriting (3.1) in the covariant form $\widetilde{\jmath}_\Sigma = (Q^T C\gamma_\mu q)\Gamma'_\mu \widetilde{Q}$, $\widetilde{\jmath}_{\Sigma^*\nu} = (Q^T C\gamma_\mu q)\Gamma'_{\mu\nu}\widetilde{Q}$, we obtain $\mathcal{B}_\Lambda = u$, $\mathcal{B}_{\Sigma\mu} = -(\gamma_\mu + v_\mu)u$, $\mathcal{B}_{\Sigma^*\mu} = u_\mu$. The spectral density of the correlator is $\left(\overline{\mathcal{B}}_2 \frac{1+\widehat{v}_2}{2}\Gamma\frac{1+\widehat{v}_2}{2}\mathcal{B}_1\right)\rho_s$ (where tensor indices may be contracted between ρ_s and $\mathcal{B}_{1;2}$); the baryons' contribution to it is $\widetilde{f}_{B_2}\widetilde{f}_{B_1}<B_2|\widetilde{J}|B_1>\delta(\omega_2 - \widetilde{\varepsilon}_2)\delta(\omega_1 - \widetilde{\varepsilon}_1)$. Hence we obtain [44, 45, 46]

$$
\begin{aligned}
<\Lambda_Q|\widetilde{J}|\Lambda_Q> &= \xi_\Lambda(\operatorname{ch}\varphi)\overline{u}_2\Gamma u_1, \\
<\Sigma_Q|\widetilde{J}|\Sigma_Q> &= \frac{1}{3}\xi_{\Sigma\mu\nu}\overline{u}_2(\gamma_\mu + v_{2\mu})\gamma_5\Gamma(\gamma_\nu - v_{1\nu})\gamma_5 u_1, \\
<\Sigma^*_Q|\widetilde{J}|\Sigma_Q> &= \frac{1}{\sqrt{3}}\xi_{\Sigma\mu\nu}\overline{u}_{2\mu}\Gamma(\gamma_\nu - v_{1\nu})\gamma_5 u_1, \\
<\Sigma^*_Q|\widetilde{J}|\Sigma^*_Q> &= \xi_{\Sigma\mu\nu}\overline{u}_{2\mu}\Gamma u_{1\nu}.
\end{aligned}
\tag{3.8}
$$

The result for Λ_Q is particularly simple because light fields have $j^P = 0^+$, and the spin of Λ_Q is carried by the heavy quark.

At the point $\varphi = 0$

$$
\xi_\Lambda(1) = \xi_{\Sigma||}(1) = \xi_{\Sigma\perp}(1) = 1 \tag{3.9}
$$

(at this point $\xi_{\Sigma ij} = \delta_{ij}$ because there are no selected directions).

Inclusive Λ_Q decays were treated in [34, 35]. With the scalar heavy quarks, the matrix element of the decay $\Lambda_Q \to X$ has the structure $M = g\varphi^*_2(X)\varphi_1$ where phi_1 is the scalar Λ_Q wave function. The Λ_Q decay rate is given by (2.22) with the structure function $w(\widetilde{\varepsilon}, \operatorname{ch}\varphi) = \sum_X \varphi^*_2(X)\varphi_2(X)\delta(\widetilde{\varepsilon}_X - \widetilde{\varepsilon})$ obeying the Bjorken sum rule (2.23). Transitions to the excited baryons and their contribution to the Bjorken sum rule were considered in [48, 49].

Polarization effects in Λ_Q decays were discussed in [50]. Heavy to light baryon transitions were considered in [45, 46]; they are described by several form factors. Inclusive heavy to light decays and sum rules for them in the deep inelastic region were investigated in [35]. At a large number of colors, baryons are bound states of a chiral soliton and a meson; this model was considered in [51].

4 Renormalization

Renormalization properties (anomalous dimensions etc.) of HQET are different from that of QCD. The ultraviolet behavior of a HQET diagram is determined by the region of loop momenta much larger than all characteristic scales of the process but much less than the heavy quark mass which tends to infinity from the very beginning. It has nothing to do with the ultraviolet behavior of the corresponding QCD diagram with the heavy quark line which is determined by the region of loop momenta much larger than the heavy quark mass. In the conventional QCD the first region produces hybrid logarithms [52, 53], and the problem of summation of these logarithmic corrections is highly nontrivial. In HQET hybrid logarithms become ultraviolet logarithmic divergencies governed by the renormalization group with corresponding anomalous dimensions. For example, correlators of QCD meson currents contain large hybrid logarithmic corrections $\alpha_s \log\frac{m}{\omega}$. Correlators of the corresponding HQET currents contain instead corrections $\alpha_s \log\frac{\omega}{\mu}$ where μ is the normalization point. The dependence on μ is determined by the currents' anomalous dimensions; the corrections are small at $\mu \sim \omega$.

HQET is closely related to the theory of Wilson lines in QCD [54]. As follows from the Lagrangian (1.2), the static quark propagator in a gluon field is the straight Wilson line

$$
\widetilde{S}(x) = -i\vartheta(x_0)\delta(\vec{x})P\exp ig\int A_\mu dx_\mu. \tag{4.1}
$$

The effective Lagrangian identical to the HQET Lagrangian (1.5) (strictly speaking, with the scalar static quark Lagrangian (1.10)) was proposed in [55] for investigation of Wilson lines. Their renormalization properties were considered in [56].

One-loop renormalization of straight Wilson lines (static quark propagators) and cusps on them (heavy-heavy velocity changing currents) are known from [54]. Two-loop calculation [57] for straight Wilson lines is incorrect; the correct result was obtained in [58]. It was also obtained in [59] starting from the on-shell renormalization of the QCD heavy quark propagator at finite m, and in [60, 61] in the HQET framework. Two-loop renormalization of a cusp on a Wilson line was first considered in [58], but the authors were unable to get rid of all double integrals. A simple result containing only simple single integrals was obtained in [62]. The attempt [63] in the HQET framework was unsuccessful: the result contains a double integral (with a variable undefined in the paper); some other integrals are in fact equal to 0 or each other.

One-loop renormalization of the heavy-light bilinear current in HQET was first considered in [52, 53]. Two-loop corrections were obtained in [60, 61] (in the second paper, a different external momentum configuration was chosen which made calculations more difficult). Four-quark operators with two static quark fields were also investigated in [61]. One-loop renormalization of baryon currents was considered in [47].

We shall use $\overline{\text{MS}}$ scheme, the space dimension $D = 4 - 2\varepsilon$. The HQET lagrangian (1.2) expressed via bare fields and couplings is

$$\begin{aligned} L &= \widetilde{Q}_b^+ i(\partial - i g_b A_b^a t^a)_0 \widetilde{Q}_b + \overline{q}_b i(\hat{\partial} - i g_b \hat{A}_b^a t^a) q \\ &- \frac{1}{4} G^a_{b\mu\nu} G^a_{b\mu\nu} + \frac{1}{2a_b}(\partial_\mu A^a_{b\mu})^2 + (\text{ghost}). \end{aligned} \qquad (4.2)$$

The bare quantities are related to the renormalized ones as

$$\begin{aligned} &\widetilde{Q}_b = \overline{\mu}^{-\epsilon} \widetilde{Z}_Q^{1/2} \widetilde{Q}, \quad q_b = \overline{\mu}^{-\epsilon} Z_q^{1/2} q, \quad A^a_{b\mu} = \overline{\mu}^{-\epsilon} A^a_\mu, \\ &g_b = \overline{\mu}^{\epsilon} Z_\alpha^{1/2} g, \quad a_b = Z_A a, \end{aligned} \qquad (4.3)$$

where Z_q, Z_A, Z_α are the same as in QCD with n_l light flavours (there are no static quark loops), and $\overline{\mu}^2 = \mu^2 e^\gamma / 4\pi$, μ is the normalization point. The static quark field renormalization constant $\widetilde{Z}_Q$ is determined from the requirement that the renormalized propagator $\widetilde{S}(\omega) = \widetilde{S}_b(\omega)/\widetilde{Z}_Q$ is finite. If we denote the sum of bare one-particle-irreducible static quark diagrams $-i\widetilde{\Sigma}_b(\omega)$, then the propagator $\widetilde{S}_b(\omega) = \widetilde{S}_0(\omega) + \widetilde{S}_0(\omega)\widetilde{\Sigma}_b(\omega)\widetilde{S}_0(\omega) + \cdots = 1/(\omega - \widetilde{\Sigma}_b(\omega))$.

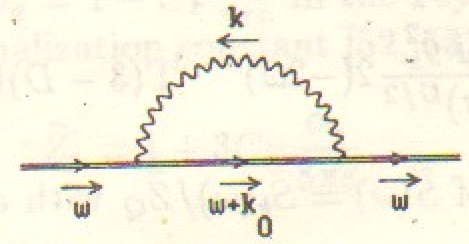

Figure 6: HQET mass operator

The bare one-loop HQET mass operator (Fig. 6) in the Feynman gauge $a_b = 0$ is

$$\widetilde{\Sigma}(\omega) = -iC_F g_b^2 \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2(\omega + k_0)} \qquad (4.4)$$

where $C_F = \frac{N_c^2 - 1}{2N_c}$. A variant of the Feynman parametrization

$$\frac{1}{a^\alpha b^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty \frac{y^{\beta-1} dy}{(a + yb)^{\alpha+\beta}} \qquad (4.5)$$

is used to combine a square denominator a with a linear denominator b; the parameter y has the dimension of mass. We have

$$\widetilde{\Sigma}(\omega) = -iC_F g_b^2 \int_0^\infty dy \int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 + yk_0 + y\omega)^2}. \qquad (4.6)$$

The denominator is equal to $k'^2 - \frac{y^2}{4} + \omega y$, $k' = k + \frac{y}{2} v$. Using the standard formula

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - a^2)^n} = \frac{i(-1)^n \Gamma(n - \frac{D}{2})(a^2)^{D/2-n}}{(n-1)!(4\pi)^{D/2}}, \qquad (4.7)$$

we obtain

$$\widetilde{\Sigma}(\omega) = \frac{C_F g_b^2}{(4\pi)^{D/2}} \Gamma\left(2 - \tfrac{D}{2}\right) \int_0^\infty \left(\frac{y^2}{4} - \omega y\right)^{D/2-2} dy. \qquad (4.8)$$

The integral

$$\int_0^\infty y^\alpha (ay + b)^\beta dy = \frac{b^{\alpha+\beta+1}}{a^{\alpha+1}} \frac{\Gamma(-1-\alpha-\beta)\Gamma(1+\alpha)}{\Gamma(-\beta)} \qquad (4.9)$$

is calculated using the substitution $y = \frac{b}{a}\left(\frac{1}{z} - 1\right)$. Finally,

$$\widetilde{\Sigma}(\omega) = \frac{C_F g_b^2}{(4\pi)^{D/2}} 2(-2\omega)^{D-3}\Gamma(3-D)\Gamma\left(\tfrac{D}{2}-1\right). \tag{4.10}$$

Requiring the finiteness of $\widetilde{S}(\omega) = \widetilde{S}_b(\omega)/\widetilde{Z}_Q$ with a minimal $\widetilde{Z}_Q = 1 + c\frac{\alpha_s}{\varepsilon}$, we find

$$\widetilde{Z}_Q = 1 + C_F \frac{\alpha_s}{2\pi\varepsilon}. \tag{4.11}$$

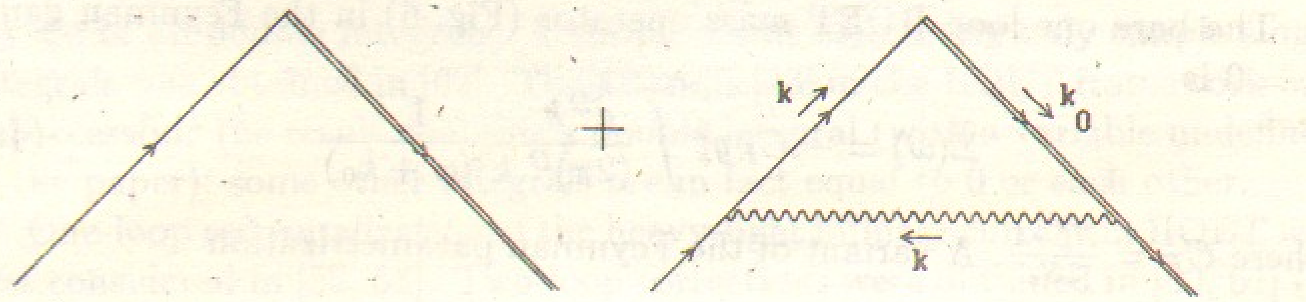

Figure 7: Heavy-light vertex

Now we shall consider renormalization of a heavy-light bilinear current. The bare current $\widetilde{\jmath}_b = \overline{\widetilde{Q}}_b \Gamma q_b$ is related to the renormalized one as $\widetilde{\jmath}_b = \overline{\mu}^{-2\varepsilon}\widetilde{Z}_j \widetilde{\jmath}$, or $\overline{\widetilde{Q}}\Gamma q = \widetilde{Z}_\Gamma \widetilde{\jmath}$ where $\widetilde{Z}_j = Z_q^{1/2}\widetilde{Z}_Q^{1/2}\widetilde{Z}_\Gamma$. Then the matrix element $\widetilde{\Gamma} = <\widetilde{Q}|\overline{\widetilde{Q}}\Gamma q|q> = \widetilde{Z}_\Gamma <\widetilde{Q}|\widetilde{\jmath}|q>$ where the matrix element of $\widetilde{\jmath}$ is finite. The vertex $\widetilde{\Gamma}$ does not include corrections on the external legs because it contains renormalized fields. We shall calculate it in the one-loop approximation (Fig. 7) in the Feynman gauge. We are interested only in the ultraviolet divergence of the one-loop diagram that does not depend on external momenta. At zero external momenta we have

$$\Gamma\left[1 - iC_F g_b^2 \int \frac{d^D k}{(2\pi)^D}\frac{\widehat{k}\gamma_0}{(k^2)^2 k_0}\right] = \Gamma\left[1 - iC_F g_b^2 \int \frac{d^D k}{(2\pi)^D}\frac{1}{(k^2)^2}\right] \tag{4.12}$$

because $\widehat{k} = k_0\gamma_0 - \vec{k}\cdot\vec{\gamma}$ and the integral with $\vec{k}$ vanishes due to the symmetry. If we started from an infrared regularized matrix element (e. g. with nonzero external momenta or gluon mass) we would obtain an integral with the same ultraviolet divergence but infrared safe. Separating the ultraviolet pole we have $\widetilde{Z}_\Gamma = 1 + C_F\frac{\alpha_s}{4\pi\varepsilon}$. Using also (4.11) and the standard QCD

renormalization constant $Z_q = 1 - C_F\frac{\alpha_s}{4\pi\varepsilon}$ in the Feynman gauge, we obtain the gauge invariant renormalization constant [52, 53, 4]

$$\widetilde{Z}_j = 1 + 3C_F\frac{\alpha_s}{8\pi\varepsilon}. \tag{4.13}$$

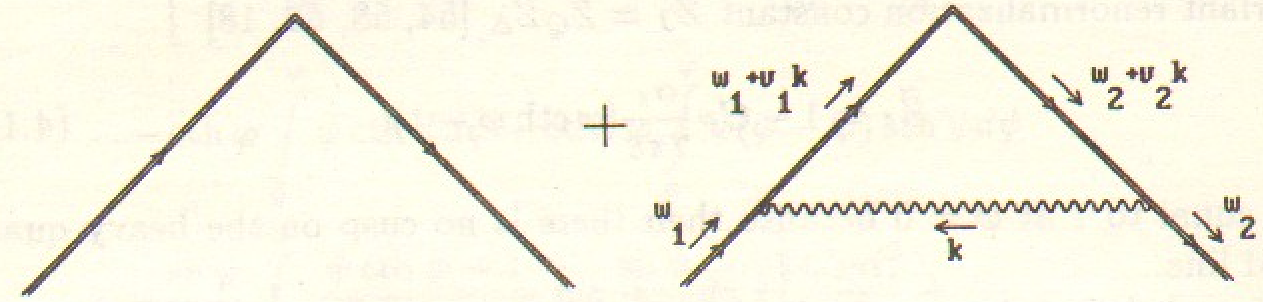

Figure 8: Heavy-heavy vertex

Similarly, the vertex $\widetilde{\Delta} = <\widetilde{Q}_2|\overline{\widetilde{Q}}_2\widetilde{Q}_1|\widetilde{Q}_1>$ in the one loop approximation (Fig. 8) in the Feynman gauge is

$$\begin{aligned}
& 1 - iC_F g_b^2 \operatorname{ch}\varphi \int \frac{d^D k}{(2\pi)^D}\frac{1}{k^2(v_1 k + \omega_1)(v_2 k + \omega_2)} \qquad (4.14)\\
= \; & 1 - 2iC_F g_b^2 \operatorname{ch}\varphi \int_0^1 dx \int_0^\infty dy \int \frac{d^D k}{(2\pi)^D} \\
& \frac{1}{[k^2 + y(xv_1 + (1-x)v_2)k + y(x\omega_1 + (1-x)\omega_2)]^3} \\
= \; & 1 - \frac{C_F g_b^2}{(4\pi)^{D/2}}\Gamma(1+\varepsilon)\operatorname{ch}\varphi \int_0^1 dx \int_0^\infty dy y^{-\varepsilon} \\
& \left[\tfrac{1}{4}(x^2 + (1-x)^2 + 2x(1-x)\operatorname{ch}\varphi)y - x\omega_1 - (1-x)\omega_2\right]^{-1-\varepsilon} \\
= \; & 1 - \frac{C_F g_b^2}{(4\pi)^{D/2}} 4\Gamma(2\varepsilon)\Gamma(1-\varepsilon)\operatorname{ch}\varphi \int_0^1 \frac{[-2\omega_1 x - 2\omega_2(1-x)]^{-2\varepsilon} dx}{[x^2 + (1-x)^2 + 2x(1-x)\operatorname{ch}\varphi]^{1-\varepsilon}}.
\end{aligned}$$

Retaining only the ultraviolet $1/\varepsilon$ pole and using the substitution $x = \frac{1}{2}(1+$

$z \operatorname{cth} \frac{\varphi}{2}$), we obtain

$$1 - C_F \frac{\alpha_s}{2\pi\varepsilon} \operatorname{cth}\varphi \int\limits_{-\operatorname{th}\frac{\varphi}{2}}^{+\operatorname{th}\frac{\varphi}{2}} \frac{dz}{1+z^2}, \qquad (4.15)$$

and finally $Z_\Delta = 1 - C_F \frac{\alpha_s}{2\pi\varepsilon} \varphi \operatorname{cth}\varphi$. Using (4.11) we obtain the gauge-invariant renormalization constant $\widetilde{Z}_J = \widetilde{Z}_Q \widetilde{Z}_\Delta$ [54, 58, 62, 18]

$$\widetilde{Z}_J = 1 - C_F \frac{\alpha_s}{2\pi\varepsilon} (\varphi \operatorname{cth}\varphi - 1). \qquad (4.16)$$

It is equal to 1 at $\varphi = 0$ because then there is no cusp on the heavy quark world line.

Knowledge of the renormalization constants Z allows us to determine the dependence of the renormalized operators on the normalization point μ using the renormalization group equations. Up to two loops, $\overline{\text{MS}}$ renormalization constants have the form

$$Z(\mu) = 1 + \frac{\alpha_s(\mu)}{4\pi} \frac{c_1}{\varepsilon} + \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 \left(\frac{c_{22}}{\varepsilon^2} + \frac{c_{21}}{\varepsilon} \right) + \cdots \qquad (4.17)$$

From $g_b = \overline{\mu}^\varepsilon Z_\alpha^{1/2} g = \text{const}$ we obtain the evolution of $\alpha_s(\mu)$: $\frac{d\log\alpha_s}{d\log\mu} = -2(\varepsilon + \beta(\alpha_s))$, $\beta(\alpha_s) = \frac{1}{2}\frac{d\log Z_\alpha}{d\log\mu} = \beta_1 \frac{\alpha_s}{4\pi} + \beta_2 \left(\frac{\alpha_s}{4\pi}\right)^2 + \cdots$. It is well known that $\beta_1 = \frac{11}{3}N_c - \frac{2}{3}n_l$, $\beta_2 = \frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_c}\right) n_l$. Substituting the form (4.17) for Z_α we see that $c_1 = -\beta_1$, $c_{22} = \beta_1^2$, $c_{21} = -\frac{1}{2}\beta_2$, i. e. c_{22} is not independent. Similarly, the μ-dependence of any operator j is usually characterized by its anomalous dimension $\gamma_j = \frac{d\log Z_j}{d\log\mu} = \gamma_1 \frac{\alpha_s}{4\pi} + \gamma_2 \left(\frac{\alpha_s}{4\pi}\right)^2 + \cdots$. Substituting the form (4.17) for Z_j we see that $c_1 = -\frac{1}{2}\gamma_1$, $c_{22} = \frac{1}{8}(\gamma_1^2 + 2\beta_1\gamma_1)$, $c_{21} = -\frac{1}{4}\gamma_2$, i. e. again c_{22} is determined by one-loop quantities. In the case of a non-gauge-invariant operator, Z also depends on the gauge parameter $a(\mu)$, and the formulae become more complicated. Solving the renormalization group equation we obtain

$$j(\mu) = \hat{\jmath}\, \alpha_s^{\gamma_1/2\beta_1}(\mu) \left[1 + \left(\frac{\gamma_2}{2\beta_1} - \frac{\gamma_1\beta_2}{2\beta_1^2} \right) \frac{\alpha_s(\mu)}{4\pi} + \cdots \right], \qquad (4.18)$$

where $\hat{\jmath}$ is a renormalization group invariant. This formula allows us to relate $j(\mu_1)$ to $j(\mu_2)$. Here we present for reference the two-loop anomalous

dimensions of heavy-light and heavy-heavy currents [60, 62]

$$\begin{aligned}
\widetilde{\gamma}_j &= -\frac{3C_F\alpha_s}{4\pi} - \left[\frac{49}{96}N_c - \frac{5}{32}C_F - \frac{5}{48}n_l + (4C_F - N_c)\frac{\pi^2}{24} \right] \frac{C_F\alpha_s^2}{\pi^2} + \cdots \\
\widetilde{\gamma}_J &= \frac{C_F\alpha_s}{\pi}(\varphi \operatorname{cth}\varphi - 1) \qquad (4.19) \\
&+ \Bigg[-n_l \frac{5}{18}(\varphi\operatorname{cth}\varphi - 1) + N_c \bigg(\frac{1}{2} + \left(\frac{67}{36} - \frac{\pi^2}{24} \right)(\varphi \operatorname{cth}\varphi - 1) \\
&\quad - \operatorname{cth}\varphi \int\limits_0^\varphi \psi \operatorname{cth}\psi\, d\psi + \operatorname{cth}^2\varphi \int\limits_0^\varphi \psi(\varphi - \psi)\operatorname{cth}\psi\, d\psi \\
&\quad - \frac{\operatorname{sh}\varphi}{2} \int\limits_0^\varphi \frac{\psi\operatorname{cth}\psi - 1}{\operatorname{sh}^2\varphi - \operatorname{sh}^2\psi} \log\frac{\operatorname{sh}\varphi}{\operatorname{sh}\psi}\, d\psi \bigg) \Bigg] \frac{C_F\alpha_s^2}{\pi^2} + \cdots
\end{aligned}$$

Until now we discussed the renormalization inside HQET. But usually we are interested in matrix elements of QCD operators (e. g. weak currents). Therefore we have to discuss the relation of QCD operators to their HQET analogues. Operators in HQET differ from those in QCD starting from the one-loop level even if written in the same form via the fields because their matrix elements are calculated using different Feynman rules. A QCD operator j matches the corresponding HQET operator $A\widetilde{\jmath}$ if they give identical physical (on-shell) matrix elements between states suitable for HQET treatment (with residual momenta much less than m). In order to calculate on-shell matrix elements we have to use the on-shell renormalization scheme in which propagators in the on-shell limit are free. For the "massless" fields q, $\widetilde{Q}$ the bare on-shell propagators get no corrections because loop integrals are no-scale (ultraviolet and infrared divergencies cancel). Therefore the on-shell renormalized fields coincide with the bare ones: $q = Z_q^{-1/2} q_{\text{os}}$, $\widetilde{Q} = \widetilde{Z}_Q^{-1/2}\widetilde{Q}_{\text{os}}$. Note that although the expressions for the renormalization constants Z_q, $\widetilde{Z}_Q$ are the same as above, all divergencies in them are infrared ones because these Z factors relate renormalized (ultraviolet-finite) fields. For the massive quark field we have $Q = Z_Q^{-1/2} Q_{\text{os}}$, $Z_Q = 1 + C_F \frac{\alpha_s}{4\pi}\left(\frac{2}{\varepsilon} - 3L + 4\right)$, where $L = \log\frac{m^2}{\mu^2}$. The infrared divergence of the on-shell massive quark propagator Z_Q is the same as that of the static quark propagator $\widetilde{Z}_Q$.

For the heavy-light bilinear currents we have $j = Z_\Gamma^{-1} Z_q^{-1/2} Z_Q^{-1/2} \overline{Q}_{\text{os}} \Gamma q_{\text{os}}$, $\widetilde{\jmath} = \widetilde{Z}_\Gamma^{-1} Z_q^{-1/2} \widetilde{Z}_Q^{-1/2} \overline{\widetilde{Q}}_{\text{os}} \Gamma q_{\text{os}}$. Hence the on-shell matrix elements are

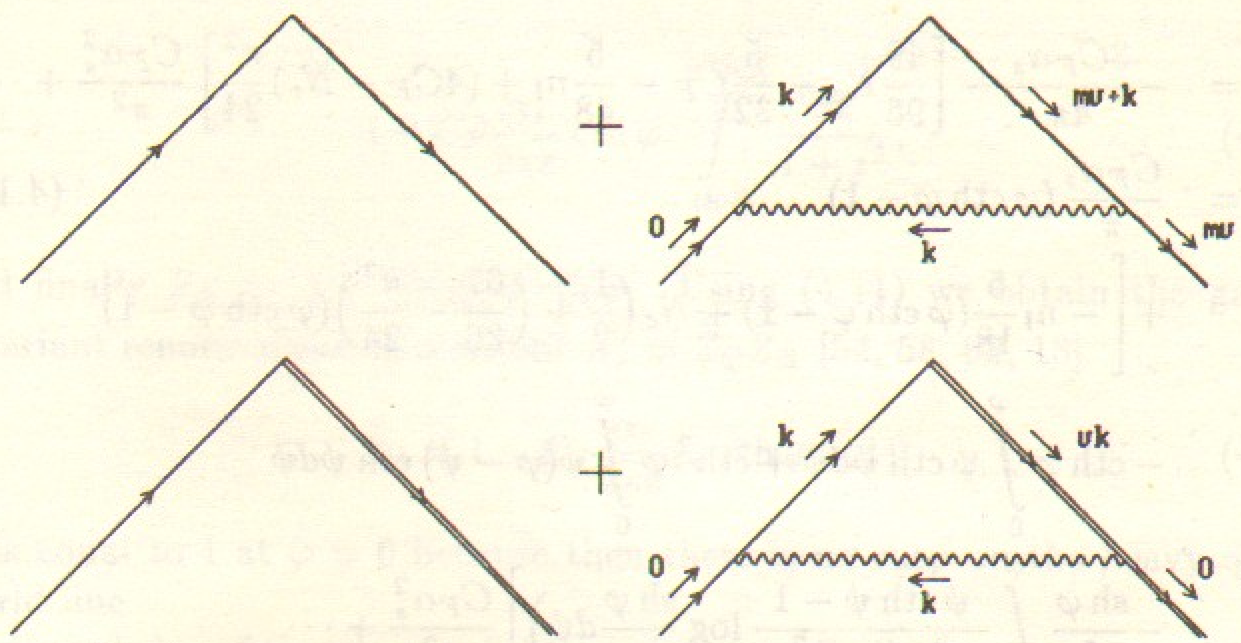

Figure 9: Light-light → heavy-light matching

$<Q|j|q> = Z_\Gamma^{-1} Z_q^{-1/2} Z_Q^{-1/2}\Gamma$, $<Q|\tilde{\jmath}|q> = \widetilde{Z}_\Gamma^{-1} Z_q^{-1/2} \widetilde{Z}_Q^{-1/2}\widetilde{\Gamma}$, where the proper vertices Γ, $\widetilde{\Gamma}$ are depicted on Fig. 9. We obtain the matching constant

$$A = A_Q \frac{\Gamma/Z_\Gamma}{\widetilde{\Gamma}/\widetilde{Z}_\Gamma}, \quad A_Q = \left(\frac{\widetilde{Z}_Q}{Z_Q}\right)^{1/2} = 1 + C_F\frac{\alpha_s}{4\pi}\left(\frac{3}{2}L - 2\right). \tag{4.20}$$

Here ultraviolet divergencies cancel in Γ/Z_Γ, $\widetilde{\Gamma}/\widetilde{Z}_\Gamma$ by definition; infrared divergencies cancel between these two expressions because the infrared behavior of QCD and HQET is identical; A_Q is finite for the same reason. We choose all quark momenta in HQET to be zero; this corresponds to the heavy quark momentum mv ($v = (1, \vec{0})$) in QCD. Then the HQET loops (Fig. 9) vanish: $\widetilde{\Gamma} = 1$. Let's calculate the QCD vertex Γ (Fig. 9) in the Feynman gauge

$$\Gamma - iC_F g_b^2 \int \frac{d^D k}{(2\pi)^D} \frac{\gamma_\mu(\hat{k} + m\hat{v} + m)\Gamma\hat{k}\gamma_\mu}{(k^2)^2(k^2 + 2mvk)} \tag{4.21}$$

$$= \Gamma - 2iC_F g_b^2 \int_0^1 dx(1-x) \int \frac{d^D k}{(2\pi)^D}$$

$$\frac{\gamma_\mu(\hat{k}' + m(1-x)\hat{v} + m)\Gamma(\hat{k}' - mx\hat{v})\gamma_\mu}{(k'^2 - m^2x^2)^3},$$

where $k' = k + mxv$. The term with two $\hat{k}'$ in the numerator gives $\frac{k'^2}{D}H^2(D)\Gamma$ where $\gamma_\mu\Gamma\gamma_\mu = H(D)\Gamma$; terms with one $\hat{k}'$ vanish. Terms without $\hat{k}'$ give $\pm m^2 x\gamma_\mu(1 - x + \hat{v})\Gamma\gamma_\mu = -m^2x(2 \pm H(D)x)\Gamma$ where the upper (lower) sign is for Γ anticommuting (commuting) with $\hat{v}$ and we have used the fact that $\hat{v}$ on the left may be replaced by 1 in the on-shell matrix element. Therefore all the terms have the common γ-matrix structure Γ; calculating the integrals, we have the vertex

$$\Gamma = 1 + C_F\frac{\alpha_s}{4\pi}\left(\frac{m^2}{\mu^2}\right)^{-\varepsilon} 2\int_0^1 dx\, x^{-2\varepsilon}(1-x)\left[\frac{H^2(D)}{4\varepsilon} + \frac{1}{x} \pm \frac{H(D)}{2}\right]$$

$$= 1 + C_F\frac{\alpha_s}{4\pi}\left(\frac{m^2}{\mu^2}\right)^{-\varepsilon}\frac{1}{(1-\varepsilon)(1-2\varepsilon)}\left[\frac{H^2(D)}{4\varepsilon} - \frac{1-\varepsilon}{\varepsilon} \pm \frac{H(D)}{2}\right] \tag{4.22}$$

The first divergence is ultraviolet:

$$Z_\Gamma = 1 + C_F\frac{\alpha_s}{4\pi}\frac{H^2}{4\varepsilon}. \tag{4.23}$$

By the way, the one-loop renormalization constant of the QCD bilinear quark currents is $Z_j = Z_q Z_\Gamma = 1 + C_F\frac{\alpha_s}{4\pi}\frac{H^2-4}{4\varepsilon}$; the vector and axial current ($H = \pm 2$) anomalous dimension vanishes. Finally we obtain from (4.20) the matching [4]

$$\overline{Q}\Gamma q = \left[1 + C_F\frac{\alpha_s}{4\pi}\left(-\frac{H^2-10}{4}L + \frac{3}{4}H^2 - HH' \pm \frac{1}{2}H - 4\right) + \cdots\right]\overline{\widetilde{Q}}\Gamma q, \tag{4.24}$$

where $H' = \frac{dH}{dD}$. This equation holds separately for QCD currents with Γ (anti-) commuting with γ_0. If it does not have this property, it can be split into a commuting and an anticommuting part; it then maps onto a combination of two HQET currents. The logarithmic part of the matching constant (4.24) is determined by the difference of anomalous dimensions of the QCD and HQET currents; the non-logarithmic part should be included account only when the two-loop anomalous dimension is also taken into account (4.18).

Now we shall consider the current $\overline{\widetilde{Q}}_2\Gamma Q_1$ in the effective theory where Q_1 is a heavy quark with the mass m. We can go to the second effective theory

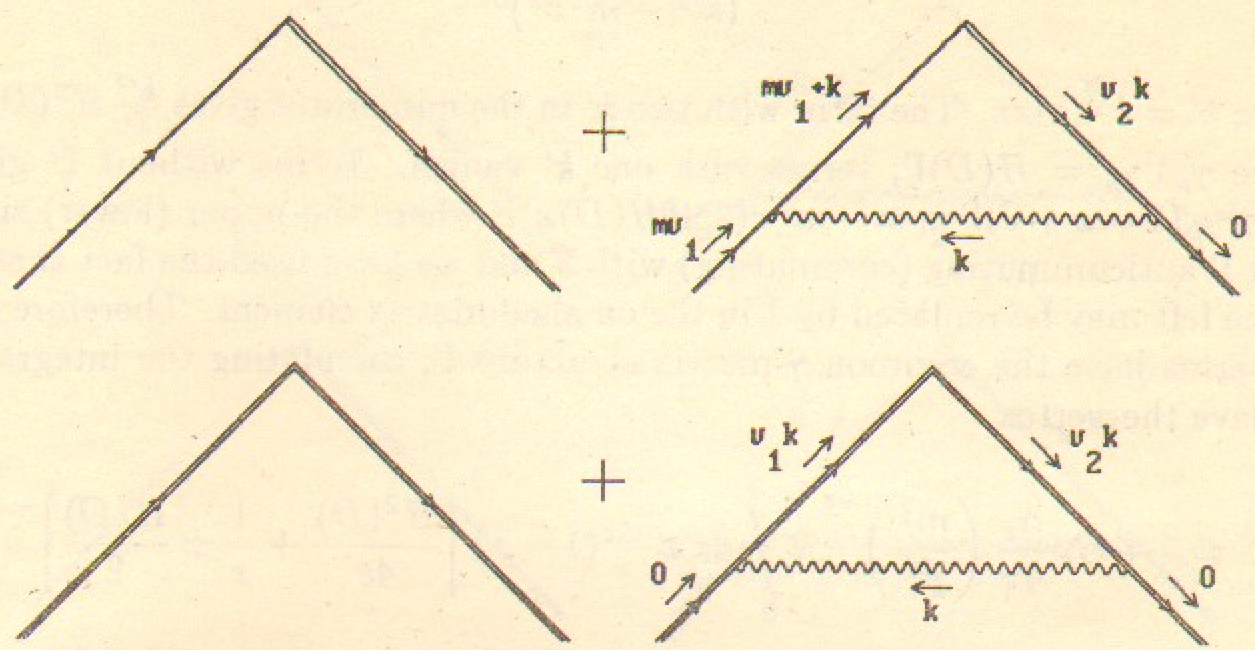

Figure 10: Heavy-light → heavy-heavy matching

in which this quark is also considered static. The matching is obtained by comparing the diagrams in Fig. 10. All external momenta are zero in the second theory, therefore the loops vanish. In the first theory, the heavy quark has the momentum mv_1. The vertex is the matrix Γ times

$$1 - iC_F g_0^2 \int \frac{d^D k}{(2\pi)^D} \frac{(\hat{k} + m\hat{v}_1 + m)\hat{v}_2}{k^2(k^2 + 2mv_1 k)v_2 k} \tag{4.25}$$

$$= 1 - 2iC_F g_0^2 \int_0^\infty dy \int_0^1 dx \int \frac{d^D k}{(2\pi)^D} \frac{-y/2 + m(1-x)\hat{v}_1\hat{v}_2 + mv_2}{(k'^2 - y^2/4 - m^2x^2 - mxy \operatorname{ch}\varphi)^3}$$

Now we calculate the loop integrals. The parametric integrals factorize after using the substitution $y = 2mxz$, and the x integrals are trivial. The upper (lower) sign is for Γ anticommuting (commuting) with $\hat{v}_2$.

$$1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{m^2}{\mu^2}\right)^{-\varepsilon} \int_0^\infty dz \int_0^1 dx\, x^{-1-2\varepsilon} \frac{xz - 2(1-x)\operatorname{ch}\varphi \pm x}{(1 + z^2 + 2z\operatorname{ch}\varphi)^{1+\varepsilon}} \tag{4.26}$$

$$= 1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{m^2}{\mu^2}\right)^{-\varepsilon} \int_0^\infty dz \frac{(z + 2\operatorname{ch}\varphi \pm 1)/(1-2\varepsilon) - \operatorname{ch}\varphi/\varepsilon}{(1 + z^2 + 2z\operatorname{ch}\varphi)^{1+\varepsilon}}$$

The first ultraviolet divergent integral can be calculated by splitting the integration region at a large A and ignoring ε in the first region and $1/z$ in the second one:

$$\int_0^\infty \frac{z\,dz}{(1 + z^2 + 2z\operatorname{ch}\varphi)^{1+\varepsilon}} = \int_0^A \frac{z\,dz}{1 + z^2 + 2z\operatorname{ch}\varphi} + \int_A^\infty \frac{dz}{z^{1+2\varepsilon}} \tag{4.27}$$

$$= \log A - \varphi \operatorname{cth}\varphi + \frac{1}{2\varepsilon} - \log A.$$

The second integral is convergent; we need it up to $O(\varepsilon)$ because it is multiplied by the infrared pole $1/\varepsilon$:

$$\int_0^\infty \frac{dz}{(1 + z^2 + 2z\operatorname{ch}\varphi)^{1+\varepsilon}} = \frac{1}{\operatorname{sh}\varphi}\left[\varphi - \frac{\varepsilon}{2}\left(F(e^{2\varphi} - 1) - F(e^{-2\varphi} - 1)\right)\right], \tag{4.28}$$

where

$$F(x) = \int_0^x \frac{\log(1+y)}{y} dy \tag{4.29}$$

is the Spence function. Two Spence functions in (4.28) are not independent: $F(e^{2\varphi} - 1) + F(e^{-2\varphi} - 1) = 2\varphi^2$. The vertex is

$$\Gamma = 1 + C_F \frac{\alpha_s}{4\pi}\left[\frac{1}{\varepsilon} - \frac{2}{\varepsilon}\varphi\operatorname{cth}\varphi - L + 2L\varphi\operatorname{cth}\varphi + 2 + 2\varphi\operatorname{cth}\varphi \pm 2\frac{\varphi}{\operatorname{sh}\varphi}\right.$$

$$\left. + \operatorname{cth}\varphi\left(F(e^{2\varphi} - 1) - F(e^{-2\varphi} - 1)\right)\right]. \tag{4.30}$$

The first divergence is ultraviolet: $Z_\Gamma = 1 + C_F \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}$; this is the renormalization constant of the heavy-light current, and it indeed agrees with what we have found before (4.13). In the denominator of (4.20), we should use the renormalization constant of the heavy-heavy current $\widetilde{Z}_\Delta$ found before (4.16). The infrared divergence cancels as it should do, and we obtain the matching [18, 64]

$$\bar{\widetilde{Q}}_2 \Gamma Q_1 = \left[1 + C_F \frac{\alpha_s}{4\pi}\left(2L\varphi\operatorname{cth}\varphi + \frac{1}{2}L + 2\varphi\operatorname{cth}\varphi \pm 2\frac{\varphi}{\operatorname{sh}\varphi}\right.\right.$$

$$\left.\left. + \operatorname{cth}\varphi\left(F(e^{2\varphi} - 1) - F(e^{-2\varphi} - 1)\right)\right) + \cdots\right] \bar{\widetilde{Q}}_2 \Gamma \widetilde{Q}_1 \tag{4.31}$$

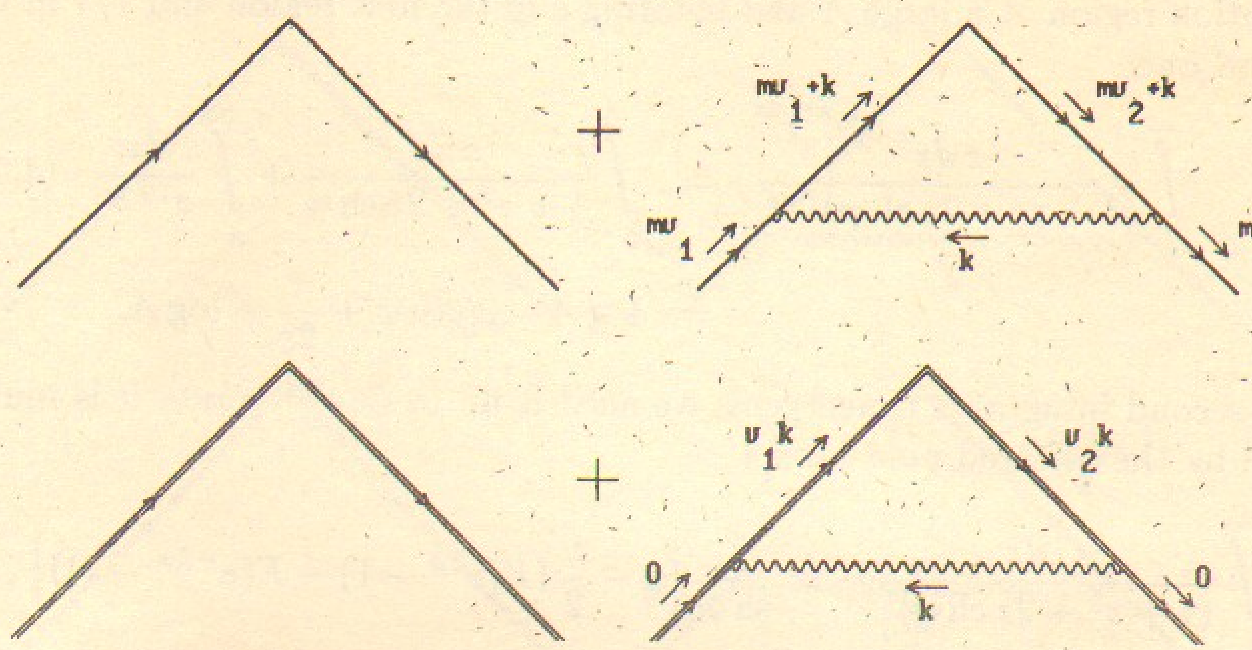

Figure 11: Light-light → heavy-heavy matching

Finally we shall consider the current $\overline{Q}_2\Gamma Q_1$ where $Q_{1,2}$ are the heavy quarks with the masses $m_{1,2}$. We can go to the effective theory in which both of them are considered static. The matching is obtained by comparing the diagrams in Fig. 11. All external momenta are zero in the effective theory, therefore the loops vanish. In QCD, the heavy quarks have the momenta $m_{1,2}v_{1,2}$. The vertex is

$$\Gamma - iC_F g_b^2 \int \frac{d^D k}{(2\pi)^D} \frac{\gamma_\mu(\hat{k} + m_2\hat{v}_2 + m_2)\Gamma(\hat{k} + m_1\hat{v}_1 + m_1)\gamma_\mu}{k^2(k^2 + 2m_1 v_1 k)(k^2 + 2m_2 v_2 k)} \tag{4.32}$$

$$= \Gamma - 2iC_F g_b^2 \int dx_1 dx_2 \frac{d^D k}{(2\pi)^D}$$

$$\frac{\gamma_\mu(\hat{k}' + m_2(1-x_2)\hat{v}_2 - m_1 x_1\hat{v}_1 + m_2)\Gamma(\hat{k}' + m_1(1-x_1)\hat{v}_1 - m_2\hat{v}_2 + m_1)\gamma_\mu}{(k'^2 - a^2)^3},$$

where $a^2 = m_1^2 x_1^2 + m_2^2 x_2^2 + 2m_1 m_2 x_1 x_2 \operatorname{ch}\varphi$. Now we calculate the loop integrals. The parametric integrals factorize after the substitution $x_{1,2} = x(1 \pm z)/2$, and the x integrals are trivial ($a^2 = m_1 m_2 x^2 a_+ a_-$, $a_\pm = \operatorname{ch}\frac{\psi\pm\varphi}{2} + z \operatorname{sh}\frac{\psi\pm\varphi}{2}$, where $\psi = \log\frac{m_1}{m_2}$):

$$\Gamma\left\{1 + C_F\frac{\alpha_s}{4\pi}\left(\frac{m_1 m_2}{\mu^2}\right)^{-\varepsilon}\int_{-1}^{+1}\frac{dz}{(a_+a_-)^\varepsilon}\left[\frac{H^2(D)}{8\varepsilon(1-\varepsilon)} + \frac{\operatorname{ch}\varphi}{\varepsilon(1-2\varepsilon)a_+a_-}\right.\right.$$

$$\left.\left. + \frac{\operatorname{ch}\psi + z\operatorname{sh}\psi}{(1-2\varepsilon)a_+a_-} - \frac{H(D)(1-z^2)}{16(1-\varepsilon)a_+a_-}\right]\right\} \tag{4.33}$$

$$-\hat{v}_1\Gamma C_F\frac{\alpha_s}{4\pi}\frac{1}{2}\int_{-1}^{+1}\frac{dz}{a_+a_-}\left[1 - z + \tfrac{1}{8}He^\psi(1+z)^2\right]$$

$$-\Gamma\hat{v}_2 C_F\frac{\alpha_s}{4\pi}\frac{1}{2}\int_{-1}^{+1}\frac{dz}{a_+a_-}\left[1 + z + \tfrac{1}{8}He^{-\psi}(1-z)^2\right]$$

$$-\hat{v}_1\Gamma\hat{v}_2 C_F\frac{\alpha_s}{4\pi}\frac{H}{16}\int\frac{1-z^2}{a_+a_-}dz.$$

At $m_1 = 0$ ($\psi \to -\infty$) this vertex reduces to (4.22), and at $m_2 = 0$ ($\psi \to +\infty$)—to the mirror symmetric expression. In order to check this, we should take the limit before $\varepsilon \to 0$. Ultraviolet divergencies are the same (4.23), but the structure of infrared $1/\varepsilon$ poles is different at zero and nonzero masses. The integrals in z are easily calculable:

$$\int_{-1}^{+1}\frac{dz}{(a_+a_-)^\varepsilon} = 2 - 2\varepsilon\left(\frac{\varphi\operatorname{sh}\varphi - \psi\operatorname{sh}\psi}{\operatorname{ch}\varphi - \operatorname{ch}\psi} - 2\right), \tag{4.34}$$

$$\int_{-1}^{+1}\frac{dz}{(a_+a_-)^{1+\varepsilon}} = \frac{1}{\operatorname{sh}\varphi}\left\{2\varphi + \varepsilon\left[\left(F\left(\frac{e^\varphi - e^{-\varphi}}{e^\psi - e^\varphi}\right) + (\psi \to -\psi)\right)\right.\right.$$

$$\left.\left. - (\varphi \to -\varphi) + 2\psi\log\frac{\operatorname{sh}\frac{\psi+\varphi}{2}}{\operatorname{sh}\frac{\psi-\varphi}{2}}\right]\right\},$$

$$\int_{-1}^{+1}\frac{z\,dz}{a_+a_-} = \frac{2}{\operatorname{sh}\varphi}\frac{\varphi\operatorname{sh}\psi - \psi\operatorname{sh}\varphi}{\operatorname{ch}\varphi - \operatorname{ch}\psi},$$

$$\int_{-1}^{+1}\frac{z^2 dz}{a_+a_-} = -\frac{4}{\operatorname{ch}\varphi - \operatorname{ch}\psi} - \frac{4\psi\operatorname{sh}\psi}{(\operatorname{ch}\varphi - \operatorname{ch}\psi)^2} + \frac{2\varphi}{\operatorname{sh}\varphi}\frac{\operatorname{sh}^2\varphi + \operatorname{sh}^2\psi}{(\operatorname{ch}\varphi - \operatorname{ch}\psi)^2}.$$

The Spence functions here are not independent: $F\left(\frac{e^{\varphi}-e^{-\varphi}}{e^{\pm\psi}-e^{\varphi}}\right)+(\varphi\to-\varphi)=\frac{1}{2}\log^2\frac{e^{\pm\psi}-e^{\varphi}}{e^{\pm\psi}-e^{-\varphi}}$. Matching is determined by the formula similar to (4.20) but with $A_{Q1}A_{Q2}$; infrared divergencies cancel as they should do, and we finally obtain [23, 64]

$$
\begin{aligned}
&\overline{Q}_2\Gamma Q_1 = A\overline{\widetilde{Q}}_2\Gamma\widetilde{Q}_1 + A_1\overline{\widetilde{Q}}_2\widehat{v}_1\Gamma\widetilde{Q}_1 + A_2\overline{\widetilde{Q}}_2\Gamma\widehat{v}_2\widetilde{Q}_1 + A_{12}\overline{\widetilde{Q}}_2\widehat{v}_1\Gamma\widehat{v}_2\widetilde{Q}_1, \qquad (4.35)\\
&A = 1 + C_F\frac{\alpha_s}{4\pi}\left[-\left(\frac{H^2}{4}+2\varphi\,\mathrm{cth}\,\varphi-3\right)L-\frac{H^2}{4}\left(\frac{\varphi\,\mathrm{sh}\,\varphi-\psi\,\mathrm{sh}\,\psi}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}-3\right)\right.\\
&\quad -HH'+\frac{H}{4}\left(\frac{\varphi}{\mathrm{sh}\,\varphi}\frac{\mathrm{ch}\,\varphi\,\mathrm{ch}\,\psi-1}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}-\frac{\psi\,\mathrm{sh}\,\psi}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}-\frac{1}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}\right)\\
&\quad +\mathrm{cth}\,\varphi\left(\left(F\left(\frac{e^{\varphi}-e^{-\varphi}}{e^{\psi}-e^{\varphi}}\right)+(\psi\to-\psi)\right)-(\varphi\to-\varphi)+2\psi\log\frac{\mathrm{sh}\,\frac{\psi+\varphi}{2}}{\mathrm{sh}\,\frac{\psi-\varphi}{2}}\right)\\
&\quad \left.-2\frac{\varphi}{\mathrm{sh}\,\varphi}\frac{\mathrm{ch}\,\varphi\,\mathrm{ch}\,\psi-2\,\mathrm{ch}^2\varphi+1}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}-2\frac{\psi\,\mathrm{sh}\,\psi}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}-4\right],\\
&A_1 = -C_F\frac{\alpha_s}{4\pi}\left[\frac{1}{4}He^{\psi}\left(\frac{\varphi}{\mathrm{sh}\,\varphi}\left(\frac{\mathrm{ch}\,\varphi\,\mathrm{ch}\,\psi-1}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}+\frac{\varphi\,\mathrm{sh}\,\psi-\psi\,\mathrm{sh}\,\varphi}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}+1\right)\right.\right.\\
&\quad \left.\left.-\frac{\psi\,\mathrm{sh}\,\psi}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}-\frac{1}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}\right)+2\frac{\varphi}{\mathrm{sh}\,\varphi}\left(1-\frac{\varphi\,\mathrm{sh}\,\psi-\psi\,\mathrm{sh}\,\varphi}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}\right)\right],\\
&A_2 = -C_F\frac{\alpha_s}{4\pi}\left[\frac{1}{4}He^{-\psi}\left(\frac{\varphi}{\mathrm{sh}\,\varphi}\left(\frac{\mathrm{ch}\,\varphi\,\mathrm{ch}\,\psi-1}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}-\frac{\varphi\,\mathrm{sh}\,\psi-\psi\,\mathrm{sh}\,\varphi}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}+1\right)\right.\right.\\
&\quad \left.\left.-\frac{\psi\,\mathrm{sh}\,\psi}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}-\frac{1}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}\right)+2\frac{\varphi}{\mathrm{sh}\,\varphi}\left(1+\frac{\varphi\,\mathrm{sh}\,\psi-\psi\,\mathrm{sh}\,\varphi}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}\right)\right],\\
&A_{12} = C_F\frac{\alpha_s}{4\pi}\frac{H}{4}\left(\frac{\varphi}{\mathrm{sh}\,\varphi}\frac{\mathrm{ch}\,\varphi\,\mathrm{ch}\,\psi-1}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}-\frac{\psi\,\mathrm{sh}\,\psi}{(\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi)^2}-\frac{1}{\mathrm{ch}\,\varphi-\mathrm{ch}\,\psi}\right),
\end{aligned}
$$

where $L=\log\frac{m_1m_2}{\mu^2}$.

The one-loop matching of the baryonic currents was considered in [47].

Now we are in a position to make some statements of Sec. 2, 3 more precise. The QCD meson constants (2.9) are related to the HQET constant by the matching (4.24):

$$
f=\frac{2\widetilde{f}(m)}{\sqrt{m}}\left(1-c\frac{\alpha_s(m)}{\pi}+\cdots\right), \qquad (4.36)
$$

where $c=\frac{4}{3}$ for vector mesons and $c=\frac{2}{3}$ for pseudoscalar mesons (if a fully anticommuting γ_5 is used). The HQET constant depends on the normalization point as (4.18–4.19):

$$
\widetilde{f}(\mu)=\hat{f}\alpha_s^{-2/\beta_1}(\mu)\left(1-k\frac{\alpha_s(\mu)}{\pi}+\cdots\right), \quad k=\frac{5}{12}-\frac{285-7\pi^2}{27\beta_1}+\frac{107}{2\beta_1^2}. \qquad (4.37)
$$

There are two approaches to the $b\to c$ weak decays: one-step matching [23, 64] and two-step matching [18, 65, 64]. We are interested in hadronic matrix elements of the vector and axial weak currents $j=\bar{c}\Gamma b$, $\Gamma=\gamma_\mu$ or $\gamma_\mu\gamma_5$. These currents are defined in QCD at a high normalization point $\mu\sim m_W$. In the first approach, we use QCD at the scales from m_W down to some not exactly definable border $\overline{m}\sim m_b\sim m_c$. By a chance, the QCD anomalous dimensions of these currents vanish, and $j(\overline{m})=j(m_W)$. At $\mu=\overline{m}$ we perform the matching (4.35) to the HQET in which both b and c quarks are considered static. The vector current becomes a combination of $\bar{\tilde{c}}\Gamma\tilde{b}$ with $\Gamma=\gamma_\mu$, $v_{b\mu}$, and $v_{c\mu}$; the axial current—of the similar currents with the extra γ_5. Then we scale down to a typical hadronic μ using the HQET heavy-heavy anomalous dimension (4.19). At this point we use the heavy quark spin symmetry, and express the matrix elements via the Isgur-Wise form factors.

In the two-step approach, we use QCD from $\mu=m_W$ down to $\mu=m_b$. At this point we perform the matching (4.24) to the HQET-1 in which b is static while c is still dynamic. The vector current becomes a combination of $\bar{c}\Gamma\tilde{b}$ with $\Gamma=\gamma_\mu$ and $v_{b\mu}$ (and the extra γ_5 in the axial case). Then we use the HQET heavy-light anomalous dimension (4.19) to scale these currents down to $\mu=m_c$. At this point we perform the matching (4.31) to the HQET-2 in which both b and c are static. We obtain a combination of $\bar{\tilde{c}}\Gamma\tilde{b}$ with $\Gamma=\gamma_\mu$, $v_{b\mu}$, and $v_{c\mu}$ (with the extra γ_5 in the axial case). Then we use the HQET heavy-heavy anomalous dimension (4.19) and the spin symmetry as before.

In the one-step approach, we can't sum the $\alpha_s\log\frac{m_b}{m_c}$ corrections; we can do it in the two-step approach (even in the subleading order). On the other hand, the first matching in the two-step method gives a series in $\frac{m_c}{m_b}$ because m_c is the largest mass scale in the intermediate HQET. In the above description all $\frac{m_c}{m_b}$ corrections were discarded, and this is not a good approximation in the real world. The first $\frac{m_c}{m_b}$ correction can be included [65] (the leading $\alpha_s\log\frac{m_b}{m_c}$ corrections are summed in this term using the one-loop anomalous dimensions), but incorporating the second term would require a large work.

The one-step matching seems more adequate in our world in which $\frac{m_c}{m_b}$ is not very small and $\log\frac{m_b}{m_c}$ is not too large. It is possible to obtain the optimal combination of both approaches [64]. We expand the result of the one-step matching in $\frac{m_c}{m_b}$. Then we extract the zeroth term from the series, and replace it by the result of the two-step matching. We also extract the first term, and replace it by the first power correction from the two-step matching. The errors of this procedure are of the order of α_s^2, or $\frac{m_c}{m_b}\alpha_s$, or $\left(\frac{m_c}{m_b}\right)^2 \alpha_s \log\frac{m_b}{m_c}$; they all are small.

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Introduction to the Heavy Quark Effective Theory Part 1

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