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# CP-ODD INTERACTION OF LIGHT QUARKS AND THE NEUTRON ELECTRIC DIPOLE MOMENT

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CP-Odd Interaction of Light Quarks
and the Neutron Electric Dipole Moment

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ABSTRACT

The experimental upper limit on the neutron electric dipole moment (NEDM) allows one to bound the parameters of CP-violation in the system of light quarks. New strict upper limits on these parameters are obtained within the chiral approach where the long-distance, logarithmic in $m_\pi$ contributions to NEDM are considered.



1. Up to now CP-violation has been observed only in the decays of neutral K-mesons [1]. One more important source of the information on this phenomenon are the searches for the NEDM. Although it has not been discovered up to now, the experimental bounds on the NEDM are extremely important: due to them a lot of models of CP-violation have become obsolete. The most strict upper limit on the NEDM $d_n$ has been obtained in refs. [2, 3]. It constitutes

$$|d_n/e| < 10^{-25} cm. \quad (1)$$

Since the origin of CP-violation is still obscure, it looks quite reasonable to interprete this result in terms of limits on the effective coupling constants of the phenomenological CP-odd quark-quark and quark-gluon operators. Such an approach was applied previously in ref. [4] where quite essential bounds on these constants were obtained by means of the QCD sum rules technique and the factorization approximation. In the present article these bounds are revised and most of them considerably improved in the chiral approach where the contributions to the NEDM singular in the $\pi$-meson mass are considered.

For the first time this approach to the NEDM calculation was used in ref. [5] to obtain the contribution of the $\theta$-term to the dipole moment. It was shown in that paper that the only diagrams singular in $m_\pi$, in fact proportional to $\log m_\pi$, are those in Figs. 1a,b. Here one $\pi NN$ vertex, $g_r\sqrt{2}i\gamma_5$, is the strong one, and the second, $\bar{g}\sqrt{2}1$, is CP- and P-odd. The strong $\pi NN$ constant, $g_r$, equals 13.6, and CP-odd one, $\bar{g}$, is induced by the

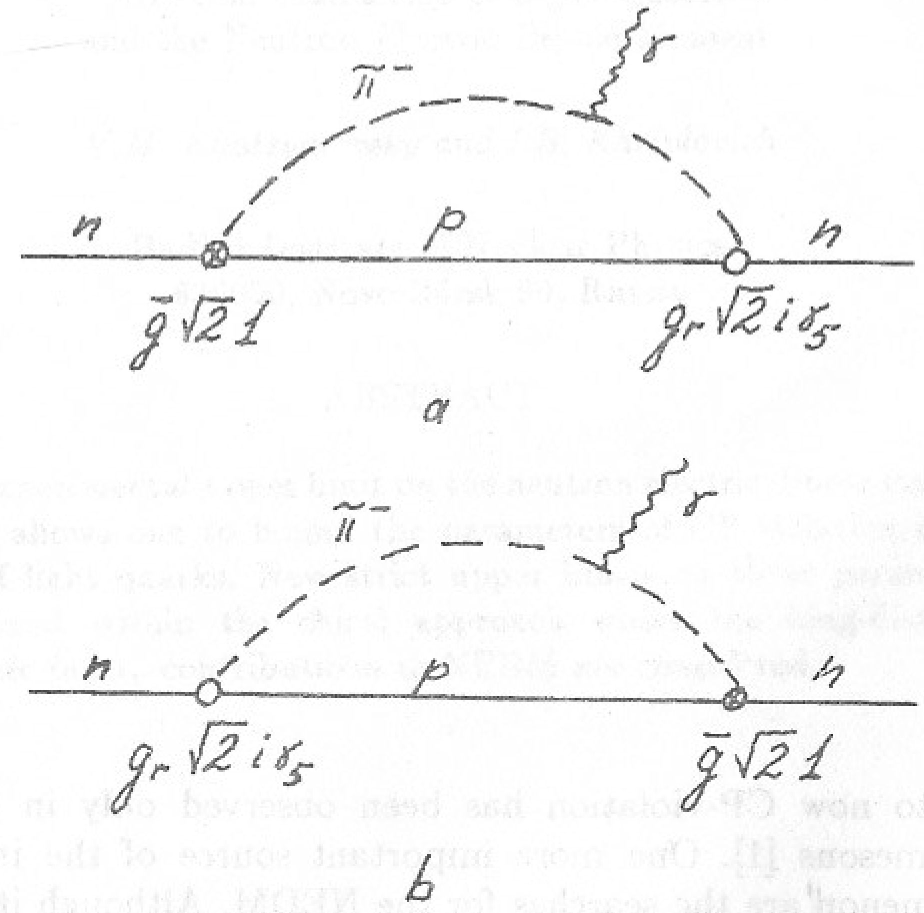


*Fig.1.* Chiral contribution to $d_n$.

discussed CP-odd interaction, $\theta$-term in ref. [5], or quark-quark and quark-qluon interactions in this paper. The NEDM induced in this way equals

$$d_n/e = \frac{1}{m_p}\frac{g_r\bar{g}}{4\pi^2}\log\frac{m_\rho}{m_\pi}, \tag{2}$$

where $m_p$ is the proton mass. The choice of the $\rho$-meson mass $m_\rho$ as the typical hadronic scale at which the logarithmic integral is cut off, is of course somewhat a matter of convention. It is proper to recall here the relativistically invariant definition of a dipole moment $d$ as the coefficient in the effective interaction

$$V = \frac{1}{2}d\bar{\psi}\gamma_5\sigma_{\mu\nu}F_{\mu\nu}\psi \tag{3}$$

of a particle with an electromagnetic field $F_{\mu\nu}$.

Expression (2) for the NEDM allows us to reformulate the experimental result (1) as an upper limit on the CP-odd constant of the charged pion interaction with a nucleon:

$$\bar{g} < 10^{-11}. \tag{4}$$



In fact, the chiral parameter, $\log(m_\rho/m_\pi)$, is not large, 1.7 only. However, due to the absence of other logarithmic terms, mutual cancellation between this contribution and other ones looks quite unnatural. So, expression (2) can serve as a rather conservative estimate for the NEDM.

2. Following ref. [4], let us write the CP-odd Hamiltonian of quark-quark interaction as

$$H = \frac{G}{\sqrt{2}}\sum_i k_i O_i \tag{5}$$

$$\begin{aligned} O_s &= \bar{q}_1 i\gamma_5 q_1 \bar{q}_2 q_2 \\ O_s^c &= \bar{q}_1 i\gamma_5 t^a q_1 \bar{q}_2 t^a q_2 \\ O_t &= \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\bar{u}\sigma_{\mu\nu}u\bar{d}\sigma_{\alpha\beta}d \\ O_t^c &= \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\bar{u}\sigma_{\mu\nu}t^a u\bar{d}\sigma_{\alpha\beta}t^a d \end{aligned} \tag{6}$$

The summation in the first line is performed over the operators with different Lorentz and colour structures enumerated in the next lines. The operators $O_s$ and $O_s^c$ refer to the cases of the same quarks ($q_1 = q_2 = u, d$) and different ones ($q_1 \neq q_2$). Our results will be formulated as upper limits on the dimensionless constants $k_i$ characterizing the magnitude of corresponding interaction in the units of the Fermi constant $G = 10^{-5}/m_p^2$.

The CP-odd vertex of interest to us can be transformed by means of the PCAC technique as follows:

$$< \pi^- p|O_i|n >= \frac{i}{f_\pi} < p|[O_i, u^+\gamma_5 d]|n > \tag{7}$$

where $f_\pi = 130\,MeV$. In the case of the quark-quark operators the arising matrix element is reduced by the vacuum factorization to

$$< p|[O_i, u^+\gamma_5 d]|n >= i < \bar{q}q >< p|\bar{u}d|n > C_i \tag{8}$$

where $C_i$ are numerical factors dependent on the concrete form of the operator $O_i$. They are

$$C_s = \frac{5}{6}, \quad C_s^c = -\frac{2}{9}, \quad C_t = 2, \quad C_t^c = \frac{8}{3} \tag{9}$$

For the quark vacuum condensate we will assume the numerical value $< \bar{q}q >= (0.25\,GeV)^3$. For the nucleon matrix element the $SU(3)$-symmetry arguments give [5]:

$$< p|\bar{u}d|n >= \bar{u}_p u_n \rho, \quad \rho = \frac{m_\Xi - m_\Sigma}{m_s} \approx 1 \tag{10}$$

At last, using the limit (4) we get the following results:

$$|k_s| < 3 \cdot 10^{-5}, \quad |k_s^c| < 1.1 \cdot 10^{-4}, \quad |k_t| < 1.2 \cdot 10^{-5}, \quad |k_t^c| < 0.9 \cdot 10^{-5} \quad (11)$$

The presented upper limits on the constants $k_s^c$ at $q_1 \neq q_2$, $k_t, k_t^c$ were obtained previously in the chiral approach in ref. [6]. (The explanation of some difference in the numbers is that here we use the value 1.7, but not 1, for the chiral logarithm.) However, the method of treating the vertex $< p\pi^-|O_i|n >$ by the direct factorization of the matrix element $< \pi^-|u\bar{d}|0 >$ used in ref. [6], naturally is not operative for identical quarks, and does not allow therefore to get the limits on $k_s, k_s^c$ at $q_1 = q_2$. As to the constants $k_s$ at $q_1 \neq q_2$, that method gives a limit much weaker than (11). The same situation holds for the vertex $< \pi^0 N|O|N >$ dominating CP-odd nuclear forces. PCAC technique with the consequent vacuum factorization leads to the limits on $k_s$ by a factor of 2 better than the direct factorization of $\pi^0$-meson used in ref. [4].

3. Now we address the quark-gluon operators

$$O_q^g = m_p \bar{q} g \sigma G \gamma_5 q, \qquad \sigma G \equiv \sigma_{\mu\nu} G_{\mu\nu}^a t^a \quad (12)$$

Let us note that the combinations

$$d_{u,d}^c = \frac{G}{\sqrt{2}} 2k_{u,d}^g m_p g \quad (13)$$

could be called chromoelectric dipole moments of u- and d-quarks. In this case matrix element (7) reduces to $< p|\bar{u}ig\sigma Gd|n > / f_\pi$. Applying the technique used previously in refs. [7, 8] and based on the QCD sum rules, the last matrix element can be expressed via vacuum expectation values:

$$< p|\bar{u}ig\sigma Gd|n > \simeq \frac{m_p}{- < \bar{q}q >} K. \quad (14)$$

The nonperturbative correlator in this formula

$$K = i \int d^4x < T\bar{d}u(x), \bar{u}ig\sigma Gd(0) > \quad (15)$$

is transformed by means of motion equations to

$$K = -\frac{2}{3} \frac{< \bar{u}u - \bar{d}d >}{m_u - m_d} \frac{< \bar{q}ig\sigma Gq >}{< \bar{q}q >}. \quad (16)$$



Let us note that in this way the breaking of the isotopic symmetry in the quark-gluon condensate $< \bar{q}ig\sigma Gq >$ is related to its breaking in the quark one $< \bar{q}q >$ [8]. To estimate the last breaking we use the relation [9, 10]

$$\Delta = \frac{< \bar{q}q > (m_u - m_d)}{< \bar{u}u - \bar{d}d >} = \frac{m_\rho^2}{m_\pi^2}(m_u + m_d) \approx 330 \ MeV. \quad (17)$$

Finally, we need one more estimate [11]:

$$m_0^2 = \frac{< \bar{q}ig\sigma Gq >}{< \bar{q}q >} \approx 0.8 \ GeV^2. \quad (18)$$

In result, we get

$$< p|\bar{u}ig\sigma Gd|n > \approx -1.5 \ GeV^2. \quad (19)$$

At last, for the constants of interest we get the following limit:

$$|k_q^g| < 2.4 \cdot 10^{-7}. \quad (20)$$

This result differs from the recent one of ref. [12] which is obtained within the QCD sum rules technique and exceeds ours by a factor of ten. The disagreement is due to different approaches adopted and to a numerical error made in ref. [12].

Let us compare these approaches. The QCD sum rules used in refs. [4], [12] lead (without numerical errors) to the result for the NEDM, as induced by the quark chromoelectric dipole moment, which is roughly an order of magnitude smaller then the present one obtained in the chiral limit. We believe that the explanation of this difference is as follows. Being treated via the direct application of the sum rules technique, our problem consists in calculating the 4-point amplitude of the nucleon interaction with a photon and spurion coupled to the CP-odd quark-gluon operator. Meanwhile, the QCD sum rules are well-known to be unreliable when applied to multiparticle amplitudes.

In the present paper we reduce the problem via the chiral approach to that of the 3-point vertex of the nucleon interaction with a spurion coupled to CP-even quark-gluon operator. Its evaluation by the QCD sum rules is more reliable than that of a 4-point vertex.

An argument of a more qualitative nature in favor of the present estimate is perhaps proper here. Let us turn to the ratio of the NEDM to the quark chromoelectric dipole moment. Since the electric charge $e$ is a parameter unrelated to the nucleon structure, it is reasonable to expect that the ratio of $d_n/e$ to $d_q^g$ should be about unity. Therefore, being formulated for this ratio, our present result 0.8 looks extremely natural.

4. Thus, the use of the chiral approach allowed us to improve considerably the limits, following from the experimental searches for the NEDM, on the effective constants of the CP-odd quark-quark and quark-gluon interaction (see Table). On the other hand, it is clear from the same Table that the main conclusion of ref. [4] remains valid (to a considerable degree, due to the essential progress in the experimental searches for the electric dipole moment of $Hg^{199}$ atom [13]): the upper limits on the constants discussed following from the neutron and atomic experiments, are quite comparable.

Table

| $k_i O_i$ | $\lvert d_n/e\rvert < 10^{-25}$ cm | $\lvert d(Hg^{199})/e\rvert < 2.5\cdot 10^{-27}$ cm |
|---|---|---|
| $k_s(\bar{q}_1 i\gamma_5 q_1)(\bar{q}_2 q_2)$ | $\lvert k_s\rvert < 3\cdot 10^{-5}$ | $\lvert k_s\rvert < 0.5\cdot 10^{-5}$ |
| $k_s^c(\bar{q}_1 i\gamma_5 t^a q_1)(\bar{q}_2 t^a q_2)$ | | |
| $q_1 = q_2$ | $\lvert k_s^c\rvert < 1.1\cdot 10^{-4}$ | $\lvert k_s^c\rvert < 2\cdot 10^{-5}$ |
| $q_1 \neq q_2$ | $\lvert k_s^c\rvert < 1.1\cdot 10^{-4}$ | $\lvert k_s^c\rvert < 3\cdot 10^{-3}$ |
| $k_t \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\bar{u}\sigma_{\mu\nu}u)(\bar{d}\sigma_{\alpha\beta}d)$ | $\lvert k_t\rvert < 1.2\cdot 10^{-5}$ | $\lvert k_t\rvert < 4\cdot 10^{-4}$ |
| $k_t^c \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\bar{u}\sigma_{\mu\nu}t^a u)(\bar{d}\sigma_{\alpha\beta}t^a d)$ | $\lvert k_t^c\rvert < 0.9\cdot 10^{-5}$ | $\lvert k_t^c\rvert < 4\cdot 10^{-4}$ |
| $k^g m_p \bar{q}\gamma_5\sigma_{\mu\nu} g G^a_{\mu\nu} t^a q$ | $\lvert k^g\rvert < 2.4\cdot 10^{-7}$ | $\lvert k^g\rvert < 10^{-7}$ |

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*REFERENCES*

[1] J.H. Christensen, J.W. Cronin, V.L. Fitch and R. Turlay, Phys.Rev.Lett. **13** (1964)138.

[2] K.F. Smith et al, Phys.Lett. **B234** (1990)191.

[3] I.S. Altarev et al, Phys.Lett. **B276** (1992) 242.

[4] V.M. Khatsymovsky, I.B. Khriplovich and A.S. Yelkhovsky, Ann.Phys. **186** (1988) 1.

[5] R.J. Crewther, P. Di Vecchia, G. Veneziano and E. Witten, Phys.Lett. **B88** (1979) 123; [Erratum: B91 (1980) 487].

[6] I.B. Khriplovich, Nucl.Phys. **B352** (1991) 385.

[7] V.M. Khatsymovsky, Nucl.Phys. **B277** (1986) 298.

[8] V.M. Khatsymovsky, I.B. Khriplovich and A.R. Zhitnitsky, Z.Phys. **C36** (1987) 455.

[9] V.L. Chernyak and A.R. Zhitnitsky, Phys.Rep. **112** (1984) 173.

[10] P. Pascual and R. Tarrach, Phys.Lett. **B116** (1982) 443.

[11] V.M. Belyaev and B.L. Ioffe, Sov.Phys.JETP **56** (1982) 493.

[12] I.I. Kogan and D. Wyler, Phys.Lett. **B274** (1992) 100.

[13] S.K. Lamoreaux, Talk at the III International Symposium on Weak and Electromagnetic Interactions in Nuclei, Dubna, June 1992.

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**CP-нечетные взаимодействия легких кварков и электрический дипольный момент нейтрона**

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