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THE KINETICS OF INHOMOGENEOUS PLASMA

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НОВОСИБИРСК

The Kinetics of Inhomogeneous Plasma

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ABSTRACT

A new concept of plasma kinetic description is developed. To demonstrate its strength the instrumentation was elaborated for describing of plasma cross diffusion via the particle interaction with drift waves. The instrumentation includes consistent equations of particle distribution evolution and of drift wave pumping. The effect of plasma nonstationarity and particle induced scattering are taken into account, just as the effect of inhomogeneity induced wave drift in space and wavenumbers.

The descriptive potential of a new concept is higher of that of a commoniy accepted concept of Bogolubov - Born - Green - Kircwood - Yven (BBGKY). The new concept do not involve the averaging over the initial data ensemble. In treating of turbulent plasma it gives possibilities to avoid the using of the wellknown Random Phase Approximation and of any renormalizations.

INTRODUCTION

There are many problems in plasma physics, the solving of which requires a kinetic plasma description.

The commonly accepted concept of kinetics is a BBGKY concept [1 - 4]. In its basement a knowledge was laid of plasma being a statistical system. For such a systems the meaningful entities are those, which are statistically averaged over the initial data ensemble. But the real plasma do not contain any initial data ensemble: only one set of initial data is represented. In spite of the fact the reliability of the BBGKY kinetics is commonly believed to be out of any doubt.

Author has developed another concept of the plasma kinetics. To demonstrate its strength the instrumentation was elaborated for the correct calculation of the drift wave contribution to the inhomogeneous plasma diffusion.

Drift waves are always represented in an inhomogeneous plasma. They are excited due to the so-called "universal" instability [5].

In the usual case of a nonlinear waves the motion of the plasma particles in cross plane is slightly irregular, and plasma - contrary to the case of linear wave field - intermingles in this plane.

The plasma with maxwellian distributions was shown to have the most unstable waves within the transverse wave-length range from electron Larmor radius to ion Larmor radius [6]. Hence the plasma motion cannot be described within the frames of the drift approximation. This is the reason for the kinetics to be relevant in the situation.

It should be stressed that the inhomogeneity of plasma plays a crucial role in the situation. Meanwhile there was no valid theory for the inhomogeneous plasma kinetics to a current moment.

The problem investigated was the following. The fully ionized high temperature plasma with low kinetic pressure is placed into a volume with strong magnetic field. The Coulomb collisions are negligibly rare. The plasma meets a condition

$$\beta = \frac{8\pi nT}{H^2} << \frac{m_e}{m} \quad (1)$$

The plasma is inhomogeneous. The typical inhomogeneity length a satisfies a restriction

$$a > \rho_{Li} / \beta^{1/2} .$$

It ensures the drift waves frequencies being small compared with ω_{Hi} up to transverse wavelengths $\lambda_\perp \sim \rho_{Le}$. Waves with smaller transverse lengths are dumped: their phase velocities along Z are small compared with the thermal velocity.

The condition (1) provides the waves being potential. Usually it does not meet, and Alfven and ion-sound waves should be considered too [7]. The condition is inserted mere to simplify the physical situation.

The thorough description of the concept and of the kinetic equations derivation is sent for publication in *Physics of Fluids B: Plasma Physics*. The given paper outlines the concept and the equations obtained.

The structure of the paper is as follows. The fist part of the article contains the definition of the basic entities of the concept. The second part presents without derivation the kinetic equations for the problem chosen.

The main entity of the kinetics is the distribution function. Its definition opens the first section. The time derivative of the function contain the so-called collision integral. To calculate the integral the notion of the two-time correlative function is defined. This function resembles the wellknown analog in the weak turbulence theory. The starting point for the derivation of corresponding evolution equations is the plasma description on the basement of Klimontovich - Dupree equation [8, 9] and Maxwell equations. In the section the scheme of the kinetics derivation is described. Some steps of its realization take advantage of the graphic formulae writing. The main notions of the corresponding diagram technic are represented, and some relevant equations are written.

The two-time correlative function can be expressed in terms of an one-time correlative entity - the spectral density. Each moment the spectral densities are strictly determined, and their evolution too.

The ideas formulated are urgent not for the plasma case only, but for all others areas too, where the BBGKY approach is used.

Section 1. The plasma kinetics: the theory outline

A main notion of plasma kinetics is the distribution function. It should be defined on the base of a microdistribution function - the entity of the Klimontovich-Dupree equation. We define the function as follows. Let us average the microdistribution function

$$N_\alpha(\mathbf{r},\mathbf{p},t) = \sum_i^N \delta^3(\mathbf{r}-\mathbf{r}_i(t))\ \delta^3(\mathbf{p}-\mathbf{p}_i(t)) ,$$

in the phase space **r**, **p** over the 6-dimensional parallelepiped with the centre at a given point (**r**,**p**). We will consider this averaged distribution as a function of variables **r** and **p**. Let us take for it a denotation

$f_\alpha(r,p,t)$.

The parallelepiped mentioned contains a great number of given type particles provided its volume is large enough ($V \gg (m_\alpha v_{T\alpha})^3/n$). In this case the distribution function f is a smooth function of its variables, and it possesses a definite statistical trustworthiness.

The ratio of different dimensions of the parallelepiped should be chosen depending on the problem under study. It means the very entity of the distribution is a relative one. For our problem we can take the parallelepiped with the spatial Y,Z dimensions up to the corresponding sizes of the plasma volume. At the expense of them the momentum dimensions of the parallelepiped can be made small compared with the thermal velocity, and X dimension - small compared with the spatial motion scale. The last two facts are of essential use in the derivation of the following equations.

It should be stressed the distribution function is always spread over its variables. We can achieve small gradations of the function in any of spatial and momentum variables, but at the expense of the remaining 5 variables. Otherwise the function is not statistically trustworthy and is useless.

To derive an equation of distribution function evolution we average the Klimontovich-Dupree equation over the very same parallelepiped. Then we will obtain the equation

$$\left[\frac{\partial}{\partial t} + v^\nu \frac{\partial}{\partial r^\nu} + \frac{e_\alpha}{c} v_i \, {}^0F^{i\nu} \frac{\partial}{\partial p^\nu}\right] f_\alpha(r,p,t) =$$

$$= -\frac{e}{c} v_i \frac{\partial}{\partial p^\nu} \left\langle \delta F^{i\nu}(r,t)\, \delta N_\alpha(r,p,t) \right\rangle \qquad (2)$$

(In this formula and further the co- and contravariant indices are used. The Einstein summation convention is implied. The latin letters are for the indexes of 4-vectors

(i = 0,...,3), the greek letters - with the exclusion of α - for the indexes of 3-vectors ($\nu,\gamma,...$ = 1,2,3). The metric tensor is as usual : its unequal to zero components are $g_{00} = - g_{11} = - g_{22} = - g_{33} = 1$). Right hand side here is called a collision integral. The tensor ${}^0F^{ij}$ is the stationary part of the electromagnetic field tensor. It correspond to the external magnetic field. The tensor δF^{ij} represents the electromagnetic field of the plasma. If the plasma does not exposed to the external electromagnetic radiation, the tensor can be expressed in terms of the microdistribution with the use of wellknown delayed potentials. The corresponding relation is

$$\delta F_{ik}(r,t) = \sum_\alpha e_\alpha \int d^3r_1\, d^3p_1 \int_{-\infty}^{t} dt_1 \times$$

$$\times\, \hat{\mathcal{F}}_{ik}\, r,t,r_1,v_1,t_1)\, N_\alpha(r_1,p_1,t_1) ,$$

with $\hat{\mathcal{F}}_{ik}$ being a definite integral differential operator.

To calculate the collision integral it is convenient to introduce a Green function $G_\alpha(r,p,t,r_1,p_1,t_1)$ - the solution of the equation

$$\left[\frac{\partial}{\partial t} + v^\nu \frac{\partial}{\partial r^\nu} + \frac{e_\alpha}{c} v_i \, {}^0F^{i\nu} \frac{\partial}{\partial p^\nu}\right] G_\alpha =$$

$$= \delta^3(r-r_1)\, \delta^3(p-p_1)\, \delta(t-t_1) ,$$

and a two-point correlative function

$$\left\langle \delta F_{i\nu}(r_1,t_1)\, N_\alpha(r,p,t) \right\rangle .$$

The averaging here is of the same type over the parallelepiped shaped neighbourhood. The variables $\mathbf{r}$ and $\mathbf{r}_1$ are being varied synchronously: the difference $\mathbf{r}-\mathbf{r}_1$ is fixed. Such an averaging - with fixed shifts of variables $\mathbf{r}$ from one point involved to another is a crucial one in the approach developed. The above introduced function is one of the range of manypoint correlative functions. They all are physically meaningful and are the averaged products of one microdistribution function and of a number of electromagnetic field tensors. All the terms of the product are taken in different points at different moments. The shifts of spatial coordinates are fixed when averaging.

To simplify the intermediate calculations it is convenient to use the graphic formulae writing. Let us take the following denotations. The Green function we denote with the solid line, the function f_α - with the oblong rectangle, the operator $\hat{\mathcal{F}}$ - with the dashed line, and the formal entity

$$\langle \delta N_\alpha(\mathbf{r},\mathbf{p},t)\ \delta N_{\alpha'}(\mathbf{r}_1,\mathbf{p}_1,t_1)\rangle$$

- with the wavy line. The given entity is empty in the statistical sense: it contain an unknown initial data. But in all equations it enters in a convolution with the operator $\mathcal{F}$, thus acquiring a physical meaning of a function

$$\langle \delta N_\alpha(\mathbf{r},\mathbf{p},t)\ \delta\tilde{F}^{ij}(\mathbf{r}_1,\mathbf{p}_1,t_1)\rangle .$$

In this definition the following functions are involved

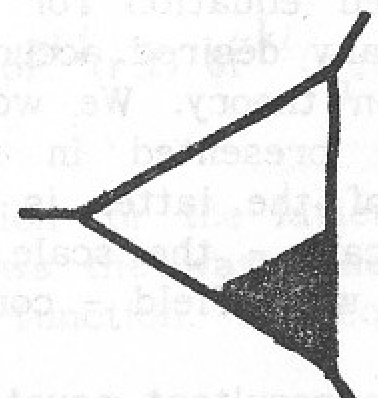
$$\delta N_\alpha(\mathbf{r},\mathbf{p},t) = N_\alpha(\mathbf{r},\mathbf{p},t) - f_\alpha(\mathbf{r},\mathbf{p},t) ,$$

$$\delta\tilde{F}^{ij}(\mathbf{r},t) = \delta\tilde{F}^{ij}(\mathbf{r},t) - \langle \delta\tilde{F}^{ij}(\mathbf{r},t)\rangle .$$

(In our problem the average $\langle\delta\tilde{F}^{ij}(\mathbf{r},t)\rangle$ can be neglected: it corresponds to the magnetic field of the plasma diamagnetic

currents and is small compared to the external magnetic field.)

Let us define also an asymmetrical vertex



The vertex we associate with the definite moment t, the space point vector $\mathbf{r}$, the momentum $\mathbf{p}$ and the coefficient $(-e_\alpha/c)$. The vertex has one entry (on the picture it is from the left) and two exits. If to rewrite (2) in the form

$$f_\alpha = G_\alpha(\text{RHS of (2)}) ,$$

then the entry of the vertex corresponds to the point of G_α operation onto right hand side of (2).

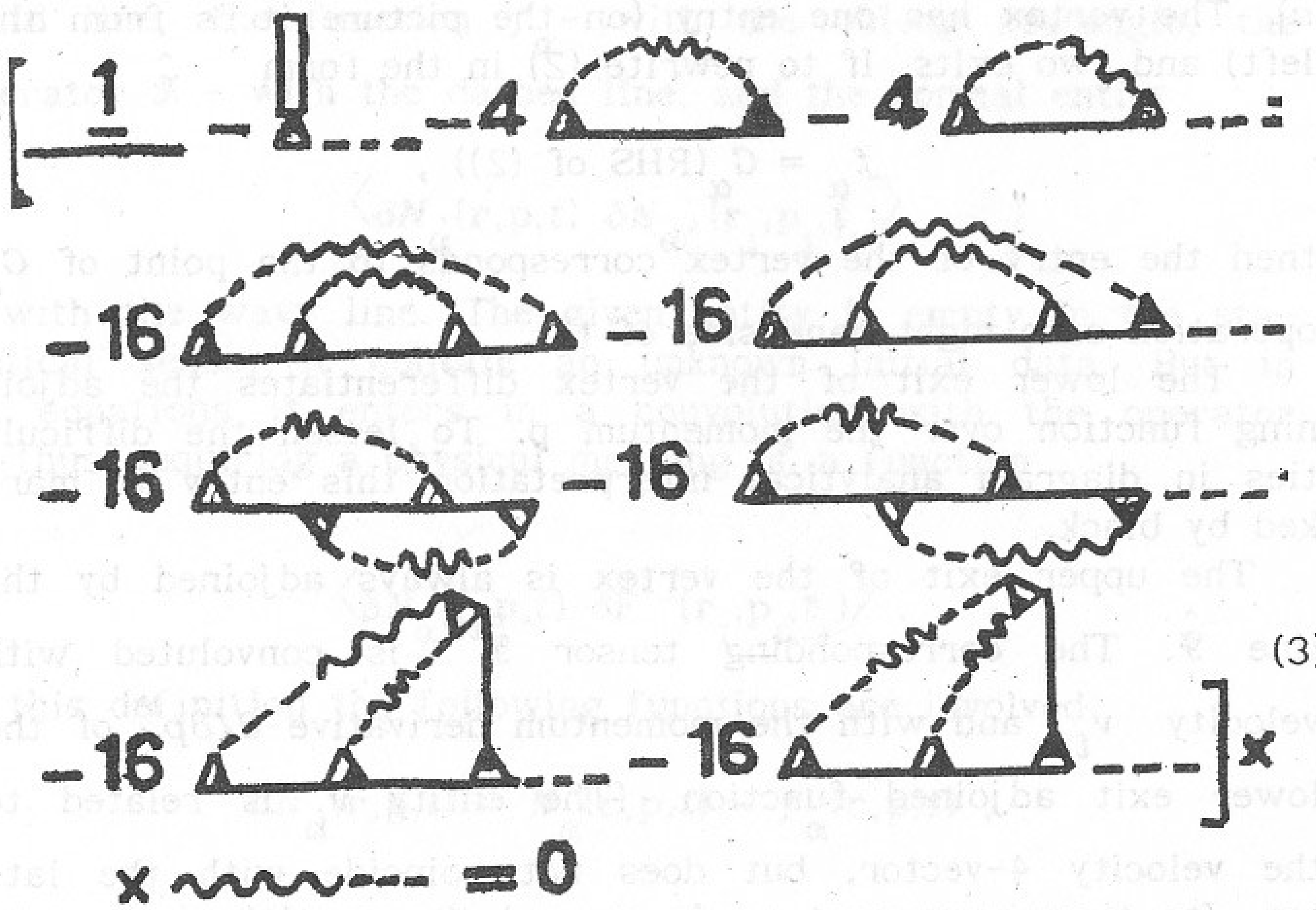
The lower exit of the vertex differentiates the adjoining function over the momentum $\mathbf{p}$. To lessen the difficulties in diagram analytical interpretation this entry is marked by black.

The upper exit of the vertex is always adjoined by the line $\hat{\mathcal{F}}$. The corresponding tensor $\hat{\mathcal{F}}^{i\nu}$ is convoluted with velocity v_i and with the momentum derivative $\partial/\partial p^\nu$ of the lower exit adjoined function. (The entity v_i is related to the velocity 4-vector, but does not coincide with the latter. Its time component v_0 is c, and the spatial components $(v^\nu,\ \nu = 1,3)$ are the usual physical components of the velocity.)

If the vertex entry is adjoined by the line G, the integration over the corresponding variables $\mathbf{r}, \mathbf{p}$ and t takes place.

The truncated closed equation for the function $\langle \delta N \delta \tilde{F} \rangle$ can be obtained up to any desired accuracy with the utilization of the perturbation theory. We would not describe the utilization: it will be presented in the journal publication. The crucial point of the latter is the smallness of the characteristic motion scale - the scale is but a correlative length of the turbulent wave field - compared to the dimensions of plasma volume.

We present here the resultant equation, which correspond to plasma description with the three-wave interactions being kept in the scope. It is



(3)

But the mentioned function $\langle \delta N_\alpha \delta \tilde{F} \rangle$ is unusual as the entity of the kinetic theory. It can be expressed with the help of

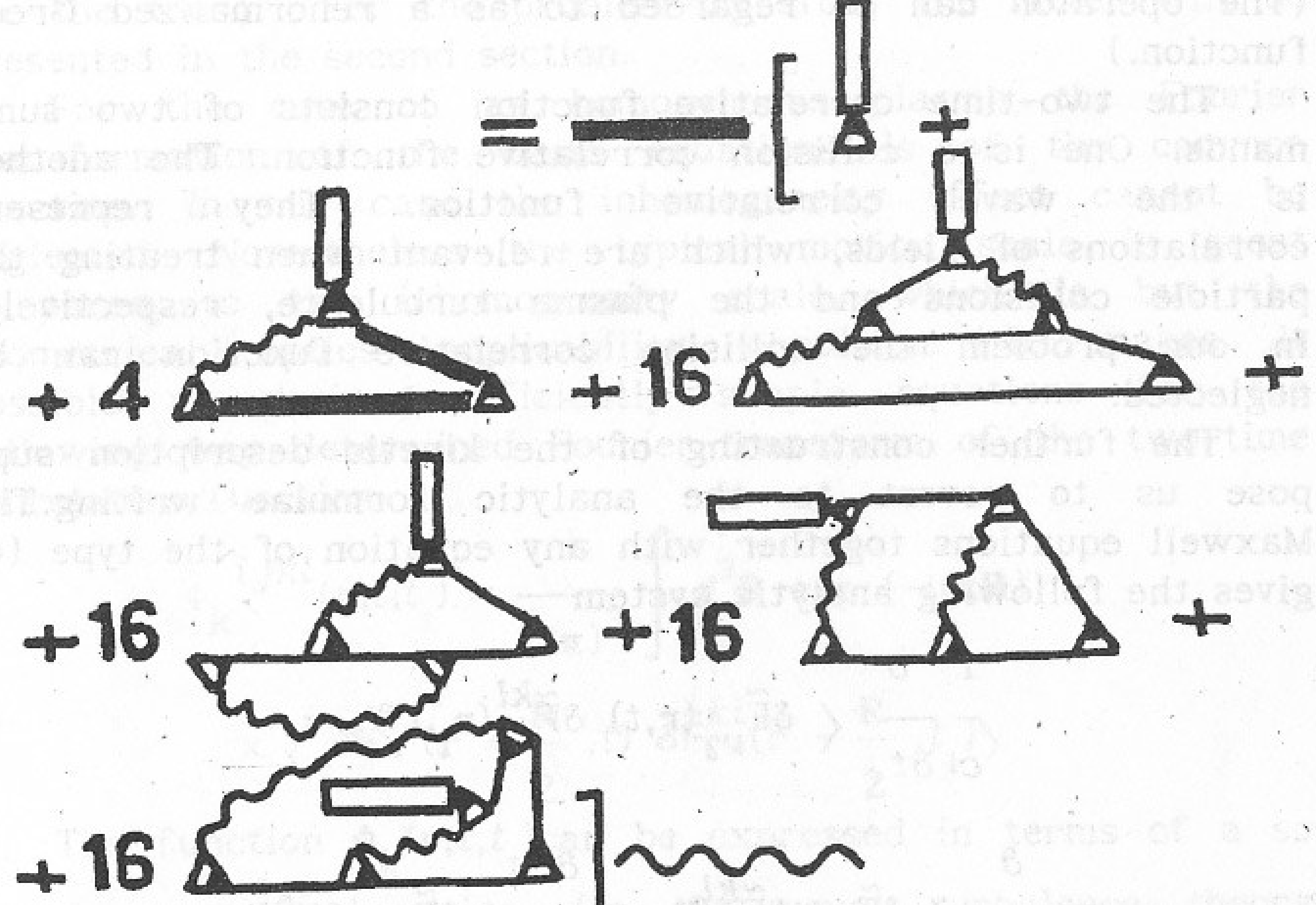
the last equation in terms of the two-time correlative function

$$\langle \delta \tilde{F}^{ij}(\mathbf{r},t)\ \delta \tilde{F}^{kl}(\mathbf{r}_1,t_1) \rangle .$$

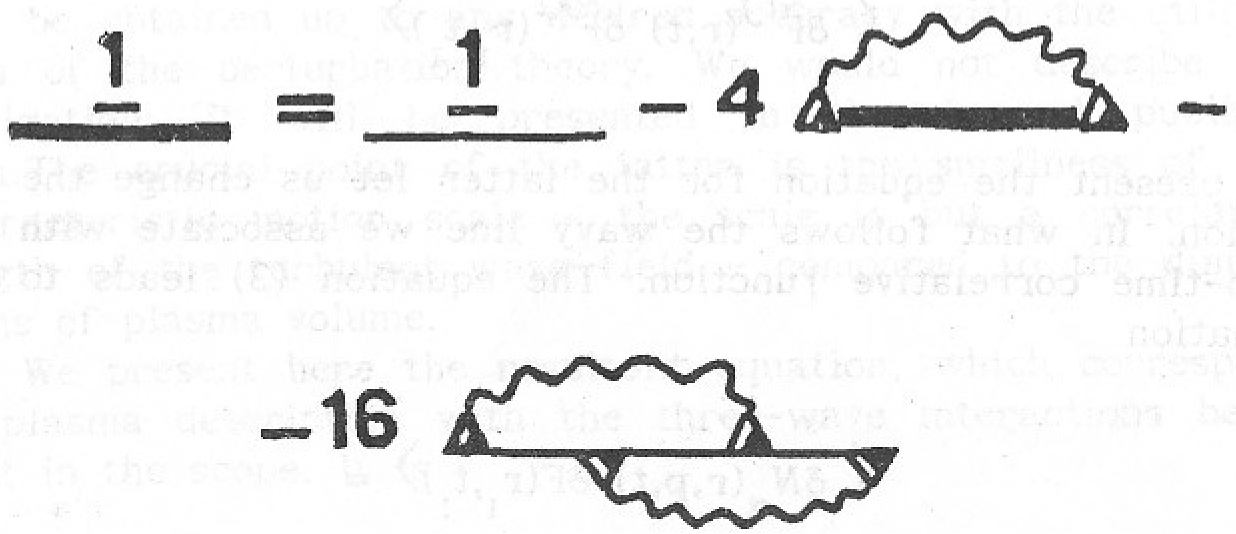
To present the equation for the latter let us change the notation. In what follows the wavy line we associate with the two-time correlative function. The equation (3) leads to the equation

$$\langle \delta N_\alpha(\mathbf{r},\mathbf{p},t)\ \delta \tilde{F}(\mathbf{r}_1,t_1) \rangle =$$

(4)



where the operator is defined by the relation



(5)

(The operator can be regarded to as a renormalized Green function.)

The two-time correlative function consists of two summands. One is a collision correlative function. The another is the wave correlative function. They represent correlations of fields, which are relevant when treating the particle collisions and the plasma turbulence, respectively. In our problem the collision correlative function can be neglected.

The further constructing of the kinetic description suppose us to revert to the analytic formulae writing. The Maxwell equations together with any equation of the type (4) gives the following analytic system

$$\frac{1}{c}\frac{\partial}{\partial t}\left\langle \delta\tilde{F}_{\nu\gamma}(r,t)\ \delta\tilde{F}^{kl}(r_1,t_1)\right\rangle =$$

$$= \frac{\partial}{\partial r^{\nu}}\left\langle \delta\tilde{F}_{\gamma 0}\ \delta\tilde{F}^{kl}\right\rangle - \frac{\partial}{\partial r^{\gamma}}\left\langle \delta\tilde{F}_{\nu 0}\ \delta\tilde{F}^{kl}\right\rangle ,$$

$$\frac{1}{c}\frac{\partial}{\partial t}\left\langle \delta\tilde{F}^{\nu 0}(r,t)\ \delta\tilde{F}^{kl}(r_1,t_1)\right\rangle =$$

$$-\frac{\partial}{\partial r^{\gamma}}\left\langle \delta\tilde{F}^{\nu\gamma}\ \delta\tilde{F}^{kl}\right\rangle - \frac{4\pi}{c}\int d^3r_2\ dt_2\ \sigma^{\nu m\cdot}_{\cdot\cdot\gamma}(r,t,r_2,t_2)\ \times$$

$$\times \left\langle \delta\tilde{F}_{m\cdot}^{\ \cdot\gamma}(r_2,t_2)\ \delta\tilde{F}^{kl}(r_1,t_1)\right\rangle . \qquad (6)$$

In the last equation the denotation $\sigma^{\nu m\cdot}_{\cdot\cdot\gamma}(r,t,r_2,t_2)$ is for the tensor with the physical meaning of a conductivity tensor. In general case its analytical expression consists of many terms corresponding to different diagrams in RHS of the eq. (4). We would not write down the explicit expression for the tensor. For the problem on study the latter will be presented in the second section.

For the case of an homogeneous plasma the Fourier transformation of the basic equations is of the common practice. In our case the inhomogeneity effect cannot be neglected. Nevertheless the typical motion scale is great compared to the inhomogeneity scale, which is but the geometrical optics applicability condition. It makes it possible to derive sufficiently simple equations for the following way determined Fourier-transform of the two-time correlative function

$$\Phi_{\mathbf{k}}^{\ ijkl}(r,t,t_1) = \frac{1}{(2\pi)^3}\int d^3\mathbf{R}\ \exp(-i(\mathbf{kR}))\ \times$$

$$\times \left\langle \delta\tilde{F}^{ij}(r + \frac{\mathbf{R}}{2},t)\ \delta\tilde{F}^{kl}(r - \frac{\mathbf{R}}{2},t_1)\right\rangle .$$

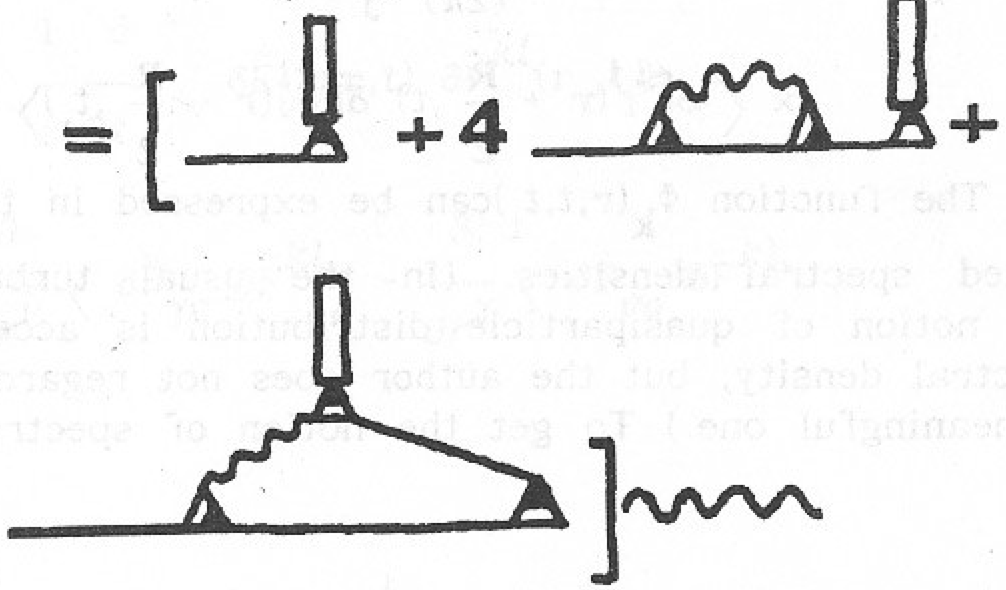
The function $\Phi_{\mathbf{k}}(r,t,t_1)$ can be expressed in terms of a so called spectral densities. (In the usual turbulence theory the notion of quasiparticle distribution is accepted for the spectral density, but the author does not regard the term as a meaningful one.) To get the notion of spectral density we should solve the problem of the two-time function expressing in terms of the autocorrelative function $\Phi_{\mathbf{k}}(\mathbf{r},t,t)$. In our case the task is simplified by the fact that the the wave polarization does not depend on space position, time and wavevector. But in the general case the dependencies mentioned must be taken into consideration. The description of the way to solve the problem both for the general case and for the problem taken will be presented in journal publication. This way utilizes the Laplace transform technic. (The very possibility of consistent expression of all relevant integrals in terms of spectral densities is connected with the fact of plasma being weakly turbulent - in a sense of an usual theory. It means the inverse evolution time of the plasma - which does not exceeds the increment of the most unstable waves, - is small compared to the spectrum frequency width. In our case this condition is satisfied. Moreover, the author suppose that in all well posed problems the plasma weakly turbulent).

The real fact is that the question of plasma turbulence description is settled. Let us proceed with the results obtained for the problem chosen.

Section 2. The results obtained.

The relevant equation for the function $\langle \delta N_\alpha \delta\tilde{F} \rangle$ can be presented in the form

$$\langle \delta N_\alpha \delta\tilde{F} \rangle = \qquad (7)$$



It adequately describes in the lowest order the effects of wave pumping, of plasma nonstationarity, of wave drift in $\mathbf{k}-\mathbf{r}$ space (remember the wave package behavior in the inhomogeneous medium) and of the wave induced scattering on plasma particles. The corresponding plasma description is sufficient to learn about the role of the drift vortices in the plasma, which some scientists suppose to be essential. If they are essential for the plasma diffusion, the equations must demonstrate the wave condensation in the region of long wavelengths.

The dispersion equation for the drift waves is

$$i\omega - 4\pi \left[\sigma_{\mathbf{k}\omega}(\mathbf{r},t) + \delta\sigma_{\mathbf{k}\omega}(\mathbf{r},t) + \frac{i}{2}\frac{\partial^2}{\partial k_\delta\, \partial r^\delta}\sigma_{\mathbf{k}\omega}(\mathbf{r},t) \right] = 0 . \qquad (8)$$

It contains a conductivity scalar. The latter consists of the linear term

$$\sigma_{\mathbf{k}\omega} = \sum_\alpha e_\alpha^2 \int d^3\mathbf{p}\, d^3\mathbf{p}_1\, v^\nu \frac{k_\nu k^\gamma}{k^2} G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1) \frac{\partial}{\partial p_1^\gamma} f_\alpha(\mathbf{r},\mathbf{p}_1,t) , \qquad (9)$$

and of the nonlinear one

$$\delta\sigma_{\mathbf{k}\omega}(\mathbf{r},t) = 4 \sum_{\alpha,s} e_\alpha^4 \frac{k_\nu k^\varepsilon}{k^2} \int d^3\mathbf{p}\, d^3\mathbf{p}_1\, d^3\mathbf{p}_2\, d^3\mathbf{p}_3\, d^3\mathbf{k}_1 \times$$

$$\times \frac{k_1^\eta k_1^\delta}{k_1^2} v^\nu G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)\, n^s_{\mathbf{k}_1}(\mathbf{r},t) \frac{\partial}{\partial p^\eta} G_{\alpha\, \mathbf{k}-\mathbf{k}_1\, \omega-\omega_1}(\mathbf{p}_1,\mathbf{p}_2) \times$$

$$\times \left[\frac{\partial}{\partial p_2^\delta} G_{\alpha k\omega}(\mathbf{p}_2,\mathbf{p}_3) \frac{\partial f_\alpha(\mathbf{r}, \mathbf{p}_3,t)}{\partial p_3^\varepsilon} + \frac{\partial}{\partial p_2^\varepsilon} G_{\alpha -\mathbf{k}_1 -\omega_1}(\mathbf{p}_2,\mathbf{p}_3) \times \right.$$

$$\left. \times \frac{\partial f_\alpha(\mathbf{r},\mathbf{p}_3,t)}{\partial p_3^\delta} \right] \Bigg|_{\omega_1 = \omega^s_{\mathbf{k}_1}(\mathbf{r},t)} \qquad (10)$$

In these formulae the function $G_{\alpha k\omega}(\mathbf{p},\mathbf{p}_1)$ is the Fourier-Laplace transform of the Green function. Its calculation contain no difficulties. We restrict ourselves only to presenting of its definition

$$G_{\alpha k\omega}(\mathbf{p},\mathbf{p}_1) = \int d^3\mathbf{r}\ dt\ G_\alpha(\mathbf{r},\mathbf{p},t,\mathbf{r}_1,\mathbf{p}_1,t_1) \times$$

$$\times \exp\left(i\omega\,(t-t_1) - i(\mathbf{k}(\mathbf{r}-\mathbf{r}_1))\right).$$

The index s in formula (10) takes values 1 and -1. The function $n^s_{\mathbf{k}}(\mathbf{r},t)$ in eq. is related to the spectral density of drift waves $n_{\mathbf{k}}(\mathbf{r},t)$. The function $\omega^s_{\mathbf{k}}(\mathbf{r},t)$ is the solution of eq. (8) and is related to the natural frequency of the drift wave $\omega_{\mathbf{k}}$(complex!). The relations can be defined as follows. We choose the index s system to embody the property Re $\omega^1_{\mathbf{k}} > 0$. Then we can take a following definitions of drift waves natural frequency and spectral density

$$\omega_{\mathbf{k}}(\mathbf{r},t) = \omega^1_{\mathbf{k}}(\mathbf{r},t) = -\left[\omega^{-1}_{-\mathbf{k}}(\mathbf{r},t)\right]^*,\quad n_{\mathbf{k}}(\mathbf{r},t) = n^s_{s\mathbf{k}}(\mathbf{r},t).$$

The evolution equation of the function $n^s_{\mathbf{k}}$ is

$$\frac{\partial n^s_{\mathbf{k}}}{\partial t} = 2\ \mathrm{Im}\ \omega^s_{\mathbf{k}}\ n^s_{\mathbf{k}} + 2\ \mathbf{Re}\ {}^sA^2_{\mathbf{k}}\ , \qquad (11)$$

where ${}^sA^2_{\mathbf{k}}$ is given by

$${}^sA^2_{\mathbf{k}} = -\frac{1}{2}\frac{\partial\omega^s_{\mathbf{k}}}{\partial r^\delta}\frac{\partial n^s_{\mathbf{k}}}{\partial k_\delta} - \frac{i}{2}\frac{\partial\omega^s_{\mathbf{k}}}{\partial k_\delta}\frac{\partial n^s_{\mathbf{k}}}{\partial r^\delta} + n^s_{\mathbf{k}}\left(\frac{i}{4\pi} - \frac{\partial\sigma_{\mathbf{k}\omega}}{\partial\omega}\right)^{-1} \times$$

$$\times \left[\frac{1}{2}\frac{\partial^2\sigma_{\mathbf{k}\omega}}{\partial\omega^2}\frac{\partial\omega^s_{\mathbf{k}}}{\partial t} + \frac{\partial^2\sigma_{\mathbf{k}\omega}}{\partial t\,\partial\omega} + \frac{1}{2}\frac{\partial^2\sigma_{\mathbf{k}\omega}}{\partial\omega\,\partial r^\delta}\frac{\partial\omega^s_{\mathbf{k}}}{\partial k_\delta} - \right.$$

$$\left. - \frac{1}{2}\frac{\partial^2\sigma_{\mathbf{k}\omega}}{\partial\omega\,\partial k_\delta}\frac{\partial\omega^s_{\mathbf{k}}}{\partial r^\delta} \right] \Bigg|_{\omega = \omega^s_{\mathbf{k}}} . \qquad (12)$$

(Here and in what follows we omit the variables r and t: they are implied.) The first two terms of RHS here correspond to the wave drift forced oscillations. The terms in square brackets are connected with the effects of medium time dispersion and of medium nonstationarity. Both the effects are out of the scope of an usual geometric optics.

The distribution function of the given type α particles evolves in accordance with the equation

$$\left[\frac{\partial}{\partial t} + v^\nu\frac{\partial}{\partial r^\nu} + e_\alpha\frac{v^\gamma}{c}\ {}^0F^{\gamma\nu}\frac{\partial}{\partial p^\nu}\right] f_\alpha = \mathrm{St}\, f_\alpha\ , \qquad (13)$$

where

$$\mathrm{St}\, f_\alpha = e^2_\alpha \sum_s \frac{\partial}{\partial p^\nu}\int d^3\mathbf{k}\,\frac{k^\nu\, k^\gamma}{k^2}\, d^3\mathbf{p}_1 \times$$

$$\times \left[G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1) \frac{\partial f_\alpha}{\partial p_1^\gamma} n_{\mathbf{k}}^s + \frac{i}{2} \frac{\partial G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)}{\partial \omega} \frac{\partial f_\alpha}{\partial p_1^\gamma} {}^sA_{\mathbf{k}}^2 + \right.$$

$$+ \frac{i}{2} \frac{\partial^2 G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)}{\partial \omega^2} \frac{\partial f_\alpha}{\partial p_1^\gamma} n_{\mathbf{k}}^s \frac{\partial \omega_{\mathbf{k}}^s}{\partial t} + \frac{i}{2} \frac{\partial G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)}{\partial k_\delta} \frac{\partial^2 f_\alpha}{\partial p^\gamma \partial r^\delta} n_{\mathbf{k}}^s +$$

$$+ \frac{i}{2} \frac{\partial G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)}{\partial k_\delta} \frac{\partial f_\alpha}{\partial p_1^\gamma} \frac{\partial n_{\mathbf{k}}^s}{\partial r^\delta} + \frac{i}{2} \frac{\partial^2 G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)}{\partial k_\delta \partial \omega} \frac{\partial f_\alpha}{\partial p_1^\gamma} n_{\mathbf{k}}^s \frac{\partial \omega_{\mathbf{k}}^s}{\partial r^\delta} -$$

$$- \frac{i}{2} G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1) \frac{\partial^2 f_\alpha}{\partial p^\gamma \partial r^\delta} \frac{\partial n_{\mathbf{k}}^s}{\partial k_\delta} - \frac{i}{2} \frac{\partial G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)}{\partial \omega} \frac{\partial^2 f_\alpha}{\partial p_1^\gamma \partial r^\delta} n_{\mathbf{k}}^s \frac{\partial \omega_{\mathbf{k}}^s}{\partial k_\delta} +$$

$$\left. + \, i \frac{\partial G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1)}{\partial \omega} \frac{\partial^2 f_\alpha}{\partial t \, \partial p^\gamma} n_{\mathbf{k}}^s \right] \Bigg|_{\omega = \omega_{\mathbf{k}}^s} +$$

$$+ 4e^4 \sum_{s,s'} \frac{\partial}{\partial p^\nu} \int d^3\mathbf{k} \frac{k^\nu k^\varepsilon}{k^2} \int d^3\mathbf{p}_1 \, d^3\mathbf{p}_2 \, d^3\mathbf{p}_3 \, d^3\mathbf{k}_1 \frac{k_1^\eta k_1^\delta}{k_1^2} \times$$

$$\times G_{\alpha\mathbf{k}\omega}(\mathbf{p},\mathbf{p}_1) n_{\mathbf{k}}^s \frac{\partial}{\partial p_1^\eta} G_{\alpha\, \mathbf{k}-\mathbf{k}_1\, \omega-\omega_1}(\mathbf{p}_1,\mathbf{p}_2) \, n_{\mathbf{k}_1}^{s'} \times$$

$$\times \left[\frac{\partial G_{\alpha\mathbf{k}\omega}(\mathbf{p}_2,\mathbf{p}_3)}{\partial p_2^\delta} \frac{\partial f_\alpha(\mathbf{p}_3)}{\partial p_3^\varepsilon} + \frac{\partial G_{\alpha\, -\mathbf{k}_1\, -\omega_1}(\mathbf{p}_2,\mathbf{p}_3)}{\partial p_2^\varepsilon} \times \right.$$

$$\left. \times \frac{\partial f_\alpha(\mathbf{p}_3)}{\partial p_3^\delta} \right] \Bigg] \Bigg|_{\omega_1 = \omega_{\mathbf{k}_1}^{s'}, \ \omega = \omega_{\mathbf{k}}^s} . \qquad (14)$$

The first square brackets represents the contribution of the first diagram in RHS of (7). Its first term correspond to the usual quasilinear diffusion [10]. All other summands of the collision integral are the corrections, which are out of the scope of an usual theory. But it is these corrections, the consideration of which is necessary to develop a picture of plasma diffusion and to learn the role of the vortices.

The last term in the braces issues from the plasma nonstationarity. The second square bracket represent the wave effect on plasma particles in the process of induced wave scattering.

The system of equations (8-14) consistently describes the plasma evolution up to the accuracy wanted. It should be admitted that these equations are not compact. But they accord to the goal desired. They would be studied more thoroughly, with reasonable simplifications. But such a study should constitute a subject for another article.

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