ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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HIGH TEMPERATURE PHASE OF 2D COULOMB GAS MODEL NEAR THE KOSTERLITZ-THOULESS PHASE TRANSITION

PREPRINT 91-103

НОВОСИБИРСК

High Temperature Phase of 2D Coulomb Gas Model Near the Kosterlitz-Thouless Phase Transition

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ABSTRACT

We show that the high-temperature phase of 2D Coulomb scalar gas model near the Kosterlitz-Thouless phase transition at a large length scale is equivalent the free fermion model and define more precisely the Kosterlitz estimate for the correlation length.

1. Kosterlitz and Thouless [1, 2] have proposed a theory for a phase transition in two-dimensional planar spin systems and helium films, and for two-dimensional theory of melting. These systems have singularities, vortices for planar spins and helium, dislocation in the case of melting, which interact with logarithmic potential, as well as charged particles in 2D. The 2D Coulomb scalar gas model (2DCSGM) is defined by the partition function

$$Z_{CG} = \sum_{s(\vec{r})=0;\pm 1} \exp\left(K \sum_{r\neq r'} s(\vec{r}) \ln \left| \frac{\vec{r}-\vec{r}'}{a} \right| s(\vec{r}') - y \sum_{r} s^2(\vec{r}) \right) , \qquad (1)$$

where ε is dielectric constant, μ is fugacity, T is temperature and a is elementary scale of the model in the lattice. The Kosterlitz-Thouless theory harmonic approximation is exact at all temperatures below the transition temperature T_{KT}, provided one only investigates fluctuations at a sufficiently large length scale and allows for a renormalization of the constants K,y of the theory due to bound pairs. Above T_{KT} correlations are expected to decay exponentially with a correlation length of order of the mean spacing between free vortices, but there isn't full-length information about the high-temperature phase.

In the high-temperature phase the Kosterlitz-Thouless renormgroup scale movement from initial elementary scale a to renormalized elementary scale R_D and, essential, from initial constant K, y to renormalized constant K_D, y_D is correct to leading order in z = **exp**(-y), $z_D \ll 1$. We show it's possible to choose renormalized scale R_D in such a way that 2DCSGM with renormalized constants K_D, y_D is equivalent the free fermion model. The latter allows to define more precisely the Kosterlitz estimate for the correlation length in high-temperature phase [2].

The Letter is organized as follows. In sec. 2 we briefly introduce the 2DCG self-consistented renormalization group and present the crude estimate of the correlation length in high temperature phase [1 - 4]. In sec. 3, we then present the equivalence between the massive Thirring, the quantum sine-Gordon and the 2DCSG models [5 - 7]. Using the renorm-group analysis from sec. 2 and the exact equivalence of the models from sec. 3, in sec. 4, the exponentially decaying connected correlation function and the correlation length in the high-temperature phase are obtained.

2. The central result of the Kosterlitz-Thouless theory is a self-consistent integral equation for $\varepsilon(r)$ the scale-dependent constant to leading order in **exp**(-y) ≡ z:

$$\varepsilon(r) = \varepsilon(a) + 2\pi \int_a^r n(r')\,\alpha(r')\,\frac{r'dr'}{a^2}\,, \qquad (2)$$

where n(r) is the density of charged pairs separated by r

$$n(r) = e^{-2y}\left(\frac{r}{a}\right)^{K(r)}, \qquad (3)$$

$\alpha(r)$ is the polarizability of a single dipole of separation r. The equation (2) is invariant about a scale transformations of the constant of the theory (K, z)

$$a \to a\,(1 + \xi)\,,$$

$$K^{-1} \to K^{-1} + 4\pi z^2\,,$$

$$z \to (1+\xi)^{2-K/2} z\,. \qquad (4)$$

So the equation (2) is equivalent the pair of coupled differential equations

$$\frac{d\,K^{-1}}{d\,\xi} = 4\pi\, z^2\,,$$

$$\frac{d\,z}{d\,\xi} = (2 - \frac{K}{2})\,z\,, \qquad (5)$$

where $\xi = \ln(r/a)$. The trajectories which follow from eqs. (5)

$$\frac{4}{K} + \ln\frac{K}{4} - 1 = 2\,\pi^2 z^2 - \frac{c}{32}\,, \qquad (6)$$

are shown schematically in Fig. The trajectories with c = 0 are long dashed lines (separatrices) in Fig., which separate the plane on three regions. Region 1 is an infinite correlation length phase, the low temperature phase of the model. Region 2 and 3 are both finite correlation phases with free charges. The separatrice with K > 4 is the phase transition line. In this Letter we discuss only the high temperature phase in region 2 near the phase transition AB trajectory in Fig., where

$$c \sim \tau = \frac{T - T_{KT}}{T_{KT}} \ll 1\,,\ c > 0\,. \qquad (7)$$

We cannot use (5) for $\xi > \xi_B$, where $z_B \sim 1$, because the approximations $z \ll 1$ used to derive it break down. However it's possible to appreciate the correlation length above T_{KT} [2]. Let us assume that higher degree term in (5) stop z growth near $z_B \sim 1$. This hypothesis looks probable, so the dissociation of the pairs begins from the largest scale

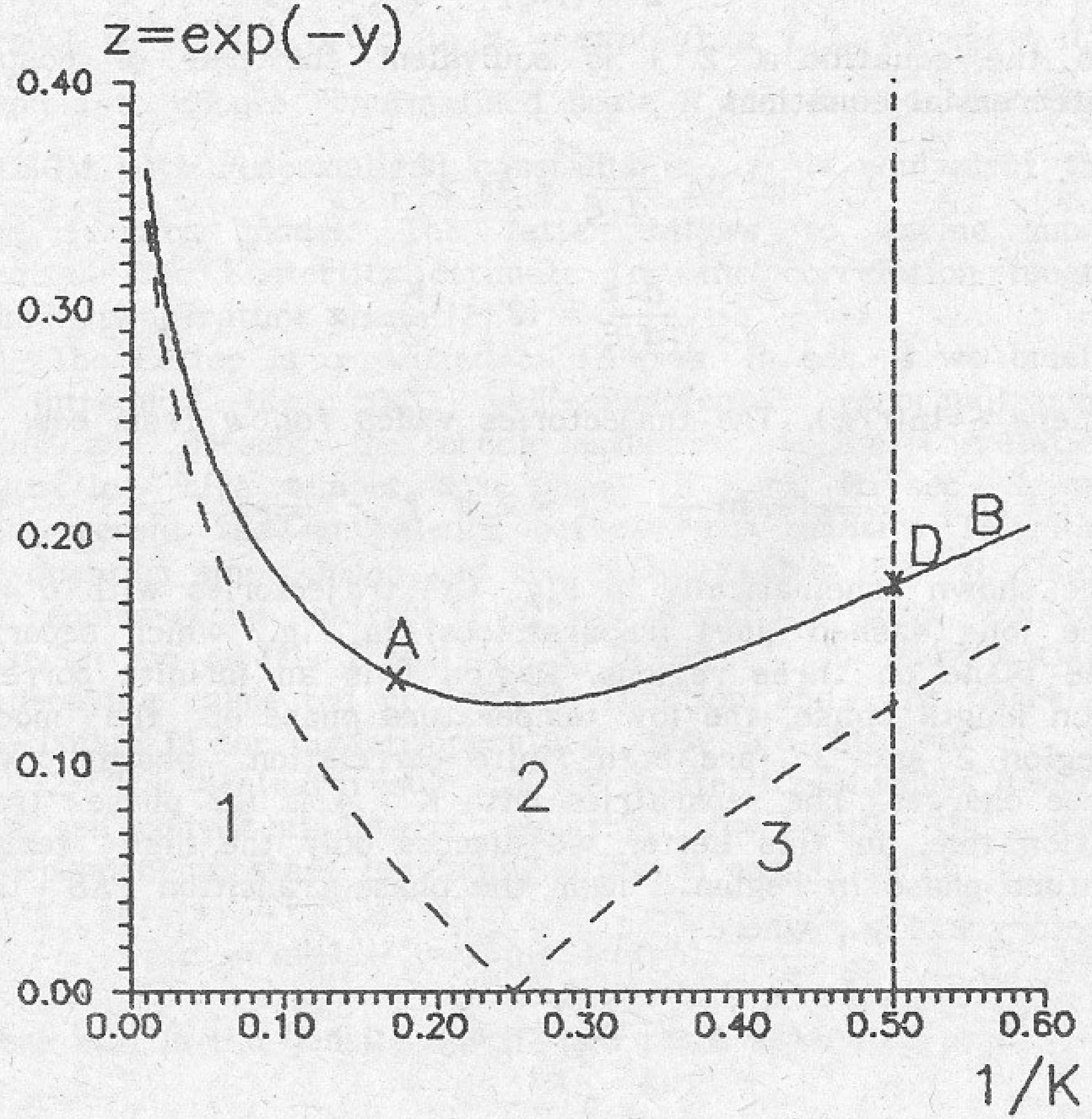

Fig. The renormgroup flow induced by the scaling equation (5). Long dashed lines are separatrices, separating the plane on three regions. Solid AB line is a typical renormgroup trajectory in high-temperature phase. Dashed vertical line is the bench mark's line (K = 2). The model (1) with constants K, y from the bench mark's line is equivalent a free fermion model.

pairs. The small scale pairs "nobody know" about the large scale pairs behavior. From (5) and (6) it's straightforward to obtain for small c

$$\ln \frac{R_B}{a} = \xi_B = \int_{4-K_A}^{4-K_B} \frac{dt}{t^2 + c} \simeq \frac{\pi}{\sqrt{c}} . \tag{8}$$

This is the Kosterlitz crude estimate of the correlation length.

In the forth section we will show, that the AB trajectory on the R_r scale intersects the bench mark's line (the dashed line on the Fig.). 2DCGM with the constants of the model (K, y) from the bench mark's line and the smallest scale R_p is equivalent the free exact solvable theory. The last permits to obtain the large scale behavior of the model and to specify the Kosterlitz estimate of the correlation length in the high-temperature phase.

3. We can represent the partition (1) as a functional integral with an action of the sine-Gordon model [5]:

$$Z_{SG} = \int D\psi \, \exp\left(- \int d^2r \, \mathfrak{L}_{SG} \right) ,$$

$$\mathfrak{L}_{SG} = \frac{1}{2} (\nabla\psi)^2 - \frac{2\, e^{-y}}{R^2} \cos\left(\sqrt{(2\pi K)}\ \psi\right) . \tag{9}$$

On the expansion (9) in powers of h it has been recognized the relation between the quantum sine-Gordon and massive Thirring partition [6 - 8]. The Thirring model is described in terms of Dirac fermions with an action

$$Z_T = \int D\bar{\varphi}\, D\varphi \, \exp\left(-\int d^2r \, \mathfrak{L}_T \right) ,$$

$$\mathfrak{L}_T = -i\, \bar{\varphi}\, \gamma_\mu \partial_\mu \varphi - \frac{1}{2} g^2 (\bar{\varphi}\, \gamma_\mu \varphi)^2 + i\, m\, \bar{\varphi}\, \varphi , \tag{10}$$

in which γ_μ is γ-matrices in 2D. The correspondence is then

$$1 + \frac{g}{\pi} = \frac{2}{K} , \qquad (11,a)$$

$$m = \frac{e^{-y}}{\pi K R^2} \left(\frac{R}{R'}\right)^{K/2} R' , \qquad (11,b)$$

where R is the minimal length scale and R′ is the arbitrary length scale of the model. The correspondence between the connected correlation functions can be calculated too:

$$\langle\langle \cos\left(\sqrt{(2\pi K)}\,\psi(r)\right) \cos\left(\sqrt{(2\pi K)}\,\psi(r')\right) \rangle\rangle_{SG} =$$

$$= - \frac{m^2 R^4}{4} e^{2y} \langle\langle \bar\psi(r)\psi(r)\bar\psi(r')\psi(r') \rangle\rangle_T . \qquad (12)$$

We observe that the value K = 2 correspondence to a free fermion field theory. Thus, the line K = 2 is the bench mark's line.

4. Trajectory AB intersects the bench mark's line in the point D. There are renormalization coordinates $K_D = 2$, y_D and minimal length scale R_D in the point D. We find z_D and R_D to leading order in c from (5), (6)

$$z_D = e^{-y_D} = \sqrt{\frac{1-\ln 2}{2\pi^2}} \left(1 + \frac{c}{64\,(1-\ln 2)}\right) \cong$$

$$\cong 0.125 \; (1 + 0.05\, c) , \qquad (13)$$

$$\ln \frac{R_D}{a} = \int_K^2 \frac{d K}{4K + K \ln\frac{K}{4} - \left(1 - \frac{c}{32}\right) K^2} . \qquad (14)$$

All the following results are true to leading order in $z_D \ll 1$.

Making use of results from sec. 3, the model (1) are equivalent the free fermion field theory , where the mass is

$$m = \frac{1}{2\pi R_D} e^{-y_D} . \qquad (15)$$

It is naturally to identify reverse mass with correlation length R_c

$$\frac{R_c}{R_D} = 2\pi\, e^{y_D} \simeq 50.2 \; (1 - c\; 0.051) \gg 1 . \qquad (16)$$

So, one obtains the dependence R_c on c, τ (14, 16), defining more precisely the Kosterlitz estimate (8).

According to (12), connected correlation function in point D takes the form

$$\langle\langle \cos\left(\sqrt{(2\pi K)}\,\psi(r)\right) \cos\left(\sqrt{(2\pi K)}\,\psi(r')\right) \rangle\rangle_{SG} =$$

$$= \frac{m^2 R_D^4}{4} e^{2y_D} \int \frac{d^2p\, d^2p'\, e^{i(\vec p + \vec p')\vec r}}{(2\pi)^4\,(p^2 + m^2)\,(p'^2 + m^2)} \mathrm{Tr}\left((m - i\hat p)(m - i\hat p')\right) =$$

$$= \frac{m^2 R_D^4}{4\pi^2} e^{2y_D} \left(K_0^2(mr) - \frac{1}{2} K_1^2(mr)\right) \simeq \frac{m^3 R^4}{8\pi} \frac{e^{-2mr}}{r} , \qquad (17)$$

where $\hat p = \gamma_\mu p_\mu$ and $K_0(r)$, $K_1(r)$ are modificated Bessel functions. So, one obtains exponentially decaying order (17) above T_{KT} on large distance in comparison with powerly decaying order in low temperature phase [8].

Finally, it should be emphasized that the equivalence of 2DCSGM above T_{KT} the free fermion model has been obtained on the expansion in powers of z_D and only in the large length scale. One differs from the exact equivalence of 2D Ising model to the free fermion model near the phase transition [10].

Author is grateful to prof.A.Z. Patashinski for the suggesting of the problem and simulating discussions.

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Высокотемпературная фаза в модели двумерного кулоновского газа

Ответственный за выпуск С.Г. Попов

Работа поступила 4 октября 1991 г.
Подписано в печать 8.10 1991 г.
Формат бумаги 60×90 1/16 Объем 0,8 печ.л., 0,7 уч.-изд.л.
Тираж 250 экз. Бесплатно. Заказ N 103

Ротапринт ИЯФ СО АН СССР,
Новосибирск, 630090, пр. академика Лаврентьева, 11.