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To the Theory of Wall Layer in Collisional Plasma

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ABSTRACT

A structure of wall layer is considered with the assumption of small mean free path of particles compared with scale length of the problem. It is shown that in the case of weakly absorbing wall the structure of wall sheath determines uniquely the plasma inflow velocity. For the conditions of the absolutely absorbing wall, solution of the kinetic equation gives the whole spectrum of values of the inflow velocity.

1. INTRODUCTION

In many types of plasma devices the plasma behavior is significantly influenced by the processes near material walls which bound the confinement volume [1, 2]. In toroidal machines plasma has a direct contact with limiters or/and divertor plates, and in mirror traps end walls absorb plasma flowing out from the mirror along the field lines. Often, particularly in large machines, the mean free path λ in the edge plasma is small compared with the connection length L (counted along field lines) between the walls. In such a situation, Coulomb collisions play an important role in formation of the wall layer.

In the simplest model of plasma-wall interaction it is usually supposed that plasma particles that impact the wall experience reconbination but the interaction of the plasma with the neutral component is ignored. Assuming that the magnetic field lines are normal to the wall, one of the main conclusions of the existing theory [1] is that the plasma must flow to the wall layer with the velocity v which is of the order of the ion-sound velocity, $v \sim c_s = (T_e + T_i)^{1/2}/m_i^{1/2}$. This conclusion is derived basing on the Bohm criterion [3] which ensures monotonicity of the potential profile in the Debye sheath near the wall together with an analysis of the plasma flow outside the wall layer due to a particle source. In our opinion, this conclusion, being qualitatively correct, does not adequately describe physics of the collisional wall layer.

A correct approach to the problem in the limit $\lambda \ll L$ has to be based on a solution of the kinetic equation near the wall with a collision term and a proper boundary condition accounting for particle absorption by the wall. This solution describes a transition in a layer of width $\sim \lambda$ from a shifted maxwellian (that corresponds to a plasma flow) to the distribution function on the wall. A solvability condition of the kinetic equation must provide the value of the inflow velocity v without any additional considerations related with the Bohm criterion. For the hydrodynamic equations that are valid far from the wall layer (at the distance much larger than λ) the inflow velocity v can be considered as a boundary condition on an absorbing wall.

An attempt to account for collisions in the edge plasma in numerical calculations has been done in Ref. 4. However, due to numerical difficulties the authors of Ref.4 have considered only a parameter range that corresponds to $\lambda \geq L$.

A complete solution to the kinetic problem requires an extensive computational work. In this paper, in order to demonstrate the main ideas, we restrict our analysis by two relatively simple considerations.

In Sec.2, we describe a model which assumes that only a small fraction of impinging particles is absorbed by the wall; the rest of particles is elastically reflected and comes back to the plasma. It turns out that solution to this problem can be found analytically. In the limit of a small electron temperature when one can neglect an ambipolar potential this solution is obtained in Sec. 2. In Sec. 3 effects of finite electron temperature T_e are taken into account. It is worth noting that the model of a weakly absorbing wall was also used in Ref. 5 for a study of the effect of a conducting wall on plasma stability in mirrors.

In Sec. 4 we consider a problem of an absolutely absorbing wall. For the sake of simplicity, we consider only a case in which plasma flows into the layer with the velocity significantly exceeding the velocity of sound. Using a model collision term we are able to reduce the kinetic problem to an integral equation whose solution is found numerically. Therefore, we show that the absolutely absorbing wall allows not only sonic but also supersonic inflow velocities.

In Sec. 5 the main results are discussed.

2. A MODEL OF A WEAKLY ABSORBING WALL

We begin from the problem of the plasma flow to a solid wall that absorbs only a small part ε, $\varepsilon \ll 1$, of the particles that reach the material surface while other particles are elastically reflected from the surface. Consider a two component plasma that consists of singly charged ions and electrons. We also assume that the magnetic field lines are normal to the surface of the wall and the plasma flows along the magnetic field. The coordinate system is chosen so that x axis is perpendicular to the wall surface and is directed to the plasma, the surface coordinate being $x = 0$.

In this section, for the sake of simplicity, we restrict ourselves by consideration of a plasma with a small electron temperature, $T_e \ll T_i$. In this approximation, the ambipolar potential arising in the plasma does not affect the dynamics of ions and therefore can be ignored. Hydrodynamic equations of particle and energy balance for ions have the following form:

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i V_i}{\partial x} = 0, \qquad (1.1)$$

$$\frac{3}{2}\frac{\partial n_i T_i}{\partial t} + \frac{\partial}{\partial x}\left(\frac{5}{2} n_i V_i T_i - \kappa \frac{\partial T_i}{\partial x}\right) = 0, \qquad (1.2)$$

where n_i, T_i, V_i are respectively density, temperature and

hydrodynamic velocity of ions, $\kappa = 1.6 T_i^{5/2}/e^4 m_i^{1/2}\Lambda$ is the ion heat conductivity (Λ is the Coulomb logarithm). In the energy balance equation (1.2) we do not take into account the term which is responsible for the electron drag. A condition of such a negligence will be given at the end of the present section.

From what follows we will see that the characteristic length l of change of n_i, T_i and V_i equals to $l \sim \lambda/\varepsilon$ and the characteristic hydrodynamic velocity equals to $V_i \sim \varepsilon(T_i/m_i)^{1/2}$. With these values of l and V_i a simple estimate shows that one can neglect ion inertia in the equation of motion and take it in the following form

$$\frac{\partial n_i T_i}{\partial x} = 0 \; . \qquad (1.3)$$

Writing down Eq.(1.3) we have also neglected ion viscosity which appears to be small in parameter ε.

The system of equations (1.1) - (1.3) must be completed by proper boundary conditions at the wall and at infinity. The boundary condition at infinity consists of the requirement that the density and the temperature of ions are to be equal to some constant values

$$n_i\Big|_{x\to\infty} = n_{i\infty} \; , \qquad T_i\Big|_{x\to\infty} = T_{i\infty} \; . \qquad (2)$$

As follows from Eq.(1.3) with accounting for Eq.(2) the ion pressure does not depend on neither coordinate x nor time:

$$n_i T_i = p_i = \text{const.} \tag{3}$$

Substituting Eq.(3) in Eq.(1.2) we find that the density of the energy flux q_i also does not depend on x:

$$q_i = q(t) = \frac{5}{2} n_i V_i T_i - \kappa \frac{\partial T_i}{\partial x} . \tag{4}$$

Proceed now to the boundary condition at the wall. In order to derive it we need to know the ion distribution function near the surface at the distance $x \ll \lambda$. In the limit, $\varepsilon \ll 1$ when almost all particles are reflected elastically from the wall the distribution function f at the wall is close to a local maxwellian, $f_w \simeq (m_i/2\pi T_{iw})^{3/2} \times \times \exp(-m_i v^2/2T_{iw})$, with the temperature T_{iw} being the ion temperature at the wall. The required boundary condition follows from the spatial continuity of the energy flux: when $x \to 0$ the quantity q_i has to match the value q_{iw} of the energy flux absorbed by the wall. The latter can be found as the product of ε times the energy flux flowing to the wall:

$$q_{iw} = \varepsilon n_{iw} \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \int_{-\infty}^{0} dv_x \, \frac{1}{2} m_i v^2 v_x f_{iw} = -\varepsilon \left(\frac{2p_i^3}{\pi m_i n_{iw}} \right)^{1/2} , \tag{5}$$

where n_{iw} is the density of ions at the wall. Performing a similar calculation for the particle flux $n_{iw} V_{iw}$ absorbed by the wall:

$$n_{iw} V_{iw} = \varepsilon n_{iw} \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \int_{-\infty}^{0} dv_x \, v_x f_{iw} = -\varepsilon \left(\frac{p_i n_{iw}}{2\pi m_i} \right)^{1/2}, \tag{6}$$

and substituting Eqs. (5) and (6) in Eq.(4), with account for Eq.(3), we find the following boundary condition

$$\left. \frac{\partial n_i}{\partial x} \right|_{x=0} = \varepsilon \frac{1}{\kappa_\infty} \left(\frac{p_i}{8\pi m_i n_{i\infty}^5} \right)^{1/2} n_{iw}^4 , \tag{7}$$

where κ_∞ is the ion heat conductivity at the infinite distance from the wall.

From Eqs. (1.1) - (1.3) one can easily derive a single equation for the function $n_i(x, t)$:

$$\frac{\partial n_i}{\partial t} - \frac{2}{5} \frac{\partial}{\partial x} \left[\frac{\kappa}{n_i} \frac{\partial n_i}{\partial x} + \varepsilon \left(\frac{2p_i}{\pi m_i} \right)^{1/2} \frac{n_i}{n_{iw}^{1/2}} \right] = 0. \tag{8}$$

Here κ is meant as a function of n_i and p_i. Solving for Eq.(8) with the boundary conditions (2) and (7) one can not only find a steady state density profile in the wall layer but also examine a transition from an initial profile to a steady state one.

To find a steady state solution we require the expression in the square brackets in Eq.(8) to be constant. This yields the following equation that implicitly determines the function $n_i(x)$:

$$\frac{1}{2\sqrt{N}}\frac{x}{l} = -\frac{1}{\sqrt{N}}\left(\frac{1}{5N^2}+\frac{1}{3N}+1\right)+\frac{1}{2}\ln\left(\frac{1+\sqrt{N}}{1-\sqrt{N}}\right)+$$
$$+\frac{1}{\sqrt{N_w}}\left(\frac{1}{5N_w^2}+\frac{1}{3N_w}+1\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{N_w}}{1-\sqrt{N_w}}\right), \qquad (9)$$

where $l = \frac{1}{\varepsilon}\kappa_{\infty}\left(\frac{\pi m_i}{2n_{i\infty}p_i}\right)^{1/2}$, $N=n_i/n_{i\infty}$, and N_w is a constant that has a meaning of the ratio $n_{iw}/n_{i\infty}$. To evaluate N_w one needs to equate the value of the expression in the square brackets in Eq.(8) at the infinity to the value of this expression at the wall taking into account the boundary conditions (2) and (7). As a result one finds that $N_w = 4/5$, i.e.

$$n_{iw} = \frac{4}{5}n_{i\infty}. \qquad (10)$$

According to Eqs. (3) and (10) the temperature T_{iw} is

$$T_{iw} = \frac{5}{4}T_{i\infty}. \qquad (11)$$

Now, it is easy to find the velocity of the plasma flow at the infinite distance from the wall. As follows from the flux continuity, $n_{iw}V_{iw} = n_{i\infty}V_{i\infty}$, and Eqs. (6) and (10):

$$V_{i\infty} = \varepsilon\left(\frac{2p_i}{5\pi m_i n_{\infty}}\right)^{1/2}.$$

This result together with the definition of l given above justify the above formulated assumption about the order of magnitudes of l and V_i.

In the case when the initial density profile differs from that given by Eq.(9), a transition process will take place during which n_i will tend to the steady state distribution (9). To illustrate this transition we considered an initial profile corresponding to a constant density along the x axis, $n_i(x, t=0) = n_{i\infty} =$ const. A numerical solution to Eq.(8) has been found; it shown in Fig. 1. The density and the length in this figure are measured in units $n_{i\infty}$ and l respectively. As is seen from the figure, the initial profile does evolve to the steady state one, the characteristic time of the relaxation being of order of $l/V_{i\infty} \sim \kappa_{\infty}m_i/\varepsilon^2 p_i$.

Returning now to the negligence of the electron drag in Eq.(1.3) one can show that it is small in comparison with the terms retained in Eq.(1.3) if the following inequality holds:

$$\varepsilon \gg \left(\frac{m_e}{m_i}\right)^{1/4}\left(\frac{T_i}{T_e}\right)^{3/4}.$$

This inequality constrains the electron temperature from below.

3. EFFECTS OF FINITE ELECTRON TEMPERATURE

In this section we consider a steady state problem of a weakly absorbing wall in the case when the electron temperature T_e is of order of the ion temperature T_i.

Equations of particle and energy conservation for the ion species can be written as follows

$$n_i V_i = j_i = \text{const}, \tag{12.1}$$

$$\frac{5}{2} j_i T_i - \kappa \frac{dT_i}{dx} + e\varphi j_i = q_i = \text{const}, \tag{12.2}$$

where φ is the ambipolar potential. The force balance equation for the ions is

$$\frac{dn_i T_i}{dx} + en_i \frac{d\varphi}{dx} = 0, \tag{12.3}$$

(here we ignore ion inertia as we did in the previous section).

For electrons we assume that their temperature is constant (which is a good approximation taking into account high electron heat conductivity):

$$n_e V_e = j_e = \text{const}, \tag{13.1}$$

$$T_e \frac{dn_e}{dx} - en_e \frac{d\varphi}{dx} = 0. \tag{13.2}$$

Suppose that far from the wall the fluxes of electrons and ions are equal; due to the flux conservation they will be equal throughout:

$$j_i = j_e = j .$$

From the equality of electron and ion fluxes together with the quasineutrality condition it follows that the hydro-

dynamic velocities of electrons and ions are also equal to each other. This means that there is no friction force between electrons and ions.

In order to derive boundary conditions for Eqs. (12.1), (12.3), (13.1), (13.2) we define the temperature $T_{i\infty}$ and the plasma density n_∞ at the infinity and take into account that the ambipolar electric field has to vanish far from the wall:

$$\varphi \Big|_{x \to \infty} = 0 . \tag{14}$$

Let us calculate the fluxes j and q_i as functions of the density n_{iw} and the temperature T_{iw} of ions at the wall. Using the same arguments as in the derivation of Eqs. (5) and (6) in the previous section we obtain:

$$j = \varepsilon n_{iw} \int_{\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \int_{-\infty}^{0} dv_x v_x f_{iw} \simeq -\varepsilon n_{iw} \left(\frac{T_{iw}}{2\pi m_i} \right)^{1/2} , \tag{15}$$

$$q_i = \varepsilon n_{iw} \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z \int_{-\infty}^{0} dv_x \frac{1}{2} m_i v^2 v_x f_{iw} \simeq 2jT_{iw} , \tag{16}$$

where f_{iw} is the ion distribution function at the wall. From the other hand, taking into account the requirement $dT_i/dx = 0$ at the infinity together with Eq.(14) we find from Eq. (12.2) that

$$q_i = \frac{5}{2} jT_{i\infty} .$$

Comparison of the last expression with Eq. (15) gives $T_{iw} = 5T_{i\infty}/4$ which coincides with the result (11) of Sec. 1.

Proceed now to analysis of the spatial profile of the ion and electron densities. Note that the plasma is quasineutral everywhere except for a thin sheath near the wall with the width of order of the Debye length. In this Debye sheath the quasineutrality breaks because electrons partially escape the plasma due to their high mobility so that the total plasma charge becomes positive. In the Debye sheath the potential drop arises that prevents further escaping of electrons and accelerates ions moving to the wall.

First, we will find the ion density outside of the Debye sheath where the quasineutrality holds. As follows from Eq.(13.2) the electron density satisfies the Boltzman equation,

$$n_e = n_\infty \exp\,(e\varphi/T_e)\ . \qquad (17)$$

Summing up Eqs. (12.3) and (13.2) yields

$$n_i = n_\infty \frac{(T_{i\infty} + T_e)}{(T_i + T_e)}\ . \qquad (18)$$

The quantities φ and n_i can be excluded from Eq. (12.2) by using Eqs. (17) and (18) and the quasineutrality condition, $n_i = n_e$. The resulting differential equation for the unknown temperature T_i can be easily integrated and gives

$$\frac{jT_{i\infty}^{5/2}}{\kappa_\infty} x = \int_{T_{iw}}^{T_i} \frac{\tilde{T}_i^{5/2} d\tilde{T}_i}{\frac{5}{2}(\tilde{T}_i - T_{i\infty}) + T_e \ln\left(\frac{T_{i\infty} + T_e}{\tilde{T}_i + T_e}\right)}. \qquad (19)$$

The ion flux j entering Eq. (19) is not yet determined and will be found later.

Consider now the Debye sheath. We assume the temperature T_i in the sheath to be constant because according to Eq. (19) the characteristic length on which T_i varies is of the order of λ/ε, where λ is the ion mean-free path. Therefore within the Debye sheath which is much thinner than λ the ion temperature is approximately constant and is equal to T_{iw}. From Eq. (12.3) one finds that in the sheath

$$n_i = n_0 \exp\,(-e\varphi/T_{iw})\ , \qquad (20)$$

where n_0 is a constant to be determined. The dependence (20) needs to be matched with the density (17) at the distance that is much larger than the Debye length (but yet much smaller than λ/ε). The value φ_0 of the potential in the matching region is defined by the quasineutrality condition. Substituting T_{iw} instead of T_i into Eq. (18) and equating the result to Eq. (17) one obtains:

$$\varphi_0 = \frac{T_e}{e} \ln\left(\frac{T_{i\infty} + T_e}{T_{iw} + T_e}\right)\ .$$

The equality of the expressions (17) and (20) in the matching region leads to the following value of n_0

$$n_0 = n_\infty \left(\frac{T_{i\infty} + T_e}{T_{iw} + T_e} \right)^{1+\frac{T_e}{T_{iw}}} .$$

The potential φ_w at the wall is determined by putting the electron flux absorbed by the wall equal to the ion one. The electron flux can be calculated with the use of the formula (15) in which the ions quantities are substituted by the electron ones. This yields

$$j=- \frac{\varepsilon}{\sqrt{2\pi}} n_0 \left(\frac{T_{iw}}{m_i}\right)^{1/2} \exp\left(-\frac{e\varphi_w}{T_{iw}}\right) = - \frac{\varepsilon}{\sqrt{2\pi}} n_\infty \left(\frac{T_e}{m_e}\right)^{1/2} \exp\left(\frac{e\varphi_w}{T_e}\right), \quad (21)$$

and

$$\varphi_w = \frac{1}{e} \left(\frac{1}{T_e} + \frac{1}{T_{iw}} \right)^{-1} \ln\left[\left(\frac{m_e T_{iw}}{m_i T_e} \right)^{1/2} \left(\frac{T_{i\infty} + T_e}{T_{iw} + T_e} \right)^{1+T_e/T_{iw}} \right] .$$

Now, the flux j can be found by putting φ_w into Eq. (21):

$$j = - \frac{\varepsilon}{\sqrt{2\pi}} n_\infty \left(\frac{T_e}{m_e} \right)^{1/2} \left(\frac{T_{i\infty} + T_e}{T_{iw} + T_e} \right) \left(\frac{m_e T_{iw}}{m_i T_e} \right)^{T_{iw}/2(T_e+T_{iw})} .$$

The calculation of j completes determination of the ion temperature profile (19).

The qualitative dependence of the potential versus x is shown in Fig 2. Note that the wall potential φ_w appears not to depend on the parameter ε.

4. SUPERSONIC PLASMA FLOW TO THE WALL

Consider a steady state plasma flow to an absolutely absorbing wall. Again we assume that, first, electrons are cold and one can neglect the influence of the ambipolar potential on the ions, and second, that the magnetic field lines are normal to the surface of the wall and plasma flows along the magnetic field. In this section, we will seek a solution in the case when the plasma flows to the wall with the velocity essentially exceeding the ion acoustic velocity.

At large distance from the wall the ion density n_i, the ion temperature T_i and the plasma flow velocity V_∞ uniquely determine a maxwellian distribution function:

$$f_\infty = \left(\frac{m_i}{2\pi T_i} \right)^{3/2} \exp\left(- \frac{m_i(\vec{v}-\vec{V}_\infty)^2}{2T_i} \right).$$

As is mentioned above, we suppose that

$$V_\infty \gg c_s \sim v_T \sim (T_i/m_i)^{1/2}. \quad (22)$$

Our approach is based on a solving of the kinetic equation for the ion distribution function in the wall layer

$$v_x \frac{\partial f}{\partial x} = \mathrm{St} f . \quad (23)$$

To this equation a boundary condition should be added which takes into account absorbing properties of the wall.

Due to the absolute absorption, ions with positive velocities v_x are absent on the wall surface:

$$f(x = 0,\ v_x > 0) = 0\ . \qquad (24)$$

We will see from what follows that near the wall, at distances $\sim\lambda$, the distribution function f is considerably distorted from the maxwellian only in the velocity region

$$v_x \sim v_T^2/V_\infty\ ,\ v_\perp \sim v_T\ , \qquad (25)$$

($v_\perp^2 = v_y^2 + v_z^2$), i.e. in the region whose scales are equal to characteristic velocities on which the function f_∞ (taken at the point $v_x = 0$) varies. However, outside of this region, and in particular where the bulk ions are located, $v_x \simeq -V_\infty$, $v_\perp \sim v_T$, the distribution f_∞ appears to be almost unperturbed when the condition (22) holds.

Since the bulk ions are maxwellian we can use the ion-ion collision term in the following form [6] :

$$\mathrm{St}f = \frac{\partial}{\partial v_k}\left(D_{kj}\frac{\partial f}{\partial v_j} + F_k f\right), \qquad (26)$$

where

$$D_{kj} = \frac{\mathrm{Ln}}{8\pi}\left(\frac{\delta_{kj}}{v} + \frac{v_k v_j}{v^2}\left(\frac{v_T^2}{v^3} - \frac{1}{v}\right)\right),$$

$$F_k = \frac{\mathrm{Ln}}{4\pi}\frac{v_k}{v^3},$$

$$\mathrm{L} = \left(\frac{4\pi e^2}{m_i}\right)^2 \Lambda\ ,\quad v_T^2 = \frac{2T}{m_i}\ .$$

The expression (26) is valid in the velocity region (25) under the condition (22). In that region the collision term can be even more simplified allowing for $|\partial f/\partial v_x| \gg |\partial f/\partial v_\perp|$ and keeping only the largest terms in the parameter v_T/V_∞:

$$\mathrm{St}f = \frac{\mathrm{Ln}}{8\pi}\left(\frac{1}{V_\infty}\Delta_\perp + \frac{v_\perp^2}{V_\infty^3}\frac{\partial^2}{\partial v_x^2} + \frac{v_T^2}{V_\infty^3}\frac{\partial^2}{\partial v_x^2} - \frac{2}{V_\infty^2}\vec{v}_\perp\nabla\frac{\partial}{\partial v_x}\right) f\ , \qquad (27)$$

In Eq. (27) $\Delta_\perp = \dfrac{\partial^2}{\partial v_y^2} + \dfrac{\partial^2}{\partial v_z^2}$ and ∇ denotes the gradient in the velocity space.

In the collision integral (27) we neglected the electron-ion collisions causing the friction between the two species. It is easy to show that the ion-electron collision term is small in comparison with the terms retained in Eq. (27) if

$$T_e \gg T_i\left(\frac{V_\infty}{v_T}\right)^2\left(\frac{m_e}{m_i}\right)^{1/3},\quad T_e \gg m_e V_\infty^2\ .$$

Even with a relatively simple collision term (27), the kinetic equation (23) is too complicated for analysis. Being a three dimensional problem its solution requires extensive numerical work. In order to proceed analytically, we substitute the multiplier $v_\perp^2$ in the second term of (27) by the factor v_T^2 :

$$\mathrm{St}f = \frac{\mathrm{Ln}}{8\pi}\left(\frac{1}{V_\infty}\Delta_\perp + \frac{2v_T^2}{V_\infty^3}\frac{\partial^2}{\partial v_x^2} - \frac{2}{V_\infty^2}\vec{v}_\perp\nabla\frac{\partial}{\partial v_x}\right) f\ . \qquad (28)$$

The new collision integral conserves particles as the original one. Since of order of magnitude $v_\perp^2 \sim v_T^2$, our substitution does not qualitatively change the behavior of the distribution function f. It seems reasonable that if a solution of Eq.(23) with the collision term (28) exists than the solution of the kinetic equation with the collision integral (27) exists as well and the two solutions are qualitatively similar.

Before turning to the analysis of Eq. (23) note that the equilibrium distribution function that vanishes the collision term (28) is

$$f_0 = \left(\frac{2}{\pi^{3/2}\, v_T^3}\right) \exp\left(-\frac{V_\infty^2}{v_T^2}\right) \exp\left(-\frac{2v_\perp^2}{v_T^2} - \frac{2V_\infty v_x}{v_T^2}\right) .$$

This function satisfies Eq. (23) and corresponds to a maxwellian with anisotropic temperature, $T_\parallel = 2T_\perp$, and the macroscopic velocity $\vec{V}_\infty$ in the x direction.

Integrating Eq. (23) over the perpendicular velocity $v_\perp$ one finds

$$v_x \frac{\partial F}{\partial x} = A \frac{\partial^2 F}{\partial v_x^2} + B \frac{\partial F}{\partial v_x} , \qquad (29)$$

where

$$F = \int d\vec{v}_\perp f \; , \; A = \frac{\text{Ln}}{4\pi} \frac{v_T^2}{V_\infty^3} \; , \; B = \frac{\text{Ln}}{2\pi} \frac{1}{V_\infty^2} .$$

A boundary condition at the wall for Eq. (29) is obtained by **integrating** Eq. (24) over $v_\perp$:

$$F(x = 0,\ v_x > 0) = 0 . \qquad (30)$$

We must also require the function F to go to a maxwellian when $x \to \infty$:

$$F\Big|_{x\to\infty} = F_\infty , \qquad (31)$$

Neglecting terms of order of v_x^2/V_∞^2 we have for F_∞

$$F_\infty = \left(\frac{1}{\pi v_T^2}\right)^{1/2} \exp\left(-\frac{2v_x V_\infty + V_\infty^2}{v_T^2}\right) .$$

In addition to Eq.(31), we require

$$F\Big|_{v_x\to\infty} = 0 . \qquad (32)$$

We have also to take into account that outside of the interval (25), in the direction of negative v_x, the function F has to match F_∞

$$F\Big|_{v_x\to -\infty} = F_\infty . \qquad (33)$$

Now Eq. (29) is to be solved subject to the boundary conditions (30)-(33). Let us transform to a new function g instead of F, $g = -(F-F_\infty)$, which clearly satisfies Eq. (29) as well. The boundary conditions (31) - (33) for the new function g have zeros on the right hand sides and the condition (30) takes the following form

$$g\ (x = 0,\ v_x > 0) = F_\infty .$$

An equation similar to Eq. (29) (it differs by the absence of the term with the first derivative on the right hand side) has been solved in Ref. 7. Following the method employed in this reference we seek a particular solution of Eq. (29) in the form

$$g\,(x,\,v_x) = f_s(v_x)\,\exp\,(-s^3x),$$

where s is a positive constant because $g|_{x\to\infty}=0$. Using dimensionless variables, $x\to xA^2/B^3$, $v_x\to v_x A/B$, and defining a new function h_s and a new variable w,

$$f_s(v_x) = h_s(v_x)\,\exp\left(-\frac{v_x}{2}\right),\qquad v_x = w + \frac{1}{4s^3}\,,$$

we obtain the following equation for h_s:

$$\frac{d^2h_s}{dw^2} + s^3wh_s = 0\,.$$

A regular solution to this equation is the Airy function $Ai(sw)$:

$$h_s(w) = Ai\,(sw)\,.$$

Now, a general solution to Eq. (29) can be written as a superposition of particular solutions:

$$g(x,v_x) = \exp\left(-\frac{1}{2}\,v_x\right)\int_0^\infty dsC(s)\exp(-\,s^3x)Ai\,\left(-\,sv_x + \frac{1}{4s^2}\right), \qquad (34)$$

where $C(s)$ is an unknown function. Solving the integral equation (34) and finding the function $C(s)$ allows to determine the distribution function $F(x,\,v_x)$.

The equation (34) has been solved numerically and using

the function g the distribution function F has been found. The dependence of F versus v_x at the point $x = 0$ (i.e. on the wall surface), is shown in Fig. 3.

5. DISCUSSION

One of the main results of our work is an establishing of a boundary condition on the plasma normal velocity near a material surface. This boundary condition arises due to the existence of the collisional wall layer near the solid wall. For a weakly absorbing wall, it is shown in Sections 2 and 3 that this velocity turns out to be uniquely determined and by order of magnitude is equal to the product of the ion thermal velocity times absorption coefficient ε. We emphasize again that this conclusion is not related with the Bohm criterion and results only from the solution of the hydrodynamic equation describing the plasma flow into the wall. In addition to the plasma velocity V, we were able to find, in Sec. 3, the plasma potential distribution. Choosing the plasma potential at large distance from the wall as zero, this potential drops to $\varphi_0=-0.12T_e/e$ in the wall layer (for $T_e= T_i$ and hydrogen plasma) and continues to decrease in the Debye sheath taking the value $\varphi_w=-2.14T_e/e$ at the wall.

Extrapolating the results of Sec. 2 in the limit $\varepsilon\to1$ (that is the case of an absolutely absorbing wall), would lead to the conclusion that the plasma velocity in that case must be of order of the sound velocity c_s. However, the situation appears to be more complicated. As is shown in Sec. 4, in this case there is a continuous spectrum of allowable velocity values V; in particular, the velocity can be arbitrarily large compared with the sound velocity. However, it is clear from simple physical arguments that this velocity can not be much smaller than c_s so V has to be bounded from below. The exact value of the lower boundary should be found from a numerical solution in the wall layer of kinetic problem with the exact collision integral. Though the constrain on V from below by a quantity of the order of c_s is similar to the Bohm criterion, the physical nature of these two criteria is quite different.

What value of V will set up in a particular problem depends on the geometry of magnetic field and plasma sources. In limiter shadow of tokamaks typically $V \approx c_s$. An example of supersonic plasma flow into the wall is given by the expanders of the gasdynamic trap [8].

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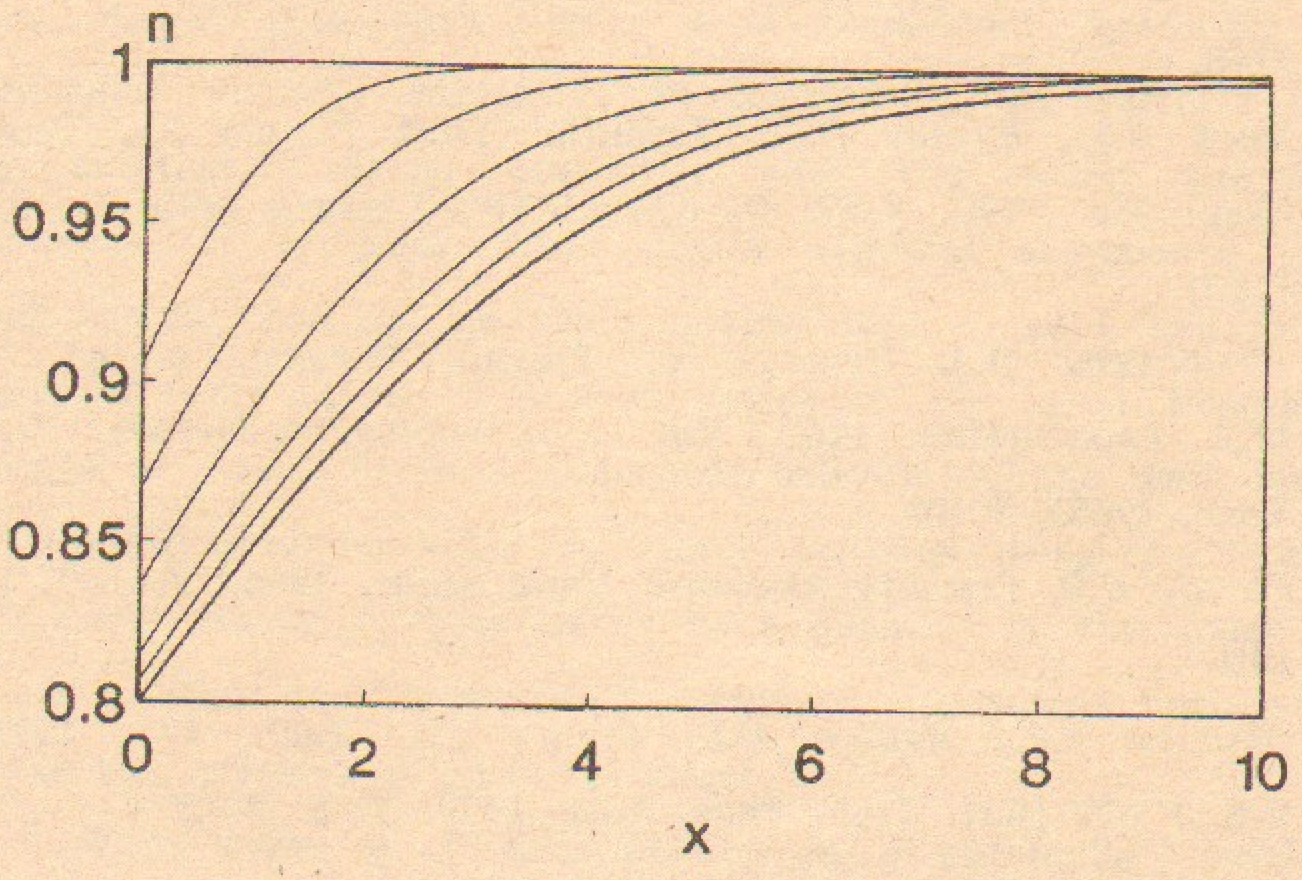

Fig. 1. The relaxation of the initial profile $n_i(x,t=0) = n_{i\infty}$ = const to the steady state. The length and the density are measured in units $l = \varepsilon^{-1}\kappa_\infty(\pi m_i/2n_{i\infty}p_i)^{1/2}$ and $n_{i\infty}$ respectively. The profiles shown by thin lines correspond to times t/τ = 1, 3, 7, 15, 23, where $\tau = 5\pi\kappa_\infty m_i/4e^2 p_i$ (they approach the steady state profile shown by the heavy line when t increases).

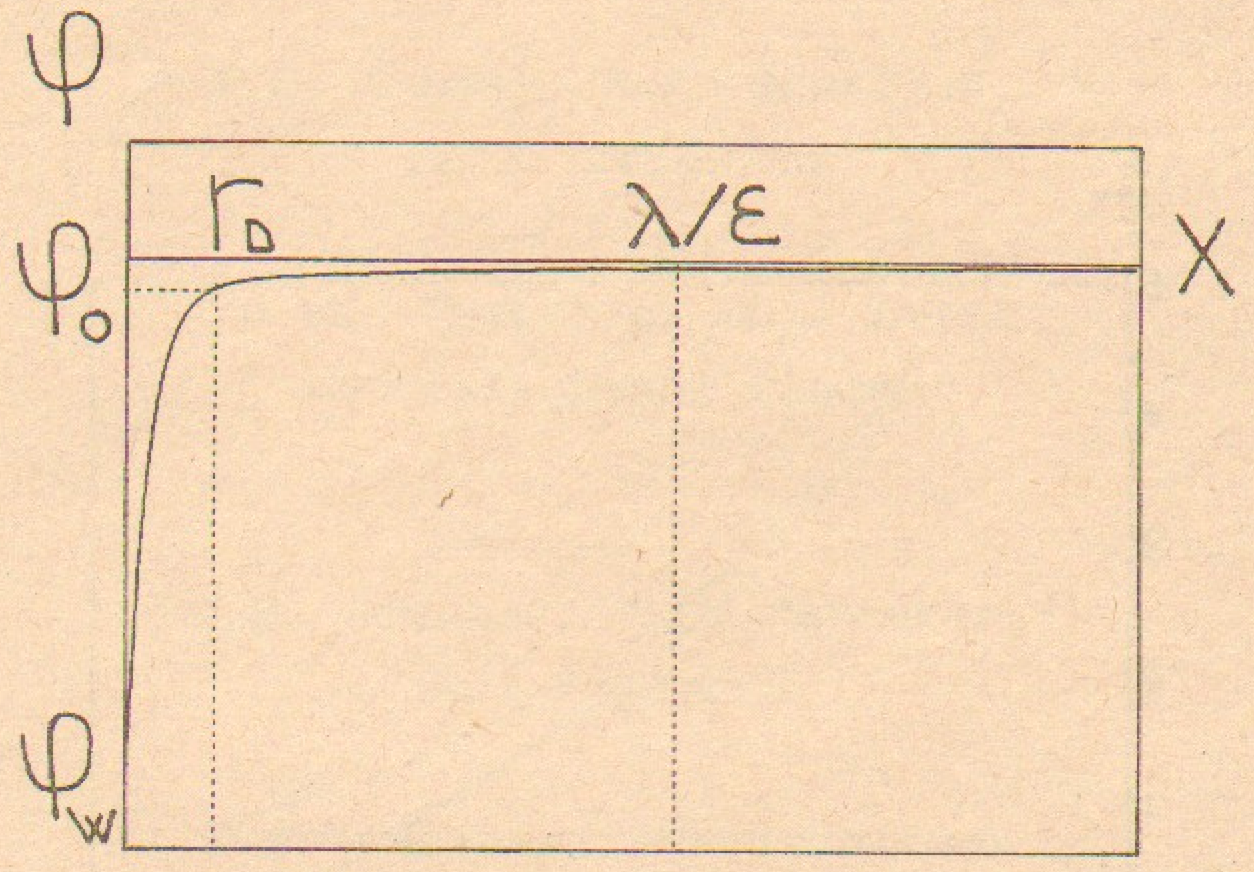

Fig. 2. The distribution of the potential near the material wall. The main potential drop occurs in the Debye sheath. Outside of the sheath the potential changes on the characteristic length $\sim \lambda / \varepsilon$, going to zero at infinity.

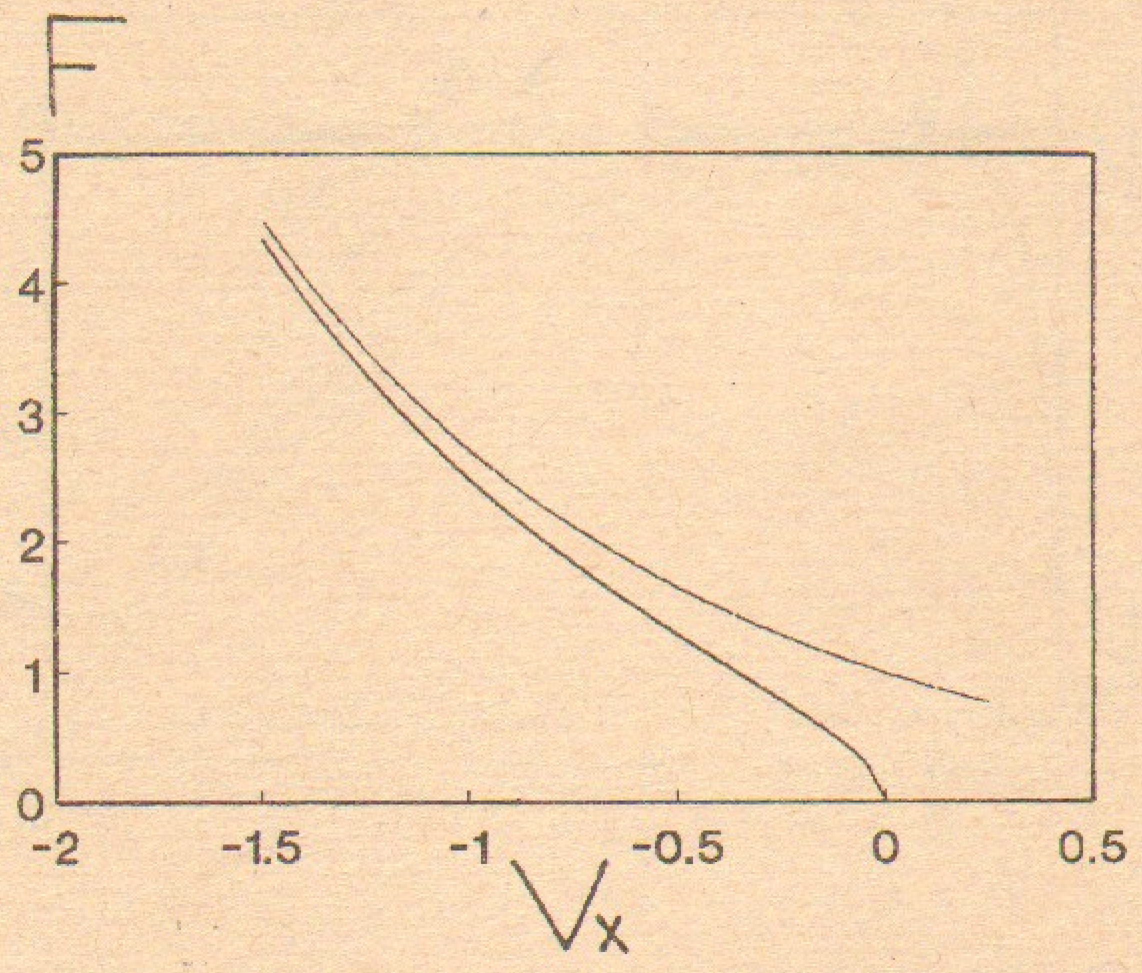

Fig. 3. The ion distribution function $F(v_x)$ on the surface of the wall in the velocity region $v_x \sim v_T^2/\nu_\infty$ (bold line). For comparison, the equilibrium ion distribution function $F_\infty(v_x)$ is also shown (thin line). The velocity v_x is measured in units $v_T^2/2\nu_\infty$ and the function F — in such units that F_∞ is equal to $\exp(-v_x)$.

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To the Theory of Wall Layer in Collisional Plasma

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