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V.N. Baier, V.M. Katkov, V.M. Strakhovenko

QUANTUM EFFECTS IN CHANNELING RADIATION

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Quantum Effects in Channeling Radiation

V.N. Baier, V.M. Katkov, V.M. Strakhovenko

Budker Institute of Nuclear Physics,
630090, Novosibirsk-90, USSR

A B S T R A C T

The quantum effects in the total intensity of radiation at channeling have been considered. It is shown that the problem can be considered in frame of magnetic bremsstrahlung limit. In a region, where quantum effects are weak, the general formulae have been obtained for quantum corrections to the total intensity of the channeling radiation. While in diamond and silicon the quantum effects become noticeable at energy of order 5 GeV, in tungsten the quantum effects lead to appreciable decreasing the radiation intensity already at energy 1 GeV: order of 25 % at room temperature and 34% at T = 77 K.

1. A radiation mechanism at motion of fast charged particles in oriented single crystals depends essentially on their energy ε. At relatively low energy (order of MeV in axial case and tens of MeV in planar case) when in the transverse potential well, which forms the channel, there are a few levels (a number of levels $\propto \sqrt{\varepsilon}$ in an axial potential well and $\propto \sqrt{\varepsilon}$ in a planar well), the channeling radiation is connected with transitions between these levels. With energy increase, when the number of levels become large, classical description both motion and radiation become valid. The character of radiation depends on parameter*) $\rho \approx 2V_0\varepsilon/m^2$ (V_0 is the value of the potential of an axis (plane), ε (m) is the energy (mass) of the particle). At $\rho \ll 1$ the radiation is dipole one and at $\rho \gg 1$ one has magnetic bremsstrahlung limit [1]. In an intermediate region where $\rho \sim 1$ the theoretical description of the

*)The system of units $\hbar = c = 1$ is used.

radiation spectrum is rather complicated problem even in the planar case [2]. At last, with further energy increase it appears that recoil at radiation become important and the nature of radiation turns out again quantum. This situation is controlled by the parameter $\chi_s=eE_s\varepsilon/m^3$, $eE_s = V_0/a_s$ (E_s is the value of the electric field, a_s is the screening radius) and at $\chi \ll 1$ the classical theory is valid. Because of $\rho/\chi_s \sim 2ma_s \sim 10^2$, at $\chi_s \sim 1$, when the radiation is essentially quantum, one has $\rho \sim 10^2$ so that the radiation is of the magnetic bremsstrahlung nature. For this case the general radiation theory was developed [3, 4].

However, one should to has in mind that the quantum effects in the magnetic bremsstrahlung are "turn on" rather early. For example, already at $\chi=0.1$ the classical value of total intensity is around 1.5 times larger, than correct result. Even at $\chi \sim 10^{-2}$ ($\rho \sim 1$) quantum corrections are getting quite noticeable. Just in this energy region (when $\rho \sim 1$) which in axial case ranges for different crystals from several hundreds MeV to a few GeV, the radiation intensity at channeling exceeds the radiation intensity of the bremsstrahlung and the radiation at channeling may be used as a powerful source of the hard directed radiation. This paper is devoted to consideration of the quantum effects in the channeling radiation.

In the classical electrodynamics the total intensity of the radiation depends on local characteristics $I(\vec{\rho}) =$

$= (2/3)e^2m^2\chi^2(\vec{\rho})$, where $\chi(\vec{\rho}) = eE(\vec{\rho})\varepsilon/m^3$, here $E(\vec{\rho})$ is the local value of the electric field at a distance $\vec{\rho}$ from an axis (for definiteness, we consider axial case). Due to this fact the total intensity of the channeling radiation is obtained by integration over $\vec{\rho}$ of the local intensity $I(\vec{\rho})$ irrespective of the set of the trajectories.

As the weight in this integration the distribution function in transverse phase space is used. This function depends on a thickness of the crystal. In thin crystals it is determined by the initial conditions. In thick crystal, the thickness of which is $L \gg l_d$ (l_d is the dechanneling length, the length at which the distribution function is changed appreciably), the distribution function in the transverse phase space become uniform one due to multiple scattering [5]. As the result, the total intensity of the channeling radiation in thick crystals has the form of the average:

$$\bar{I} = \int I(\vec{\rho})\, d^2\rho/S, \qquad (1)$$

where S is the area of a cell in the transverse plane which contains the projection of one atomic chain, $S = \pi r_0^2$, then $n_\perp = 1/S$ is the density of the chains of atoms (axes). Integral in eq. (1) is taken over this cell. This circumstance is particularly valuable in the axial case, where the description of the spectral distribution of the radiation still wait for solution. However, the formula of the type of eq. (1) in nondipole region ($\rho \geq 1$) is much simpler than a calculation which is based on spectral distribution in planar case too. In the frame of this approach the radiation yield at axial channeling was considered in the paper [6] using classical electrodynamics. The experimental data on the radiation yield [7, 8] agree quite satisfactory with the theory predictions.

2. However, the quantum corrections should be taken into account in the upper part of the energy interval considered in [6]. Generally speaking, in the quantum region the simple formula of the type (1) for the total intensity does not exist. But in magnetic bremsstrahlung limit ($\rho \gg 1$) the description of the radiation is getting again local. So, for us the interval $1 \leq \rho \leq 10^2$ is most interesting, because at $\rho \leq 1$ the quantum correction are very small and classical electrodynamics is applicable and at $\rho \geq 10^2$ ($\chi \geq 1$) the magnetic bremsstrahlung (local) description is valid. This means that the most strong effects of nonlocality in the total intensity of radiation (or in total energy loss) become apparent just inside of the discussed interval of ρ, i.e. in the region where one can put $\chi \ll 1$. For analysis of this problem we will use the general quasi-classical equation for energy losses (see eq. (2.46) in [4] and eqs. (2.3), (2.5) in [9]). If one represents the particle velocity in the form $V(t)=v_0+v(t, v_0)$, where v_0 is the mean particle velocity, then the energy losses E are

$$E = \frac{ie^2m^2}{2\pi}\int_0^\infty \frac{u\,du}{(1+u)^3}\int dt \int_\infty^\infty \frac{d\tau}{\tau - i0}\left[1 + \gamma^2 f(u)(v_1 - v_2)^2/2\right] e^{-iu\phi}, \qquad (2)$$

where

$$\phi = \frac{\varepsilon\tau}{2}\left[\frac{1}{\gamma^2} + \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} ds\, v^2(t+s) - \left(\frac{1}{\tau}\int_{-\tau/2}^{\tau/2} ds\, v(t+s)\right)^2\right] \qquad (3)$$

$u = \omega/(\varepsilon - \omega)$, ω is the photon energy, $\gamma = \varepsilon/m$, $f(u) = 1 + u^2/(2(1+u))$, $v_{1,2} = v(t_{1,2})$, $t_{1,2} = t \mp \tau/2$. Expanding the integrand in eq. (2) into a series in powers of u (the main contribution is given by the region $u \sim \chi \ll 1$) and retaining the terms which give the contribution up to χ^4, we obtain after integration over u:

$$E=E_0 + E_1 + E_2 = \frac{e^2m^2}{2\pi}\int dt \int \frac{d\tau}{\tau - i0}\left\{\left[1 + \gamma^2(v_1 - v_2)^2/2\right] \times\right.$$

$$\left.\times[-i/\phi^2 + 6/\phi^3] + (3i/\phi^4)\left[12 + (13/2)\gamma^2(v_1 - v_2)\right]\right\}. \qquad (4)$$

In this expression the term $1/\phi^2$ gives the classical formula for the energy losses. Because the function $\phi(\tau)$ is odd the integral over τ may be taken as half-residue in the point $\tau = 0$. As a result we obtain known local expression

$$I_{c1} \equiv dE_0/dt = (2/3)e^2 \dot{v}^2\gamma^4 = (2/3)\, e^2m^2\chi^2. \qquad (5)$$

Precisely in the same way one can calculate the term with $1/\phi^4$ which gives the second quantum correction ($\propto \chi^4$). It is also universal and local:

$$I_2 \equiv dE_2/dt = 32e^2m^2\Big[(\dot{v}\gamma^2/m)^4 + (\gamma^6/160m^4)\Big(8(d^2v/dt^2)^2 - $$
$$- 41\,(dv/dt\cdot d^3v/dt^3)\Big)\Big]. \qquad (6)$$

But the term containing $1/\phi^3$ is, generally speaking, non-local. For it calculation one have to substitute into (4) the given trajectory (the explicit expression for v(t)). We consider here rather general situation of quasi classical motion following an elliptic trajectory discussed in [9] (Section 4). For this case one has for the first quantum correction in terms of mean radiation intensity: $\langle I_1\rangle \equiv$ $\equiv(1/T)\int_0^T dt\ldots$, where the explicit representation of the integrand is given in eq. (4):

$$\langle I_1\rangle = -\,(55e^2m^2/\,8(3)^{1/2}\lambda)\langle\chi^3\rangle k_1(\rho) \qquad (7)$$

here ω_0 is the motion frequency, $T = 2\pi/\omega_0$ is the period of the motion

$$k_1(\rho) = -\frac{B}{\rho^{3/2}}\int_0^\infty \frac{d\tau}{\tau^4}\times$$

$$\times\left\{\frac{\Big[(1+\rho\sin^2\tau)(2g_1^2+\alpha^2g_2^2)+3\rho\alpha^2g_1g_2\sin^2\tau\Big]}{(g_1^2-\alpha^2g_2^2)^{5/2}} - 2 - 2\rho\tau^2\right\} \qquad (8)$$

where the components of the transverse velocity of the particle are $v_x = a\cos\omega_0 t$, $v_y = b\sin\omega_0 t$,

$$\rho = \gamma^2(a^2 + b^2),\ \alpha = (a^2 - b^2)/(a^2 + b^2),$$
$$g_1 = 1+\rho(1-\sin^2\tau/\tau^2)/2,\ g_2 = (\rho/2)[\sin 2\tau/2\tau - \sin^2\tau/\tau^2],$$
$$B = (48(6)^{1/2}/55\pi)\lambda,\ \lambda = \langle v_\perp^2\rangle^{3/2}/\langle|v_\perp|^3\rangle,$$

<..> means avaragin g over time, two last terms in brace in eq. (4) are subtraction terms. Let us stress, that eqs. (5)-(7) are obtained as expansion in terms of χ, but are exact functions of the parameter ρ characterizing multipolarity of the radiation.

In eq. (7) at $\alpha = 0$ we have circular motion and at $\alpha=1$ we have planar motion. For these cases the function $k_1(\rho)$ (eq. (8)) is presented in Fig. 1. We use such normalization of this function that $k_1(\rho) \Rightarrow 1$ when $\rho \gg 1$, what corresponds to magnetic bremsstrahlung limit. The curve 1 is for circular motion and the curve 2 is for planar motion. It's seen, that deviation from magnetic bremsstrahlung limit is larger for circular motion, but the interval between curves 1 and 2 is rather narrow and intermediate values of α is lying inside this interval. For circular motion the

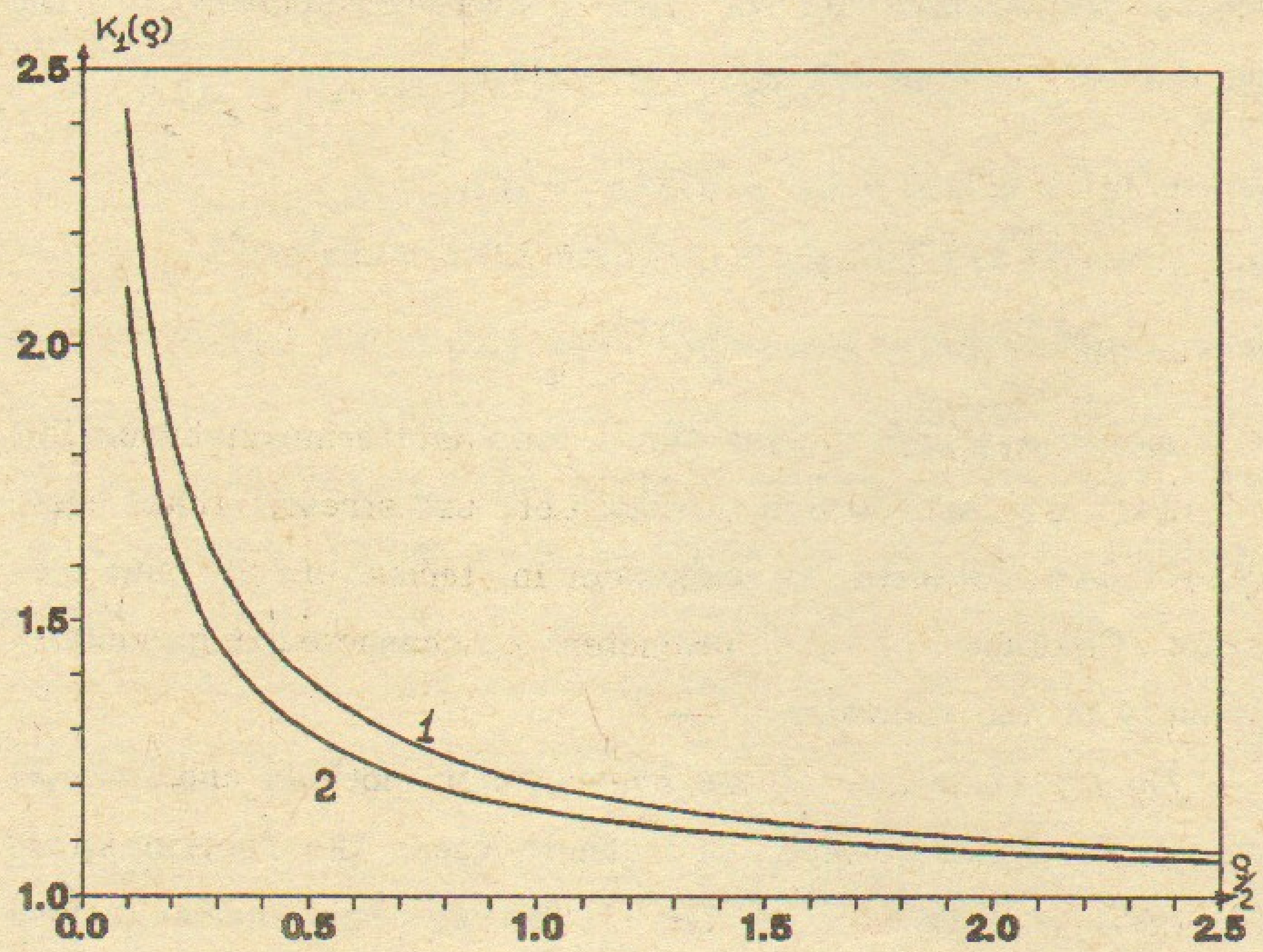

Fig. 1. The function $k_1(\rho)$ (eq. (8)) for circular(1) and planar (2) motion.

asymptotic expansions of the function $k_1(\rho)$ are.

$$k_1(\rho) = \begin{cases} (112(6)^{1/2}/275\rho^{1/2}) & \rho \ll 1 \\ 1 + (114/275\rho) & \rho \gg 1 \end{cases} \qquad (9)$$

Let us note, that the asymptotic expansion at $\rho \gg 1$ is valid actually up to $\rho \approx 1$ (at $\rho=1$ the accuracy is better than 2 %).

Let us note also, that for arbitrary ρ the function $k_1(\rho)$ may be presented by simple interpolation formulae: $k_1(\rho)=\left[1+1/\rho\right]^{1/2}$ for the circular motion (the accuracy is better than 2%) and $k_1(\rho)=\left[1+\ln\ 2/\rho\right]^{1/2}$ for planar motion (the accuracy is better than 1%). This may be useful for numerical calculations.

Finally, we present the explicit form of the mean intensity at the elliptic quasi-classical motion as a series in powers of χ which is valid for arbitrary ρ

$$\langle I\rangle = \frac{2}{3} e^2 m^2 \left[\langle\chi^2\rangle - \frac{55(3)^{1/2}}{16} \langle\chi^3\rangle k_1(\rho) + 48 \langle\chi^4\rangle k_2(\rho) \right] , \qquad (10)$$

where the function $k_1(\rho)$ is given by eqs. (8), (9), and

$$k_2(\rho) = 1 + (49/40\rho(2+\alpha^2)) , \qquad (11)$$

what follows from eq. (6) with allowance for eq. (8).

3. Now, we will use different approach. We will start from the quantum radiation theory at quasi-periodic motion [9] which may be used for description of the channeling radiation if one knows the set of the trajectories. In this paper it's shown also that when $\rho \gg 1$ the general results turn into formulae of the magnetic bremsstrahlung with corrections $\propto 1/\rho$ which control a transition to this limit. Let us take into account that the classical radiation intensity is proportional to the square of acceleration (quantum corrections contain higher powers of acceleration). So, on the strongly stretched trajectories the particle radiates mostly in the region where acceleration is maximal, i.e. on the small part of the trajectory. In another words, the radiation is quasilocal. The least favorable situation is in this sense for circular trajectories, as we have already seen at the discussion of the function $k_1(\rho)$ (see Fig. 1). Therefore we consider the radiation intensity with $1/\rho$ corrections just for circular motion (see eq. (4.16) in [9]):

$$dI = dI_m - dI_c/\rho, \tag{12}$$

where

$$dI_c = \frac{4e^2m^2 udu}{15\sqrt{3}\,\pi\kappa(1+u)^3}\left[\frac{7}{3}f(u)\frac{d}{d\kappa}\left(\kappa K_{1/3}(\kappa)\right) + (f(u)-1/2)\frac{d}{d\kappa}\left(\kappa K_{2/3}(\kappa)\right)\right], \tag{13}$$

here dI_m is the radiation intensity in the magnetic bremsstrahlung limit, $u=\omega/(\varepsilon-\omega)$, ω is the photon energy, $f(u)=1+u^2/2(1+u)$, $\rho=2\gamma^2 v_\perp^2$, $v_\perp$ is the transverse particle velocity, $\varepsilon=\gamma/m$, $K_\nu(z)$ is the MacDonald functions, $\kappa = 2u/3\chi$, χ is the quantum parameter in our case $\chi=\sqrt{(\rho/2)}\,\omega_0\varepsilon/m^2$, ω_0 is the motion frequency in l-system. Let us stress, that this expression is valid for any value of the parameter χ, but contains only the first term of the expansion over $1/\rho$. Taking in (13) the integral over u we obtain the correction to the total intensity:

$$I_c = \frac{2e^2m^2}{15\sqrt{3}\,\pi}\int_0^\infty \left[7\chi u(3+2u+u^2)\,K_{1/3}(2u/3\chi) + u^2(3+u+2u^2)\,K_{2/3}(2u/3\chi)\right]\frac{du}{(1+u)^5}, \tag{14}$$

where integration by parts was carried out. The integral in this expression is of the type encountered in the theory of the magnetic bremsstrahlung. So, using the standard methods (see e.g. [4]) one can obtain the asymptotic expansion of I_c at small and large χ. For $\chi \ll 1$ one obtains from eq. (14)

$$I_c = e^2m^2\chi^3\ [(19\sqrt{3}/20) - (98/5)\chi + \ldots]. \quad (15)$$

Using known asymptotic expansion of the intensity of the magnetic bremsstrahlung at $\chi \ll 1$ (see e.g. [4]) we find with allowance for (12) and (15) the expression for total intensity at $\chi \ll 1$, which agrees, naturally, with (10), (11) and (9):

$$I = \frac{2}{3} e^2m^2\chi^2\left[1 - \frac{55\sqrt{3}}{16}\chi\left(1 + \frac{114}{275\rho}\right) + 48\chi^2\left(1 + \frac{49}{80\rho}\right) + ..\right] \quad (16)$$

As known, the series (16) is asymptotic series, but the corrections of the order $1/\rho$, which control the transition to the magnetic bremsstrahlung limit, have the coefficients smaller 1.

4. The parameters χ and ρ entering eq. (16) are dependent on local values of the field. So, for the calculation of the radiation in crystals one has to do averaging as in eq. (1). We will use the expression of the axis potential in the form (for details see [3], [4]):

$$U(x) = V_0\left[\ln(1+1/(x+\eta)) - \ln(1+1/(x_0+\eta))\right], \quad (17)$$

where $x=\vec{\rho}^{\,2}/a_s^2$, $V_0 \simeq Ze^2/d$, d is the average distance between the atoms in the chain, $\eta \simeq 2u_1^2/a_s^2$, u_1 is the amplitude of thermal vibrations. Then for the current values of the parameters χ and ρ one has:

$$\chi(x) = \chi_s 2\sqrt{x}\ g(x),\ \rho(x) = 4V_0\varepsilon\ xg(x)/m^2 \quad (18)$$

here $g(x) = 1/[(x+\eta)(1+x+\eta)]$, the value χ_s is defined at the beginning of the paper. After averaging of eq. (16) we obtain:

$$\bar{I} = \frac{8e^2m^2\chi_s^2}{3x_0}\left[S(\eta) - \frac{55\sqrt{3}}{16}\chi_s R(\eta) - \frac{57\sqrt{3}\pi}{160ma_s}A(\eta) + 48\chi_s^2 T(\eta)\right] \quad (19)$$

where $x_0 = r_0^2/a_s^2$,

$$S(\eta) = (1 + 2\eta)\ln.((1+\eta)/\eta) - 2,$$

$$R(\eta) = \frac{3\pi}{4\sqrt{\eta(1+\eta)}}\ \frac{1}{(\sqrt{\eta} + \sqrt{(1+\eta)})^5},$$

$$A(\eta) = \eta^{-1/2} + (1+\eta)^{-1/2} + 4(\sqrt{(1+\eta)} - \sqrt{\eta}), \quad (20)$$

$$T(\eta) = 4\left[(1/\eta + 1/(1+\eta))/3 - 4\ \ln((1+\eta)/\eta)\left[5\eta(1+\eta)+1\right] + 20\eta + 10\right].$$

The terms with the functions S, R and T are written with the same coefficients as in the decomposition (16), the linearly increasing with the energy parameter χ_s depends on the field on the screening radius. These functions are presented in Fig. 2 in the interval of η within which are all the used crystals. It's seen that the functions R and T are larger appreciably than S particularly for small amplitudes of thermal vibration (η). This means that the distances smaller

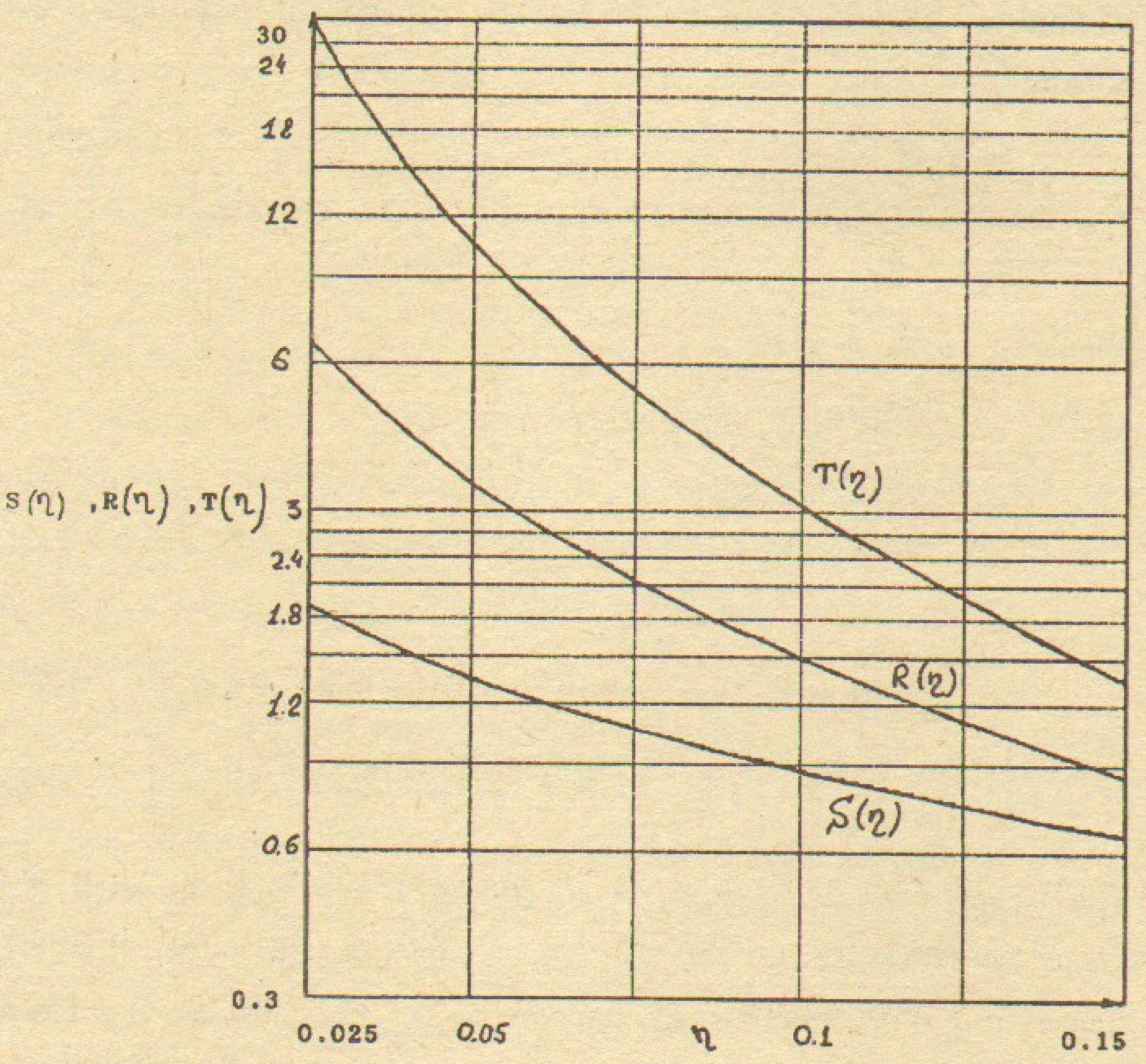

Fig. 2. The functions S(η), R(η) and T(η) in interval of the parameter η within which all the used crystals are lying.

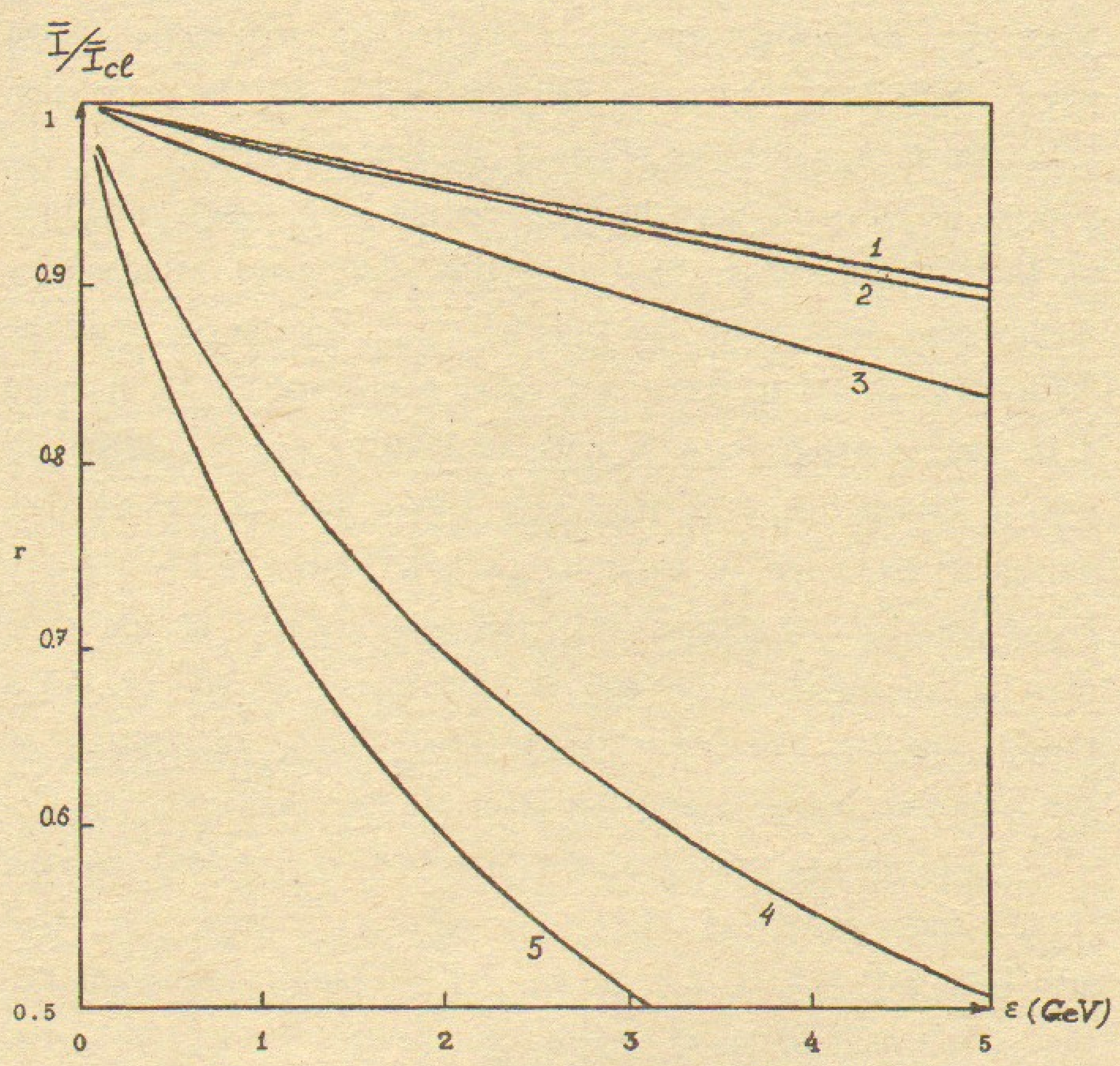

Fig. 3. The mean radiation intensity at channeling divided by the mean classical intensity versus electron energy at room temperature in single crystals: curve 1 is for Si (axis <110>), curve 2 is for diamond (axis <111>), curve 3 is for Ge (axis <110>). The curve 4 is for W (axis <111>) at room temperature and curve 5 at T=77 K.

than the screening radius are contribute noticeable. From above it follows that the validity region of the expansion (19), which is quite satisfactory if the corrections are not exceed 10-15%, reduces in the crystals with small amplitude of thermal vibrations. The correction $\propto 1/\rho$ gives the energy independent term with the function $A(\eta)$. Comparing with the main term ($S(\eta)$) it forms (at room temperature): in diamond 4.7% and in tungsten 4.5% (for axis <111>) and in silicon 3% and in germanium 2.8% (for axis <110>). Let us remind that expansion over $1/\rho$ is valid up to ρ=1. Thus, in the validity region of the expansion (12) the contribution of the term $\propto 1/\rho$ is smaller than 5% and is independent on energy. As a result one can use magnetic bremsstrahlung description of the radiation intensity and $1/\rho$ term can be estimated.

With energy increase the quantum corrections become large and expansion (16) is getting invalid. In this situation one has to put in eq. (1) complete expression for the radiation intensity valid at arbitrary χ (see, i.e. [4]). One can use also the approximate expression for the radiation intensity (the accuracy is better than 2% for any χ):

$$I(\chi)=(2/3)e^2m^2\chi^2\left[1+4,8(1+\chi)\ln(1+1,7\chi)+2,44\chi^2\right]^{-2/3}. \qquad (21)$$

The result of averaging of this expression is shown in Fig. 3 where the ratio $\bar{I}/\bar{I}_{cl}$ is presented, where $\bar{I}_{cl}$ is the

mean classical intensity, as the function of the electron energy. The curve 1 is for Si, axis <110>; 2 is for diamond, axis <111>; 3 is for Ge, axis <110>; 4 is for W, axis <111>. It is seen, that the quantum corrections exceed 10% at the energy near 5 GeV in Si and diamond and at the energy near 3 GeV in Ge. So, for the energy up to 5 GeV considered in [6] the quantum corrections in these crystals are rather small. The experimental data for Si and diamond [7, 8] are known for the energy near 1 GeV, where the corrections are a few percent what is less than accuracy of both the theory and data.

The situation is different for tungsten where already at the energy 1 GeV (axis <111>, room temperature) the correction dependent on energy is 19% (the correction $\propto 1/\rho$ is up to 4.5%). As the result, at the energy 1 GeV the radiation intensity of electrons in W for this conditions is 77% from the classical one. For energies 2 GeV and 5 GeV one has 66% and 47% respectively. Thus, in W the quantum effects in the radiation are very strong. They are still more strong at low temperature (axis <111>, T=77 K, curve 5), where the energy dependent correction is 27% at 1 GeV (the correction $\propto 1/\rho$ is up to 6.5%). The experimental situation in tungsten is still uncertain.

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V.N. Baier, V.M.Katkov, V.M. Strakhovenko

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В.Н. Байер, В.М. Катков, В.М. Страховенко

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