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A POSSIBLE MECHANISM FOR ENHANCED ABSORPTION OF P–MODES IN SUNSPOT AND PLAGE REGIONS

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НОВОСИБИРСК

Abstract. Magnetic regions on the Sun's surface are observed to absorb large fractions of the p-mode (acoustic) wave power incident upon them. We propose a mechanism to explain the absorption, based on the idea that sunspots are assembled from many individual flux tubes with highly variant physical conditions. Strong gradients in the (perturbed) parameters of a wave propagating through such an inhomogeneous medium result in enhanced absorption of the wave power. The gradients in the wave parameters occur on the scale of the background flux tubes which is much smaller than the wavelength.

1. Introduction

Solar p-mode oscillations are normally treated as global resonant modes, with lifetimes much longer than their periods. While many modes are longlived, all p-modes have finite lifetimes and thus source and sinks. A series of papers by Braun, Duvall, and LaBonte (1987, 1988, 1989, 1990) studying the interaction of p-modes with magnetic regions on the Sun showed that sunspot and plages both are sinks of p-mode wave power. Compared to the nonmagnetic regions of the Sun, the magnetic regions show enhanced absorption of p-mode (acoustic) waves. The mechanism that absorbs the p-mode power is unknown.

Many regularities in the observed properties of the p-mode absorption are seen.

1) The absorbing regions are spatial coincident with sunspots and plage seen on the solar surface, with some differences in detail.

2) The fraction of the incident p-mode power absorbed is zero at low wavenumbers ($k < 0.1\ Mm^{-1}$), then rises to a high value at high wavenumbers ($k > 0.4\ Mm^{-1}$). The fraction absorbed remains constant at the high value to the observational limit ($k \sim 1\ Mm^{-1}$).

3) The onset of absorption occurs at lower wavenumbers and the absorbed fraction reaches a higher value in larger sunspots than in smaller sunspots. Typical isolated sunspots absorb 40% of the high wavenumber ($k > 0.4\ Mm^{-1}$) p-mode power incident upon them. Giant spots absorb up to 70%, while small spots absorb only 20%.

4) The absorbed fraction is larger in spots than in plages, but the "acoustic opacity", the absorption per unit magnetic field appears to be saturated in spots, relative to plages.

5) There is no observed variation of the absorbed fraction with temporal frequency, or with radial order of the p-modes.

Any mechanism for the p-mode absorption should explain all these properties.

In this paper we propose a mechanism for the enhanced absorption of p-modes based on the idea that in sunspots the magnetic field has a fine filamentary structure (for example, Livingston 1991). We consider the sunspot to be a dense conglomerate of closely packed magnetic flux tubes of typical radius $R = 50$ km. Physical parameters such as magnetic field strength and plasma density, vary from one flux tube to another by the order of unity. The spot is then a strongly inhomogeneous medium. Lites *et al.* (1991) made direct observations of magnetic fields in spot umbras to test this idea. They found only small variation in the magnetic field strength at the surface but argue that the high temperature of the spot umbra indicates that the spot must be highly inhomogeneous just below the visible surface. Such inhomogeneity should hold in the deeper layers traversed by the acoustic waves.

The physical mechanism responsible for the enhanced absorption of acoustic waves propagating in such an inhomogeneous medium is easily understood. The perturbations of all parameters in a propagating wave will be different inside the different flux tubes. In a dense conglomerate, flux tubes have common boundaries. Near those boundaries, strong local gradients of all physical parameters will appear. The equations of motion then contain a large vortex component of the perturbed quantities. The characteristic scale of the perturbations is naturally the size of the background inhomogeneities R. The presence of the strong small-scale gradients results in the enhanced absorption of the wave energy with the properties specified above.

Rosenthal (1990) has also considered the absorption of acoustic waves in an inhomogeneous sunspot. However, he focused on the trapping of waves in resonant layers in the steep gradients in the boundaries between the individual fluxtubes. He shows that such an inhomogeneous system provides a better match to observations than a homogeneous sunspot, but still falls short of explaining the observations in detail.

In Section 2 we review the properties of wave propagation in an inhomogeneous medium. In section 3 we derive the response of the sunspot plasma to the wave perturbation (3.1), the dissipation caused by thermal conduction (3.2) and viscosity (3.3), and the total anomalous dissipation (3.4). In Section 4 we compare the results of our model with the observed properties listed above.

2. Wave Propagation in an Inhomogeneous Medium

We consider p-modes with length scale $\lambda\!\!\!^{-} = 1/k$ (where k is the wavenumber) much larger than the size of the magnetic field inhomogeneities, $\lambda\!\!\!^{-} >> R$. To visualize in the simplest way the mechanism of the enhanced absorption, we restrict ourselves to the case of an acoustic wave propagating across the magnetic field. As will be seen from our analysis, the same mechanism acts for an arbitrary propagation angle and, to order of magnitude, the enhanced dissipation rate should not depend on angle.

The general idea of our present theoretical approach is very similar to that described in the paper by Ryutova and Persson (1984) which is devoted to the study of the dispersion properties and enhanced dissipation of MHD waves in a plasma containing closely packed random inhomogeneities of plasma density and magnetic field. There are two main results in that paper. First, the procedure which allows one to find the linear equations of the evolution of plasma parameters in the presence of small scale random inhomogeneities was described. It was shown that the problem can be reformulated in terms of equations for averaged quantities (pressure, density, velocity, etc.). Averaging is being made over an intermediate scale L satisfying the condition $R << L << \lambda\!\!\!^{-}$. Unlike the propagation of linear MHD waves in a homogeneous plasma, in the problem under consideration the vortex component of the equations of motion is essential. For harmonic waves, these equations result in a linear dispersion relation with some renormalized phase velocity, which, generally speaking, depends on the propagation angle in the xy-plane (magnetic field is directed along the z-axis):

$$\omega^2 = 2\left\langle \frac{1}{\left(\frac{\gamma}{2} - 1\right)P + \mathbb{P}} \right\rangle^{-1} \left\{ \left\langle \frac{1}{\rho} \right\rangle k^2 + Q_{\alpha\beta} k_\alpha k_\beta \right\} \qquad (1)$$

Here $\rho = \rho(x,y)$ and $P = P(x,y)$ are the unperturbed plasma density and gas-kinetic pressure respectively, $\mathbb{P} = P(x,y) + \frac{B^2(x,y)}{8\pi}$ is the total pressure, and $Q_{\alpha\beta}$ is a tensor whose symmetry is determined by the statistical properties of medium, in other words, by the field of background density and magnetic field variations. Note that this dispersion relation can be directly used for the diagnostic goals; the measured $\omega(k_x,k_y)$ diagram together with (1) can give the morphological map of the observed region.

The renormalized phase velocity corresponding to the dispersion relation (1) can be represented as follows:

$$v_{ph} = \frac{\omega}{k} = \left\langle \frac{1}{\rho\left[c_{s_i}^2 + v_A^2\right]} \right\rangle^{-1/2} \left\{ \left\langle \frac{1}{\rho} \right\rangle + Q_{\alpha\beta}\frac{k_\alpha k_\beta}{k^2} \right\}^{1/2} \tag{2}$$

where $c_s^2 = \gamma\frac{P}{\rho}$ and $v_A^2 = \frac{B^2}{4\pi\rho}$ are the sound speed and Alfvèn velocity, respectively,and which also change from one tube to other. To order of magnitude, the phase velocity of sound waves propagating inside the sunspot becomes that of the fast MHD mode (cf., for example, Thomas 1985). Indeed, for a homogenously magnetized region this expression gives simply $\omega/k = \sqrt{c_s^2 + v_A^2}$. If we use the values given by Thomas (1988), we get a phase velocity for the homogeneous case that is significantly different from the 25 km s^{-1} slope found in the ω-k diagram by Abdelatif, Lites, and Thomas (1986). By contrast, the additional term in (2) gives a better agreement with the observed slope. We should bear in mind, of course, that this is the component of phase velocity surface (across the magnetic field). If beneath the surface sound waves are propagating with some angle θ with respect to magnetic field then the phase velocity is as follows:

$$v_{ph} = \frac{v_{ph}^{\parallel}}{\cos\theta} . \tag{3}$$

In the final section Ryutova and Persson studied the dissipative effects in plasma with random inhomogeneities, and have found the damping rates corresponding to viscosity, thermal conductivity and Ohmic losses. It was shown that the strong enhancement of the absorption of the wave energy takes place due to the presence of small scale inhomogeneities and is provided mostly

by the viscosity and thermal conductivity. The Ohmic losses appeared to be of the same order as those in a homogeneously magnetized region. The dissipative coefficients are derived by Ryutova and Persson for the case completely ionized and strongly magnetized plasma with $\omega_i\tau_i >> 1$, where τ_i is ion collision time and $\omega_i = eB/m_ic$, the ion gyrofrequency. In that case the transverse transport coefficients are strongly suppressed by the magnetic field. By contrast, the observational data show that in the sunspot interior $\omega_i\tau_i < 1$, and the magnetic field must have a weak effect on the transport coefficients. Thus although there is good qualitative agreement of the observed features of acoustic absorption with the theoretical results of Ryutova and Persson (1984), for a quantitative analysis of the dissipation rate we need a modified approach that is appropriate to the case with $\omega_i\tau_i < 1$, which we present in the next Section.

It should be noted that the theory for collisionless plasmas ($\omega_i\tau_i \gg 1$) applies in higher layers of the solar atmosphere and we expect analogous phenomena to the acoustic absorption that is observed in the photosphere. Any inhomogeneous region should behave as a sink for MHD wave energy, and linear damping rates from the Ryutova and Perssön (1984) can be used directly.

Recently Ryutova, Kaisig, and Tajima (1991) studied the propagation of nonlinear magneto-acoustic waves in the solar atmosphere, with random inhomogeneities in density and magnetic field. They found that in the nonlinear regime the energy transfer from acoustic waves to the randomly magnetized region can occur through different scenarios depending on the statistical properties of the region. If the process of enhanced absorption of the incident wave does not stay in the linear regime, it goes either through the formation of shocks or through the storage of energy in a system of solitons that is later damped away. The results of the nonlinear studies are valid for arbitrary values of $\omega_i\tau_i$, except those processes where the usual dissipative effects (thermal conductivity, viscosity, and Ohmic losses) play the direct role. The possible observational manifestation of nonlinear dynamics and a corresponding qualitative analysis will be presented elsewhere.

3. Absorption Coefficient in Sunspots

3.1 Spatial scale of the perturbations

In the present paper we use the results of Ryutova and Persson for the averaged quantities and derive the procedure for the description of the absorption of sound waves for the case of $\omega_i \tau_i < 1$. We are considering p-mode oscillations incident on a sunspot so the perturbation of the sunspot is the pressure variation over the spatial scale of the wavelength. We consider modes with wavelengths that are much larger than the spatial scale of the sunspot inhomogeneities. We will assume that there is also a temporal scale difference, with the individual flux tubes responding in such a way as to preserve the pressure perturbation set up by the wave. Given the small spatial scale of the flux tubes, the perturbation of the total pressure

$$\delta\mathbb{P} \equiv \delta P_m + \delta P \qquad (4)$$

then remains effectively constant over the cross-section of the individual flux tube (Figure 1).

The plasma conductivity in sunspots is high enough that the magnetic field may be considered as frozen into plasma and the magnetic pressure perturbation δP_m is related to the density perturbation $\delta\rho$ by the equation

$$\frac{\delta P_m}{P_m} = 2\frac{\delta\rho}{\rho}. \qquad (5)$$

The gas-kinetic pressure perturbation for an adiabat is

$$\frac{\delta P}{P} = \gamma\frac{\delta\rho}{\rho}. \qquad (6)$$

From equations (4)-(6) we find the density response to the total pressure perturbation $\delta\mathbb{P}$:

$$\frac{\delta\rho}{\rho} = \frac{\delta\mathbb{P}}{\gamma P + 2P_m}. \qquad (7)$$

Rearranging the definition of the total pressure

$$P = \mathbb{P} - P_m, \qquad (8)$$

we find that

$$\frac{\delta\rho}{\rho} = \frac{\delta\mathbb{P}}{\mathbb{P}}\frac{1}{\gamma + (2-\gamma)\frac{P_m}{\mathbb{P}}}. \qquad (9)$$

As the magnetic pressure P_m is varying on the small scale $R \ll \bar{\lambda}$, so does the density perturbation. Again using an adiabatic relation, the temperature perturbation is directly related to the density perturbation

$$\frac{\delta T}{T} = (\gamma - 1)\frac{\delta\rho}{\rho} = \frac{\delta\mathbb{P}}{\mathbb{P}}\frac{(\gamma-1)}{\gamma + (2-\gamma)\frac{P_m}{\mathbb{P}}}. \qquad (10)$$

Thus, when the p-modes propagate in a region with a small scale structure in the magnetic field, there are always small scale temperature perturbations with a scale R which is much less than $\bar{\lambda}$ (Figure 1). The enhanced dissipation is caused by these steep temperature gradients.

3.2 Dissipation by thermal conduction

The dissipation caused by thermal conductivity is described by the expression

$$q_T = \frac{\mathcal{K}}{T}\left\langle (\nabla\delta T)^2 \right\rangle \qquad (11)$$

where q_T is the dissipation rate, $\mathcal{K}$ is the thermal conductivity, and the average is taken over a length which is much larger than R. For δT given by Eq. (10), we obtain

$$q_T = \frac{\mathcal{K}\, T}{2} \left(\frac{\gamma - 1}{\gamma} \right)^2 \frac{\delta \mathbb{P}_o^2}{[\, \mathbb{P} + (\frac{2}{\gamma} - 1) P_m]^2} \left\{ k^2 + \left\langle \frac{(\frac{2}{\gamma} - 1)^2 \; (\nabla P_m)^2}{[\, \mathbb{P} + (\frac{2}{\gamma} - 1) P_m]^2} \right\rangle \right\} \qquad (12)$$

where we have assumed a sinusoidal dependence of $\delta\mathbb{P}$ on x

$$\delta\mathbb{P} = \delta\mathbb{P}\, sin\, (kx - \omega t) \qquad (13)$$

and have taken into account that $k << 1/R$. The first term in the square brackets corresponds to the usual dissipation ot a p-modes in a homogeneous medium while the second term describes the additional damping.

The spatial damping rate Im k_T caused by thermal dissipation (see, for example, Landau and Lifshitz, Fluid Dynamics) is

$$\text{Im}\, k_T = \frac{q_T}{2\, c_s\, \varepsilon} \qquad (14)$$

where $\varepsilon = \dfrac{\rho\ \delta v^2}{2}$ is the energy density of the initial acoustic wave. Using the general MHD equations we can express the velocity perturbations in terms of the total pressure perturbations (see Ryutova and Persson 1984, equation 14):

$$\delta v = \frac{\delta\mathbb{P}}{\rho\ v_{ph}} \qquad (15)$$

where v_{ph} is determined by (1). For simplicity let us choose an isotropic distribution of inhomogeneities in the sunspot. In this case $Q_{\alpha\beta} = 0$ and, approximately,

$$v_{ph}^2 = \frac{2}{\rho} [\, \mathbb{P} + (\frac{\gamma}{2} - 1)\, P\,]$$

or

$$v_{ph}^2 = \frac{\gamma}{\rho} [\, \mathbb{P} + (\frac{2}{\gamma} - 1)\, P_m]\, . \qquad (16)$$

Using equations (12) - (16) we can write for the damping rate

$$\text{Im}\, k_T = \frac{\mathcal{K}}{2\, c_s} (\gamma - 1) \frac{T\ (\gamma - 1)}{\gamma\, [\, \mathbb{P} + (\frac{2}{\gamma} - 1)\ P_m]} \left\{ k^2 + \left\langle \frac{(\frac{2}{\gamma} - 1)^2\ (\nabla P_m)^2}{[\, \mathbb{P} + (\frac{2}{\gamma} - 1)\ P_m]^2} \right\rangle \right\} . \qquad (17)$$

From eq. (10)

$$\frac{T\ (\gamma - 1)}{\gamma\, [\, \mathbb{P} + (\frac{2}{\gamma} - 1)\ P_m]} = \frac{\delta T}{\delta \mathbb{P}} . \qquad (18)$$

Relating the temperature and gas pressure with an adiabatic relation and using the equation of state for a perfect gas we can write

$$\frac{\delta T}{\delta P} = \frac{\gamma - 1}{\gamma} \frac{1}{n} . \qquad (19)$$

It is more convenient to use the specific heat at constant volume c_V rather than the number density n, so using $c_V = n / (\gamma - 1)$,

$$\frac{\delta T}{\delta P} \simeq \frac{\delta T}{\delta \mathbb{P}} = \frac{1}{\gamma\, c_V} . \qquad (20)$$

Combining equations (17), (18) and (20) the spatial damping rate is

$$\text{Im}\, k_T = \frac{\chi}{2 c_s} \frac{\gamma - 1}{\gamma} \left\{ k^2 + \left\langle \frac{(\frac{2}{\gamma} - 1)^2\ (\nabla P_m)^2}{[\, \mathbb{P} + (\frac{2}{\gamma} - 1)\ P_m]^2} \right\rangle \right\} \qquad (21)$$

where $\chi = \mathcal{K} / c_V$ is the thermal diffusivity.

Note that in denominator of the second term in eq. (21) the last term is numerically small compared to the first term; for $\gamma = 5/3$ it is only 0.2 of the first term even when $P_m / \mathbb{P} = 1$. This allows us to use the expansion

$$\frac{1}{[\mathbb{P} + (\frac{2}{\gamma} - 1) P_m]^2} \simeq \frac{1}{\mathbb{P}^2} [1 - 2(\frac{2}{\gamma} - 1) P_m]$$

and write

$$\mathrm{Im}\, k_T = \frac{\chi}{2c_s} \frac{\gamma - 1}{\gamma} \left\{ k^2 + \left\langle \frac{(\frac{2}{\gamma} - 1)^2 (\nabla P_m)^2}{\mathbb{P}^2} \right\rangle \right\} . \qquad (22)$$

For tubes with known internal structure we can easily take the average of $<(\nabla P_m)^2>$. As an example, suppose the magnetic field decreases from the axis according to, say, a Gaussian law,

$$B = B_{max}\, exp\, (- \frac{r^2}{R^2}) \qquad (23)$$

Assuming that the characteristic distance between flux tube centers is l and defining the flux tube areal filling factor to be

$$\varphi = \frac{R^2}{l^2} \qquad (24)$$

then

$$\left\langle \frac{(\nabla P_m)^2}{\mathbb{P}^2} \right\rangle = \frac{\varphi}{R^2} \frac{P^2_{m,max}}{\mathbb{P}^2} . \qquad (25)$$

The spatial damping rate caused by thermal losses has the final form:

$$\mathrm{Im}\, k_T = \frac{\chi}{2\, c_s} \left(\frac{\gamma - 1}{\gamma}\right) \left[k^2 + (\frac{2}{\gamma} - 1)^2 \frac{\varphi}{R^2} \frac{P^2_{m,max}}{\mathbb{P}^2} \right] . \qquad (26)$$

For a gaussian magnetic field profile internal to a flux tube, the average magnetic pressure of the flux tube is $<P_m> = \varphi P_{m,max} / 2$. Even in the case of close packed fluxtubes, with $\varphi \approx 1$, the average magnetic pressure will be reduced below its maximum value, which should be taken into account in constructing equilibrium sunspot models.

<u>3.3 Dissipation by viscosity</u>

A very similar mechanism leads to enhancement of the viscous dissipation. The plasma densities inside and outside the individual flux tube are, generally speaking, different and the total pressure perturbation $\delta\mathbb{P}$ gives rise to a relative motion of the flux tube and the external medium (see, Ryutova and Persson 1984). The characteristic scale of these motions is of the order of R (and much smaller than λ). The viscous dissipation (Landau and Lifshitz, Fluid Dynamics)

$$q_V = \frac{\eta}{2} \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik}\, \mathrm{div}\, \vec{v} \right)^2 + \zeta (\mathrm{div}\, \vec{v})^2 \qquad (27)$$

can greatly exceed its "homogeneous" value. Here η and ζ are the viscous coefficients entering in the Navier-Stokes equations. Calculations similar to the ones by Ryutova and Persson show that

$$q_V \simeq \frac{1}{2} \left(\frac{4}{3} \eta + \zeta \right) c_s^2 \left(\frac{\delta\mathbb{P}}{\mathbb{P}} \right)^2 \left\{ k^2 + \frac{\eta}{\frac{4}{3} \eta + \xi} \left\langle [\nabla (1 - \frac{\rho}{<\rho>})]^2 \right\rangle \right\} . \qquad (28)$$

Assuming that the temperature of the gas in the unperturbed state (without the wave) is more or less uniform, we can write that

$$1 - \frac{\rho}{<\rho>} = \frac{P_m}{\mathbb{P}} . \qquad (29)$$

This allows us to obtain the expression for $\mathrm{Im}\, k_v$, the spatial damping rate caused by viscous dissipation

$$\mathrm{Im}\, k_v = \frac{7}{6} \frac{\nu}{c_s} \left\{ k^2 + \frac{3}{7} \frac{\varphi}{R^2} \frac{P^2_{m,max}}{\mathbb{P}^2} \right\}, \tag{30}$$

where we assumed that $\zeta \simeq \eta$, and have introduced a kinematic viscosity $\nu = \eta / \rho$.

3.4 Total dissipation rate

The total spatial damping rate which includes both thermal and viscous losses is simply the sum of expressions (26) and (30). Both these expressions contain two parts: the usual absorption of sound waves in medium with finite viscosity and thermal conductivity, and the absorption caused by the presence of small scale inhomogeneities. Let us denote the spatial damping rate corresponding to the usual losses as $\mathrm{Im}\, k_1$ and the damping rate caused by the strong local gradients by $\mathrm{Im}\, k_2$. Then the total damping rate can be written as follows

$$\mathrm{Im}\, k = \mathrm{Im}\, k_1 + \mathrm{Im}\, k_2 \tag{31}$$

where

$$\mathrm{Im}\, k_1 = \frac{k^2}{2\, c_s} \left[\frac{7}{3} \nu + (1 - \frac{1}{\gamma}) \chi \right] \tag{32}$$

and

$$\mathrm{Im}\, k_2 = \frac{\varphi}{2\, c_s R^2} \frac{P^2_{m,max}}{\mathbb{P}^2} \left[\nu + (1 - \frac{1}{\gamma})(1 - \frac{2}{\gamma})^2 \chi \right] \tag{33}$$

For pure molecular transport the kinematic viscosity ν and the thermal diffusivity χ are of the same order of magnitude. In this case, because of the small numerical factor in the second term in Eq. (33), the enhanced dissipation is dominated by viscosity. The same conclusion holds in the case of a turbulent viscosity and thermal conductivity, when

$$\nu \sim \chi \sim \tilde{v}\, \tilde{l} \tag{34}$$

with $\tilde{v}$ being a characteristic velocity of turbulent elements and $\tilde{l}$ their characteristic scale. Thermal dissipation may become more important in the case of a large radiative heat transfer where $\chi >> \nu$.

4. Comparison with Observations

For comparison with the observed properties of the p-mode absorption, we adopt the following typical values: flux tube radius R = 50 km; p-mode wave temporal frequency $\omega = 2\pi \times 3$ mHz; p-mode wavenumber $k = 0.5\ \mathrm{Mm}^{-1}$; sunspot radius L = 25 Mm; sound speed $c_s = 10\ \mathrm{km\ s}^{-1}$. We can then address each of the observational findings listed in the Introduction.

1) Acoustic absorption is cospatial with magnetic fields. In the situation where viscous processes are dominant, we conclude that $\mathrm{Im}\, k$ in a nonuniform medium increases with respect to the uniform medium by the enhancement factor

$$f = \frac{3\, \varphi}{7\, k^2 R^2} \frac{P^2_{m,max}}{\mathbb{P}^2}. \tag{35}$$

For a quantitative estimate, we use the observed quantities for k and R, take the filling factor $\varphi = 1$ as appropriate to a sunspot, and assume that $P^2_{m,max} / \mathbb{P}^2 = 1/2$, a value consistent with the models of Maltby et al.(1986) for a sunspot with a photospheric magnetic field of 2 kG. This gives an enhancement factor of

$$f \simeq 3 \times 10^2. \tag{36}$$

Thus, the enhancement of absorption is quite large. The absorbing regions should be distinct from the background. The lower filling factor φ in plage compared to sunspots explains the reduced absorption seen in the observations, despite the larger size of the plage.

2) The absorbed fraction of the acoustic power rises with increasing wavenumber at low wavenumbers but then levels off at a constant value at high wavenumbers. From equations 32 and 33 we see that Im k does not depend on the wave number k. However, at low wavenumbers,

$$k_c \leq \frac{v}{R^2 c_s} \qquad (37)$$

the viscous forces inhibit the relative motions of the fluxtubes and the ambient gas ("sloshing" mode), thus reducing Im k. For our typical values, this corresponds to a wavenumber of $k_c = 0.2$ Mm^{-1}, in good agreement with observation.

3) The absorption level increases with sunspot size and reaches tens of percents. We find the local damping rate has no dependence on the sunspot size; the total absorption of a spot then scales simply with the path length through the spot, that is, the sunspot dimension L.

If α is the total absorption of the wave propagating through the sunspot, the spatial damping rate can be evaluated as

$$\mathrm{Im}\, k = \frac{1}{L} \ln \left(\frac{1}{1-\alpha}\right). \qquad (38)$$

The observational data indicate that $\alpha \sim 0.5$, so that

$$\mathrm{Im}\, k \simeq \frac{0.7}{L}. \qquad (39)$$

Neglecting the thermal conductivity contribution we find that, in order to explain the observed absorption, the kinematic viscosity should be of the order of 10 $\mathrm{km}^2\mathrm{s}^{-1}$. This is much larger than the kinematic viscosity caused by "molecular" transport (the thermal diffusivity would also be larger than that provided by radiative transport). Therefore, we have to assume that within the spot some turbulent viscosity is present. If one uses equation (34), then, to fit the calculated value v, we use the observed limit on turbulent velocities in sunspots (Beckers, 1976) of $\tilde{v} = 2$ km s^{-1} to find the length scale of the turbulence $\bar{l} \sim 5$ km, much smaller than the fibril scale but consistent with

Beckers's observed limit on turbulent scales..

4) The absorption per unit magnetic field (acoustic opacity) appears to saturate in sunspots compared with plages. The dependence of the enhancement factor (equation 35) on the magnetic field strength is $f \sim B^4$ for weak magnetic fields; at large values of B, when $P^2_{m,max}$ becomes of the order of $\mathbb{P}^2$ the enhancement factor saturates.

5) There is no observed variation of the absorption with temporal frequency. Taking the form of k determined from equations (33) and (39), we find that the enhanced damping begins to quench at frequencies

$$\omega_c \leq \frac{1.4}{\varphi} \frac{\mathbb{P}^2}{P^2_{m,max}} \frac{c_s}{L}. \qquad (40)$$

For our standard values, $\omega_c \leq 10^{-3}\ \mathrm{s}^{-1}$. This corresponds to a frequency of 0.2 mHz, below the lowest frequency p-modes observed by Braun *et al.*. Therefore no variation is expected in the observations to date.

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Figure Caption

1) Sketch of the spatial variation of the plasma conditions in the presence of the p-mode wave. The initial state of the sunspot has large amplitude density and magnetic field strength variation on the scale of the flux tubes, R. The incident wave sets up a pressure perturbation δP on the much larger scale λ. The temperature and velocity respond to the wave with amplitudes δT and δv that depend on the small scale structure, and thus have many small regions of high gradient.