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SELF-SIMILAR REGIMES OF SUBSONIC LANGMUIR WAVE COLLAPSE

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НОВОСИБИРСК

Self-Similar Regimes of Subsonic Langmuir Wave Collapse

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ABSTRACT

The first examples of regular and singular self-similar solutions of the 3-d nonlinear Schroedinger equation for potential vector field are found explicitly. Regular solutions relate to the process of the "singularities" formation. Singular solutions relate to an early evolution of the "singularities". Together, the solutions describe behavior of Langmuir wave field in the spatiotemporal vicinities of its "singularities" and provide an effective mechanism of energy dissipation from the Langmuir condensate.

In the framework of a weak turbulence theory, the basic nonlinear processes lead to accumulation of long Langmuir waves. As a result so-called Langmuir condensate is formed. The mechanism for the energy dissipation from the Langmuir condensate remained unclear for a long time. Since 1972, the subsonic Langmuir collapse was considered to provide such a mechanism [1]. However, this kind of collapse appeared to be "weak", so that an infinitely small energy gets into arising singularity (where the absorption could take place). An effective mechanism for the energy dissipation was revealed recently in the framework of the scalar collapse model. The singularities of the scalar field were shown not vanish immediately after the absorption occurs of the waves that produced them, but exist a long time and "suck-in" new waves [2–5]. As for the true Langmuir wave field (which is a potential vector and not a scalar), its behavior in the spatiotemporal vicinity of the singularities is unclear still. To advance in the problem, the true equation for Langmuir condensate, namely, the 3-d nonlinear Schroedinger equation for potential vector field is to be studied below. This equation can be written in the form

$$\nabla \left(\imath \frac{\partial}{\partial t} + \Delta + |\nabla \psi|^2 \right) \nabla \psi = 0. \tag{1}$$

Substitution

$$\psi(\vec{r}, t) = f\left(\frac{\vec{r}}{a}, \tau\right) \exp(\imath \tau), \tag{2}$$

$$\tau \equiv \limsup_{r \to 0} \arg \psi(\vec{r}, t), \qquad a^{-2} \equiv |\dot{\tau}| \tag{3}$$

turns equation (1) to

$$\nabla\left(\imath\frac{\partial}{\partial\tau}+\Delta+|\nabla f|^2+\sigma-\imath a\dot{a}\frac{\partial}{\partial\rho}\rho\right)\nabla f=0. \tag{4}$$

$$\limsup_{\rho\to 0}\arg f(\vec{\rho},\tau)=0, \qquad \sigma\equiv -sign\dot{\tau}. \tag{5}$$

Here the dot above a or t signify the time derivative and the operator ∇ acts on the variable $\vec{\rho}=\vec{r}/a$ already.

Equation (4) has solutions looking as

$$f(\vec{\rho},\tau)=f^{\sigma}(\vec{\rho})=\begin{cases} f^-(\vec{\rho}), & \sigma=-1\\ f^+(\vec{\rho}), & \sigma=+1\end{cases}, \tag{6}$$

$$a\dot{a}=\gamma^{\sigma}=const. \tag{7}$$

Time-independent function $f^{\sigma}(\vec{\rho})$ satisfies the equation

$$\nabla\left(\Delta+|\nabla f^{\sigma}|^2+\sigma-\imath\gamma^{\sigma}\frac{\partial}{\partial\rho}\rho\right)\nabla f^{\sigma}=0. \tag{8}$$

The functions $f^-(\vec{\rho})$ and $f^+(\vec{\rho})$ describe the shape of the electrostatic potential $\psi(\vec{r},t)$ in the similarity range before and after the singularity arising at the point $\vec{r}=0$. These function are called below as regular and singular self-similar solutions respectively.

While the singularity is formed at the point $\vec{r}=0$, the spatial scale, a, of the field $\nabla\psi(\vec{r},t)$ variation decreases. In the range $r\gg a$ the field is "frozen", as the typical time of its variation there is much larger than at $r\lesssim a$. While a tends to zero, the field is "frozen" at any finite r, corresponding at $a\to 0$ to $\rho\to\infty$. This entails some relationship between asymptoticses of the functions $f^-(\vec{\rho})$ and $f^+(\vec{\rho})$ at $\rho\to\infty$. Basically, the function $f^{\sigma}(\vec{\rho})$ looks at $\rho\to\infty$ as

$$f^{\sigma}(\vec{\rho})\cong F^{\sigma}(\vec{n})\rho^{-\imath\sigma/\gamma^{\sigma}}, \qquad \vec{n}\equiv\frac{\vec{\rho}}{\rho}. \tag{9}$$

The "freezing" condition implies that, at the moment t_s of the singularity appearance, the function

$$\psi(\vec{r},t_s)=F^{\sigma}(\vec{n})r^{-\imath\sigma/\gamma^{\sigma}}\lim_{t\to t_s}a^{\imath\sigma/\gamma^{\sigma}}e^{\imath\tau} \tag{10}$$

do not depend on σ. This is possible at

$$\gamma^{\sigma}=\sigma\gamma \tag{11}$$

only. Then, in view of (3), (5) and (7),

$$a^2=2\gamma\,|t-t_s|, \qquad \tau=-\frac{1}{2\gamma}\ln|t-t_s|+\tau_0^{\sigma}, \tag{12}$$

where τ_0^{σ} $(\sigma=\pm 1)$ are constants. Hence, the "freezing" condition takes the form

$$F^+(\vec{n})e^{\imath\tau_0^+}\cong F^-(\vec{n})e^{\imath\tau_0^-}. \tag{13}$$

The solutions of equations (8) can be obtained by means of an expansion in associated Legendre polynomials:

$$f^{\sigma}(\vec{\rho})=\sum_{l,m}R^{\sigma}_{lm}(\rho)P_l^m(\cos\theta)\exp(\imath m\phi). \tag{14}$$

The radial functions behave at $\rho\to\infty$ as

$$R^{\sigma}_{lm}(\rho)\cong F^{\sigma}_{lm}\rho^{-\imath/\gamma}+C^{\sigma}_{lm}\rho^{-l-1}+D^{\sigma}_{lm}\rho^{-2}\exp\left(\imath\sigma\gamma\rho^2/2\right). \tag{15}$$

Here only the main terms of all kinds are presented, though the first item — corresponding to the "frozen" field — is dominant and small corrections to it may be much larger than a whole other item.

For a regular solution, $f^-(\vec{\rho})$, the last item in the formula (15) corresponds to the converging wave, that can be created only artificially and do not arise at the initial conditions of general kind. Hence,

$$D^-_{lm}=0. \tag{16}$$

The coefficients F^-_{lm} and C^-_{lm} are to be chosen to secure the regular behavior of the solution at $\rho\to 0$:

$$R^-_{lm}(\rho)=A_{lm}\rho^l+B_{lm}\rho^{l+2}+O(\rho^{l+4}). \tag{17}$$

At infinitely small perturbation of the parameters F^-_{lm} and C^-_{lm}, the addition to the regular self-similar solution arises which has the following asymptotics at $\rho\to 0$:

$$\delta R^-_{lm}(\rho)\cong\delta A_{lm}\rho^l+\delta B_{lm}\rho^{l+2}+E_{lm}\rho^{-l-1}+G_{lm}\rho^{-l+1}. \tag{18}$$

Here again only the main terms of all kinds are presented. The third and forth items give electric field non-analytical at $\rho\to 0$. In the asymptotics of the self-similar solution $f^-(\vec{\rho})$ both these items are to be absent:

$$E_{lm}=0, \qquad G_{lm}=0. \tag{19}$$

The number of conditions (19) coincides with the number of indefinite parameters F^-_{lm} and C^-_{lm} at any finite approximation of the expansion (14). However, the solution differing each other only by a constant phase are in fact identical, and the phase of the function $f^\sigma(\vec{\rho})$ was fixed above by the first of relationships (5). To satisfy this additional condition, the parameter γ must be used. Hence, the values of γ can constitute only a discreet set.

The actual number of the relevant parameters at any finite approximation of the expansion (14) depends on the symmetry of the solution. Further, axisymmetric solutions odd with respect to the space reflection, $\vec{r} \to -\vec{r}$, are considered. For such solutions only one value of the "momentum projection" m and only odd values of the "momentum" l are present in the expansion (14).

The most interesting are, probably, the solutions with $m = 0$ and $m = \pm 1$. Those have non-zero pressure of high-frequency field $P(\vec{\rho}) = |\nabla f^-(\vec{\rho})|^2$ at $\rho = 0$, in contrast to the solutions with larger values of $|m|$. Noteworthy, that non-zero field $\nabla f^-(\vec{\rho})$ at the center of the cavity is produced only by dipole terms of the expansion (14). At a relatively small weight of such terms in the expansion, the high-frequency pressure at the center of cavity would be reduced. This looks hardly compatible with the stable collapse to the point $r = 0$ (though the stability problem is not the subject of the current study). That is why the solutions of dipole kind are to be studied first.

Such solutions appear to be well computable by taking into account the first three terms in the expansion (14), namely the terms with $l = 1$, 3 and 5. The basic parameters of thus found solutions of dipole kind are presented in the table 1 (the cases $m = 1$ and $m = -1$ are identical, as they are turned one into another by reflection in a plane containing symmetry axis).

Figure 1 shows isolines of the pressure $P(\vec{\rho})$, computed for one, two and three terms taken into account in the expansion (14). In the vicinity of the center the pressure is given by the formula

$$P(\vec{\rho}) = P(0)\left(1 - \frac{\rho_\perp^2}{b_\perp^2} - \frac{\rho_\parallel^2}{b_\parallel^2}\right). \tag{20}$$

The values of the parameters $P(0)$, $b_\perp$ and $b_\parallel$ are also presented in the table 1. As seen from the figure 1 and formula (20), the cavity is oblate at $m = 0$ and prolate at $m = 1$.

After the singularity has formed, the singular self-similar solution $f^+(\vec{\rho})$ is relevant. At $\rho \gg 1$ its radial function, $R^+_{lm}(\rho)$, depend on three complex

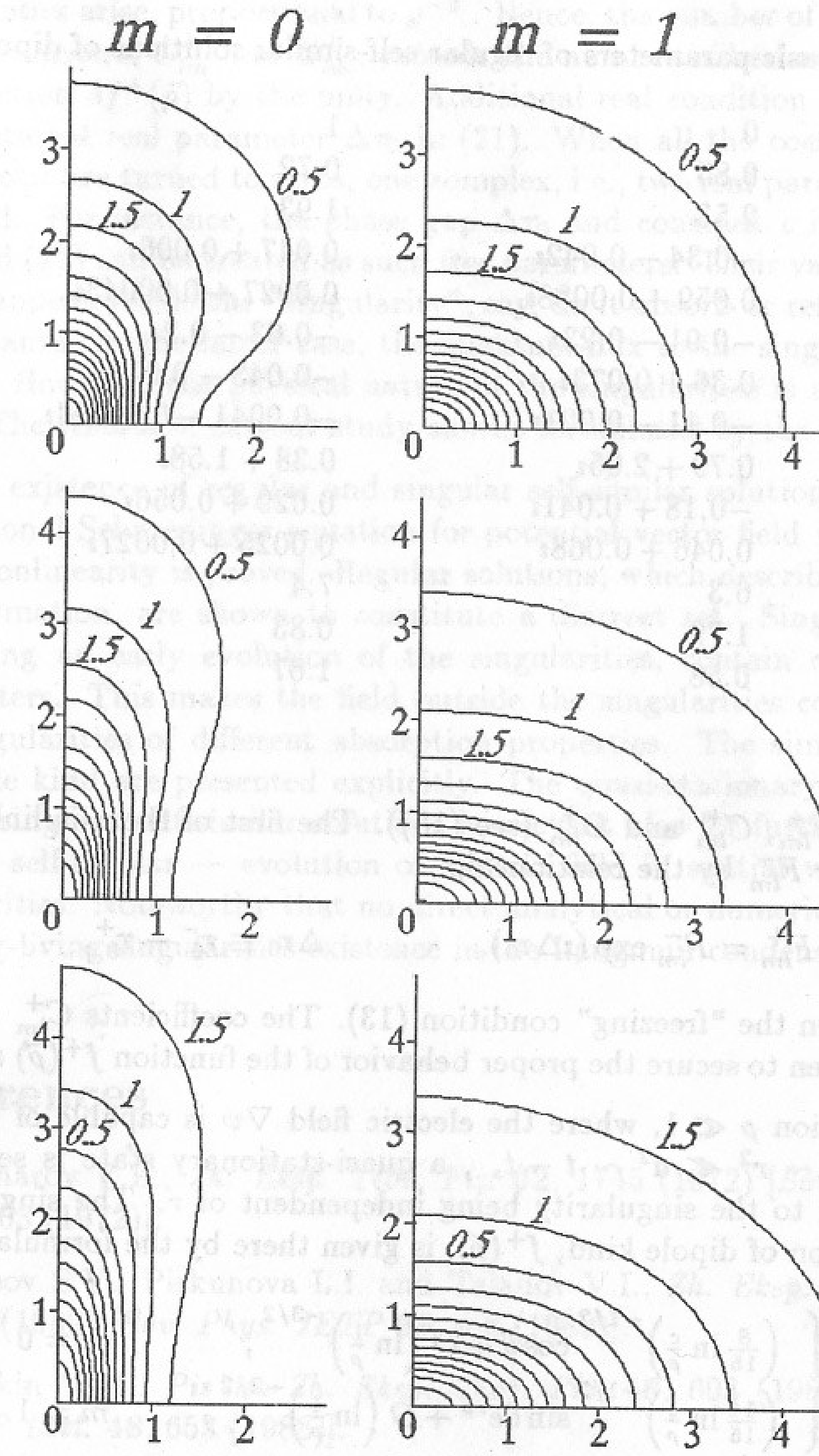

Figure 1: Isolines of the pressure $P(\vec{\rho})$ for regular self-similar solutions of dipole kind in the plane $(\rho_\parallel, \rho_\perp)$. The left figures correspond to $m = 0$ and the right ones — to $|m| = 1$. The upper, middle and lower figures are computed, respectively, for the first, third and fifth harmonics taken into account in the expansion (14).

Table 1: The basic parameters of regular self-similar solutions of dipole kind.

m	0	1
γ	0.85	0.72
A_{1m}	2.53	1.93
A_{2m}	$-0.34-0.042\imath$	$0.047+0.006\imath$
A_{3m}	$0.059+0.0088\imath$	$0.0027+0.00043\imath$
B_{1m}	$-0.91-0.23\imath$	$-0.63-0.2\imath$
B_{2m}	$0.36+0.073\imath$	$-0.043-0.011\imath$
B_{3m}	$-0.11-0.021\imath$	$-0.0041-0.00094\imath$
F^-_{1m}	$0.79+2.05\imath$	$0.38+1.58\imath$
F^-_{2m}	$-0.18+0.041\imath$	$0.029+0.056\imath$
F^-_{3m}	$0.046+0.068\imath$	$0.0025+0.0027\imath$
$P(0)$	6.3	7.4
$b_\perp$	1.79	0.83
$b_\parallel$	0.58	1.67

parameters: F^+_{lm}, C^+_{lm} and D^+_{lm} (see (15)). The first of them is linked with the parameter F^-_{lm} by the relationship

$$F^+_{lm} = F^-_{lm} \exp(\imath\Delta\tau_0), \qquad \Delta\tau_0 \equiv \tau_0^- - \tau_0^+, \tag{21}$$

as follows from the "freezing" condition (13). The coefficients C^+_{lm} and D^+_{lm} are to be chosen to secure the proper behavior of the function $f^+(\vec{\rho})$ at $\rho \to 0$.

In the region $\rho \ll 1$, where the electric field $\nabla\psi$ is capable of changing in a time $\delta\tau \sim r^2 \ll a^2 \sim t - t_s$, a quasi-stationary state is set in, the quantum flux to the singularity being independent of r. The singular self-similar solution of dipole kind, $f^+(\vec{\rho})$, is given there by the formulae

$$f^+(\vec{\rho}) = \begin{cases} \left(\frac{8}{15}\ln\frac{c}{\rho}\right)^{-1/2}\cos\theta + O\left(\ln\frac{1}{\rho}\right)^{-3/2}, & m=0 \\ \left(\frac{14}{15}\ln\frac{c}{\rho}\right)^{-1/2}\sin\theta e^{\imath\phi} + O\left(\ln\frac{1}{\rho}\right)^{-3/2}, & m=1 \end{cases} . \tag{22}$$

At infinitely small perturbation of the parameters C^+_{lm} and D^+_{lm} , an addition to the solution $f^+(\vec{\rho})$ arises which, generally speaking, has bad asymptotics at $\rho \to 0$. For each $l > 1$ two bad asymptoticses in the function δR^+_{lm} arise, proportional to ρ^{-l-1} and ρ^{-l+1} (as in (18)). For $l = 1$ only one bad

asymptotics arise, proportional to ρ^{-2} . Hence, the number of indefinite complex parameters, C^+_{lm} and D^+_{lm}, exceeds the number of bad asymptoticses in the function $\delta f^+(\vec{\rho})$ by the unity. Additional real condition (5) is balanced by additional real parameter $\Delta\tau_0$ in (21). When all the coefficients at bad asymptotic are turned to zeros, one complex, i.e., two real parameters remain free still. For instance, the phase gap $\Delta\tau_0$ and constant c in the formulae (21) and (22) can be treated as such free parameters. Their values depend on what happens inside the "singularity", say, do it absorb or reflect all coming to it quanta. In the latter case, the quantum flux to the singularity is to be absent. However, the physical nature of the singularities is a quite another topic. The results of current study can be formulated by the following way.

The existence of regular and singular self-similar solutions of the three-dimensional Schroedinger equation for potential vector field with attracting cubic nonlinearity is proved. Regular solutions, which describe the singularities formation, are shown to constitute a discreet set. Singular solutions, describing an early evolution of the singularities, contain continuous free parameters. This makes the field outside the singularities compatible with the singularities of different absorption properties. The simplest solutions of dipole kind are presented explicitly. The quasi-stationary asymptoticses (22) of singular self-similar solutions is relevant also for further — not necessarily self-similar — evolution of electric field in spatial vicinities of the singularities. Noteworthy that no direct analytical or numerical evidence on the long-living singularities existence in the Langmuir condensate were given earlier.

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