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LOGARITHMIC CORRECTIONS IN THE TWO-BODY QED PROBLEM

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НОВОСИБИРСК

Logarithmic Corrections in the Two-Body QED Problem [1]

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ABSTRACT

The logarithmic part of the Lamb shift, the contribution of the relative order $\alpha^3\log(1/\alpha)$ to the atomic state energy, is related to the usual infrared divergence. This fact allows one to calculate easily such corrections in positronium, and derive the recoil and electron-electron terms in the Lamb-shift Hamiltonian in many-electron atoms. Logarithmic energy corrections of the next order, $\alpha^4\log(1/\alpha)$, are of a different, relativistic nature. Their calculation is reduced to the ordinary perturbation theory for the nonrelativistic Schrödinger equation. The perturbation operators have the Breit-type structure and are found by the calculation of on-mass-shell diagrams. For positronium, the calculated logarithmic correction does not vanish only in n^3S_1 states and constitutes $\frac{5}{24}m\alpha^6\log(1/\alpha)/n^3$. Logarithmic corrections of the relative order $\alpha^2\log(1/\alpha)$ to the positronium decay rate are also of the relativistic origin and can be easily computed within the same approach. Arguments are presented in favor of a large numerical factor in the $(\alpha/\pi)^2$ correction to the positronium decay rate.

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1 Introduction

The increasing precision of the spectroscopic measurements in hydrogen, muonium and positronium demands higher accuracy of the theoretical calculations for the QED two-body bound states. Certainly, those problems are also of an independent theoretical interest.

The generally accepted theoretical approach to them goes back to Refs. [1, 2, 3]. Its starting point is the introduction of a relativistic two-body wave equation, which can be solved exactly, and in the nonrelativistic limit reduces to the Schrödinger equation. Then a perturbation series is developed about the exact solution.

Our approach is different. The corrections logarithmic in α originate from the effective operators which can be considered local with the accepted accuracy. These operators are found via the calculation of on-mass-shell diagrams. The corrections discussed are then computed in the standard perturbation theory for the nonrelativistic Schrödinger equation.

The logarithmic contribution to the Lamb-shift, i.e., the energy correction of the relative order $\alpha^3\log(1/\alpha)$, is related to the usual infrared divergence. It allows one to calculate easily this contribution not only in hydrogen, but in positronium and many-electron atoms as well. Those considerations are presented in detail in the second section of the article.

The next logarithmic energy corrections, of the relative order $\alpha^4\log(1/\alpha)$, are of different, relativistic origin. This fact is demonstrated in the next sections of the paper where those corrections are calculated explicitly. Meanwhile, in the Introduction, we restrict to a somewhat formal argument in favor of the relativistic origin of the contributions discussed: these corrections of high order in α do not have any power of π in the denominator, as distinct from the usual QED expansion. This is why we avoid in our paper the common adjective "radiative" at the noun "corrections".

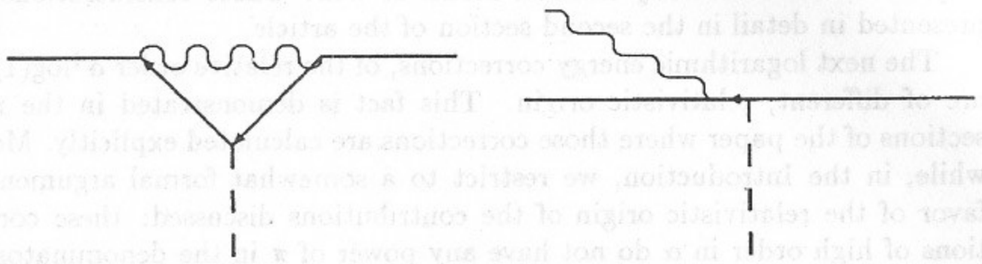
This approach in its simplest form was used previously by two of us to calculate the corrections of relative order $\alpha^2\log(1/\alpha)$ to the para- and or-

thopositronium decay rates [4]. Unfortunately, these corrections are insufficient to reconcile the theoretical prediction for the orthopositronium decay rate with its experimental value [5]. We argue in the present paper that the factor in the nonlogarithmic correction $\sim (\alpha/\pi)^2$ can well turn out large enough to bring the theoretical number into the agreement with the experimental one.

The calculation of the corrections of the relative order $\alpha^4\log(1/\alpha)$ to the energy levels is much more complicated problem than that of the logarithmic corrections to the positronium decay rate. The complete result for this energy correction in the two-body problem was presented in our note [6], where for brevity we restricted to a formal scheme of calculations. Here we present in detail somewhat different approach to this problem. It naturally leads to the same result, but allows for an insight into the physical meaning of different contributions. Our main object is again positronium. However, at least to have an extra check on our results we consider the more general case of particles with different masses, m and M.

2 Infrared divergence, Thomson amplitude and Lamb-shift

The origin of the Lamb-shift in hydrogen is closely related to the infrared divergence in the electron scattering on the Coulomb centre. Indeed, at the regularization via the introduction of the photon mass λ, the logarithmic dependence on it of the vertex part (Fig.1a) is cancelled by the analogous dependence of the Bremsstrahlung (Fig.1b). (We use the Coulomb gauge; the dashed line here and below refers to the Coulomb field, the wavy one to a transverse photon.)



(a) (b)

Fig. 1

If there is no acceleration, i.e., at the vanishing momentum transfer q, the radiation vanishes also. Therefore, it is only natural that the infrared part of the vertex correction is proportional to q^2. Indeed, with the account for this correction the potential of the electron interaction with a Coulomb centre is in the momentum representation (see, e.g., [7], §117)

$$V(\vec{q}) = -\frac{4\pi\alpha}{q^2}\left(1 - \frac{\alpha q^2}{3\pi m^2}\log\frac{m}{\lambda}\right). \qquad (1)$$

Of course, in the bound state problem there is no infrared radiation. But the electron here is not on the mass shell, the deviation from it coinciding in the order of magnitude with the binding energy, $\sim m\alpha^2$. On the other hand, the role of the photon mass in the Bremsstrahlung is in fact to fix the minimum possible deviation of the final state invariant mass from that of the free electron. So, in the bound state problem one can put $\lambda \to m\alpha^2$ in formula (1) with the logarithmic accuracy.

Since the typical atomic momentum transfer is $q \sim m\alpha$, the relative magnitude of the correction to the potential, and that of the energy correction as well, constitutes $\alpha^3\log(1/\alpha)$.

More accurately, the discussed radiative correction to the potential with the logarithmic accuracy is in the momentum representation

$$\delta V = \frac{8}{3}\frac{\alpha^2}{m^2}\log\frac{1}{\alpha}. \qquad (2)$$

In the coordinate representation it equals evidently

$$\delta V(\vec{r}) = \frac{8}{3}\frac{\alpha^2}{m^2}\log\frac{1}{\alpha}\delta(\vec{r}). \qquad (3)$$

From it we find with the logarithmic accuracy the known result for the Lamb-shift in hydrogen

$$\delta E_{nl} = \frac{8}{3}\frac{\alpha^2}{m^2}\log\frac{1}{\alpha}|\psi_{n,l}(0)|^2 = \frac{8}{3}\frac{m\alpha^5}{\pi n^3}\log\frac{1}{\alpha}\delta_{l0}. \qquad (4)$$

Here n and l are the principal and orbital quantum numbers of the atomic state.

Let us turn now to a more general case of the Lamb-shift in a system of charged particles of different masses. The essential constituent of this problem is to find the Lamb interaction of two particles with charges e_1, e_2 and masses m_1, m_2. The infrared divergent radiative corrections to the scattering

amplitude for these particles are described by the diagrams of the type 2, 3. In other words, the virtual transverse photon can be absorbed both by the same particle that has emitted it, as well as by another one. For instance, in positronium it could be expected naively that the arising perturbation operator will turn out 4 times larger than that in hydrogen, and the corresponding energy correction (with the account for the two times smaller reduced mass and, correspondingly, eight times smaller value of $|\psi(0)|^2$) will turn out two times smaller.

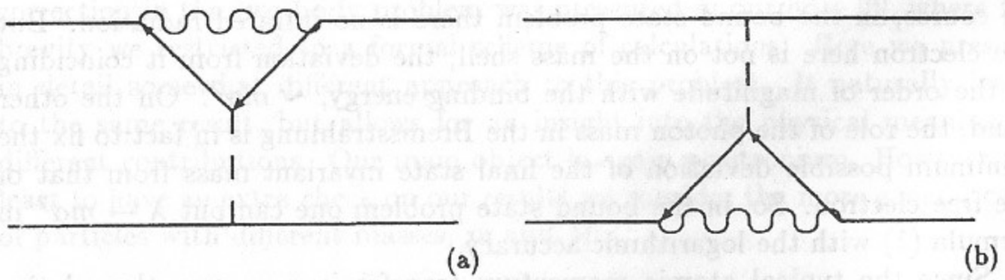


Fig. 2

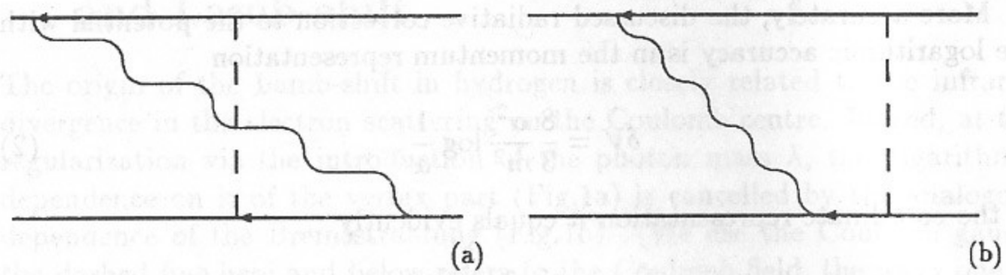


Fig. 3

However, the situation is different by two reasons. The first of them allows for a quite simple physical explanation. The arguments below constitute a modification, as applied to the two-body problem, of the intuitive description of the Lamb-shift in hydrogen ascending to Ref.[8]. Let, due to the vacuum fluctuations of the electromagnetic field, the radius-vector $\vec{r}$ of a charged particle fluctuates as follows: $\vec{r} \to \vec{r} + \vec{\rho}$. Then the interaction potential of the particles 1 and 2, averaged over the fluctuations, constitutes

$$\langle V(\vec{r}_1 - \vec{r}_2 + \vec{\rho}_1 - \vec{\rho}_2)\rangle = V(\vec{r}_1 - \vec{r}_2) + \frac{1}{6}\langle(\vec{\rho}_1 - \vec{\rho}_2)^2\rangle \Delta V(\vec{r}_1 - \vec{r}_2). \quad (5)$$

The mean square fluctuation of the i-th particle coordinate $\langle\rho_i^2\rangle$ is proportional with the logarithmic accuracy to (see [8])

$$\frac{e_i^2}{m_i^2}\int_{m\alpha^2}^{m}\frac{d\omega}{\omega} = 2\frac{e_i^2}{m_i^2}\log\frac{1}{\alpha}, \quad (6)$$

(with the accepted accuracy one can put the binding energy equal to $m\alpha^2$). The corresponding contribution to the interaction operator is described in fact by diagrams 2a,b. Let us note that the correction originating from the mean square fluctuation of the nucleus coordinate is suppressed by the inverse nucleus mass squared, $\sim 1/M^2$, and will be neglected below.

Let us consider now the average $-2\langle\vec{\rho}_1\vec{\rho}_2\rangle$. It does not vanish only for the fluctuations of the wave length exceeding the size of the atomic system which is $m\alpha$ with the logarithmic accuracy. For smaller wave lengths, or higher frequencies, $\omega > m\alpha$, the coordinate fluctuations are uncorrelated, i.e., $\langle\vec{\rho}_1\vec{\rho}_2\rangle = 0$. In other words, the upper limit for the integration over frequencies of the virtual quanta in the correlator $\langle\vec{\rho}_1\vec{\rho}_2\rangle$ is not m, as in the formula (6), but $m\alpha$. Therefore, with the logarithmic accuracy the contribution of this correlator is

$$-2\langle\vec{\rho}_1\vec{\rho}_2\rangle \propto -2\frac{e_1e_2}{m_1m_2}\int_{m\alpha^2}^{m\alpha}\frac{d\omega}{\omega} = -2\frac{e_1e_2}{m_1m_2}\log\frac{1}{\alpha}. \quad (7)$$

We will be interested in the recoil corrections to the Lamb-shift, $\sim 1/M$, so this contribution can refer to the electron–nucleus interaction also. It can be easily seen that the discussed correlator $-2\langle\vec{\rho}_1\vec{\rho}_2\rangle$ corresponds to diagrams 3a,b.

Thus, the perturbation operator δV_C, generated by diagrams 2, 3 with the Coulomb interaction, equals

$$\delta V_C(\vec{r}) = 8\frac{\alpha^2}{m^2}\log\frac{1}{\alpha}\delta(\vec{r}) \quad (8)$$

for positronium;

$$\delta V_C(\vec{r}_{12}) = -\frac{8}{3}\frac{\alpha^2}{m^2}\log\frac{1}{\alpha}\delta(\vec{r}_{12}), \quad (9)$$

for atomic electrons;

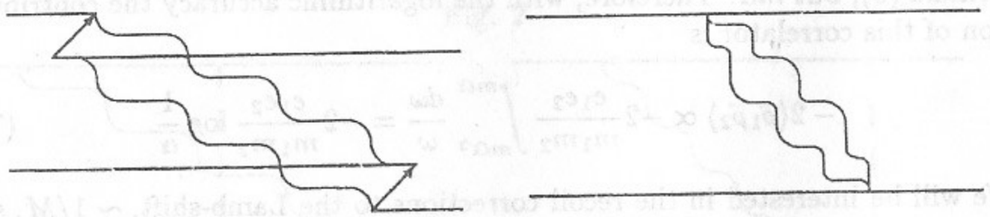
$$\delta V_C(\vec{r}) = \frac{8}{3}\frac{Z\alpha^2}{m^2}\log\frac{1}{\alpha}(1 + Zm/M)\delta(\vec{r}) \quad (10)$$

for the electron-nucleus interaction up to the first order in $1/M$.

Certainly, to the given order in α we have considered all the diagrams with a true infrared divergence which is cut off at $m\alpha^2$. The above arguments, however, demonstrate that in diagrams 2, 3 with a double exchange there is a contribution cutting them off effectively at the frequencies larger than the typical momentum transfer $q \sim m\alpha$. It is natural therefore to consider in the same region $m\alpha < \omega < m$ diagrams with the exchange by two magnetic quanta. With the accepted accuracy one can neglect in these diagrams the three-dimensional external momenta of both particles. It is well-known that in this case, in the totally nonrelativistic limit, the scattering of a transverse photon is described by the contact operator

$$V = \frac{e_i^2}{2m_i}\vec{A}^2. \tag{11}$$

Correspondingly, the double magnetic exchange is reduced to the simple diagram 4 with the vertices generated by the operator above. The computation



(a) (b)

Fig. 4

of the effective interaction arising in this way is of no difficulty. In the coordinate representation it is equal with the logarithmic accuracy to

$$\delta V_M(\vec{r}_{12}) = -\frac{2e_1^2e_2^2}{m_1m_2}\log\frac{1}{\alpha}\delta(\vec{r}_{12}). \tag{12}$$

which is

$$-\frac{2\alpha^2}{m^2}\log(1/\alpha)\delta(\vec{r}_{12}) \tag{13}$$

for electron-electron and electron-positron interaction and

$$-\frac{2Z^2\alpha^2}{mM}\log(1/\alpha)\delta(\vec{r}) \tag{14}$$

for the electron-nucleus one.

The total Lamb-shift operator in positronium reads

$$\delta V(\vec{r}) = \delta V_C(\vec{r}) + \delta V_M(\vec{r}) = \frac{6\alpha^2}{m^2}\log\frac{1}{\alpha}\delta(\vec{r}), \tag{15}$$

and the corresponding energy shift for a level with quantum numbers n, l equals

$$\delta E_{nl} = \frac{6\alpha^2}{m^2}\log\frac{1}{\alpha}|\psi_{nl}(0)|^2 = \frac{3}{4}\frac{m\alpha^5}{\pi n^3}\log\frac{1}{\alpha}\delta_{l0}. \tag{16}$$

The answer obtained reproduces the logarithmic in α part of the exact result for the Lamb-shift in positronium found many years ago in Ref. [9] (see also [10, 11]). The numerical difference of the logarithmic result (16) from the exact one is small for parapositronium (the total spin $S = 0$): $\log 1/\alpha = 4.9$ is substituted by 4.7 in the exact result. The difference for orthopositronium ($S = 1$) is larger: the coefficient analogous to $\log 1/\alpha = 4.9$, constitutes 3.0 in the exact result.

Within the presented approach to the logarithmic contribution to the Lamb-shift in positronium, this contribution independence of S becomes quite obvious. For the correction $\delta V_C(\vec{r})$ related to the Coulomb interaction it is spin-independence of the infrared radiation. For the correction $\delta V_M(\vec{r})$ due to the double magnetic exchange it is spin-independence of the Thomson amplitude, i.e., of the nonrelativistic limit of the Compton scattering.

The total atomic Lamb-shift operator up to the first order in $1/M$ with the logarithmic accuracy reads

$$\begin{aligned}\delta V \;=\; & \frac{8}{3}\frac{Z\alpha^2}{m^2}\log(1/\alpha)\sum_i\delta(\vec{r}_i) - \frac{14}{3}\frac{\alpha^2}{m^2}\log(1/\alpha)\sum_{i<j}\delta(\vec{r}_{ij}) \\ & +\frac{2}{3}\frac{Z^2\alpha^2}{mM}\log(1/\alpha)\sum_i\delta(\vec{r}_i).\end{aligned} \tag{17}$$

We assume here that Z is not too large, so that with our accuracy we still can neglect $\log Z$. The electron-electron term in this expression was obtained many years ago for the helium [12, 13, 14], the recoil term $\sim 1/M$ is known for the hydrogen [15].

In conclusion of this section let us emphasize that the logarithmic part of the Lamb-shift is completely described by the quantum electrodynamics of nonrelativistic particles. In other words, it is a true radiative correction to the nonrelativistic bound-state problem.

3 Corrections to the positronium decay rate

3.1 Logarithmic corrections

The theoretical value for the orthopositronium decay rate, as calculated in Refs. [16, 17, 18, 19], constitutes

$$\Gamma_{th} = m\alpha^6 \frac{2(\pi^2-9)}{9\pi}\left[1 - 10.28\frac{\alpha}{\pi} - \frac{1}{3}\alpha^2 \log\frac{1}{\alpha}\right] = 7.03830\mu s^{-1}. \qquad (18)$$

What is the origin of the logarithmic term in this expression? If it were of the infrared nature, its relative magnitude would be $\alpha^3\log(1/\alpha)$, as it was argued above, but not $\alpha^2\log(1/\alpha)$. One more possible source of logarithmic corrections is the singularity of the relativistic wave function at $r \to 0$, well-known from the solution of the Dirac equation for the electron in a Coulomb field. Indeed, at the total electron angular momentum $j = 1/2$ this solution behaves at $r \to 0$ as $\psi \sim (m\alpha r)^{\sqrt{1-\alpha^2}-1} \approx 1 - \frac{\alpha^2}{2}\log(m\alpha r)$ (see, e.g., Ref. [7], §36). Of course, we cannot solve exactly the relativistic two-body problem for positronium. But the logarithmic nature of the correction we are looking for allows us to restrict to the momenta range $m\alpha \ll p \ll m$ where relativistic effects can be treated as perturbations. So, we will consider these $(v/c)^2$ corrections via the Breit equation (cf. Ref.[7], §83).

The part of the Breit Hamiltonian (BH) that corresponds to the relativistic corrections to the dispersion law of the particles and to their Coulomb interaction,

$$V_1 = -\frac{p^4}{4m^3} + \frac{\pi\alpha}{m^2}\delta(\vec{r}), \qquad (19)$$

can be easily transformed to

$$V_1 \to -\frac{\alpha^2}{4mr^2} - \frac{\alpha}{2m^2r^2}\partial_r \qquad (20)$$

In (20) we retain only those terms which are sufficiently singular at $r \to 0$ to influence the behavior of the wave function at small distances. Let us note that

$$\partial_r R_{n,0}|_{r=0} = -\frac{1}{a}R_{n,0}|_{r=0}. \qquad (21)$$

where $R_{n,0}$ is the radial wave function of the Coulomb state with the principal quantum number n and vanishing orbital angular momentum l. The Bohr radius a in positronium is $2/m\alpha$, so that with our accuracy V_1 vanishes. The conclusion that this part of relativistic effects does not work in the $\alpha^2\log(1/\alpha)$ corrections to the positronium decay rate, was made already in [18].

The next spin-independent term in the BH

$$V_2 = -\frac{\alpha}{2m^2r}\left(p^2 + \frac{1}{r^2}\vec{r}(\vec{r}\vec{p})\vec{p}\right), \qquad (22)$$

describes the magnetic electron-positron interaction due to the orbital motion. At $l = 0$ it can be easily transformed to

$$V_2 = -\frac{\alpha}{m^2r}\left(p^2 + \frac{1}{r}\partial_r\right). \qquad (23)$$

Using the same substitution (see (21)), $\partial_r \to -m\alpha/2$, and retaining only the terms singular as r^{-2}, we get

$$V_2 \to -\frac{\alpha^2}{2mr^2}. \qquad (24)$$

Now one can show in a straighforward way that the S-wave radial function, instead of being constant at $r \to 0$, behaves as

$$\psi|_{r\ll a} \propto \left(\frac{r}{a}\right)^{-\alpha^2/2} \approx 1 - \frac{\alpha^2}{2}\log(m\alpha r). \qquad (25)$$

The corresponding relative correction to the particle density at the distances $r \sim 1/m$, where the annihilation takes place, and therefore to the decay rate itself is

$$\alpha^2\log(1/\alpha). \qquad (26)$$

The spin-orbit part of the BH is irrelevant to our problem since it does not work at all in S-states. The part of the BH that describes the tensor spin-spin interaction does not contribute to the decay rate to the accuracy considered. Indeed, being applied to the S-state, this interaction either annihilates it (in the singlet case) or transforms it (in the triplet one) into the D-state with the same total angular momentum. But the annihilation from a D-state is strongly hampered. So, we are left with the contact spin-spin interaction

$$V_3 = \frac{\pi\alpha}{m^2}\left(\frac{7}{3}S(S+1) - 2\right)\delta(\vec{r}), \qquad (27)$$

It originates from both the magnetic spin-spin interaction, and from the one-quantum annihilation contribution which does not vanish in the triplet case only.

We will solve the corresponding wave equation

$$\left\{-\frac{1}{m}\Delta-\frac{\alpha}{r}-E+A\frac{\pi\alpha}{m^2}\delta(\vec{r})\right\}\psi(\vec{r})=0\,, \tag{28}$$

where

$$A=\frac{7}{3}S(S+1)-2=\begin{cases}-2, & S=0\\ 8/3, & S=1\end{cases}$$

by iterations. In its turn in the inhomogeneous equation

$$\left\{\Delta+\frac{m\alpha}{r}-\frac{m^2\alpha^2}{4}\right\}\delta\psi(\vec{r})=A\frac{\pi\alpha}{m}\delta(\vec{r})\psi(0) \tag{29}$$

we omit the term $-m^2\alpha^2/4$, regular at $r\to 0$, and treat $m\alpha/r$, which is less singular than Δ, as a perturbation. Simultaneously this procedure is also an expansion in α.

The solution of the zeroth order in α,

$$\delta_0\psi=-A\frac{\alpha}{4mr}\psi(0)\,, \tag{30}$$

at the distances $r\sim 1/m$ of interest to us leads evidently to the correction $\sim\alpha$ to the decay rate. The calculation of the corresponding numerical factor is beyond our accuracy. The first-order solution,

$$\delta_1\psi=A\frac{\alpha^2}{4}\log(m\alpha r)\psi(0)\,, \tag{31}$$

gives evidently at $r\sim 1/m$ the relative correction $A\alpha^2\log\alpha/4$ to $\psi(\vec{r})|_{r\sim 1/m}$ and the correction

$$\alpha^2\log(1/\alpha)\begin{cases}1, & S=0\\ -4/3, & S=1\end{cases}. \tag{32}$$

to $|\psi(\vec{r})|^2_{r\sim 1/m}$ and to the decay rate.

The total correction to the decay rate includes both (26) and (32) and thus constitutes

$$\frac{\delta\Gamma}{\Gamma}\approx\alpha^2\log(1/\alpha)\begin{cases}2, & S=0\\ -1/3, & S=1\end{cases}. \tag{33}$$

One may feel dissatisfied with the above treatment of the spin-spin interaction (27). Have not we buried some contribution $\sim\alpha^2\log(1/\alpha)$ in the

linearly divergent, at $r\to 0$, term $\delta_0\psi$? To reject the suspicion we will calculate the discussed conribution of V_3 in a straightforward way, following the line of reasoning close to that of Refs.[20, 21]. Let us consider the diagram of Fig.5. Its left vertex corresponds to the interaction (27); in the momentum

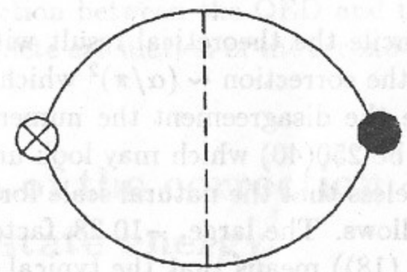


Fig. 5

representation this amplitude is $-A\pi\alpha/m^2$. The right vertex is the annihilation one. In the momentum representation it is, to our accuracy, a constant, correction to which we are looking for. The Coulomb attraction (the dashed vertical line) supplements the imaginary part of the corresponding one-loop diagram by the well-known Coulomb factor $\pi\alpha/(2v)$ where $2v$ is the relative velocity of the particles. So, the imaginary part of the diagram becomes a constant:

$$ImM=-A\frac{\pi\alpha}{m^2}\frac{v}{16\pi}4m^2\frac{\pi\alpha}{2v}=-\frac{\pi\alpha^2}{8}A. \tag{34}$$

The dispersion integral

$$ReM=\frac{1}{\pi}\int_0^m\frac{dE\,ImM}{E-E_0}\bigg|_{E_0\approx-\frac{m\alpha^2}{4}}\approx-\frac{A}{4}\alpha^2\log\frac{1}{\alpha} \tag{35}$$

is in fact the correction to the annihilation amplitude we are looking for. The correction to the decay rate is evidently twice as large and coincides with (32).

Our result for orthopositronium coincides with that found earlier. As for the disagreement with earlier results for parapositronium [18, 22], its origin can be easily elucidated in the case of Ref.[18]. In that paper the result for parapositronium was obtained from the result for orthopositronium merely by neglecting the single-photon annihilation contribution without changing the contact magnetic interaction.

3.2 Nonlogarithmic aside

The theoretical result (18) for the orthopositronium decay rate is in a strong disagreement with its experimental value [5]

$$\Gamma_{exp} = 7.0482(16)\mu s^{-1}. \qquad (36)$$

Can one hope to reconcile the theoretical result with the experimental one by including into (18) the correction $\sim (\alpha/\pi)^2$ which has not been calculated until now? To resolve the disagreement the numerical factor at $(\alpha/\pi)^2$ in this correction should be 250(40) which may look unreasonably large.

We believe nevertheless that the natural scale for this factor is about 100. The argument is as follows. The large, -10.28, factor at the α/π correction to the decay rate (see (18)) means that the typical magnitude of the factor at the α/π correction to the decay amplitude is roughly 5. Correspondingly, this correction squared contributes about $30(\alpha/\pi)^2$ to the decay rate.

In fact, quite recent numerical calculations [23] have given factor 28.8 ± 0.2 at $(\alpha/\pi)^2$ in this contribution.

This contribution is described by diagram 6a for the probability. Here the wavy lines refer to the decay photons, thin vertical line means the unitarity cross-section, wavy lines to the left and to the right of this cross-section denote symbolically the whole set of the first-order corrections to the decay amplitude.

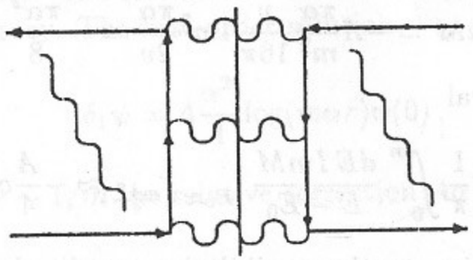


Fig. 6a

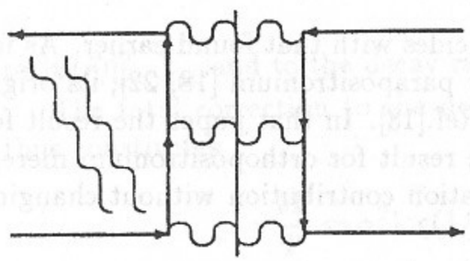


Fig. 6b

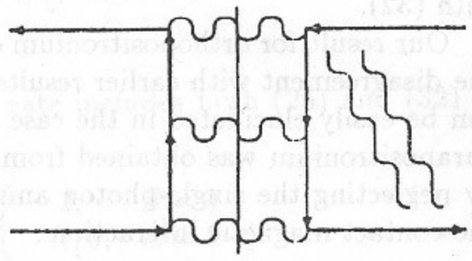


Fig. 6c

It is only natural to expect that each of the diagrams 6b,c where both radiative-correction quanta are to the left or to the right of the cross-section, makes about the same contribution, $30(\alpha/\pi)^2$, to the decay rate. In other words, the natural scale for the total second-order radiative correction to the decay rate is indeed $100(\alpha/\pi)^2$. We believe therefore that it is premature to talk about the contradiction between the QED and the experimental result (36) now, until the complete calculation of the second-order correction to the decay rate is done.

4 Calculation of the correction $\sim \alpha^4\log(1/\alpha)$ to the bound state energy

In this section we will calculate the correction $\sim \mu\alpha^6\log(1/\alpha)$ to the energy of a bound state of the two particles with masses m and M (here and below $\mu = mM/(m+M)$ is the reduced mass). Considering particles of different masses allows us to have an additional control of the calculations, in particular by means of the comparison with the results for the hydrogen atom [24] obtained to the first order in m/M at $m \ll M$.

The direct approach to the problem consists in the solution of the Bethe-Salpeter equation with the expansion in v/c in the momenta region which contributes to the energy correction of interest to us. Thus an equation arises which resembles the Breit equation, but includes as compared to the latter not only the corrections to the Hamiltonian of the order v^2/c^2, but those $\sim v^4/c^4$ as well. Let us emphasize that such an expansion is possible since we are interested in the logarithmic energy corrections which can be calculated in the nonrelativistic momenta region $p \ll \mu$. This equation is satisfied by the solution of the Bethe-Salpeter equation projected onto the positive-energy states and integrated over the relative energy of the two particles.

However, the simplest way to derive the equation discussed and the corresponding energy correction is to extend the approach used above for the derivation of the $\alpha^2\log(1/\alpha)$ correction to the positronium decay rate. Indeed, if we take for instance the contact magnetic interaction which gives a correction $\sim \alpha^2$ to the positronium energy, in the momentum region $m\alpha \ll p \ll m$ it is as contact as the annihilation kernel. So, proceeding along the lines used in Sec. 3.1, we will get an energy correction $\sim \alpha^4\log(1/\alpha)$. It corresponds to the iteration of the Breit corrections $\sim (v/c)^2$. Certainly, we have to include also the corresponding contributions $\sim (v/c)^4$, as well as those of the irreducible diagrams for the scattering amplitude. Our regular approach to the

problem will follow in fact the usual derivation of the Breit equation (cf. [7], §83). In this method the correction to the effective interaction Hamiltonian is found by the expansion in v/c of the on-mass-shell scattering amplitude. Such an approach is in our opinion not only simpler, but allows also for the transparent physical interpretation of different contributions to the correction discussed.

In the nonrelativistic region of interest to us it is convenient to use noncovariant perturbation theory and the Coulomb gauge. Since this technique is not too common, let us present the corresponding Feynman rules. We assume that the particles have opposite charges. Then the exchange by a Coulomb quantum of a momentum $\vec{q}$ is described by a factor $-4\pi\alpha/q^2$, the exchange by a magnetic quantum gives the factor $-\alpha_i \otimes \alpha_j S_{ij}(\vec{q}) 4\pi\alpha/2q$, where $S_{ij}(\vec{q}) = \delta_{ij} - q_i q_j/q^2$ is the photon polarization density matrix. Let us recall that in the noncovariant perturbation theory the frequency of an intermediate photon equals its momentum. The projectors onto the positive and negative energy states of a fermion with a momentum $\vec{p}$ are correspondingly

$$\Lambda_\pm(\vec{p}) = \frac{1}{2}\left(1 \pm \frac{\vec{\alpha}\vec{p} + \beta m}{\omega_p}\right),$$

$\omega_p = \sqrt{m^2 + p^2}$. The projector Λ_- enters an expression for the effective potential with the minus sign. Any intermediate state introduces factor $(E - E_n + i0)^{-1}$ where E_n is the energy of an intermediate state and E is the energy of the system. As distinct from Sec.3.1, all the calculations are performed in the momentum representation.

4.1 Pure Coulomb exchange

Let us start with the correction due to the Coulomb exchange. We will work in the centre of mass frame where the particle of a mass m has a momentum $\vec{p}$, that of a mass M momentum $-\vec{p}$, $E \approx m + M$. Presenting the Dirac spinors as

$$u = \sqrt{\frac{\omega_p + m}{2\omega_p}}\left(1 + \frac{\vec{\alpha}\vec{p}}{\omega_p + m}\right)\begin{pmatrix}\phi\\0\end{pmatrix},$$

we find easily that the correction to the Hamiltonian can be written as

$$V_C(\vec{p},\vec{p}') = V_C^{(2)}(\vec{p},\vec{p}') + V_C^{(4)}(\vec{p},\vec{p}').$$

The second-order Breit correction $V_C^{(2)}(\vec{p},\vec{p}')$ including in particular the relativistic correction to the dispersion law is

$$\begin{aligned} V_C^{(2)}(\vec{p},\vec{p}') &= -\frac{p^4}{8}\left(\frac{1}{m^3} + \frac{1}{M^3}\right)(2\pi)^3\delta(\vec{q}) + \frac{\pi\alpha}{2}\left(\frac{1}{m^2} + \frac{1}{M^2}\right) \\ &\quad -\frac{i\pi\alpha}{q^2}\vec{p}\times\vec{p}'\left(\frac{\vec{\sigma}}{m^2} + \frac{\vec{\sigma}'}{M^2}\right) \end{aligned} \tag{37}$$

where $\vec{\sigma}$ and $\vec{\sigma}'$ are the Pauli matrices for the first and second particles, $\vec{q} = \vec{p}' - \vec{p}$. The next in v^2/c^2 relativistic correction to the Breit Hamiltonian due to the Coulomb exchange constitutes

$$\begin{aligned} V_C^{(4)}(\vec{p},\vec{p}') &= -\frac{\pi\alpha}{16}\left[\frac{q^2}{m^2M^2} + 3\left(\frac{1}{m^4} + \frac{1}{M^4}\right)(p^2 + p'^2)\right] \\ &\quad -\frac{\pi\alpha}{32q^2}\left[5\left(\frac{1}{m^4} + \frac{1}{M^4}\right)(p^2 - p'^2)^2\right. \\ &\quad \left. -\frac{8}{m^2M^2}(\vec{\sigma},\vec{p}\times\vec{p}')(\vec{\sigma}',\vec{p}\times\vec{p}')\right]. \end{aligned} \tag{38}$$

The energy correction $\delta E'_C$ due to the operator $V_C^{(4)}$ equals its mean value over nonrelativistic wave functions. Calculating the integral with the logarithmic accuracy in the region $\mu\alpha \ll p, p' \ll \mu$, we obtain

$$\delta E'_C = \epsilon\mu^2\left[\frac{5}{4}\left(\frac{m}{M^3} + \frac{M}{m^3}\right) + \frac{\vec{\sigma}\vec{\sigma}'}{6mM}\right], \tag{39}$$

wher $\epsilon \equiv \alpha^6(\mu^3/mM)\log(1/\alpha)\delta_{l0}/n^3$. The logarithmic contribution arises only from the last two terms in (38). Let us note that at the expansion in p/m we get operators which do not lead to contributions logarithmic in α, but lead to momentum integrals diverging at the upper limit linearly, not logarithmically (e.g., the term $\sim p^6$ from the expansion of ω_p). Those operators give rise to corrections $\sim \alpha^5$ and hence can be omitted.

The contribution to the energy $\sim \mu\alpha^6\log(1/\alpha)$ from the Breit Hamiltonian $V_C^{(2)}$ arises in the second order perturbation theory, i.e. at the iteration of $V_C^{(2)}$:

$$\delta E''_C = \int \frac{d\vec{p}d\vec{p}'d\vec{\mathcal{P}}d\vec{\mathcal{P}}'}{(2\pi)^{12}}\psi^*(\vec{p})V_C^{(2)}(\vec{p},\vec{\mathcal{P}})G(\vec{\mathcal{P}},\vec{\mathcal{P}}'|E)V_C^{(2)}(\vec{\mathcal{P}}',\vec{p}')\psi(\vec{p}'). \tag{40}$$

Here $G(\vec{\mathcal{P}},\vec{\mathcal{P}}'|E)$ is the nonrelativistic Coulomb Green's function. In fact we need the zeroth and the first terms of its expansion in α:

$$G^{(0)}(\vec{\mathcal{P}},\vec{\mathcal{P}}'|E) = \frac{(2\pi)^3\delta(\vec{\mathcal{P}}-\vec{\mathcal{P}}')}{E-P^2/2\mu}, \tag{41}$$

$$G^{(1)}(\vec{\mathcal{P}},\vec{\mathcal{P}}'|E) = \frac{1}{E-P^2/2\mu}\,\frac{-4\pi\alpha}{(\vec{\mathcal{P}}-\vec{\mathcal{P}}')^2}\,\frac{1}{E-P'^2/2\mu}. \tag{42}$$

Simple counting of the momenta powers in the integrand of (40) demonstrates that only the function $G^{(1)}$ contributes to the logarithm. So, we find with the logarithmic accuracy:

$$\delta E_C'' = -\epsilon\frac{\mu^2}{4m^3M^3}(M-m)^4. \tag{43}$$

This contribution is evidently spin-independent, since the only spin-dependent term in the expression (37) is the spin-orbit interaction which is absent in the s-state.

Then, there is the contribution due to the negative-energy intermediate states. It is described by diagrams of the kind of Fig. 7. For "heavy" intermediate states, where $|E-E_n|\sim 2m, 2M \gg p$, the line of the inverse

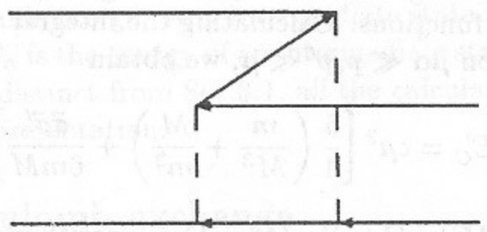


Fig. 7

direction contracts into a point. Diagrams like that on Fig.7 lead to the effective operator

$$\tilde{V}_C(\vec{p},\vec{p}') = -\frac{(\pi\alpha)^2}{8}\left(\frac{1}{m^3}+\frac{1}{M^3}\right)q, \tag{44}$$

while all other diagrams with heavy intermediate states can be shown to be non-logarithmic. Perturbation (44) gives rise to the energy shift

$$\delta\tilde{E}_C = -\epsilon\left(\frac{m}{M}+\frac{M}{m}-1\right). \tag{45}$$

Adding up (39), (43) and (45), we get the total energy correction due to the Coulomb exchange:

$$\delta E_C = \epsilon\frac{\mu^2}{mM}\left(-\frac{3}{2}+\frac{\vec{\sigma}\vec{\sigma}'}{6}\right). \tag{46}$$

In complete accord with the exact solution of the Dirac equation in the Coulomb field, this correction, logarithmic in α, vanishes when one of the particles becomes infinitely heavy. The spin-dependent part of expression (46) has been found previously in Refs. [25, 2, 26].

4.2 Single magnetic exchange

Let us consider now the contribution to the energy due to the exchange by one magnetic photon. Before taking the expectation value over the Dirac spinors the interaction operator looks as follows

$$V_M = -\frac{4\pi\alpha}{2q}S_{ij}(\vec{q})\alpha_i\otimes\alpha_j\left[\frac{1}{E-\Omega_{p'}-\omega_p-q}+\frac{1}{E-\Omega_p-\omega_{p'}-q}\right]. \tag{47}$$

We are interested in the region of the variables $q \gg |\Delta E| = |E-\Omega_{p'}-\omega_p|$. The energy corrections arise both from expanding in v^2/c^2 of the expectation value of the operator $\alpha_i\otimes\alpha_j$, as well as from the expansion of the denominators in the ratio $\Delta E/q$. Taking the leading terms in the numerator and denominator, we obtain the well-known magnetic contribution to the Breit Hamiltonian:

$$V_M = \frac{\pi\alpha}{mMq^2}\left[4\frac{(\vec{p}\times\vec{p}')^2}{q^2}-q^2\vec{\sigma}\vec{\sigma}'+(\vec{q}\,\vec{\sigma})(\vec{q}\,\vec{\sigma}')+2i(\vec{p}\times\vec{p}',\vec{\sigma}+\vec{\sigma}')\right]. \tag{48}$$

To find the contribution of this operator to the energy shift, one has to calculate the second order of the perturbation theory taking V_M as one of the perturbations and $V_C^{(2)}$ as the other. Now the logarithmic contributions arise due to the functions $G^{(0)}$ and $G^{(1)}$. These energy corrections are

$$\delta E_{1M}^{(1)} = -2\epsilon\left(1-\frac{4\mu^2}{mM}\right), \tag{49}$$

$$\delta E_{1M}^{(2)} = \frac{2}{3}\epsilon\vec{\sigma}\vec{\sigma}'\left(1-\frac{4\mu^2}{mM}\right). \tag{50}$$

Retaining in the matrix element of the operator $\alpha_i\otimes\alpha_j$ the next term of the expansion in v^2/c^2, we easily get for this relativistic correction:

$$\delta E_{1M}^{(3)} = 4\epsilon\left(1+\frac{\vec{\sigma}\vec{\sigma}'}{6}\right)\left(1-\frac{2\mu^2}{mM}\right). \tag{51}$$

Let us turn to the expansion in $\Delta E/q$ of the denominator in (47). Its first term gives the contribution to the Lamb-shift $\sim \mu\alpha^5\log(1/\alpha)$. This correction originates from the region $p, p' \sim \mu\alpha$, $\mu\alpha^2 \ll q \ll \mu\alpha$. The next term of the expansion gives the necessary contribution due to the region $\mu\alpha \ll p, p' \ll \mu$:

$$\delta E^{(4)}_{1M} = 2\epsilon\frac{\mu^2}{mM}\left(1+\frac{2}{3}\vec{\sigma}\vec{\sigma}'\right). \qquad (52)$$

Now we have to consider the Coulomb exchange between the emission and absorption of the magnetic photon. Both fermions stay in positive-energy states here. Counting of the momenta powers in the integrand demonstrates that the logarithmic contribution originates from diagrams 8a,b with the exchange by one and two Coulomb quanta in the intermediate state. The

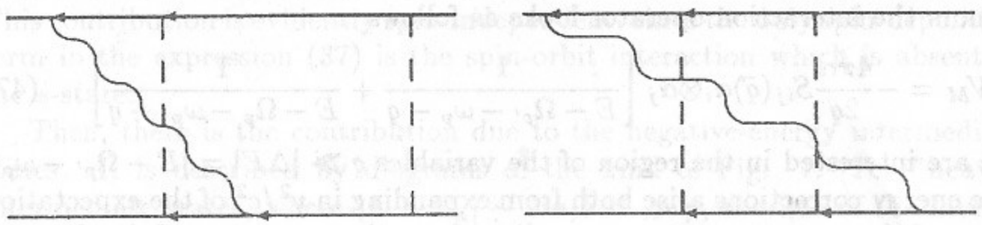


Fig. 8

energy denominators here are of the type $E - E' - k$ where E' is the energy of the particles in the intermediate state and k is the photon energy. The logarithmic contribution arises from the energy region $k \gg |E - E'|$. Therefore the denominators can be expanded in the ratio $\Delta E/k$. In the case of the single Coulomb exchange the leading term of the expansion gives the Lamb-shift correction $\sim \mu\alpha^5\log(1/\alpha)$. One could expect that the energy correction of the necessary order of magnitude, $\mu\alpha^6\log(1/\alpha)$, arises if the next term of the expansion in $\Delta E/k$, is included. It would correspond evidently to the correction of the first order in v/c to the Lamb-shift . However, there is one more correction of the same order of magnitude originating from diagrams 8b. Its meaning is the expansion of the Green's function not in $\Delta E/k$, but in the ratio of the Coulomb potential to k, which is also $\sim v/c$. Meanwhile a relativistic correction should start from $v^2/c^2 \sim \alpha^2$. Therefore the total contribution of diagrams 8a and 8b to the energy correction we are interested in, vanishes. The vanishing of this contribution is also confirmed by the direct calculation [6]. (Unfortunately, in our paper [6] Figs.2d and 2c

were interchanged).

At last, let us consider the energy shift due to the transitions into the negative-energy states. Corresponding diagrams of the noncovariant perturbation theory are presented in Fig.9. As it was mentioned above, the line corresponding to a particle of negative energy can be contracted into a

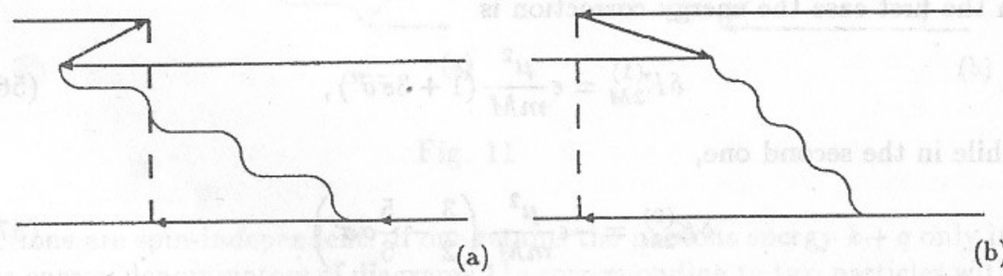


Fig. 9

point. In this case an effective vertex arises corresponding to the emission of a Coulomb and magnetic quanta. Such an interaction is evidently of a spin nature. After simple calculations we find the effective operator corresponding to diagrams in Fig.9,

$$V^{(5)}_M = -\frac{(\pi\alpha)^2 q}{6\mu mM}\vec{\sigma}\vec{\sigma}', \qquad (53)$$

and the energy correction induced by it,

$$\delta E^{(5)}_{1M} = -\frac{4}{3}\epsilon\vec{\sigma}\vec{\sigma}'. \qquad (54)$$

Adding up the corrections $\delta E^{(1-5)}_{1M}$ we get the total contribution to the energy from the single magnetic exchange:

$$\delta E_{1M} = \epsilon\left(2 + 2\frac{\mu^2}{mM} - \frac{8}{3}\frac{\mu^2}{mM}\vec{\sigma}\vec{\sigma}'\right). \qquad (55)$$

Its spin-dependent part was calculated previously [2].

One can check also that the anomalous magnetic moment of the heavy particle does not influence the spin-independent term in this correction to the first order in $1/M$ which means that this contribution refers also to hydrogen. Indeed, the first term in the brackets agrees with the corresponding correction for hydrogen found recently in Ref.[24].

4.3 Double magnetic exchange

Our consideration of the double-magnetic-exchange contribution to the energy shift will start from the second-order perturbation theory in V_M (cf. formula (40), where V_M should be substituted now for $V_C^{(2)}$). Again we have to take into account in the Green's function G the terms $G^{(0)}$ and $G^{(1)}$ only. In the first case the energy correction is

$$\delta E_{2M}^{(1)} = \epsilon \frac{\mu^2}{mM}(1+3\vec{\sigma}\vec{\sigma}'), \tag{56}$$

while in the second one,

$$\delta E_{2M}^{(2)} = -\epsilon \frac{\mu^2}{mM}\left(\frac{3}{2} - \frac{5}{6}\vec{\sigma}\vec{\sigma}'\right). \tag{57}$$

Let us consider now the contributions of the negative-energy states, starting from the case presented in Fig.10 when only one particle goes over into

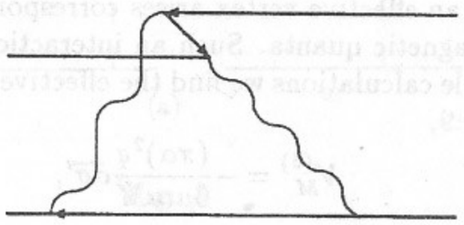


Fig. 10

the negative-energy state. To our approximation such a zigzag in a diagram contracts into a vertex described by the two-photon operator $e^2\vec{A}^2/2m$ of the nonrelativistic electrodynamics. Since this vertex is spin-independent, the perturbation operator originating from the diagrams presented in Fig.10 is also spin-independent:

$$V_{2M}^{(3)} = -\frac{(\pi\alpha)^2 q}{8\mu m M}. \tag{58}$$

The corresponding energy correction equals

$$\delta E_{2M}^{(3)} = -\epsilon. \tag{59}$$

Let us consider now the case when both particles go over into the negative-energy states (see diagrams 11). By the same reasons as above, these contri-

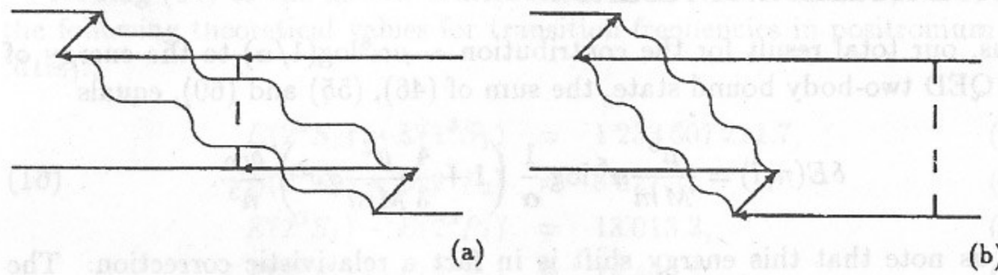


Fig. 11

butions are spin-independent. If one retains the photons energy $k+q$ only in the energy denominators of diagrams 11a corresponding to two particles with positive energies, the Lamb-shift contribution (12) of the order $\mu\alpha^5\log(1/\alpha)$ arises. As well as in the case of the single magnetic excnange, the corrections to the Lamb-shift of the first order in v/c arising due to the next term of the expansion of diagrams 11a in $\Delta E/(k+q)$, are exactly cancelled with the next correction of the same order of magnitude originating from diagrams 11b, its meaning being the expansion of the Green's function in the ratio of the Coulomb potential to $k+q$. Vanishing of the total contribution of diagrams 11 to the energy correction $\sim \mu\alpha^6\log(1/\alpha)$ is also confirmed by the direct calculation (see [6]).

Besides, it has been checked that the diagrams where magnetic quanta are emitted and absorbed by the same particle, do not contribute to the correction of interest to us.

Therefore, the total contribution of the double magnetic exchange equals

$$\delta E_{2M} = \epsilon\left(-1 - \frac{1}{2}\frac{\mu^2}{Mm} + \frac{23}{6}\frac{\mu^2}{Mm}\vec{\sigma}\vec{\sigma}'\right). \tag{60}$$

The spin-dependent part of this expression was found previously in Ref.[2].

Again, the spin-independent term to the first order in $1/M$ refers also to hydrogen since the diagrams with $\vec{A}^2$ are evidently independent both of the spin and of the possible anomalous magnetic moment. Indeed, the first term in the brackets, -1, agrees with the result for hydrogen obtained in Refs. [24, 27].

4.4 Discussion of results

Thus, our total result for the contribution $\sim \mu\alpha^6\log(1/\alpha)$ to the energy of the QED two-body bound state, the sum of (46), (55) and (60), equals

$$\delta E(n,l) = \frac{\mu^3}{Mm}\alpha^6 \log\frac{1}{\alpha}\left(1+\frac{4}{3}\frac{\mu^2}{Mm}\vec{\sigma}\vec{\sigma}'\right)\frac{\delta_{l,0}}{n^3}. \qquad (61)$$

Let us note that this energy shift is in fact a relativistic correction. The relativistic origin of the contributions arising to the second order in the terms $\sim v^2/c^2$ in the Breit equation and to the first order in the corrections $\sim v^4/c^4$, is self-evident. For other contributions, due to negative-energy states, this assertion is somewhat more a matter of convention.

As for positronium, one should add to (61) the annihilation contribution [18],

$$\delta E_a(n,l) = \frac{1}{96}m\alpha^6 \log\frac{1}{\alpha}(3+\vec{\sigma}\vec{\sigma}')\frac{\delta_{l,0}}{n^3}. \qquad (62)$$

Since the annihilation operator is reduced with the necessary accuracy to the same form as the contact magnetic spin-spin interaction in the Breit equation (see [7], §83), the calculation of this correction can be also easily performed within the approach used in the present paper.

The final result for positronium is

$$\delta E(n,l) = \frac{5}{96}m\alpha^6 \log\frac{1}{\alpha}(3+\vec{\sigma}\vec{\sigma}')\frac{\delta_{l,0}}{n^3}. \qquad (63)$$

which can be rewritten also in a more compact form:

$$\delta E(n,l,s) = \frac{5}{24}m\alpha^6 \log\frac{1}{\alpha}\frac{\delta_{l,0}\delta_{s,1}}{n^3}. \qquad (64)$$

In other words, this correction in positronium does not vanish only for triplet S-states.

Our result (64) differs from the recent one [28],

$$\delta E(n,l) = \frac{1}{96}m\alpha^6 \log\frac{1}{\alpha}(3+5\vec{\sigma}\vec{\sigma}')\frac{\delta_{l,0}}{n^3}, \qquad (65)$$

obtained via a relativistic two-particle equation for positronium. The absence of the results for separate contributions in Ref. [28], as well as the difference in the technique of calculations, hampers the elucidation of reasons of the disagreement. However, according to the private communication by R. Fell, our results agree for the pure Coulomb and single-magnetic exchanges.

Adding (64) to the known contributions of orders lower in α, we obtain the following theoretical values for transition frequencies in positronium (in MHz):

$$E(2^3S_1) - E(1^3S_1) = 1\,233\,607\,211.7, \qquad (66)$$
$$E(2^3S_1) - E(2^3P_2) = 8\,627.7, \qquad (67)$$
$$E(2^3S_1) - E(2^3P_1) = 13\,013.3, \qquad (68)$$
$$E(2^3S_1) - E(2^3P_0) = 18\,498.5. \qquad (69)$$

Correction (64) contributes -16.7 MHz to the first of these frequencies and 2.4 MHz to all others.

A comparison with the experimental values

$$E(2^3S_1) - E(1^3S_1) = 1\,233\,607\,218.9\,(10.9)[29], \qquad (70)$$
$$E(2^3S_1) - E(2^3P_2) = 8\,628.4\,(2.8)[30],\ 8\,619.6\,(2.7)(0.9)[31], \qquad (71)$$
$$E(2^3S_1) - E(2^3P_1) = 13\,001.3\,(3.9)(0.9)[31], \qquad (72)$$
$$E(2^3S_1) - E(2^3P_0) = 18\,504.1\,(10.0)(1.7)[31], \qquad (73)$$

demonstrates that the calculated correction will be essential for the next generation of experiments.

In conclusion, let us add some words on the analogous logarithmic correction to the electron-electron interaction. The corresponding effective operator is

$$V_{ee} = -\frac{\pi\alpha^3}{3m^2}(3+\vec{\sigma}_1\vec{\sigma}_2)\log\frac{1}{\alpha}\delta(\vec{r}_{12}). \qquad (74)$$

It corresponds to the energy correction (61) at $m = M$ with the change of the overall sign. The last prescription becomes especially obvious in the approach used in Ref. [6] where all the effective operators leading to formula (61) arise from the three-photon exchange diagrams. Perturbation (74) is operative in the triplet states only where the coordinate wave function is antisymmetric. It means that the corresponding energy correction vanishes due to $\delta(\vec{r}_{12})$.

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Logarithmic Corrections in the Two-Body QED Problem

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И.Б. Хриплович

Логарифмические поправки в задаче связанных состояний в квантовой электродинамике

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