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I.B. Khriplovich and M.E. Pospelov

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НОВОСИБИРСК

Quadrupole P-odd Electron-Nucleus Interaction

I.B. Khriplovich and M.E. Pospelov

Institute of Nuclear Physics
630090 Novosibirsk, USSR

ABSTRACT

The quadrupole distribution of the weak nuclear charge causes *P*-odd mixing of $s_{1/2}$ and $p_{3/2}$ atomic states. The measurement of the corresponding *P*-odd effects in atoms would allow to determine the neutron quadrupole moment of a nucleus. The experiments with rare earths are of a particular interest in this respect. A simple derivation is presented for the imitating effect which originates from the combined action of the total weak charge and the quadrupole hyperfine interaction.

1. Parity nonconservation in atoms is now a firmly established phenomenon. Its investigations bring a first-class quantitative information about the weak interactions of elementary particles (see, e.g., book [1]). Not only nuclear-spin-independent effects have been studied, but the first evidence has been obtained of much smaller nuclear-spin-dependent ones, caused by the so-called nuclear anapole moment [2]. Due to this progress it becomes reasonable to consider more subtle *P*-odd effects in atoms which depend on the nuclear spin, to be more exact, on the quadrupole distribution of the weak nuclear charge which is close to the quadrupole distribution of neutrons. These effects have been previously mentioned in Ref.[1].

One more proposal should be mentioned here [3]. The idea is, by using the accurate value of $\sin^2\theta$ obtained at *Z*-boson peak at electron-positron colliders, to extract from the atomic experiments with different rare-earth isotopes the information on the neutron distribution in their nuclei.

2. The Hamiltonian of the P-odd electron-nucleon interaction which does not depend directly on nuclear spin is

$$H = -\frac{G}{\sqrt{2}}\cdot\frac{Q}{2}\,\gamma_5\cdot\rho(\vec{r}) \, , \qquad (1)$$

where $G=10^{-5}/m_p^2$ is the Fermi weak interaction constant; Q is the weak nuclear charge close numerically to $-N$, N being the neutron number. The quadrupole component of the nuclear density $\rho(\vec{r})$ introduces in fact the dependence on nuclear spin into this interaction and causes the mixing of the electronic states $s_{1/2}$ and $p_{3/2}$. The relativistic wave functions behave at small r as $r^{\gamma_{2j}}$ where $\gamma_{2j}=\sqrt{(j+1/2)^2-Z^2\alpha^2}$. Therefore, the mixing discussed is suppressed as compared to the usually considered mixing of the states $s_{1/2}$ and $p_{1/2}$, due to the spherically-symmetric part of the nuclear density $\rho(\vec{r})$, by factor close to Zr_0/a where a/Z is the Bohr radius for the unscreened nucleus, Z the nuclear charge, r_0 its radius. At $Z \sim 80$ this suppression constitutes about 10^{-2}. Evidently, we loose here also the factor Q as compared to the nuclear-spin-independent mixing. But some enhancement of the effect should occur in deformed nuclei.

3. We start our quantitative consideration from the case of deformed nuclei not only due to this enhancement, but because this case is slightly simpler technically. The

density of nucleons in a nucleus can be considered with a good accuracy as a constant. Therefore, for spherical nuclei we can present this density as

$$\rho_0(r) = \frac{3}{4\pi r_0^3}\cdot\theta(r_0-r). \qquad (2)$$

The density of a deformed nuclei is conveniently parameterized in its rest frame as [4]

$$\rho(\vec{r}) = \rho_0\left(\frac{r}{1-\frac{\beta^2}{4\pi}+\beta Y_{20}}\right). \qquad (3)$$

Since the deformation β is small, we can use the expansion of this expression:

$$\rho(\vec{r})= \rho_0(r) + \frac{3}{4\pi r_0^2}\cdot\beta Y_{20}\,\delta(r-r_0). \qquad (4)$$

The nuclear rotation is fast as compared to the electron motion. So we have to average the quadrupole part of this expression over the rotation. This procedure is carried out at the fixed projection Ω of the nuclear spin $\vec{I}$ onto the spheroid axis. As a result the quadrupole P-odd Hamiltonian is presented as

$$H_q = -\frac{G}{\sqrt{2}}\cdot\frac{Q}{2}\,\gamma_5\cdot\frac{3}{4\pi r_0^2}\cdot\beta\delta(r-r_0)\cdot A\cdot\sum_m Y_{2m}S_{2m}\, , \qquad (5)$$

where $A=\dfrac{3\Omega^2-I(I+1)}{I(I+1)(2I-1)(2I+3)}$; S_{2m} are the spherical components of the tensor $\hat{I}_i\hat{I}_j+\hat{I}_j\hat{I}_i-\frac{2}{3}\delta_{ij}I(I+1)$; $S_{20}=\hat{I}_z^2-\frac{2}{3}I(I+1)$.

It is convenient to choose the wave functions of the states $p_{3/2}$ and $s_{1/2}$ as

$$\Psi_{p_{3/2}}=\begin{pmatrix}-f_p(r)\cdot(\vec{\sigma}\vec{n})\Omega_{3/2,2}\\ ig_p(r)\cdot\Omega_{3/2,2}\end{pmatrix};\ \Psi_{s_{1/2}}=\begin{pmatrix}f_s(r)\cdot\Omega_{1/2,0}\\ -ig_s(r)\cdot(\vec{\sigma}\vec{n})\Omega_{1/2,0}\end{pmatrix}. \quad (6)$$

Here Ω_{jl} are the spherical wave functions with spin corresponding to the total angular momentum j and orbital angular momentum l. Using for the radial wave functions f and g the following expressions at small r (see, e.g., [1])

$$f_{njl}=\frac{k}{|k|}\cdot(k-\gamma_{2j})\left(\frac{Z}{a^3\nu^3}\right)^{1/2}\cdot\frac{2}{\Gamma(2\gamma_{2j}+1)}\left(\frac{a}{2Zr}\right)^{1-\gamma_{2j}}, \quad (7)$$

$$g_{njl}=\frac{k}{|k|}\cdot Z\alpha\left(\frac{Z}{a^3\nu^3}\right)^{1/2}\cdot\frac{2}{\Gamma(2\gamma_{2j}+1)}\left(\frac{a}{2Zr}\right)^{1-\gamma_{2j}};$$

$$k=(-1)^{j+1/2-\ell}(j+1/2),$$

we get for the mixing matrix element

$$\left\langle p_{3/2}\middle|H_q\middle|s_{1/2}\right\rangle = ig\cdot\frac{Z^2R_1}{(\nu_s\nu_p)^{3/2}}\cdot\frac{Q}{2}\cdot Ry\times$$

$$\frac{Zr_0}{a}\cdot\frac{R_3^{1/2}}{R_1^{1/2}}\cdot\frac{\beta}{6\sqrt{5\pi}}\cdot\frac{3\Omega^2-I(I+1)}{I(I+1)}\cdot\begin{Bmatrix}\sqrt{I/(2I+3)}\\ -\sqrt{(I+1)/(2I-1)}\end{Bmatrix}. \quad (8)$$

Here $g=\frac{Gm^2\alpha^2}{\sqrt{2\pi}}=3.65\cdot10^{-17}$; m is the electron mass; ν_s, ν_p are the principal quantum numbers of the s and p states; Rydberg $Ry=m\alpha^2/2$; R_1 and R_3 are relativistic enhancement factors:

$$R_3^{1/2}=\frac{4!}{\Gamma(2\gamma_3+1)}\cdot\left(\frac{a}{2Zr_0}\right)^{2-\gamma_3},$$

$$R_1^{1/2}=\frac{2}{\Gamma(2\gamma_1+1)}\cdot\left(\frac{a}{2Zr_0}\right)^{1-\gamma_1}. \quad (9)$$

Here and below the upper line in the curly bracket corresponds to the total atomic angular momentum $F=I+1/2$, the lower to $F=I-1/2$. In the second case **the** mixing is always larger. The best nuclear situation corresponds to $\Omega=I$, i.e., to a nonrotating nucleus.

In the first line of formula (8) we single out the factor equal to the mixing matrix element $\langle p_{1/2}|H|s_{1/2}\rangle$. Then the second line is a kind of a suppression factor which constitutes about 10^{-4} at $\beta\sim0.3$ and $Z\sim60$.

If the nucleus is described by the shell model, the mixing matrix element for the atomic states is

$$\left\langle p_{3/2}\middle|H_q\middle|s_{1/2}\right\rangle = -ig\cdot\frac{Z^2R_1}{(\nu_s\nu_p)^{3/2}}\cdot\kappa_{n,p}Ry\times$$

$$\times \frac{Zr_0}{a}\cdot\frac{R_3^{1/2}}{R_1^{1/2}}\cdot\frac{1}{16}\cdot\frac{(2I-1)(2I+3)}{I(2I+2)}\cdot\begin{cases}\sqrt{I/(2I+3)}\\ -\sqrt{(I+1)/(2I-1)}\end{cases} \qquad (10)$$

Here κ is the weak charge of the valence nucleon. It constitutes $\kappa_n=-1/2$ for a neutron, and is much smaller numerically for a proton: $\kappa_p = (1/2)(1-4\sin^2\theta) = 0.04$ at the experimental value of the mixing parameter $\sin^2\theta=0.23$. Even for a valence neutron the mixing constitutes about 10^{-5} only as compared to the nuclear-spin-independent effect.

Matrix element (10) was calculated under the assumption that the radial density of the valence nucleon follows the same law as the total one, i.e., is proportional to $\theta(r_0-r)$. It simplifies the calculations since now they can be reduced by integration by parts again to the δ-function kernel which in its turn allows us to use the usual Coulomb radial wave functions outside the nucleus. According to our estimates, the accuracy of this approximation is about 30-50%. If necessary, it can be easily improved in this respect by numerical calculations.

One should have in mind of course that in atoms with some outer electrons formulae (8) and (10) cannot be applied directly since at first we have to add the angular momenta of these electrons to form the total electronic one J which in its turn couples to the nuclear spin I.

4. The effect discussed can be singled out in experiments on the parity nonconservation in atoms through its specific dependence on the total angular momenta F, F' of the initial and final atomic states. Unfortunately, in the atoms being investigated now, caesium, thallium, bismuth, nuclei are not deformed and within the naive shell model the valence nucleon is a proton. So, the effect is additionally suppressed. As to lead, its only stable isotope ^{207}Pb has spin 1/2 and hence cannot possess a quadrupole density distribution.

But of a particular interest in this respect could be rare earth atoms. First, deformed nuclei are quite common here. Second, one can find in these atoms opposite-parity levels with very small energy separation. If the difference of the total electronic angular momenta of such levels is ΔJ=2, they can be mixed by the interaction discussed only, and P-odd effects in the corresponding transitions can be strongly enhanced. The analogous phenomenon for the P-odd interaction of an electron with a nuclear anapole moment and the close atomic levels with ΔJ=1 in rare earths is discussed in Ref.[5].

Some examples of such close opposite-parity levels in rare earths taken from Ref.[6] are presented below. The levels' energies are given in cm^{-1}.

Sm	30753.72 (J=1)	30755.28 (J=3) ,
	31246.30 (J=2)	31234.62 (J=4) ;
Er	25861.232 (J=7)	25863.453 (J=9);
Dy	18472.71 (J=8)	18462.65 (J=10),
	22061.29 (J=7)	22045.79 (J=9) ,
	23529.01 (J=6)	23534.50 (J=8) .

5. The effect discussed can be imitated by the combined action of the usual, spherically-symmetric part of Hamiltonian (1) and the hyperfine quadrupole interaction. An analogous effect takes place for the nuclear-spin-dependent interaction caused by a nuclear anapole moment: it is imitated by the same part of Hamiltonian (1) combined with the magnetic hyperfine interaction. In that case the imitating effect turned out not only parametrically different from the anapole interaction, but also numerically smaller [7, 8, 1].

To derive the effective operator of the imitating interaction we shall use here an approach considerably simplified as compared to that used in Refs.[7, 8, 1]. The version of the perturbation theory used there consists in the iteration in one of the interactions which is equivalent in fact to the use of the Coulomb Green's function. A simple observation is that the intermediate states of importance

here are high-energy ones. So, using instead of the Coulomb Green's function the free one, we introduce an error at the most of the order $Z\alpha$. Such an accuracy is here sufficient for us. But with a free electron Green's function we can use the usual Feynman technique in external field to calculate the corresponding scattering amplitude which is equivalent to effective operator for the imitating interaction.

In the approximation of a point-like nucleus interaction (1) simplifies in the momentum representation to

$$H_s = - \frac{G}{\sqrt{2}} \cdot \frac{Q}{2} \cdot \gamma_0 \gamma_5 \; . \tag{11}$$

The matrix γ_0 is introduced in this formula and in the next one since we shall use the covariant Feynman technique. The electric quadrupole interaction is in the momentum representation

$$H_{hf} = 4\pi \, \frac{e}{6} \cdot Q_{ij} \cdot \frac{p_i p_j}{p^2} \cdot \gamma_0 \, , \tag{12}$$

where Q_{ij} is the nuclear quadrupole moment. Since the distances of the order of the nuclear radius r_0 are of importance now, we can neglect in the free electron Green's function both its mass and energy, simplifying its covariant form as follows:

$$S(q) = \frac{q_\mu \gamma_\mu + m}{\varepsilon^2 - \vec{q}^{\,2} - m^2} \approx \frac{q_i \gamma_i}{\vec{q}^{\,2}} \; . \tag{13}$$

Then the effective interaction induced in this way (see diagrams 1, 2) is

$$H_{eff}=\frac{G}{\sqrt{2}}\cdot\frac{Q}{2}\cdot Q_{ij}\frac{2\pi}{3}\,e\cdot\int\frac{d^3q}{(2\pi)^3}\cdot\frac{q_iq_j}{q^2}\left(\frac{(p_2-q)_k\gamma_k}{(p_2-q)^2}+\frac{(p_1-q)_k\gamma_k}{(p_1-q)^2}\right)\gamma_5\gamma_0. \qquad (14)$$

To simplify further the calculations we shall expand the integrand in the ratio of $p_{1,2}/q$. Although in fact all the momenta are of the same order of magnitude, such an accuracy is sufficient for us now. Finally, we come to the following closed form for the effective interaction:

$$H_{eff}(r)=-\frac{G}{\sqrt{2}}\cdot\frac{Q}{2}\cdot\frac{\alpha}{r_0}\cdot Q_{ij}\Sigma_i(p_j\delta(r)+\delta(r)p_j)\cdot\frac{2}{45}. \qquad (15)$$

Here $\vec{\Sigma}=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}$ is the relativistic spin operator. The accurate integration over q when going over from formula (14) to (15) requires the introduction of nuclear formfactors in both vertices (11) and (12). We have performed this procedure in a simplified manner, introducing a cut-off momentum q_m. Clearly, it is close to the inverse nuclear radius r_0. In fact, the comparison of the results obtained in Refs.[7, 8, 1] for other induced interactions with their momentum representation derivation analogous to presented here shows that the correspondence is $2q_m = \pi r_0$. Just this relation was used to get $1/r_0$ in formula (15).

Simple estimates confirmed by explicit calculations demonstrate that the mixing caused by this induced interaction is much smaller numerically, because of the factor 2/45, than that due to the weak quadrupole interaction. The only possible exception is the case of a nondeformed nucleus with a valence proton since the proton weak charge is very small numerically.

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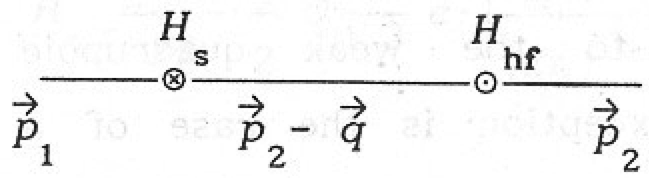

Figure 1

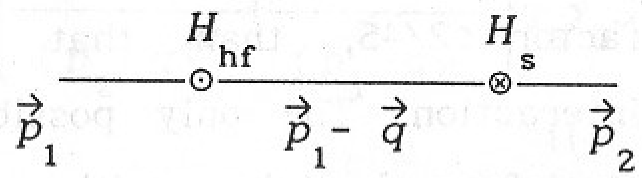

Figure 2

I.B. Khriplovich and M.E. Pospelov

Quadrupole P-odd Electron-Nucleus Interaction

И.Б. Хриплович, М.Э. Поспелов

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