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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

V.A. Lebedev, V.V. Parchomchuk,
V.D. Shiltsev, A.N. Skrinsky

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НОВОСИБИРСК

## Suppression of Emittance Growth Caused by Mechanical Vibrations of Magnetic Elements in Presence of Beam-Beam Effects in the SSC

*V.A. Lebedev, V.V. Parchomchuk,*
*V.D. Shiltsev, A.N. Skrinsky*
Budker Institute of Nuclear Physics,
630090, Novosibirsk, USSR

ABSTRACT

A ground motion produces shifts of storage ring quadrupoles. It strongly influences on the beam behavior in large proton (anti)proton colliders due to the closed orbit distortion and due to the transverse emittance growth. Calculations of both effects are presented in this paper. An active feedback system is useful for the emittance growth suppression. It is shown that in this case the main parameter which determines the emittance growth is the betatron tune spread due to the beam-beam effects. A simple analytical model is considered which results are in good coincidence with computer simulations. All calculations are adapted to the Superconductive Super Collider (SSC).

АННОТАЦИЯ

Колебания земли приводят к смещениям квадруполей в накопительных кольцах, что в случае больших протон-(анти)протонных коллайдеров существенно влияет на пучки из-за искажения замкнутых орбит и роста поперечного эмиттанса. В работе представлены аналитические и численные расчеты этих эффектов. Для подавления роста эмиттанса требуется активная система корректировки положения сгустков, эффективность которой ограничивается эффектами встречи, вносящими разброс частот бетатронных колебаний. Рассмотренная в статье простая аналитическая модель хорошо согласуется с результатами компьютерного моделирования. Результаты расчетов ориентированы на применение к коллайдеру SSC.



CONTENTS

## INTRODUCTION

The noise fluctuations of currents and transverse positions of the storage ring magnetic elements can lead to a luminosity reduction which value depends on the an amplitude of the fluctuations and their spectrum. There can be distinguished two classes of frequencies.

To the first class belong all frequencies close to the betatron resonance frequencies $f_n = f_0 \cdot |kQ-n|$, where n=0, ±1, ± 2,···, k = 1, 2, 3,···, $f_0$ is the revolution frequency and Q is the betatron tune. These frequencies lead to a continuous emittance growth being linear in time. Most dangerous is the dipole mode with k = 1 which determines, as a rule, the emittance growth rate.

To the second class belong all non resonance frequencies. We can distinguish here a small frequency range ($f \ll f_0$), which produces a closed orbit distortion equal for all bunches, and a high frequency range, which produces a closed orbit distortion different for different bunches. It looks similar to the excitement of coherent betatron oscillations. In reality the spectral power of fluctuations grows fast with a frequency decrease and therefore the main influence of non resonance frequencies occurs due to the contribution of small frequencies. In the first approximation the non resonance frequencies do not any contribution to the emittance growth rate. Nevertheless, if the value of the beam separation at interaction points is comparable with beam sizes the influence of beam-beam effects is considerably increased due to the appearance of odd resonances.

As a result of a small revolution frequency in the SSC these noises have a great influence on the beam dynamics and can lead to a drastic decrease in luminosity. In the absence of a strong damping of the coherent betatron oscillation the emittance will be doubled in few minutes after injection. If the feedback system damps this coherent oscillation faster than the time of decoherention the emittance growth can be strongly suppressed. In this case the beam-beam effects determine the main limitation on the achievable suppression of the emittance growth. As our study shows, to reach the design luminosity in the SSC a feedback system with ultimate decrements should be used.

It seems reasonable that it should be the same system which damps the beam coherent instabilities due to the beam interaction with the vacuum chamber walls. The feedback



system consists of a beam position monitor (BPM) which measures bunch position and a kicker which corrects the bunch transverse velocity. The betatron phase advance between the BPM and the kicker should be approximately $(n+0.5)\pi$ and the time delay for a signal should be equal to the time delay for a bunch. The frequency band of this system should be large enough to damp the movements of separated bunches.

To simplify the requirements to the fast feedback system it's frequency band should be limited from below therefore the correction system for the closed orbit control should be fast enough. This system should consist of a large quantity of beam position monitors and correctors distributed along the ring.

In this paper we study the influence of the ground motion on the beam dynamics in a collider. In our investigation we consider principal limitations on the suppression of the emittance growth therefore we take into account only the fast feedback system and neglect by an influence of the correction system. In the numerical simulations the spectral power of fluctuations does not depend upon frequency ("white noise"). The calculations and the analysis in application to the ground motion presented here can be easily adapted to any other external noises (fluctuation of current in dipoles, kickers etc.).

The present study is rather attempt to peep into a very complicated problem. It's results set forward strong confinements on the design of the SSC collider but nevertheless give rather an optimistic glance on the problem. To ensure a successful operation of the collider the problem should be further studied and developed.

## 1. CLOSED ORBIT DISTORTION (COD)

At the beginning an influence of slow earth movements on the beam behavior is considered. In this case the particle motions are adiabatic with respect to the external excitements and one can consider a slow change in the closed orbit corresponding to a slow earth movement.

The transverse shifts of quadrupoles produce a displacement of the closed orbit. The value of this displacement at the point A produced by the angular kick $\vartheta_i$ at the point i is given by a well known formula [1]:

$$\Delta X_A = \frac{\sqrt{\beta_i \cdot \beta}\ \cdot \cos(\phi_i - \pi\nu)}{2 \cdot \sin(\pi\nu)}\,\vartheta_i \quad , \tag{1}$$

where $\nu$ is the collider tune, $\phi_i = \mu_i - \mu$ is the betatron phase advance between the points A and i, $\beta_i$ and $\beta$ are beta functions at points i and A. The displacement $\delta_i$ of a quadrupole with a focusing length $F_i$ produces the kick:

$$\vartheta_i = \frac{\delta_i}{F_i}. \tag{2}$$

Summing the contributions of different quadrupoles we



finally have:

$$\Delta X_A = \sum_i \frac{\sqrt{\beta_i \cdot \beta}\ \cdot \cos(\phi_i - \pi\nu)}{2 \cdot \sin(\pi\nu)}\,\frac{\delta_i}{F_i} \quad , \tag{3}$$

We should note that in general experimentally observed spectra of power of seismic noises increase with decreasing the frequency [2]. It means that the main contribution in COD is determined by low frequency ground motion.

To estimate the resulting transverse r.m.s. beam displacement in time one should take into account that for two points spaced at $l \geq 90$ m (SSC FODO lattice half cell length) a practically zero correlation takes place for the periods over 10 s and their relative movement dispersion during an interval $\tau$ is equal to [3]:

$$\langle(\delta_i - \delta_j)^2\rangle = \mathcal{B} \cdot l \cdot \tau \tag{4}$$

where constant $\mathcal{B} \simeq 10^{-4}\ \mu m^2/(m \cdot s)$. This formula is valid for all low frequency movements ( when $C \cdot \tau / 4 \gg l$, C is a seismic wave velocity) which is usually of diffusive character.

To simplify the estimations one can consider that the movements of the quadrupoles are independent with the r.m.s. value of relative displacements $\Delta_i = \delta_{i+1} - \delta_i$:

$$\langle\Delta_i^2\rangle = \frac{1}{2}\,\mathcal{B} \cdot l \cdot \tau \tag{5}$$

where l is the distance between neighboring quadrupoles. The coefficient 1/2 in (5) is taken such as to obtain a result coinciding with the result of accurate calculations. After these remarks one can obtain the following expressions for a closed orbit displacement after an interval $\tau$:

$$\langle \Delta X_A^2 \rangle = \frac{\beta}{4\sin^2(\pi\nu)} \sum_{i,j} \frac{\sqrt{\beta_i \cdot \beta_j}}{F_i F_j} \times$$

$$\times \cos(\phi_i - \pi\nu)\cos(\phi_j - \pi\nu)\langle \delta_i \delta_j \rangle \approx$$

$$\approx \frac{\beta}{4\sin^2(\pi\nu)} \sum_i \frac{\beta_i}{F_i^2} \cos^2(\phi_i - \pi\nu)\langle \delta_i^2 \rangle \approx \frac{\mathfrak{B}\,\mathfrak{C}\,\tau}{4\sin^2(\pi\nu)} \frac{\beta}{\langle\beta\rangle}. \qquad (6)$$

where $\mathfrak{C} = N \cdot l$ is the storage ring circumference and N is the total number of quadrupoles. One should also take into account that for the FODO lattice $\langle\beta\rangle \approx 2 \cdot F$ and $\sum \cos^2(\phi_i - \pi\nu) \approx N/2$. The COD was also studied numerically with a more accurate displacement distribution (4). Results of these calculations are in a good coincidence with (6). In these estimations we neglect the final focus quadrupoles, as simple estimations for SSC show that their contribution to the COD is about 15 % for each high luminosity IP.

In general, there are two aspects of COD significance for the collider run: beam-beam separation at IPs and dynamic aperture limitations. As a rule the first one is essentially rigid because the beam-beam separation at IP only by about 0.1-0.2 $\sigma$ [3] increases the beam-beam effects and decreases the collider luminosity. From this point of



view the beam-beam separation at IP has to be smaller than (0.5-1) $\mu$m. The dynamic aperture limitations give an acceptable COD of about 4 mm in a regular lattice. Note, that for the SSC beam the r.m.s. normalized emittance is equal to $\varepsilon_n \approx 10^{-4}$ cm·rad (emittance $\varepsilon = \varepsilon_n/(\beta\gamma) \approx 5.3 \cdot 10^{-9}$ cm·rad), r.m.s. beam sizes are equal to 5 $\mu$m at IP and ≈100 $\mu$m in a regular lattice. Using (6) with parameters $\nu$=123.78, $\beta^*$=0.5 m, $\langle\beta\rangle \simeq 170$ m and $\mathfrak{C}$=87120 m one can calculate the values of COD at IP $\Delta_{IP}$ and in a regular lattice $\Delta_{RL}$ for different time intervals $\tau$ ( see Table 1 ). One can see that in the absence of the feedback system of the closed orbit control a significant displacement will occur a few minutes after the beam injection in the ring. Long time CODs in Table 1 determine minimal requirements to the correction system.

**Table 1**

| $\tau$ | 1 s | 1 min | 1 hour | 1 day | 1 week | 1 month | 1 year |
|---|---|---|---|---|---|---|---|
| $\Delta_{IP}$ | 0.12 $\mu$m | 1 $\mu$m | 7.5 $\mu$m | 37 $\mu$m | 100 $\mu$m | 0.2 mm | 0.72 mm |
| $\Delta_{RL}$ | 2.3 $\mu$m | 18 $\mu$m | 138 $\mu$m | 680 $\mu$m | 1.8 mm | 3.8 mm | 13 mm |
| $\Delta_{bb}$ | 0.03 $\mu$m | 0.3 $\mu$m | 2.0 $\mu$m | 11 $\mu$m | 30 $\mu$m | 0.06 mm | 0.2 mm |

Nevertheless, a relative beam-beam displacement in the collider can be considerably smaller, because both the beams move together. Really, in the SSC both the storage rings have equal tunes and similar lattices in arcs (equal $\beta$-functions and betatron phase advances). As the quadrupoles of both the rings move practically together (especially at low frequencies) they produce the same kicks and beam displacements at the IP (as can be seen from (3)). It does not hold true for the quadrupoles of the interaction region (IR) straight line where the focusing structure is different for different rings. Therefore, beam separations at the IP $\Delta_{bb}$ will be produced mainly by this quadrupoles and for the estimation we can suppose $\Delta_{bb} = \sqrt{N_{sl} / N}\, \Delta_{IP}$ , where $N_{sl}$ is the number of quadrupoles with different gradients in the IR straight line (N = 968, $N_{sl}$ = 40). Taking into account of higher value of $\beta$-functions at the final focus quadrupoles this value should be increased by 50%.

Note, that besides usual movements of lens assembles when both lenses of different storage rings are moved together there also exist tilts of these assemblies producing independent movements of the lenses in the horizontal plane. Which, in its turn, will increase the beam separation at IPs in the horizontal plane. Here we do not take into account this process because of the lack of experimental information on probable tilt values. The estimations show that the value of this additional separation will not be much higher than the given above.

All the previous estimations set a lower limit for COD due to a general diffusive character of the ground motion. But there are some factors that have a controllable character and can also lead to COD. One of the most



significant factors is the influence of atmosphere on the ground motion as it was shown in [3]. There was suggested a simple estimation of the spectral power of the atmosphere pressure $P$:

$$<\delta P^2>_f \;\approx\; \frac{\rho^2\, U^{16/3}}{4\, L^{4/3}\, f^{7/3}}\,, \tag{7}$$

where $\rho$ is the air density, U is the average velocity of wind and L is the atmosphere thickness (L $\approx$ 5 km). The frequency of oscillation f and size l of pressure inhomogeneity have the following relationship: $f \approx U/l$. Hence, the movements of quadrupoles spaced at l are independent for frequencies higher than U/l. In this case their relative displacement is about

$$<\delta_l^{\,2}> \approx \left(\frac{l}{E}\right)^2 <\delta P^2>_f f = \left(\frac{\rho\, U^2\, l}{2\, E} \cdot \left(\frac{l}{L}\right)^{2/3}\right)^2. \tag{8}$$

where E is Young's modulus. Then COD is about:

$$<\delta X_A^2> \approx \sum_i \frac{\beta\, \beta_i}{8\, F_i^2\, \sin^2(\pi\nu)} <\delta_l^{\,2}> \approx \frac{\beta\, \mathfrak{C}\, l}{2\, <\beta>\, \sin^2(\pi\nu)} \left(\frac{\rho\, U^2}{2\, E} \left(\frac{l}{L}\right)^{2/3}\right)^2. \tag{9}$$

Let's take a usual value $E \approx 10^8 N/m^2$ (the same as in [3]) and using formula (9) calculate the COD for different weather conditions:

Table 2

| weather | characteristic frequency of COD | COD at IP | COD at lattice |
|---|---|---|---|
| ≈no wind (U≈1 m/s) | 0.01 Hz | 0.08 μm | 1.6 μm |
| light wind (3 m/s) | 0.03 Hz | 0.7 μm | 15 μm |
| wind (10 m/s) | 0.1 Hz | 8 μm | 160 μm |
| storm (30 m/s) | 0.3 Hz | 74 μm | 1.5 mm |

Table 2 clearly points at the importance of the atmosphere influence on the SSC operation. Of course, more detailed measurements at the SSC site are required.

## 2. EMITTANCE GROWTH DUE TO LENSES MOTION

To understand the influence of lens motions on the beam behavior we start with the simplest task when only one particle is moving in the storage ring. It is convenient to change in the equation of betatron movement

$$\frac{d^2X}{d\vartheta^2} + g(\vartheta)\left(X - X_o(\vartheta)\right) = 0 \qquad (10)$$

the variables $x = X/f(\vartheta)$, $\psi = \nu\vartheta + \xi(\vartheta)$. Here $g(\vartheta) = eGR^2/Pc$, R is the average storage ring radius, $G \equiv dB/dX$ is the lens gradient, P is the particle momentum, $X_o(\vartheta)$ is a lens displacement, $f(\vartheta)$, $\xi(\vartheta)$ are the Floquet function amplitude and phase. As a result, one obtains a linear oscillator equation:



$$\frac{d^2x}{d\psi^2} + x = \mathfrak{F}(\psi) \qquad (11)$$

where a right-hand part $\mathfrak{F}(\psi) = g(\vartheta)\cdot X_o(\vartheta)\cdot f(\vartheta)$. While the length of lenses is much smaller than the betatron oscillation period one can consider quadrupoles as thin lenses with focusing distances $F_i = Pc/eG_i l_i$, where $G_i, l_i$ are the gradient and length of lens with index i. Now one can write

$$\mathfrak{F}(\psi) = \sum_{i=1}^{N} \delta_{2\pi\nu}(\psi - \mu_i)\mathfrak{F}_i(\psi), \quad \mathfrak{F}_i(\psi) = \frac{\sqrt{\beta_i R}}{F_i}\cdot X_{oi}(\psi) \qquad (12)$$

where $\delta_{2\pi\nu}(\psi - \mu_i)$ is the $\delta$-function of a period $2\pi\nu$, $\mu_i$, $\beta_i$ are the betatron phase and the $\beta$-function at lens i and $X_{oi}$ is the displacement of lens i. The solution of equation (11) with initial condition $x(0)=dx/d\psi(0)=0$ is:

$$x(\psi) = \int_0^{\psi} \mathfrak{F}(\tau)\sin(\psi - \tau)\,d\tau\ , \qquad (13)$$

and mean squared value:

$$\langle x^2(\psi)\rangle = \iint_0^{\psi} \sum_{i,j=1}^{N} \langle\mathfrak{F}_i(\tau_1)\cdot\mathfrak{F}_j(\tau_2)\rangle\, \delta_{2\pi\nu}(\tau_1 - \mu_i)\times$$

$$\times\, \delta_{2\pi\nu}(\tau_2 - \mu_i)\sin(\psi - \tau_1)\sin(\psi - \tau_1)\,d\tau_1 d\tau_2 =$$

$$= \frac{1}{(2\pi\nu)^2}\sum_{i,j=1}^{N}\int_{-\infty}^{\infty}\mathfrak{P}_{ij}(w)\,dw \sum_{n,m=-\infty}^{\infty}\exp\left(-i\,\frac{n\mu_i - m\mu_j}{\nu}\right)\times$$

$$\times\int_0^{\psi}\sin(\tau_1-\psi)\exp\left(i\tau_1\frac{w\nu+n}{\nu}\right)d\tau_1\cdot\int_0^{\psi}\sin(\tau_2-\psi)\exp\left(-i\tau_2\frac{w\nu+m}{\nu}\right)d\tau_2 \quad (14)$$

Here

$$\mathcal{K}_{ij}(\tau_1-\tau_2) = \langle\mathfrak{F}_i(\tau_1)\cdot\mathfrak{F}_j(\tau_2)\rangle = \int_{-\infty}^{\infty}\mathfrak{P}_{ij}(w)\,\exp[iw(\tau_1-\tau_2)]\,dw \quad (15)$$

is the correlation function and we use

$$\delta_{2\pi\nu}(\tau-\tau_0) = \frac{1}{2\pi\nu}\sum_{n=-\infty}^{\infty}\exp\left(-in\,\frac{\tau-\tau_0}{\nu}\right) \quad . \quad (16)$$

After integration in (14) one can obtain:

$$\mathfrak{I}_n = \int_0^{\psi}\sin(\tau-\psi)\exp\left(i\tau\,\frac{w\nu+n}{\nu}\right)d\tau =$$

$$= \frac{\exp\left(i\psi\,\frac{w\nu+n}{\nu}\right)-\cos(\psi)-i\left(\frac{w\nu+n}{\nu}\right)\sin(\psi)}{\left(\frac{w\nu+n}{\nu}\right)^2-1} . \quad (17)$$

For a large enough time ($\psi\to\infty$) the main contribution to sum (14) is made by addends with n=m for which the next asymptotic can be used:

$$|\mathfrak{I}_n^2| \xrightarrow{\psi\to\infty} \frac{\pi\psi}{2}\left(\delta(\alpha-1)+\delta(\alpha+1)\right), \qquad \alpha = \left(\frac{w\nu+n}{\nu}\right). \quad (18)$$

After a simple calculation and a transfer to usual variables one finally has (close results were obtained in [10]):



$$\frac{\langle X^2(t)\rangle}{\beta} = \frac{c^2 t}{4\pi R^2}\sum_{i,j=1}^{N}\sum_{n=-\infty}^{\infty}\mathcal{P}_{ij}\left(\omega_o(\nu-n)\right)\times$$

$$\times\frac{\sqrt{\beta_i\,\beta_j}}{F_i\,F_j}\cos\left(n\,\frac{\mu_i-\mu_j}{\nu}\right) . \quad (19)$$

Here $\omega_o$ is the revolution frequency, R is the average storage ring radius

$$\mathfrak{K}_{ij}(t_1-t_2) \equiv \langle X_i(t_1)\cdot X_j(t_2)\rangle = \int_{-\infty}^{\infty}\mathcal{P}_{ij}(\omega)\,e^{i\omega t}\,d\omega, \quad (20)$$

is the intercorrelation function for movements of lenses i and j.

One can see from (19) that only resonant harmonics influence the amplitude of the particle betatron oscillation (in fact, the spectral power $\mathcal{P}_{ij}(\omega)$ is averaged over a small frequency interval $\Delta\omega \approx 1/T$). For the SSC the first dangerous harmonics are $(1-0.78)\cdot f_o = 0.75$ kHz, $0.78\cdot f_o = 2.6$ kHz. Due to the beam-beam effects particles in the storage ring have a large enough tune spread $\sqrt{\langle\delta\nu^2\rangle}\approx 4\cdot 10^{-4}$ and different particles interact with different noise harmonics. While harmonics of the random process are independent, movements of the particles during time interval $\Delta t \gg (f_o\Delta\nu)^{-1}$ are also independent and after a large enough time the particles will be smoothly distributed over betatron phase. In this case the r.m.s. beam emittance growth is given in (19) ($\varepsilon(t)=\varepsilon(0)+\langle X^2(t)\rangle/\beta$).

It should be noted it is suggested here that $X_o(\vartheta)$ is the stationary function, i.e. $\int\mathcal{P}(\omega)d\omega$ has a finite value and does not depend on the value of the time interval T. It is not true for the earth motions which amplitude grows with time according to expression (4) and the spectral power at low frequencies grows with the time interval as:

$$\mathcal{P}(\omega) \propto \sin^2(\omega T/2)/\omega^2 .$$

Nevertheless, these low frequencies do not produce an emittance growth because of the adiabaticity of motion. They produce, as discussed above, the closed orbit distortion which can be easily compensated by the correction system.

The experiments described in [3] showed that for high frequencies and large distances between lenses the lens movements are independent. This allows to simplify further estimations. One can also consider that all lenses have the same amplitudes of oscillations $\mathcal{P}_{ij}(\omega) = \mathcal{P}(\omega)\cdot\delta_{ij}$ (here $\delta_{ij}$ - Cronecker's symbol) and take into account the relation justified for the FODO structure:

$$\sum_{i=1}^{N} \frac{\beta_i}{F_i^2} = \frac{4\,N}{l}\cdot tg\,\frac{\mu}{2} . \qquad (21)$$

Here l is the distance between neighboring lenses, N is the full number of lenses in the standard cells and $\mu$ is the phase advance per cell. As a result one has



$$\varepsilon(t)=\varepsilon(0)+\frac{c^2 t}{4\pi R^2}\left[\frac{4N}{l}\,tg\left(\frac{\mu}{2}\right) + \sum_{i=1}^{N^*}\frac{\beta_i}{F_i^2}\right]\sum_{n=-\infty}^{\infty}\mathcal{P}\big(\omega_o(\nu-n)\big) . \qquad (22)$$

Here the sum over i includes only the lenses of the nonstandard cells. For the SSC parameters ($2\pi R$ = 87.12 km, N = 968, l = 90 m, $tg(\mu/2)$=1, $\varepsilon(0)$ = $5.3\cdot10^{-9}$cm) the main contribution is made by standard cells and for the time of the emittance doubling about 10 h ( ≃cooling time due to SR of protons in the SSC) one has a limitation on the acceptable spectral power:

$$\sum_{n=-\infty}^{\infty}\mathcal{P}\big(\omega_o(\nu-n)\big) \le 0.92\cdot10^{-12}\ \mu m^2/Hz .$$

It is a very small value which is by two orders of magnitude smaller than the best experimentally observed value in [3]. For comparison note that for the "white" noise ($\mathcal{P}(\omega)$ = const) the r.m.s. displacement of lenses should be as small as $\sqrt{\omega_o \mathcal{P}} \simeq 1.4\cdot10^{-4}\mu m$.

As was mentioned above, the emittance growth occurs due to a betatron frequency spread in the bunch. It is necessary to add that even in the assumption of a zero betatron frequency spread the correlations in the movement of separate bunches drop fast with the increase in bunch spacing. To investigate the movement correlation of two bunches spaced in the betatron phase by $\psi_o$ let's write their movement equations (following from (11) and (12)):

$$\frac{d^2x_1}{d\psi^2} + x_1 = \mathfrak{F}_1(\psi) \quad , \qquad \frac{d^2x_2}{d\psi^2} + x_2 = \mathfrak{F}_2(\psi), \tag{23}$$

and subtract one from the other:

$$\frac{d^2}{d\psi^2}(x_1 - x_2) + (x_1 - x_2) = (\Im(\psi_1) - \mathfrak{F}_2(\psi)), \tag{24}$$

where

$$\mathfrak{F}_1(\psi) = \sum_{i=1}^{N} \delta_{2\pi\nu}(\psi - \mu_i)\, \mathfrak{F}_i(\psi) \;,$$

$$\mathfrak{F}_2(\psi) = \sum_{i=1}^{N} \delta_{2\pi\nu}(\psi - \mu_i - \psi_\circ)\, \mathfrak{F}_i(\psi) \;. \tag{25}$$

Comparing (11) and (24) one can see that it is necessary to replace the correlation function $<\mathfrak{F}_i(\tau_1)\, \mathfrak{F}_j(\tau_2)>$ in (14) by

$$<(\mathfrak{F}_i(\tau_1) - \mathfrak{F}_i(\tau_2))\cdot(\mathfrak{F}_j(\tau_2-\psi_\circ) - \mathfrak{F}_j(\tau_2-\psi_\circ)> =$$

$$= 2\int_{-\infty}^{\infty} \mathcal{P}_{ij}(w) \Big(\exp[iw(\tau_1-\tau_2)] - \exp[iw(\tau_1-\tau_2-\psi_\circ)]\Big)\, dw \;. \tag{26}$$

After simple calculations and a transfer to usual variables one finally has:

$$\frac{<(X_1-X_2)^2>}{\beta} = \frac{c^2 t}{4\pi R^2} \sum_{i,j=1}^{N} \sum_{n=-\infty}^{\infty} \times$$

$$\times\, 4\sin^2\!\left(\frac{\omega_\circ(\nu-n)\ell}{2c}\right) \mathcal{P}_{ij}\big(\omega_\circ(\nu-n)\big)\frac{\sqrt{\beta_i\beta_j}}{F_i F_j}\cos\!\left(n\,\frac{\mu_i-\mu_j}{\nu}\right) . \tag{27}$$

where $\ell$ is the distance between bunches. Comparing (17) and (27) one can see that in the case when the spectral power is



decreasing quickly with frequency there is a good correlation in the movement of closely spaced bunches and the correlation goes down if $\ell \approx \mathfrak{C}/2$.

It also means that for its effective operation the feedback correction system should have rather wide frequency band.

## 3. ACTIVE SUPPRESSION OF THE EMITTANCE GROWTH

The use of the feedback system for damping the coherent dipole oscillation allows to suppress the beam deviations from the closed orbit and, consequently, strongly suppress the emittance growth. In this case, the emittance growth occurs only due to the betatron tune spread.

Let's consider the divergence of two particles with different betatron tunes $\nu_1$, $\nu_2$ $(\delta\nu = \nu_2 - \nu_1)$ after they have passed the phase $\vartheta$. The particle coordinates are related to the beam center which is shifted in the phase space at $x_c(\vartheta)$, $dx_c/ds(\vartheta)$.

$$<(X_1-X_2)^2> = \frac{1}{2}<\Big|\big((Z_1+Z_c)\,e^{i\nu_1\vartheta} - (Z_2+Z_c)\,e^{i\nu_2\vartheta}\big)^2\Big|> =$$

$$= \frac{1}{2}<|Z_1^2|+|Z_2^2|+2|Z_c^2|[1-\cos(\vartheta(\nu_2-\nu_1))]> =$$

$$= 2\big(<X^2>+<x_c^2>[1-\cos(\delta\nu\vartheta)]\big), \tag{28}$$

where $Z=X+i\beta\cdot dX/ds$, $<Z_1Z_2>=0$, $<Z_{1,2}Z_c>=0$, $<Z_1^2>=<Z_2^2>=2<X^2>$,

$\langle Z_c^2\rangle = 2\langle x_c^2\rangle$. For $\delta\nu\vartheta \ll 1$ one can write down the value of the emittance increase:

$$\delta\varepsilon = \delta\left(\frac{1}{2}\langle(X_1-X_2)^2\rangle/\beta\right) = \frac{1}{2}\langle x_c^2\rangle\cdot(\delta\nu\vartheta)^2/\beta . \qquad (29)$$

The main source of the beam tune spread are the beam-beam effects at the IP which determine the dependence of the tune on the betatron oscillation amplitude $a$. The beam-beam tune shift can be easily calculated by means the perturbation theory for the round beam and the Gaussian density distribution with the r.m.s. size $\sigma$. In the first approximation:

$$\delta\nu(a) = \frac{\xi}{(2\pi)^2 a}\int_0^{2\pi} f(a\cdot\cos\psi)\cdot\cos\psi\, d\psi =$$

$$= \frac{4\,\xi\,\sigma^2}{a^2}\left[1 - I_0\left(\frac{a^2}{4\sigma^2}\right)\cdot\exp\left(-\frac{a^2}{4\sigma^2}\right)\right], \qquad (30)$$

where

$$f(r) = \frac{8\,\pi\,\sigma}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right] \qquad (31)$$

is a dimensionless interaction force,

$$\xi = \frac{e^2 N\beta^*}{4\pi P c\sigma^2} = \frac{e^2 N}{4\pi P c\varepsilon} \qquad (32)$$

is the parameter of the interaction (the linear beam-beam tune shift) and $I_0(x)$ is the modified Bessel function. An averaging of (30) with 2D Gaussian distribution was made numerically and gave the next result:



$$\langle\delta\nu^2\rangle \approx (0.1974\,\xi)^2 . \qquad (33)$$

Note that the interaction at parasitic interaction points produces mostly a linear tune shift with a negligible dependence on the amplitude and therefore one should take into account only the interaction at the main IPs. As can be seen from (32) for a round beam both IPs make the equal contributions and for the SSC parameters one has

$$\sqrt{\langle\delta\nu^2\rangle} \approx 2\cdot 0.1974\,\xi \approx 2\cdot 0.1974\cdot 0.0009 \approx 3.6\cdot 10^{-4}.$$

It means a full beam decoherence in SSC after about 1 second.

Other excitement factors, such as the tune spread due to the final value of chromaticity, produce an essentially smaller tune spread and their influence can be neglected.

As can be seen below, a successful SSC operation requires ultimate parameters of the feedback system. We consider here an idealized system with a high enough frequency band such that the motions of different bunches are independent. Let be for a transverse bunch displacement equal $x_1$ at the pick-up electrode point (point 1) the bunch gets a kick in x direction:

$$\delta\vartheta = \frac{g\,x_1}{\sqrt{\beta_1\beta_2}} \qquad (34)$$

at the point 2 located a quarter of betatron wave length later. Here $g$ is the dimensionless amplifications of the feedback system, $\beta_1$ and $\beta_2$ are $\beta$-functions at the pick-up and the kicker locations. Expressing $x_1$ in terms of the bunch coordinates at point 2

$$x_1 = -\alpha_2\sqrt{\beta_1/\beta_2}\, x_2 - \sqrt{\beta_1\beta_2}\,\vartheta_2 , \qquad \alpha \equiv -\frac{1}{2}\, d\beta/ds \tag{35}$$

one can get the transition matrix at point 2:

$$|M_2| = \begin{vmatrix} 1 & 0 \\ -g\alpha_2/\beta_2 & 1-g \end{vmatrix} , \tag{36}$$

and the full transition matrix of the ring:

$$|M| = |M_2| \cdot \begin{vmatrix} C+\alpha_2 S & \beta S \\ -\gamma_2 S & C-\alpha_2 S \end{vmatrix} = \begin{vmatrix} C+\alpha_2 S(1-g) & (1-g)\beta_2 S \\ -\gamma_2 S - g\alpha_2(C-\alpha_2 S)/\beta_2 & (1-g)(C-\alpha_2 S) \end{vmatrix} , \tag{37}$$

where $\gamma \equiv (1+\alpha^2)/\beta$, $C = \cos(2\pi\nu)$, $S = \sin(2\pi\nu)$. The solution of the characteristic equation:

$$\|M - \Lambda E\| = 0$$

determines eigen numbers of the matrix:

$$\Lambda_{1,2} = C\,(1 - g/2) \pm i\sqrt{S^2(1-g) - g^2C^2/4} . \tag{38}$$

For $\det \equiv S^2(1-g) - g^2C^2/4 \geq 0$ one can find their modules:

$$|\Lambda_{1,2}| = \sqrt{1-g} . \tag{39}$$

The dependence of modules of the eigen number on $g$ is shown in Fig.1. One can see that the maximum of decrement (minimum



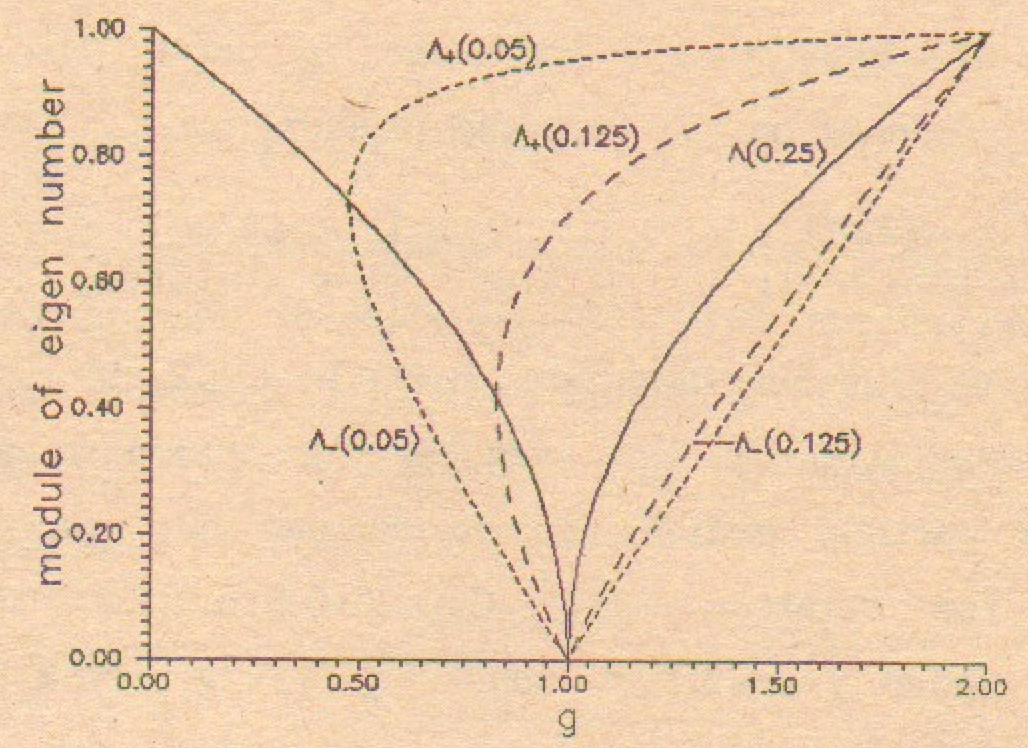


*Fig. 1.* The dependence of module of the eigen number on the dimensionless gain of the feedback system $g$.

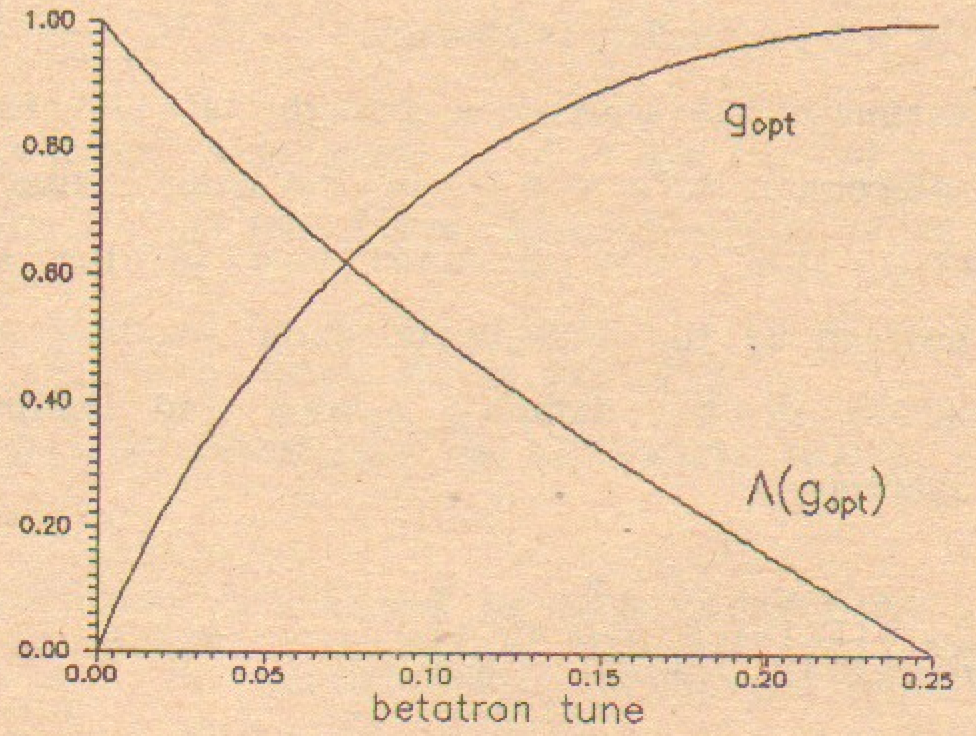


*Fig. 2.* Optimal values of $g$ and $\Lambda$ versus tune $\nu$.

of $\Lambda$) for the given tune $\nu$ is reached at det=0. The optimal value of $g$ depends on the tune and is equal to:

$$g_{opt} = \left| \frac{2\sin(2\pi\nu)}{1+\sin(2\pi\nu)} \right| , \qquad |\Lambda(g_{opt})| = \left| \frac{\cos(2\pi\nu)}{1+\sin(2\pi\nu)} \right| . \qquad (40)$$

The plots of this functions are shown in Fig.2. If $g > 2$ the motion looses stability. One sees that the maximum damping, when the particle motion can be damped per one turn, is achieved for betatron tunes Q=0.25+0.5·n. For the SSC working point $\nu$=122.78 one gets $g_{opt} \approx 0.999$ and $\Lambda(g_{opt}) \approx 0.063$. For $g \ll 1$ one can define the damping decrement to be equal to

$$\lambda = f_0(1 - \Lambda) \approx f_0 \frac{g}{2} (1 + \frac{g}{4}) \qquad (41)$$

where $f_0$ is the revolution frequency.

To estimate the emittance growth rate let us find the r.m.s. displacement $\langle x_c^2 \rangle = \varepsilon_c \beta$ of the beam center from the closed orbit. This value is determined by the equilibrium between heating by the quadrupole jitter $(d\varepsilon/dt)_0$ and by the final accuracy of the damping system and cooling by the damping system:

$$\varepsilon_c = \frac{1}{2\lambda} \left[ \left(\frac{d\varepsilon}{dt}\right)_0 + f_0 \frac{g^2}{2\beta_1} \langle x_p^2 \rangle \right] , \qquad g \ll 1 . \qquad (42)$$

Here $\langle x_p^2 \rangle$ is the r.m.s. accuracy of measurements of the beam position and one takes into account that the error in the position measurement produces an additional kick in the



kicker:

$$\langle \delta\vartheta_2^2 \rangle = g^2 \langle x_p^2 \rangle / (\beta_1 \beta_2).$$

Now we can easily estimate the emittance growth. Let us take into account in equation (29) that the correlation in the bunch movement is conserved during the time $\approx 1/\lambda$ and using (33) one can write down that the emittance increase during the time $1/\lambda$ is equal to:

$$\delta\varepsilon \approx \frac{1}{2} \varepsilon_c \cdot (0.197\xi)^2 \left( \frac{2\pi f_0}{\lambda} \right)^2 , \quad \varepsilon_c = \left\langle \frac{x_c^2}{\beta} \right\rangle \qquad (43)$$

And finally after substituting (41) and (42) in (43) one can estimate the emittance growth rate:

$$\frac{d\varepsilon}{dt} \approx \frac{A\,\xi^2}{g^2(1 + g/2)} \left[ \left(\frac{d\varepsilon}{dt}\right)_0 + f_0 \frac{g^2}{2\beta_1} \langle x_p^2 \rangle \right] , \quad \xi \ll g \ll 1, \qquad (44)$$

Here the constant $A$ is equal to $A \approx (2\pi \cdot 0.1974)^2 \approx 1.53$. The computer simulations (see below) gives a higher value $A \approx 3$.

## 4. BEAM BEAM EFFECTS IN THE PRESENCE OF EXTERNAL NOISE

To test these estimations we have carried out numerical simulations with the particle motion being influenced by the external noise and the feedback system. Two models have been used. In the first case we consider that every particle has a linear motion, with the constant betatron tune determined by the initial particle amplitude (see (30)). This tune spread produces the emittance smear and consequently the emittance growth. In the second case we use direct calculations of the nonlinear interaction with the counter bunch field given in (31) (strong-weak approximation). From the common point of view this case differs strongly from the previous one because the movement of particles becomes nonlinear and many nonlinear resonances influence the particle motion.

In both the cases the movement was two dimensional and the initial particle distribution in the phase space was Gaussian. There was an assumption that counter beam has also 2D Gaussian density distribution and does not change its size. All simulations have been performed for one IP and equals betatron tunes $Q_x=Q_z=Q$. The synchrotron motion and the finite bunch length were neglected. The noise was modeled by random additions (equal for all particles) to the transverse particle velocities and their positions after passing each IP. This corresponds to many quadrupoles distributed along the circumference and moved independently. The r.m.s. value of this additions was equal to $\langle\delta x^2\rangle=\langle\delta\theta^2\rangle=\frac{1}{2}\,\Delta^2$ and all the sizes were measured in units of r.m.s. beam sizes at IP. The distribution of hit values was Gaussian. The damping was modeled as described above in section 3 (see (34)). The kicker of the damping system was located just after the IP. There was also a possibility to



study the effects of the measurement errors of the feedback system. In this case after the interaction the particles get random additions only in velocity. We used many particles (5000-15000) to prevent the stochastic cooling which decrement, in the case of a large enough betatron tune spread, is equal to

$$\lambda_{stc} \approx g \,/\, (2N_{particle})\ . \qquad (45)$$

But in our simulations because of a small betatron frequency spread the real decrements of cooling tested numerically were much smaller and, as a rule, it was possible to neglect the effects of cooling.

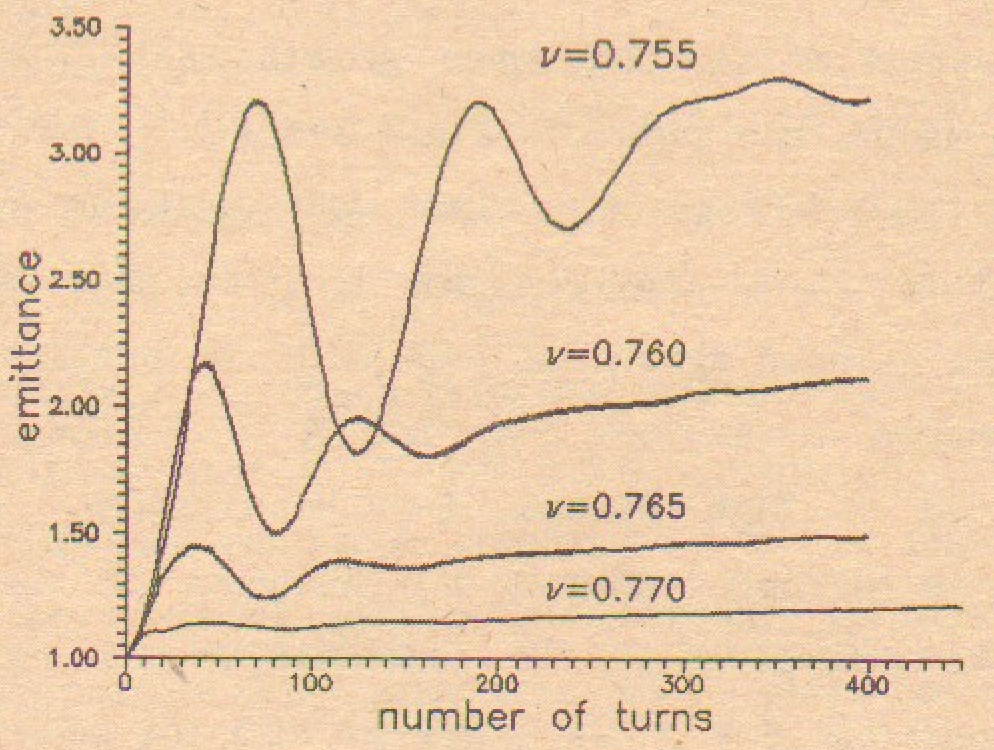


*Fig. 3.* The dependence of emittance on time for different betatron tunes in the vicinity of a resonance 3/4 in the nonlinear model; $\xi$=0.03, $\Delta$=0.05, $g$=0.2.

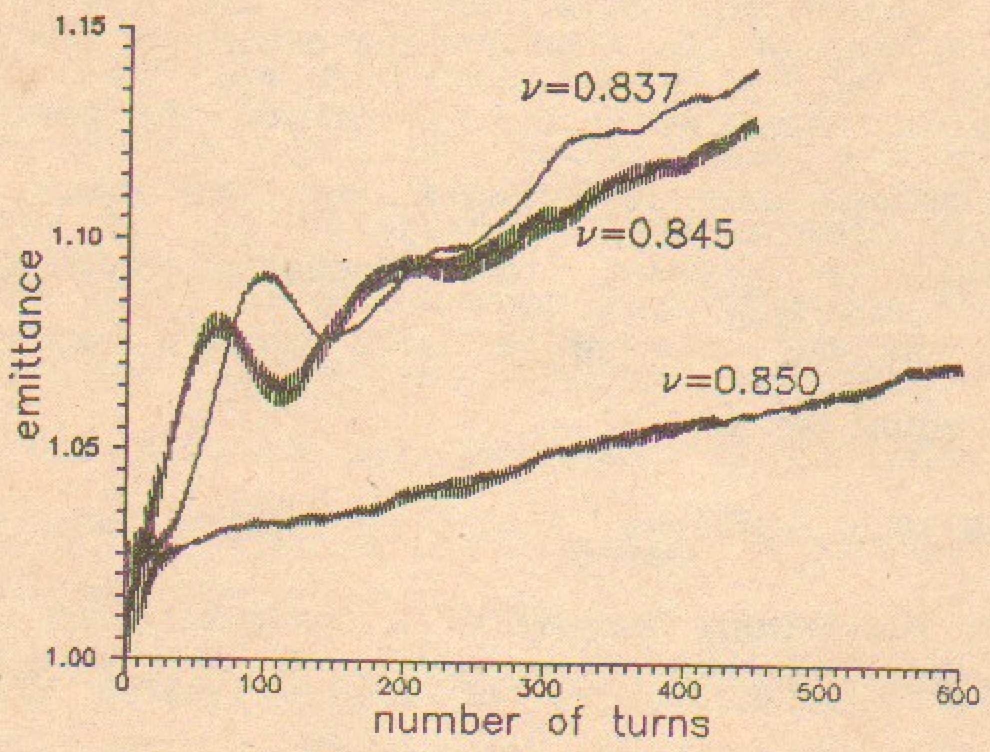


*Fig. 4.* The dependence of the emittance on time for different betatron tunes in the vicinity of a resonance 5/6 in the nonlinear model; $\xi$=0.03, $\Delta$=0.05, $g$=0.2.

Examples of the emittance growth calculations in the nonlinear model are shown in Fig.3 and Fig.4. The calculated emittance growth rates for linear and nonlinear models are presented in Fig.5. One can see that for $\xi$ < 0.03 both models are in good agreement both with one another and with the suggested estimations (44) for $A$ = 3. The divergence appears at a high enough $\xi$-value ($\xi \geq 0.03$), when the nonlinearity strongly enforces the influence of the external noise. The saturation of the emittance growth rate in the linear model for $\xi \geq g$ is bound up with the achievement of its maximum value equal to the growth rate without suppression. For small enough values of kicks $\Delta$ the emittance growth rate is proportional to $\Delta^2$ but for their



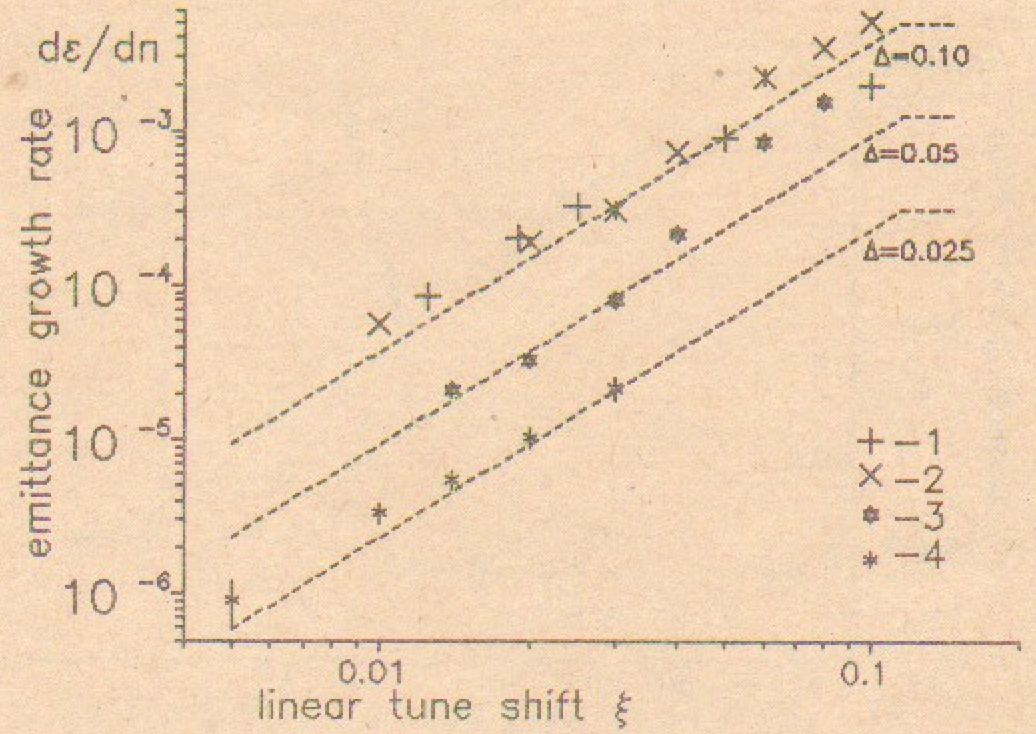


*Fig. 5.* The dependence of the emittance growth rate on the beam-beam tune shift $\xi$ for different values of noise $\Delta$; $g$=0.2, $\nu$=0.78. The solid lines are plotted via formula (44) with $A$=3:
*1* - linear model, $\Delta$=0.1; *2* - nonlinear model, $\Delta$=0.1; *3*- nonlinear model, $\Delta$ = 0.05; *4* - nonlinear model, $\Delta$=0.025.

large values ($\Delta$=0.1) the emittance growth rate increases faster. It results from large amplitude of the coherent motion $\sqrt{\langle \delta x^2 \rangle} \approx 0.1$ which produces a larger tune spread and consequently a large emittance growth rate. In Fig. 6 the dependence of the emittance growth rate on the dimensionless gain of the feedback system $g$ is shown. Likewise in the previous figure the dependence goes out on plato for $g \leq \xi$.

To understand why $\xi \approx 0.03$ is limited by a nonlinear interaction we scanned betatron tunes in the vicinity of the suggested SSC working point. The results of this scanning

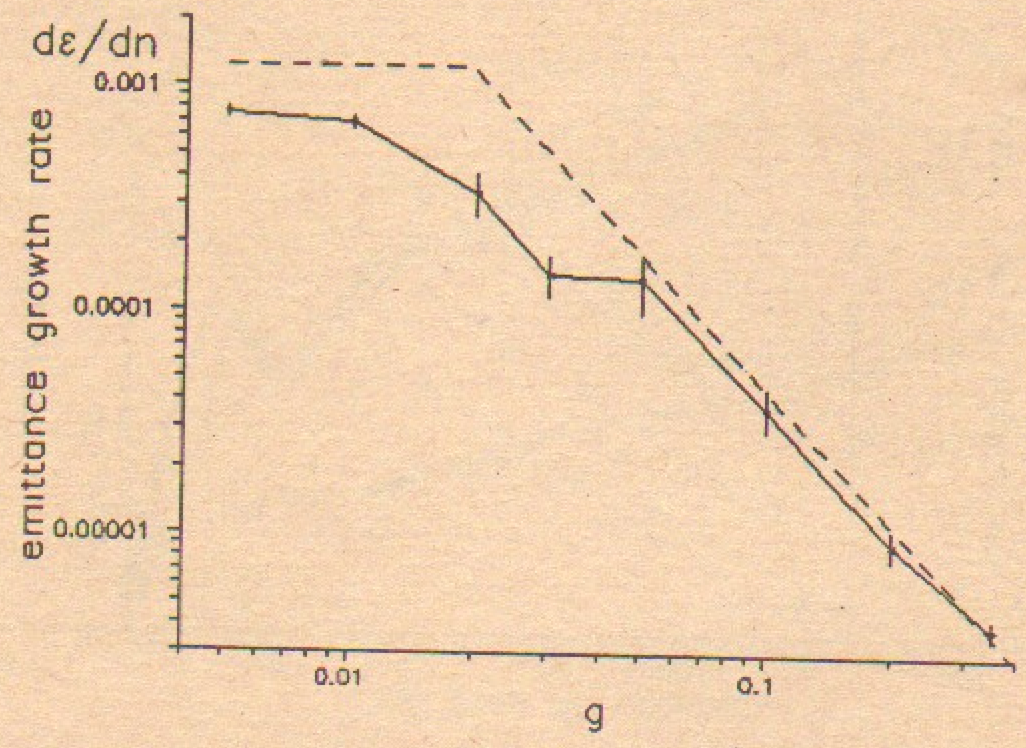


*Fig. 6.* The dependence of the emittance growth rate on the dimensionless gain $g$ of the feedback system in nonlinear model; $\Delta$=0.05 ,$\xi$=0.01, $\nu$=0.78. The dashed line is plotted with the help of formula (44) with $A$=3.

are shown in Fig.7. One can see a drastic influence of resonances 3/4 and 5/6 on the emittance growth rate. At these tunes there was observed an appearance of non-Gaussian tails in the particle density distribution. The resonances 8/10 and 11/14 can hardly be seen. This figure clearly demonstrates that the additional increase in the emittance growth rate for $\nu$=0.78 and $\xi\geq$0.03 is bound up with the influence of the powerful resonance 3/4. This shows that the nonlinearity of movement (which is not in the first model) strongly affects the particle motion only in case of a high enough $\xi$-value. For small enough $\xi$-values one can neglect



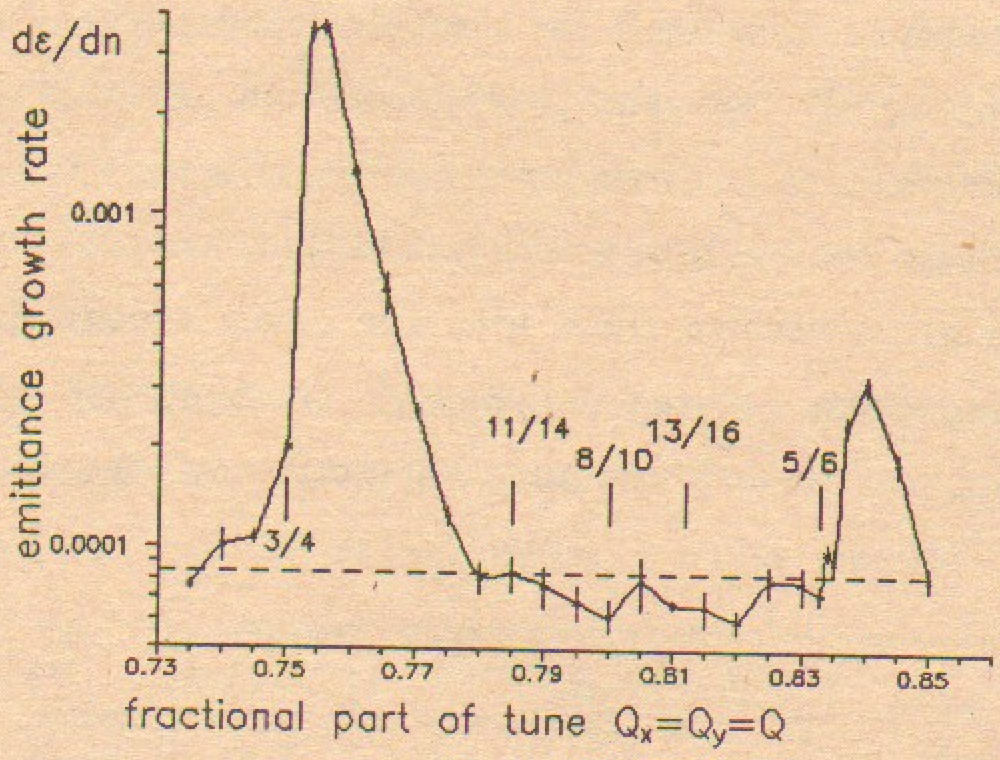


*Fig. 7.* The dependence of the emittance growth rate on the tune $\nu$; $g$=0.2, $\Delta$=0.05, $\xi$=0.03. The dashed line is plotted with the help of formula (44) with $A$=3.

the influence of nonlinear resonances. In this case, as can be seen from Fig.5, the dependence of the emittance growth rate on $\xi$-value is: $d\varepsilon/dt \propto \xi^{-2}$ for $\xi \ll g$, $d\varepsilon/dt \propto \xi^{-1}$ for $\xi \approx g$, $d\varepsilon/dt \propto$ const for $\xi \gg g$. The dependence of $d\varepsilon/dt \propto \xi^{-1}$ was predicted in ref. [6] and [7]. The influence of the feedback system was not taken into account which fact led to incorrect conclusions.

The experimental study of the beam-beam effects at proton-antiproton colliders has shown that the measured $\xi$-values are much smaller than these for the electron-positron colliders and do not surpass 0.005 for one IP. The main difference between the proton-(anti)proton and electron-positron colliders is that for the first one there is no damping of separate particle motion. In this case the beam-beam effects can be very strongly affected by the external noise which produces an additional emittance smear. The above given results show that for small enough $\xi$ and for the working point located far enough from strong resonances one can neglect a very complicated picture of the motion and

Table 3

| | | SppS [5] | Tevatron [5] | LHC [11] | SSC [12] |
|---|---|---|---|---|---|
| Energy | $E$ [GeV] | 315 | 900 | 8000 | 20000 |
| Circumference | $C$ [km] | 6.93 | 6.29 | 26.66 | 87.12 |
| Revolution frequency | $f_0$ [kHz] | 43.3 | 47.7 | 11.25 | 3.44 |
| Betatron tunes | $Q_x$ | 26.685 | 19.405 | 71.28 | 123.78 |
| | $Q_z$ | 26.675 | 19.415 | 70.31 | 123.78 |
| Number of IPs | N | 3 | 12 | 3 | 2 |
| Head-on beam-beam tune shift per collision: $(\bar{p}/p)$ $\xi_x$ | $[10^{-3}]$ | 5.0/5.3 | 1.5/2.1 | 3 | 0.9 |
| $\xi_z$ | $[10^{-3}]$ | 3.3/3.3 | 1.5/2.1 | 3 | 0.9 |
| Summed head-on beam-beam tune shift $\xi^* = N\xi$: $(p/\bar{p})$ $\xi^*_x$ | $[10^{-3}]$ | 15/16 | 18/25 | 10 | 1.8 |
| $\xi^*_z$ | $[10^{-3}]$ | 11/11 | 18/25 | 10 | 1.8 |
| Normalized emittance $\varepsilon_n = \beta\gamma\varepsilon$: $p/\bar{p}$ | [ μm ] | 3/1.75 | 4/2 | 3.75 | 1 |
| r.m.s. emittance $p/\bar{p}$ | $\varepsilon$ [nm] | 8.9/5.2 | 4.2/2.1 | 0.46 | 0.047 |
| The first resonance frequencies $f=f_0(Q-n)$, [kHz] | $f_{1x}$ | 13.6 | 19.3 | 3.1 | 0.76 |
| | $f_{2x}$ | 29.6 | 28.4 | 8.0 | 2.68 |
| | $f_{3x}$ | 56.9 | 67.0 | 14.0 | 4.20 |
| | $f_{4x}$ | 72.9 | 76.1 | 19.4 | 6.12 |
| $\beta$-functions at the feedback pick-up locations | [m] | ≈40 | ≈50 | ≈60 | ≈110 |
| Luminosity lifetime | [hours] | 20 | 20 | 11 | 10 |
| Acceptable values of : emittance growth rate | [μm/s] | $7\cdot10^{-8}$ | $3\cdot10^{-8}$ | $4\cdot10^{-9}$ | $1.3\cdot10^{-9}$ |
| $\mathcal{P}_{eff}\cdot10^{13}$, for $\xi > g$, | $[\mu m^2\cdot s]$ | 4.7 | 2.7 | 2.3 | 8.0 |
| $\mathcal{P}_{eff}/g^2\cdot10^{10}$, for $\xi < g$, | $[\mu m^2\cdot s]$ | 4.6 | 1.1 | 7.7 | 610 |
| r.m.s. coherent betatron oscilations | [μm] | $0.24\sqrt{g}$ | $0.16\sqrt{g}$ | $0.23\sqrt{g}$ | $1.8\sqrt{g}$ |
| accuracy of BPM | [μm] | 0.39 | 0.26 | 0.37 | 2.9 |



consider the external noise as the main source of the emittance growth. A strong influence of external noise on the beam lifetime was demonstrated at the Tevatron [8]. Acceptable external noise values for different colliders are shown in Table 3.

It is difficult to compare accurately these predictions with the experimental data because of lack of information and large spread in experimental results in the available

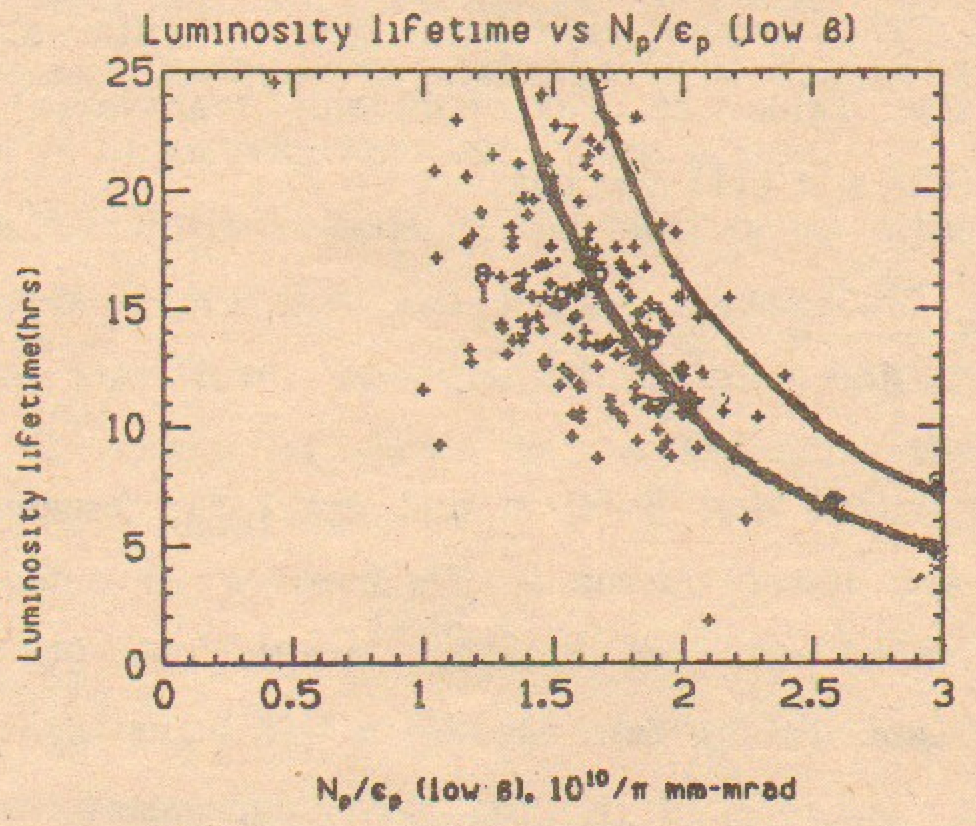


*Fig. 8.* The luminosity lifetime on the beam-beam tune shift received on the Tevatron [5]. Solid lines are plotted by using the dependence $\tau \propto \xi^{-2}$; $\xi$=(number of crossings)$\cdot r_p \cdot (N_p/\varepsilon_p)$.

publications. Nevertheless, we tried to estimate the external noise value for the Tevatron using the results from ref. [5] presented in Fig.8. There are two curves plotted by using the asymptotic $d\varepsilon/dt \propto \xi^{-2}$(see (44)): one for best points and the other for the worst ones. One can see very good agreement of theoretical dependence $d\varepsilon/dt$ with all the best points. We consider that these points correspond to good conditions and a good installation tuning and therefore, they are in a good agreement with the theoretical predictions. The known emittance growth rate allows to calculate the value of the coherent transverse motion. Combining (42) and (44) one has

$$\varepsilon_c \equiv \frac{\langle x_c^2 \rangle}{\beta} = \frac{g}{4\xi^2 f_0} \cdot \frac{d\varepsilon}{dt} \quad . \qquad (46)$$

In the supposition that $\beta$=50 m and $g$=0.1 the parameters of the upper and lower curves in Fig.2 allow to calculate the value of r.m.s. coherent betatron oscillations $(\langle x_c^2 \rangle)^{1/2} \approx$ 0.05 - 0.07 $\mu$m. Taking into account a bad knowledge of some values this is not in a too bad agreement with the experimentally measured in ref. [8] value $\approx$0.3 $\mu$m. Note that in this work the movement of bunches is assumed to be independent which is not exactly true and therefore a real amplitude of oscillations should be smaller. The results of work [13] performed later gave a smaller value after a thorough experimental study of noise sources.



A comparison of main parameters for different storage rings is given in Table 3. For simplicity the values of $\beta$-functions at locations of the kicker and the beam position monitor are taken to be equal a value of an average $\beta$-function in the arcs.

## 5. COHERENT BEAM-BEAM EFFECTS IN THE PRESENCE OF AN EXTERNAL NOISE

In addition to the kicks produced by the lens displacements the beam is also kicked by counter beam fields which leads to an additional emittance growth.

The calculations made in the hard bunch model with it's bunch-dipole interaction and in the absence of damping do not make significant contribution to the emittance growth rate. In this case, as shown in the Appendix, the emittance growth rate is equal to:

$$\frac{d\varepsilon}{dt} = \left(\frac{d\varepsilon}{dt}\right)_0 \left(1 + \frac{(4\pi\xi_s)^2}{2s[s - 4\pi\xi_s c - (4\pi\xi_s)^2 s]}\right) \quad . \qquad (47)$$

Here $\xi_s$ is the total linear tune shift with taking into account an interaction in the parasitic IPs, s=sin(2$\pi\nu$), c=cos(2$\pi\nu$). The second addend in brackets correspond to the contribution of the coherent dipole interaction. The SSC has two IPs. If tune advances between them are equal one can simply substitute 2$\pi\nu$ by $\pi\nu$ in (47) and then for $\xi \ll 1$ one has:

$$\frac{d\varepsilon}{dt} = \left(\frac{d\varepsilon}{dt}\right)_0 \left[1 + \frac{(4\pi\xi_s)^2}{2\sin^2(\pi\nu)}\right] . \qquad (48)$$

For the SSC parameters $\nu$=123.78 and $\xi_s$=0.01 there is a correction due to the coherent beam-beam interaction about $1.3\cdot10^{-2}$ which value is negligible.

To test this result we carried out direct numerical simulations of the emittance growth rate in the "hard-soft"

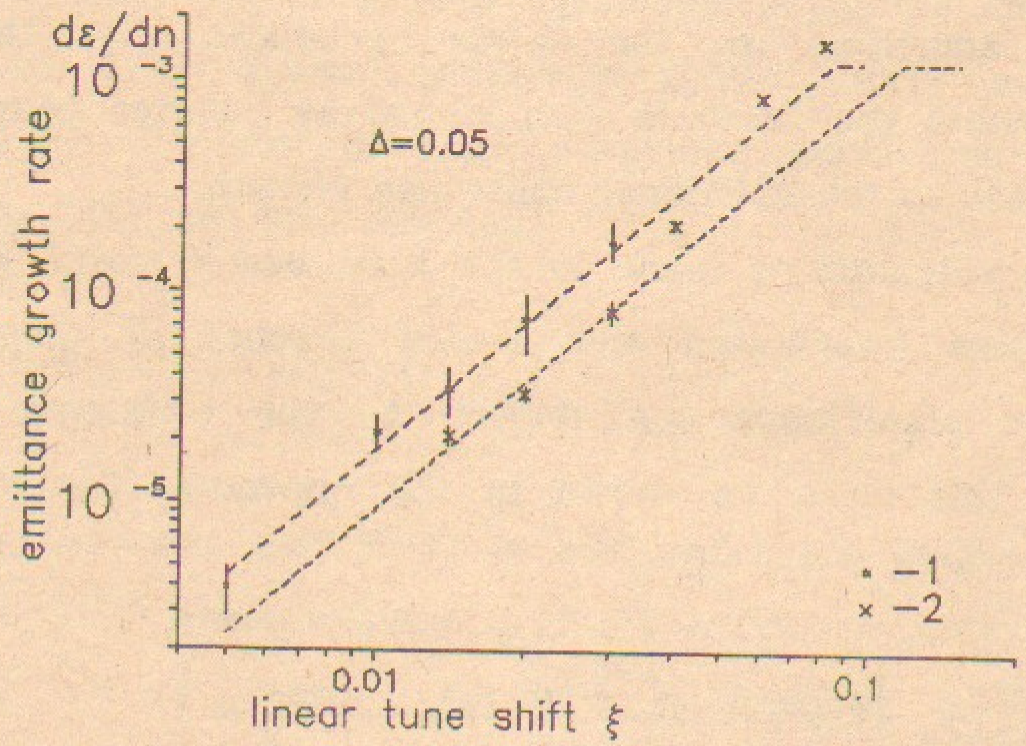


*Fig. 9.* The dependence of the emittance growth rate on the beam-beam tune shift $\xi$ with (upper curve) and without (lower curve) a coherent movement of the hard bunch; $g$=0.2, $\Delta$=0.05, $\nu$=0.78.



bunches model. In this case one bunch was modeled by many particles ($N_{particle}$=15300) and the other one was hard but, unlike in the model described in the previous Section, it was provided a two beam betatron movement. Both the bunches were influenced by an external noise and the damping system. The results of these simulations are shown in Fig.9. One can see that the dependence of the emittance growth rate on $\xi$ is similar to the one described above but its value is almost doubled. The r.m.s. coherent betatron amplitude didn't change becouse of the influence of a "hard" beam movement. The coherent amplitudes of both the beams were equal. These calculations redetermined the value of the constant $A$ in expression (44) to be equal to 5.8 in the presence of a coherent beam-beam motion.

It seems plausible that this additional increase in the emittance growth rate is bound up with the parametric excitement of particle motion by fields of a hard counter bunch. It results from the change of the counter bunch focusing strength due to it's transverse movement which is of a random character. In this case the value of the angle kick by the quadrupole fields is about:

$$\delta\vartheta \approx 4\pi\xi \, \frac{3x^2 x_c}{\beta \, \sigma^{*2}} , \qquad (49)$$

where x, $x_c$ are the particle and the bunch coordinates at the IP, $\sigma^*$ is the r.m.s. beam size at the IP. Taking into account that time of an interaction is an order of inverse decrement $\lambda^{-1}$, assuming $\langle x^2\rangle \approx \sigma^2$, using expression (42), squaring and averaging (49) one obtains an additional contribution to the emittance growth rate the similar to the one estimated in (44).

## 6. ESTIMATIONS OF THE FEEDBACK SYSTEM

Common principles and requirements to the feedback system are very close to the stochastic cooling ones. Here we shortly consider the main limitations of this system determined by the beam dynamics.

One can see from Table 3 that the accuracy of the beam position monitor (BPM) should be better than 2.9 $\mu$m and also the really attained accuracy at the Sp$\bar{p}$S and the Tevatron is much better. But it is necessary to note that in the SSC case the distance between bunches is much smaller therefore the frequency band width of the feedback system should be increased. This consequently determines a larger value of noise. The principal limitation on the BPM resolution is determined by the preamplifier's thermal noise:

$$\langle U_R^2\rangle = 4kTZ_1\cdot\Delta f \qquad (50)$$

which in comparison with BPM signal

$$U_{input} = Z_1 I_0 \frac{\delta x_{c1}}{a} \qquad (51)$$



defines the ultimate accuracy of BPM

$$\delta x_{BPM} \approx \frac{a}{I_0}\sqrt{\frac{4kT\cdot\Delta f}{Z_1}}\,. \qquad (52)$$

Here $a$ is the vacuum chamber aperture, $Z_1$ is the BPM impedance, $\Delta f$ is the band width of the feedback system, $I_0$ is the beam current and k,T are the Boltzman constant and temperature. In the worst case when the system allows to measure positions of separated bunches spaced at l=5 m for a=2 cm, $Z_1$=50 Ω, $I_0$=70 mA, T=300 K and $\Delta f$=c/l = 70 MHz one gets $\delta x_{BPM} \approx 0.06$ $\mu$m which is by an order magnitude better the required value. Thus, one can see that there are no principal limitations on the necessary BPM accuracy.

Another and more strict limitation is the limitation on the output amplifier power. To estimate it let us consider a simple model of the feedback system. Multiplying the voltage excited by the beam on the pick-up electrodes (see (51)) by the gain of amplifier K one obtains the value of the kick produced by the feedback system:

$$\delta\vartheta = \frac{e\,l_k U_{out}}{P\,c\,a} = \frac{e\,l_k}{P\,c\,a} K Z_1 I_0 \frac{\delta x_c}{a}\,, \qquad (53)$$

where $l_k$ is the full length of kickers and P is the momentum of particles. Comparing this equation with (34) one obtains the dimensionless decrement:

$$g = \frac{e I_0 Z_1 K l_k}{P c a^2} \sqrt{\beta_1 \beta_2} \quad (54)$$

and the total output power of amplifiers is equal to:

$$P_{out} = N_k \frac{U^2_{out}}{Z_2} = \frac{N_k \langle \delta x^2_c \rangle}{\beta_1 \beta_2 Z_2} \left(\frac{Pca}{e l_k}\right)^2 g^2 \ . \quad (55)$$

where $Z_2$ is the kicker impedance and $N_k$ is the number of kickers. Expressing $\langle \delta x^2_c \rangle$ through the spectral power of lens motions and using (22) and (42) we have

$$P_{out} = \frac{N_k}{\beta_1 \beta_2 Z_2} \left(\frac{Pca}{e l_k}\right)^2 g^2 \cdot \frac{\beta_1}{g f_0} \left(\frac{d\varepsilon}{dt}\right)_0 =$$

$$= \frac{N_k}{\beta_2 Z_2} \left(\frac{Pca}{e l_k}\right)^2 g \, \frac{2Nc}{Rl} \, tg\left(\frac{\mu}{2}\right) \sum_{n=-\infty}^{\infty} \mathcal{P}\left(\omega_o(\nu - n)\right) \ . \quad (56)$$

Even for the smallest experimentally observed spectral power [3]:

$$\sum_{N=-\infty}^{\infty} \mathcal{P}\left(\omega_o(\nu - n)\right) = 3 \cdot 10^{-9} \mu m^2 \cdot s \approx 10^{-8} \ \mu m^2/Hz \ ^{1)}$$

and for $l=90$ m, $2\pi R=87.12$ km, $Pc=20$ TeV, $N=968$, $\mu=90^0$, $g=0.5$, $l_k=10$ m, $N_k=4$, $\beta_2=100$ m, $Z_2=50\ \Omega$, $a=2$ cm and $\xi=0.0018$ one has $(d\varepsilon/dt)_0=4.8 \cdot 10^{-10}$cm/s, $d\varepsilon/dt=1.8 \cdot 10^{-14}$cm/s, $P_{out} = 1$

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1) *This dimension is used for the next spectral power normalization:* $\int_0^\infty \mathcal{P}(f) df = \langle x^2 \rangle$.



kW and the luminosity lifetime is ≈80 hours. It seems reasonable that under real experimental conditions the spectral power increases by an order of magnitude as minimum. In this case the luminosity lifetime will be approximately 10 hours and $P_{out} \approx 10$ kW. As the preamplifier noise can be made small enough we neglected it in these estimations. To decrease the power of the feedback system its kickers can be located at the place with a large $\beta$-function.

Since the emittance growth suppression by the feedback system can be made very strong it is necessary to study other heating mechanisms. We note here that the contributions made by trembles of sextupoles

$$\frac{d\varepsilon}{dt} \approx \frac{\beta\varepsilon}{\psi^2} \cdot \left(\frac{d\varepsilon}{dt}\right)_0 \approx 10^{-8} \left(\frac{d\varepsilon}{dt}\right)_0 \quad (57)$$

and the particle energy spread

$$\frac{d\varepsilon}{dt} \approx \frac{\Delta P^2}{P^2} \cdot \left(\frac{d\varepsilon}{dt}\right)_0 \approx 5 \cdot 10^{-9} \left(\frac{d\varepsilon}{dt}\right)_0 \quad (58)$$

are, as a rule, negligible. Here $\psi$ is the dispersion function at the arcs, $(\Delta P/P)^2$ is the relative r.m.s. energy spread.

## CONCLUSION

The experimentally observed spectra of the ground motion drop with frequency, thus, the main contribution to the closed orbit distortion in a large collider is made by low-frequency movements with large amplitudes. At the same time the transverse emittance growth occurs only due to the resonance frequencies. They are equal fractional parts of the betatron tune $f_n=(Q-n)f_0$ (for the SSC: 760 Hz, 2.68 kHz, 4.2 kHz etc.) and are high enough. Estimations point out that in the SSC in the absolute absence of an active feedback system or, which is the same, with a small decrement of the feedback system ($g \ll \xi$) the sum of the spectral powers of vibrations at resonant frequencies (which only determines the beam sizes blow up) should be less than $10^{-12} \mu m^2 s$ ($3 \cdot 10^{-12} \mu m^2/Hz$). (It corresponds to r.m.s. amplitude of a "white noise" of about 0.13 nm at frequency band $\Delta f=f_0$ =3.4 kHz). The lowest observed levels are three orders of magnitude higher and in this case the emittance will be doubled a few minutes after the injection.

A strong feedback system, with a damping time being only few turns, allows one to decrease the emittance growth over four orders of magnitude. In this case the main obstacle to the full damping of the "beam heating" are the beam-beam effects which lead to the betatron frequency spread in the beam and consequently to the emittance smear.



Obtaining of acceptable values of the emittance growth depends both on the "vibroclimate" in the SSC tunnel and on the mechanical properties of magnetic elements and their supports which can considerably amplify vibrations in case of poor design. It is also important to have a high precision of the beam position monitor in the feedback system (0.2 - 0.5 $\mu$m). But even in the case of ultimate damping for only one turn there are strong limitations for an acceptable spectral power of the lens motion $\sum \mathcal{P}(\omega_n) \ll 6 \cdot 10^{-8} \mu m^2/Hz$ (It corresponds to r.m.s. amplitude of the "white noise" of about 0.03 $\mu$m at frequency band $\Delta f = f_0 =$ 3.4 kHz). This is a very small value and can be easily exited by technical noises of accelerator systems such as movement of cooling water, helium movement, its evaporation, etc.

It is important to note that for a given external noise an acceptable $\xi$ value determined by the beam-beam effects increases proportionally to the emittance square root $\xi \propto \sqrt{\varepsilon}$. It means that the considered in ref.[12] coalescing ten bunches in one allows to increase the luminosity $\sqrt{10} \approx 3$ times for the same current. To have the same strength in the beam-beam effects the beam emittance has also to be increased 10 times.

Further serious experimental and theoretical investigations of the problems under consideration in the present paper are undoubtedly necessary, as they may solve many questions and make exact predictions about the collider operation.

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## Appendix: Coherent beam-beam effects in the presence of external noise in the hard bunch model

To estimate this influence let's consider a simple linear model in an assumption of rigid bunches and one IP. In this case after an interaction the bunch changes its angle by:

$$\delta\vartheta_1 = 4\pi\xi_s (x_1 - x_2) \quad . \qquad \text{(A1)}$$

Here the same dimensionless variables as in (11) are used, indexes 1 and 2 correspond to both colliding beams and $\xi_s$ is the total linear tune shift (with taking into account an interaction at parasitic IPs). We shall omit index s for simplicity.

One can write down the matrix of transition through IP

$$X' = M_{IP} \cdot X \equiv \begin{vmatrix} 1 & 0 & 0 & 0 \\ 4\pi\xi & 1 & -4\pi\xi & 0 \\ 0 & 0 & 1 & 0 \\ -4\pi\xi & 0 & 4\pi\xi & 1 \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ \vartheta_1 \\ x_2 \\ \vartheta_2 \end{vmatrix} \quad . \qquad \text{(A2)}$$

and through the whole ring



$$\bar{X}_{n+1} = M \cdot \bar{X}_n = M_R \cdot M_{IP} \cdot \bar{X}_n \equiv \begin{vmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 & 0 \\ 4\pi\xi & 1 & -4\pi\xi & 0 \\ 0 & 0 & 1 & 0 \\ -4\pi\xi & 0 & 4\pi\xi & 1 \end{vmatrix} \cdot \begin{vmatrix} x_{1n} \\ \vartheta_{1n} \\ x_{2n} \\ \vartheta_{2n} \end{vmatrix} . \qquad \text{(A3)}$$

where both the storage rings have equal tunes $\nu_1 = \nu_2 = \nu$ and

$$c = \cos(2\pi\nu), \quad s = \sin(2\pi\nu). \qquad \text{(A4)}$$

Let be the beam is influenced by random kicks:

$$\bar{\Delta}_n = \begin{vmatrix} \delta x_{1n} \\ \delta\vartheta_{1n} \\ \delta x_{2n} \\ \delta\vartheta_{2n} \end{vmatrix} , \qquad \text{(A5)}$$

where

$$\langle \delta x_{1n}^2 \rangle = \langle \delta\vartheta_{1n}^2 \rangle = \langle \delta x_{2n}^2 \rangle = \langle \delta\vartheta_{2n}^2 \rangle = \sigma^2/2 \qquad \text{(A6)}$$

which corresponds to the fluctuations of N lenses ($N \gg 1$) distributed along circumference and moved with r.m.s. shifts equal to $\sigma/\sqrt{N}$. After n turns, the beam coordinates will be:

$$\bar{X}_n = \sum_{k=1}^{n} M^{n-k} \bar{\Delta}_k = \sum_{k=1}^{n} \sum_{l=1}^{4} a_{kl} \bar{V}_l \Lambda_l^{n-k} \quad . \qquad \text{(A7)}$$

Here $\bar{V}_l$ and $\Lambda_l$ are eigen vectors and eigen numbers of the transition matrix M:

$$M = \begin{vmatrix} c+4\pi\xi s & s & -4\pi\xi s & 0 \\ 4\pi\xi c - s & c & -4\pi\xi c & 0 \\ -4\pi\xi s & 0 & c+4\pi\xi s & s \\ -4\pi\xi c & 0 & 4\pi\xi c - s & c \end{vmatrix} , \qquad \text{(A8)}$$

and coefficients $a_{kl}$ are defined by equat

$$\bar{\Delta}_k = \sum_{l=1}^{4} a_{kl}\, \bar{V}_l \ . \tag{A9}$$

The r.m.s. beam deviation from the closed orbit after n turns is equal to:

$$\langle x_n^2 \rangle = \frac{1}{4}\langle(\bar{X}_n^*,\bar{X}_n)\rangle =$$

$$= \frac{1}{4}\sum_{k=1}^{n}\ \sum_{l=1}^{4}\ \sum_{l=1}^{n}\ \sum_{j=1}^{4} \langle a_{kl}^*\ \Lambda_l^{*n-k} a_{lj}\ \Lambda_j^{n-l} (\bar{V}_l^*,\bar{V}_j)\rangle =$$

$$= \frac{1}{4}\sum_{l=1}^{4}\ \sum_{j=1}^{4} \langle a_l^* a_j\rangle\ (\bar{V}_l^*,\bar{V}_j)\ \sum_{k=1}^{n} \Lambda_l^{*n-k}\Lambda_j^{n-k} =$$

$$= \frac{n}{4}\sum_{l=1}^{4} \langle a_l^* a_l\rangle\ (\bar{V}_l^*,\bar{V}_l)\ , \tag{A10}$$

where * is a sign of complex conjugation and it was taken into account that:

$$\langle a_{kl}^* a_{lj}\rangle = \langle a_l^*\ a_j\rangle\ \delta_{kl}. \tag{A11}$$

and that only members with i=j make contribution which is proportional to n because $\Lambda_l^{*n-k}\Lambda_l^{n-k}=1$.

After a simple calculation one can find out that the eigen vectors and eigen numbers of the transition matrix M are equal to:

$$\lambda_{1,2}=c-is\equiv e^{\pm 2\pi i\nu},\ \lambda_{3,4}=c+4\pi\xi s\pm i\sqrt{1-(c+4\pi\xi s)^2}\equiv e^{\pm i\delta}, \tag{A12}$$



$$\bar{V}_{1,2}=\begin{vmatrix} 1 \\ \pm i \\ 1 \\ \pm i \end{vmatrix},\ \bar{V}_{3,4}=\begin{vmatrix} 1 \\ -4\pi\xi\pm i\sqrt{1-(c+4\pi\xi s)^2} \\ -1 \\ 4\pi\xi\mp i\sqrt{1-(c+4\pi\xi s)^2} \end{vmatrix}=\begin{vmatrix} 1 \\ (c-e^{\mp i\delta})/s \\ -1 \\ -(c-e^{\mp i\delta})/s \end{vmatrix}, \tag{A13}$$

and the scalar multiplications of eigen vectors are:

$$(\bar{V}_l,\bar{V}_j) = \begin{vmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & p & q \\ 0 & 0 & q^* & p \end{vmatrix}, \tag{A14}$$

$$p = \frac{4}{s^2}\ (1-c\cdot\cos\ \delta),\quad q = \frac{2}{s^2}\ (1+e^{-2i\delta}\ -2c\cdot e^{-i\delta})\ . \tag{A15}$$

The solution of equation (A9) determines the coefficients $a_{ik}$

$$a_{k1}= \frac{1}{4}\ (\bar{\Delta}_k,\bar{V}_1^*)\ ,\qquad a_{k2}= \frac{1}{4}\ (\bar{\Delta}_k,\bar{V}_2^*)\ ,$$

$$a_{k4}= a_{k3}^* = \frac{p(\bar{\Delta}_k,\bar{V}_4^*)\ -\ q(\bar{\Delta}_k,\bar{V}_3^*)}{p^2\ -\ q\ q^*} \tag{A16}$$

Substitute (A14) and (A16) in (A10) and after average:

$$\langle a_1^*\ a_1\rangle = \langle a_2^*\ a_2\rangle = \frac{1}{8}\,\sigma^2\ ,$$

$$\langle a_3^*\ a_3\rangle = \langle a_4^*\ a_4\rangle = \frac{p}{p^2-qq^*}\cdot\ \frac{\sigma^2}{2} \tag{A17}$$

one finally has the emittance growth rate

$$\frac{d\varepsilon}{dn} = \frac{\sigma^2}{4}\left(1 + \frac{p^2}{p^2-qq^*}\right) = \frac{\sigma^2}{4}\left(1 + \frac{(4\pi\xi)^2}{2s[s-4\pi\xi c-(4\pi\xi)^2 s]}\right). \quad (A18)$$

The second addend in brackets is connected with contribution of the coherent dipole interaction. It tends to infinity near the boundary of stability [9]:

$$s-4\pi\xi c-(4\pi\xi)^2 s = 0 \quad (A19)$$

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*V.A. Lebedev, V.V. Parchomchuk,*

*V.D. Shiltsev, A.N. Skrinsky*

**Suppression of Emittance Growth Caused by Mechanical Vibrations of Magnetic Elements in Presence of Beam-Beam Effects in the SSC**

*В.А. Лебедев, В.В. Пархомчук,*

*А.Н. Скринский, В.Д. Шильцев*

**Подавление роста эмиттанса, вызываемого механическими вибрациями магнитных элементов накопителя с учетом эффектов встречи**

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