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MINIMAL EMBEDDING OF VECTOR MEZONS INTO THE LOW ENERGY EFFECTIVE CHIRAL LAGRANGIAN

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НОВОСИБИРСК

Minimal Embedding of Vector Mesons into the Low Energy Effective Chiral Lagrangian

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Pseudoscalar mesons play somewhat special role among hadrons, being would-be Goldstone bosons associated with the dinamical breaking via quark-antiquark condensation of the $SU_L(3)\times SU_R(3)$ chiral symmetry, which is good enough for QCD because *u*-, *d*- and *s*-quark masses are small compared to the strong interaction scale ~ 1 Gev. So one can expect that in the low energy region it is possible to develop the effective theory of pseudoscalars only and the chiral symmetry of the underlying QCD will to great extent dictate the properties of such a theory [1, 2].

The low energy effective action for pseudoscalars containing the minimal possible number of derivatives and correctly reproducing current algebra low energy theorems has the following form [3, 4, 5] (the notations of [5] are used):

$$\Gamma = \frac{F_\pi^2}{8}\int_{M^4} dx\ \mathrm{Sp}\ (\partial_\mu U\ \partial^\mu U^+) - \frac{i}{80\pi^2}\int_M \mathrm{Sp}\ (\alpha^5)\ , \qquad (1)$$

where $U = \exp \frac{2i}{F_\pi}\Phi$, $F_\pi = 135$ MeV is the pion decay constant, $\alpha = (\partial_\mu U)U^{-1}dx^\mu = (dU)U^{-1}$ (it is very useful to use the differential form language [5, 6]), and the pseudoscalar meson matrix is

$$\Phi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} . \quad (2)$$

The necessity of the second term in (1) is dictated [3] by a requirement of the correct description of QCD chiral anomaly [7] consequences. Besides, the effective lagrangian without this term will possess greater symmetry than QCD [4].

It is clear that (1) can't describe all low energy meson physics. First of all it is necessary to include the electromagnetic interactions and this is usually done by replacing ordinary derivatives by covariant ones:

$$dU \longrightarrow DU = dU - ieA\,[Q,\,U]\,, \quad (3)$$

where $A = A_\mu dx^\mu$ is electromagnetic 1-form and

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} .$$

Doing so the Wess - Zumino - Witten term $\sim \int_{M^5} \mathrm{Sp}\,(\alpha^5)$ needs some care because we have an integral over 5-dimensional manifold M^5 whose boundary is Minkowski space- time M^4. As a rule this difficulty is overcome by the use of Noetherian trial and error methods for gauging [4, 5] or by developing some global integration techniques for chiral anomaly [8]. But the usual method of introduction covariant derivatives can also be applied [9]. Let us briefly demonstrate this.

Under interchange (3) differential forms $\alpha = (dU)U^{-1}$ and $\beta = U^{-1}dU = U^{-1}\alpha\, U$ undergo the transformation

$$\alpha \longrightarrow \alpha - ieAT\,; \qquad \beta \longrightarrow \beta + ieAS\,, \quad (4)$$

where $T = Q - UQU^{-1}$ and $S = Q - U^{-1}QU = -U^{-1}TU$.

Taking into account that $A^2 = 0$ and $A\alpha = -\alpha A$ we get for the Wess - Zumino - Witten term

$$\varepsilon_{WZW} = C\int_{M^5} \mathrm{Sp}(\alpha - ieAT)^5 = C\int_{M^5} \mathrm{Sp}\,(\alpha^5) - 5ieC\int_{M^5} \mathrm{Sp}(\alpha^4 AT)\,, \quad (5)$$

where $C = \dfrac{-i}{80\pi^2}$. As to the second term in (5), it can be re-written in such a way

$$\mathrm{Sp}\,(\alpha^4 AT) = A\,\mathrm{Sp}\,\{Q(\alpha^4 - \beta^4)\} = A\,\mathrm{Sp}\,\{Q\, d(\alpha^4 + \beta^4)\} =$$

$$= -\,d\,[A\,\mathrm{Sp}\,\{Q(\alpha^3 + \beta^3)\}] + (dA)\,\mathrm{Sp}\,\{Q(\alpha^3 + \beta^3)\}\,,$$

where we have used the fact [5] that even powers of α and β are exact forms: $\alpha^{2n} = d\alpha^{2N-1}$, $\beta^{2N} = -\,d\beta^{2n-1}$.

By the use of Stokes' theorem $\int_{M^5} d\omega = \int_{M^4} \omega$ we get the following expression

$$\Gamma_{WZW} = C\int_{M^5} \mathrm{Sp}\,(\alpha^5) + 5ieC\int_{M^4} A\,\mathrm{Sp}\,\{Q(\alpha^3 + \beta^3)\} -$$

$$-\,5ieC\int_{M^5} (dA)\,\mathrm{Sp}\,\{Q(\alpha^3 + \beta^3)\} \quad (6)$$

Let us further transform the last term:

$$\mathrm{Sp}\,\{Q(\alpha^3 + \beta^3)\} = \mathrm{Sp}\,\{(Q + UQU^{-1})\,\alpha^3\} = \mathrm{Sp}\,\{(Q + UQU^{-1}) \times$$

$$\times\,(\alpha - ieAT)^3\} + ieA\,\mathrm{Sp}\,\{(Q + UQU^{-1})\,(\alpha^2 T - \alpha T\alpha + T\alpha^2)\}$$

but due to anticommutativity property of 1-forms

$$\mathrm{Sp}\,(Q\alpha\; Q\alpha) = \mathrm{Sp}\,(\beta Q\; \beta Q) = 0$$

and therefore

$$\mathrm{Sp}\,\{(Q + UQU^{-1})\,(\alpha^2 T - \alpha T\alpha + T\alpha^2)\} =$$

$$= \mathrm{Sp}\,\{2Q^2(\alpha^2 - \beta^2) - Q\alpha\, Q\alpha + 2Q\alpha\, UQU^{-1}\alpha - \beta Q\, \beta Q\} =$$

$$= 2d\,[\mathrm{Sp}\,\{Q^2(\alpha + \beta)\}] - 2\,\mathrm{Sp}\,\{Q(dU)\,Q(dU^{-1})\}$$

the second term is an exact form too, because

$$\mathrm{Sp}\,\{Q(dU)\,Q(dU^{-1})\} =$$

$$= -d\,[\mathrm{Sp}\,\{a\,QU^{-1}QdU - b\,Q(dU^{-1})\,QU\}] \quad \text{if} \quad a + b = 1.$$

Thus, we have

$$\int_{M^5} (dA)\,\mathrm{Sp}\,\{Q(\alpha^3+\beta^3)\} = \int_{M^5} (dA)\,\mathrm{Sp}\,\{(Q+UQU^{-1})(\alpha - ieAT)^3 +$$

$$+ 2ie \int_{M^5} A(dA)\, d[\mathrm{Sp}\,\{Q^2(\alpha + \beta) + aQU^{-1}QdU - bQ(dU^{-1})\,QU\}]$$

and, since $A(dA)(dX) = -\,d[A(dA)X] + (dA)(dA)X$, we get for (5)

$$\Gamma_{WZW} = C \int_{M^5} \mathrm{Sp}(\alpha^5) + 5ieC \int_{M^4} A\,\mathrm{Sp}\,\{Q(\alpha^3 + \beta^3)\} -$$

$$- 10e^2C \int_{M^4} A(dA)\,\mathrm{Sp}\,\{Q^2(\alpha + \beta) + aQU^{-1}QdU - bQ(dU^{-1})\,QU\} -$$

$$- 5ieC \int_{M^5} (dA)\,\mathrm{Sp}\,\{(Q + UQU^{-1})\,(\alpha - ieAT)^3\} +$$

$$+ 10e^2C \int_{M^5} (dA)(dA)\,\mathrm{Sp}\{Q^2(\alpha + \beta) + aQU^{-1}QdU - bQ(dU^{-1})\,QU\}\,.$$

The last two terms are gauge invariant. For the first one this is obvious because it is expressed through covariant derivatives only, as (4) shows (note that $dA = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$ is gauge invariant). As to the last term, its invariance becomes evident if one rewrites it like that $(a + b = 1)$

$$\mathrm{Sp}\{Q^2(\alpha + \beta) + aQU^{-1}QdU - bQ(dU^{-1})\,QU\} =$$

$$= \mathrm{Sp}\{Q^2(\alpha - ieAT + \beta + ieAS) + aQU^{-1}Q(dU - ieA\,[Q,\,U]) +$$

$$+ bQU^{-1}(dU - ieA\,[Q,\,U])\,U^{-1}QU\} +$$

$$+ ieA\,\mathrm{Sp}\{Q^2(T - S) + QU^{-1}Q^2U - Q^2U^{-1}QU\}\,.$$

In the first term all derivatives have been replaced by covariant ones and the second term equals zero, as is easily checked by substituting T and S .

We are interested in a minimal gauge invariant extension of (1), so one can drop gauge invariant terms in (7). Note also that the remaining part of (7) doesn't depend on arbitrary constants a and b, such that $a + b = 1$. Indeed [5]:

$$\mathrm{Sp}\{aQU^{-1}QdU - bQ(dU^{-1})\,QU\} =$$

$$= \frac{1}{2}\,\mathrm{Sp}\{QU^{-1}QdU - QUQ(dU^{-1})\} + \frac{a-b}{2}\,d\,[\mathrm{Sp}(QU^{-1}QU)]$$

and the last term vanishes after integration over M^4.

Thus photon and pseudoscalar meson world at low energies can be described by the effective action

$$\Gamma_{e.m.}(U,\,A) = \Gamma^N(U,\,A) + \Gamma^{WZW}_{e.m.}(U,\,A)\,, \tag{8}$$

where

$$\Gamma^N(U,\,A) = \frac{F_\pi^2}{8} \int_{M^4} dx\,\mathrm{Sp}\{(D_\mu U)\,(D^\mu U)^+\} \tag{9}$$

and

$$\Gamma^{WZW}_{e.m.} = -\frac{i}{80\pi^2}\int_{M^5} Sp(\alpha^5) + \frac{e}{16\pi^2}\int_{M^4} A\ Sp\{Q(\alpha^3+\beta^3)\} + \tag{10}$$

$$+ \frac{ie^2}{8\pi^2}\int_{M^4} A(dA)\ Sp\{Q^2(\alpha+\beta) + \frac{1}{2}QU^{-1}QdU - \frac{1}{2}QUQ(dU^{-1})\} .$$

It was shown [10] that (8) correctly reproduces all current algebra low energy theorems [11].

But even (8) can't be a complete story for low energy meson physics because pseudoscalar mesons, except pions, aren't considerably lighter than other hadrons, so at least vector mesons must be included in a more realistic effective lagrangian.

To include vector (and in general axial-vector) mesons into the game, usually they gauge more wide than $U(1)$ subgroup of $SU_L(3)\times SU_R(3)$ global symmetry group and consider the corresponding gauge bosons as vector mesons. However this procedure can hardly be considered as satisfactory because QCD doesn't possess so large local symmetry and it is very unexpected for this symmetry to appear in the low energy limit of QCD.

The main idea of this work is that for including vector mesons into the low energy effective lagrangian it is quite sufficient to gauge a $U(1)$ subgroup (to turn on electromagnetism) and to use a phenomenologically well established principle of vector meson dominance [12].

For effective action $\Gamma_{eff}(U, A, V)$ to reproduce the low energy theorems as well as $\Gamma_{e.m.}(U, A)$, let us demand [13, 14]

$$\exp\{i\Gamma_{e.m.}(U, A)\} = \int DV_\mu \exp\{i\Gamma_{eff}(U, A, V)\} \tag{11}$$

(we omit normalization factors in functional integrals).

The vector meson dominance suggests the following form for Γ_{eff} $U, A, V)$ [13]

$$\Gamma_{eff}(U, A, V) =$$

$$= \Gamma(U, V) - \frac{m^2}{2}\int_{M^4} dx\ Sp\left\{\left(V_\mu - \frac{e}{g}A_\mu Q\right)\left(V^\mu - \frac{e}{g}A^\mu Q\right)\right\} , \tag{12}$$

where $g = \frac{g\rho\pi\pi}{\sqrt{2}}$, for $g_{\rho\pi\pi}$ defined from the ρ - meson width

$$\Gamma(\rho \longrightarrow 2\pi) = \frac{2}{3}\frac{g^2_{\rho\pi\pi}}{4\pi}\frac{|\vec{q}_\pi|^3}{m^2_\rho} = \frac{g^2_{\rho\pi\pi}}{4\pi}\frac{m_\rho}{12}\left(1 - \frac{4m^2_\pi}{m^2_\rho}\right)^{3/2} .$$

As to the vector meson matrix V, we don't take into account the unitary symmetry breaking and assume ideal ω - φ mixing

$$V = \begin{pmatrix} \rho/\sqrt{2} + \omega/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & -\rho/\sqrt{2} + \omega/\sqrt{2} & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\varphi \end{pmatrix} . \tag{13}$$

Note that (11) and (12) have the infinite number of solutions

$$\exp\{i\Gamma(U, V)\} = \int DZ_\mu \exp\{i\Gamma_0(U, Z) + i\frac{m^2_V}{2}\int_{M^4} dx\ Sp\{(Z_\mu - V_\mu)(Z^\mu - V^\mu)\} . \tag{14}$$

The only restriction on $\Gamma_0(U, Z)$ is

$$\Gamma_0\left(U, \frac{e}{g}AQ\right) = \Gamma_{e.m.}(U, A) . \tag{15}$$

Clearly, one gets a minimal embedding of the vector mesons if

$$\Gamma_0(U, Z) = \Gamma_{e.m.}(U, \frac{g}{e}Q^{-1}Z) \tag{16}$$

that is

$$\Gamma_0(U, Z) = \Gamma_0^{N}(U, Z) + \Gamma_0^{WZW}(U, Z) ,$$

where

$$\Gamma_0^{N}(U, Z) = \frac{F_\pi^2}{8} \int_{M^4} dx \, Sp\{(\partial_\mu U - ig[Z_\mu, U])(\partial^\mu U^+ - ig[Z^\mu, U^+])\}$$

and (we have symmetrized ZdZ term)

$$\Gamma_0^{WZW}(U, Z) = -\frac{i}{80\pi^2} \int_{M^5} Sp(\alpha^5) + \frac{g}{16\pi^2} \int_{M^4} Sp\{Z(\alpha^3 + \beta^3)\} +$$

$$+ \frac{ig^2}{16\pi^2} \int_{M^4} Sp\{[ZdZ + (dZ)Z](\alpha + \beta) + \frac{1}{2} ZU^{-1}(dZ)(dU) +$$

$$+ \frac{1}{2} (dZ)U^{-1}Z(dU) - \frac{1}{2} ZU(dZ)(dU^{-1}) - \frac{1}{2} (dZ)UZ(dU^{-1})\} .$$

Then we have a Gaussian integral in (14), but it is calculated approximately [14] considering Wess - Zumino - Witten term as perturbation.

$$\Gamma(U, V) \approx \Gamma_0^{WZW}(U, Z^{(sp)}) + \Gamma_0^{N}(U, Z^{(sp)}) +$$

$$+ \frac{m_V^2}{2} \int_{M^4} dx \, Sp\{(Z_\mu^{(sp)} - V_\mu)(Z^{(sp)\mu} - V^\mu)\} , \qquad (17)$$

where $Z_\mu^{(sp)}$ saddle point is defined from the normal part.

We also expand U relative to Φ and omit vertices which contain more than five particles. Then it is sufficient to know a saddle point with accuracy of terms of order Φ^2.

Expanding $Z_\mu = Z_\mu^a \lambda^a/\sqrt{2}$, λ^a being Gell-Mann matrices, we get

$$\frac{\delta}{\delta Z_\mu^a} \left[\Gamma_0^{N}(U, Z_\mu) + \frac{m_V^2}{2} \int_{M^4} dx \, Sp\{(Z_\mu - V_\mu)(Z^\mu - V^\mu)\} \right] \Bigg|_{Z_\mu = Z_\mu^{(sp)}} =$$

$$= A^{ab} Z_\mu^{b\,(sp)} + B_\mu^a = 0 , \qquad (18)$$

where with the needed accuracy

$$A^{ab} = m_V^2 \delta^{ab} - \frac{1}{2} g^2 Sp\{[\lambda^a, \Phi][\lambda^b, \Phi]\} ,$$

$$B_\mu^a = - m_V^2 V_\mu^a - \frac{ig}{\sqrt{2}} Sp\{[\lambda^a, \Phi] \partial_\mu \Phi\} ,$$

therefore we get for the saddle point

$$Z_\mu^{(sp)} = V_\mu - \frac{g^2}{m_V^2} (V_\mu \Phi^2 + \Phi^2 V_\mu) +$$

$$+ \frac{2g^2}{m_V^2} \Phi V_\mu \Phi + \frac{ig}{m_V^2} [\Phi \partial_\mu \Phi - (\partial_\mu \Phi)\Phi]$$

this expression can be rewritten more compactly by the introduction of a covariant derivative $D_\mu \Phi = \partial_\mu \Phi - ig[V_\mu, \Phi]$

$$Z_\mu^{(sp)} = V_\mu - \frac{ig}{m_V^2} [D_\mu \Phi, \Phi] . \qquad (19)$$

Substituting this into (17), we get the effective Lagrangian in the above described approximation

$$L_{eff} = L^{N} + L^{WZW} + L^{VDM} , \qquad (20)$$

where

$$L^{VDM} = - \frac{m_V^2}{2} Sp \left\{ (V_\mu - \frac{e}{g} A_\mu Q)(V^\mu - \frac{e}{g} A^\mu Q) \right\} ,$$

$$L^{N} = \frac{1}{2}\,\mathrm{Sp}\{(D_{\mu}\Phi)(D^{\mu}\Phi)\} +$$

$$+ \frac{1}{F_{\pi}^{2}}\left(\frac{1}{3} + \alpha_{K}\right)\mathrm{Sp}\{\Phi\,(D_{\mu}\Phi)\,\Phi\,(D^{\mu}\Phi) - \Phi^{2}(D_{\mu}\Phi)(D^{\mu}\Phi)\}$$

and

$$L^{WZW} = \frac{\varepsilon^{\mu\nu\lambda\sigma}}{\pi^{2}}\left\{\frac{2}{F_{\pi}^{5}}\left(\frac{1}{5} + \alpha_{K} + \frac{3}{2}\alpha_{K}^{2}\right) \times\right.$$

$$\times\,\mathrm{Sp}\{\Phi(\partial_{\mu}\Phi)(\partial_{\nu}\Phi)(\partial_{\lambda}\Phi)(\partial_{\sigma}\Phi)\} - \frac{3g^{2}}{4F_{\pi}}\,\mathrm{Sp}\{(\partial_{\mu}V_{\nu})(\partial_{\lambda}V_{\sigma})\Phi\} -$$

$$- \frac{ig}{F_{\pi}^{3}}(1 + 3\alpha_{K})\,\mathrm{Sp}\{V_{\mu}(\partial_{\nu}\Phi)(\partial_{\lambda}\Phi)(\partial_{\sigma}\Phi)\} +$$

$$+ \frac{g^{2}}{4F_{\pi}^{3}}\,\mathrm{Sp}\Big\{V_{\mu}(\partial_{\nu}V_{\lambda})[\Phi^{2}(\partial_{\sigma}\Phi) - \Phi(\partial_{\sigma}\Phi)\Phi + (1 + 3\,\alpha_{K})(\partial_{\sigma}\Phi)\Phi^{2}] +$$

$$+ (\partial_{\mu}V_{\nu})\,V_{\lambda}[(1 + 3\alpha_{K})\,\Phi^{2}(\partial_{\sigma}\Phi) - \Phi(\partial_{\sigma}\Phi)\,\Phi + (\partial_{\sigma}\Phi)\Phi^{2}] -$$

$$- (1 + 3\alpha_{K})[V_{\mu}\Phi(\partial_{\nu}V_{\lambda})(\partial_{\sigma}\Phi)\,\Phi + (\partial_{\mu}V_{\nu})\,\Phi V_{\lambda}(\partial_{\sigma}\Phi)\,\Phi] +$$

$$+ V_{\mu}\Phi^{2}(\partial_{\nu}V_{\lambda})(\partial_{\sigma}\Phi) + (1 + 6\alpha_{K})(\partial_{\mu}V_{\nu})\,\Phi^{2}V_{\lambda}(\partial_{\sigma}\Phi) -$$

$$\left. - (1 - 3\alpha_{K})\,V_{\mu}\Phi(\partial_{\nu}V_{\lambda})\,\Phi(\partial_{\sigma}\Phi) - (1 + 9\alpha_{K})(\partial_{\mu}V_{\nu})\,\Phi V_{\lambda}\Phi(\partial_{\sigma}\Phi)\Big\}\right\}.$$

In these formulas $\alpha_{K} = \frac{g^{2}F_{\pi}^{2}}{m_{V}^{2}}$. Note that $\alpha_{K} = \frac{1}{2}$ corresponds to well known KSRF relation [15].

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