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INTERMITTENCY OF THE TURBULENCE AND THE VORTON HYPOTHESIS

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Intermittency of the Turbulence

and the Vorton Hypothesis

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ABSTRACT

Turbulence is treated as a collection of separated vortical structures of different scales - vortons. A kinetic equation for the vorton system is written. A simple mean-vorton model of the inertial interval is considered using symmetry arguments.

A turbulent flow of an incompressible fluid has a large number N of excited freedoms that increases as a power of the Reynolds number Re for large values of Re. One hardly could expect to get a theory of such a turbulent flow as an exact general solution of Navier-Stokes equations. Attempts are made to investigate the turbulence rather qualitatively in order to understand main features of turbulent flows.

The study of laminar flows gives at least a satisfactory explanation of the source of large scale turbulent motion - large scale instabilities arouse for large Reynolds numbers $Re>Re_c$ - see [1]. The smaller scale eddys are then believed to be created by the larger scale ones via the Richardson cascade. The problem is the creation mechanism. The picture depends on the number of space dimensions. For *3D* flows the most important results of theoretical and experimental study are summarized in the Kolmogorov theory: there exists an inertial range of scales up to Kolmogorov scale $R_K=(k_K)^{-1} \propto Re^{-\beta}$, $\beta>0$, and the spect- ral density of energy obey in this range a scaling law

$$E\ (k) = \langle|v(k)|^2\rangle\ k^2 \propto k^{-\alpha}. \qquad (1)$$

In (1), v_k denotes the Fourier - harmonics of the velocity field. From physical arguments the value $\alpha = 5/3$ is found. Experiments rather confirm this value within the accuracy of measurements [2]. For the vorticity $\omega = \text{rot } v$ it leads to $<|\omega|^2> \propto k^{-\alpha}$.

The behavior of small scale component shows, that for Re ⇒ infinity the instantaneous realization of vorticity (as well as of velocity) is a singular field. To study this singularity, let us consider the average value ω_R of vorticity in a sphere centered in r_a

$$\omega_R(r_a) = (3/4\pi R)^{-3} \int \rho_R \omega\ (r - r_a) \partial^3 r. \qquad (2)$$

The integration in (2) is performed over a spherical volume centered in point r_a, R is the radius of the sphere. One obtains

$$S(R) = <|\omega|^2_R> \propto R^{-3+\alpha}. \qquad (3)$$

For $R \infty R_K \Rightarrow 0$ this quantity diverges. A property of the turbulence in the Kolmogorov inertial range, that is related to mentioned problems, is the intermittency of the vorticity field $\omega\ (r)$.

The space intermittency of turbulence is not studied in details at the present time. In what follows the basic assumption is that at every scale R there are $3D$ islands (vortons) of high vorticity in a $3D$ see of relatively low vorticity of the same scale. The mean value of the ratio

$$\Pi = R/L(R) \qquad (4)$$

of vorton size R to intervorton spacing $L(R)$ is assumed to be a small number or decreases with R: $\Pi << 1$.

To scan the scales the smoothed velocity and vorticity fields has to be defined. The definition (2) is an example of smoothing. A more general definition is

$$\omega_R(r) = Sm_R\omega\ (r) = \int \rho_R(r - r^+)\ \omega\ (r^+)\ \partial^3 r^+, \qquad (5)$$

with an appropriate smoothing function $\rho_R(r)$. The smoothing procedure is discussed in the renormalisation group theory in phase transitions - see for example [3]. If one study the smoothed field $\omega(r)$ then details of scales $L<R$ are smoothed out and R becomes the smallest scale left unchanged. According to (3) the magnitude of vorticity increase with decreasing scale, so the highest values of smoothed vorticity belong to the scale R. The operation $Al_{R,\lambda}$ that allocates the scale R, $0<\lambda<1$, may be defined as

$$Al_R\ \omega\ (r) = Sm_R\ \omega\ (r) - Sm_{\lambda R}\ \omega\ (r). \qquad (6)$$

The field $Al_R\ \omega(r)$ is the vorticity at the scale R mentioned above. Let us describe the properties of a turbulent fluid according to intermittence hypothesis. For very small $R<<R_K$ in the dissipation range the vorticity decreases exponentially. At the Kolmogorov length R_K one sees the vortons of smallest scale. At larger scales one sees new vortons as well as smoothed contributions of vortons of smaller scales. In the inertial range the spacing between regions of high vorticity of a given scale is large relative to the scale. At the longest scale in the inertial range the overlapping

of high vorticity regions become statistically significant. The role of coherent structures in turbulence and the possibility to consider the turbulent flow as a collection of spatially discrete vortical structures is widely discussed - see [4]. We use the term <vorton> to stretch the particle-like properties of the structure at every scale. A single vorton is characterized by the vorton location r_a, inverse scale $k_a = 1/R_a$ as well as other characteristics $C(a)$. The corresponding vorticity field is denoted as $f_{k,C}(r-r_a)$. The hypothesis formulated above allows to express the vorticity field as a superposition of vortons of different scales and positions

$$\omega(r) = \sum f_{k,C}(r-r_a). \qquad (7)$$

The key question is how a vorton of a smaller scale is produced from the larger ones. In a *2D* flow,the energy is transferred from small scales to large scale structures. The specific *3D* mechanism that invert the sign of the energy flux in the scale coordinate is the extension of vortex tubes. If the influence of viscosity is negligible one has for the enstropy ω^2 of an element [1]

$$d\omega^2/dt = \omega_i \omega_j e_{ij}, \qquad (8)$$

e_{ij} is the rate - of strain tensor. The extension effective in enstropy production at a given scale is due to vortex structures of nearest scales [2].

Attempts to explain the mentioned singularity of the flow realization in the limit Re ⇒ infinity as a superposition of local collapses gives at the present time rather negative results - see [5]. Here, the local collapse is meant as an evolution of a single vorton towards a singularity without essential interactions with other vortons. No solution is found of that type. In what follows a mechanism is described of a cooperative collapse in the system of many vortons. A vorton of a given scale R moves through the surrounding fluid interacting with other vortons. The most effective is the interaction with high deformation speed regions of near scales. The proposed low concentration of those regions (vortons) leads to the picture of a vorton gas and vorton - vorton collisions at every scale. A collision changes the scales of the vortex structures of colliding vortons and in general the number of vortons. A random walk of vortons in the real space, in scale and in number leads to a flux of vortons from largerto smaller scales. At scales $R > R_K$ the viscosity damp the motion. The density $W(k, C, r)$ of vortons with given k, C in a point r obey a kinetic equation one obtain by counting the changes of the number of vortons in an elementary volume of vorton characteristics space. Changes are due to single vorton motions and vorton - vorton collisions. If one takes into account only two-vorton collisions, the general form of the equation is

$$\partial W(\mu)/\partial t = L_1 + L_2,$$
$$L_1 = \int L_1(\mu, \mu') W(\mu') \partial\mu', \qquad (9)$$
$$L_2 = \int L_2(\mu, \mu', \mu'') W(k') W(\mu'') \partial\mu' \partial\mu''.$$

In (9), the whole set (k,C,r) of characteristics of a vorton is denoted as μ. Equation (9) reduces the problem of the statistical theory of turbulence to those of finding the quantities C and operators L_1 and L_2. The most important problem in such a treatment is the structure of a vorton.

Little is known about localized flows of a fluid. Vortex rings and vortex clouds like the Hill vortex are known examples of those flows [6]. If vortons are of that nature of generalized vortex rings than turbulence is a gas of vortons of different scales. An alternative picture is a random network of vortex tubes. To describe and to recognize vortons, the tensor moments technique may be applied - see [7]. In equation (9) the term L_1 describe the individual evolution of a vorton. This term is the most important in theories based on the ideas of local collapse of vortons mentioned above.

To proceed with the theory the detailed study of single vortons and vorton-vorton collision is needed. It involve in the theory the basic equation of the flow. In what follows we discuss a highly simplified model of a vorton gas. In the model, vortons are characterized by their inverse size k only. A vorton at every scale has a velocity v_k that characterize both the motion of the vorton relative to surrounding fluid and the inner motion, and v_k is a given function of k.

The number ∂N of vortons in a space region of a volume V having their inverse scales in the scale interval $\partial\lambda=\partial k/k$ is written as

$$\partial N = Vkn\ (k)\ \partial k/k,$$

$n(k)$ is the density of vortons in k-coordinate. The equation for $n(k)$ is a simplified version of (9):

$$\partial n\ (k)/\partial t = L_1 + L_2,$$

$$L_1 = \int L_1(k;\ k')\ n\ (k'')\ \partial k', \tag{10}$$

$$L_2 = \int L_2(k;\ k',\ k'')\ n\ (k')\ n\ (k'')\ \partial k'\partial k''.$$

As mentioned above, one expects the interaction to be important only for vortons with near scales. As a consequence, the functions $L_1(k;\ k')$ and $L_2(k;\ k',\ k'')$ are essentially nonzero only for small relative differences $(k - k')/k$, $(k - k'')/k$. Let us expand $n(k'')$ in series of $(k - k')$:

$$n(k') = n(k)+(k'-k)\ \partial n/\partial k+(1/2)(k'-k)^2\ \partial^2 n/\partial k^2 + \ldots \tag{11}$$

With the help of (11) eq.(10) may be written in the form of a differential equation

$$\partial n/\partial t = \Sigma c_n \partial^n n/\partial k^n + \Sigma\ g_{mn} \partial^n n/\partial k^n\ \partial n^m/\partial k^m, \tag{12}$$

$c_n(k)$ and $g_{mn}(k)$ are the moments of L_1 and L_2

$$c_0 = \int L_1(k;\ k')\ \partial k',\quad c_1 = \int L_1(k;\ k')\ (k-k')\ \partial k', \ldots \tag{13}$$

$$g_{00} = \int L_2(k;\ k',\ k'')\ \partial k'\partial k'',$$

$$g_{01} = g_{10} = \int L_2(k;\ k',\ k'')(k - k')\ \partial k'\partial k'' \ldots \tag{14}$$

The moment c_0 changes the number of vortons in single vorton evolution, as viscosity does. We suppose this term to be important only in the dissipation range. The moments c_1, c_2, ... lead to a change of scale of a vorton between collisions. We expect this change to be negligible in the inertial range, and take $c_i = 0$. The simplified model is written in order to imitate and to illustrate the properties of a system of vortons described by equation (9). The connection of the model to the Navier-Stokes equation is in a special choice of quantities c_i, g_{ij} as functions of k and v_k. Namely, we demand the model to have the same symmetry as the turbulence in the inertial range. An important role in the Kolmogorov theory plays the similarity of the turbulence at different scales in the inertial range. The proposed intermittency is in contradiction with the self-similarity of a turbulent flow at different scales- the intervorton spacing $L(R)$ in unites of vorton size R is Π^{-1} and depends on the scale R. The similarity of turbulent flows is based on the symmetry of the Navier-Stokes equation. Namely, the equations

$$\partial v/\partial t + (v \text{ grad})\, v = - \text{grad}\,(p/\rho) + \nu\, \Delta v, \quad \text{div}\, v = 0. \tag{15}$$

are invariant under similarity transformation

$$r = Z_r r', \quad k = Z_r^{-1} k', \quad t = Z_t t', \quad v = Z_v v', \tag{16}$$

$$(p/\rho) = Z_v^2 (p'/\rho'), \quad Z_v Z_t = Z_r ;$$

$$\nu = Z_v Z_r \nu'. \tag{17}$$

It is assumed that all the quantities defining the flow are transformed simultaneously. For vortons in the inertial range one expects the viscosity ν to be an irrelevant quantity- fluids with different viscosities have coinciding

sets of vorton solutions. If condition (17) is neglected, the symmetry (16) results in two symmetries -the Euler equations are invariant under (v-t) transformation (Z_r=1) and under (k-t) transformation (Z_v=1). For the simplified model, the demanded (v-t) and (k-t) symmetries determine the dependence of quantities g_{ij} upon k and the characteristic velocity v_k of vortons at scale $R = k^{-1}$. The resulting equation is

$$\partial n(k)/\partial t = (v_k/k)\, \Sigma\, h_{ij}\, [(k\, \partial/\partial k)^i\, n]\, [(k\, \partial/\partial k)^j\, n], \tag{18}$$

h_{ij} are numbers. In a stationary case ($\partial n/\partial t = 0$):

$$n(k) = n_0 k^{\gamma} \tag{19}$$

$$\Sigma\, h_{ij}\, \gamma^{i+j} = 0. \tag{20}$$

Equation (20) has a set of solutions γ_i, $i = 1, 2, \ldots$ Corresponding solutions of (18) may differ in stability and in other characteristics. Further restrictions on the constants h_{ij} comefrom the energy conservation argument. A vorton of scale $R = 1/k$ is a region of the size $R=1/k$, volume $V_k \propto k^{-3}$, characteristic velocity v_k and kinetic energy $\varepsilon(k) \propto v_k^2 k^{-3}$. The conservation of the energy in vorton collisions leads to the equation for the energy density in k-space E (k) = = ε (k) n (k) of the form

$$\partial E\,(k)/\partial t = \partial I/\partial k, \tag{21}$$

where $I(k)$ is the energy flux in k-axis. Multiplying (18) on $\varepsilon(k)$ and comparing with (21) one finds

$$\partial I/\partial k = (v_k^3\, k^{-4})\, (h_{00} n^2 + 2h_{01} nk\, \partial n/\partial k + \ldots). \tag{22}$$

The expected form of the energy flux $I\{n(k)\}$ as a local functional of $n(k)$ is

$$I\{n\} = (v_k^3 k^{-3})(a_{00} n^2 + 2a_{01} nk\, \partial n/\partial k + \ldots) \quad (23)$$

with new constants a_{00}, $a_{01}=a_{10}$, a_{11} .... For the form of the flux (23) to exist, constants h_{ij} and the velocity v_k as a function of k mast obey some conditions. The easiest way to get this conditions is to consider the stationary case $\partial n/\partial t = 0$. For that case $\partial I/\partial k = 0$ and $I = I_0$ = const., and for the vorton density the solution of (20) gives $n = n_0 k^{\gamma}$. The condition $I \propto (v_k^3 k^{-3}) k^{2\gamma} = I_0$ determine the velocity as a function of k :

$$v_k = v_0 k^{1-(2\gamma/3)}. \quad (24)$$

The way vortons are described in the model resemble on mean field theories in statistical physics, equation (24) plays the role of a self-consistency condition in a mean-vorton approximation. The final form of the mean vorton model is

$$\partial n/\partial t = (v_k^{-2} k^{3})\, \partial I/\partial k,$$

$$I\{n\} = (v_k^3 k^{-3}) \Sigma a_{ij} [(k\, \partial/\partial k)^i n]\, [(k\, \partial/\partial k)^j n], \quad (25)$$

$$a_{ij} = a_{ji}, \quad i, j = 0, 1, 2, \ldots,$$

$$v_k = v_0 k^{1-(2\gamma/3)}.$$

For $n(k) = n_0 k^{\sigma}$ one has for $I\{n\}$

$$I\{n\} = v_0^3 n_0^2 k^{2(\sigma-\gamma)} \Sigma a_{ij} \sigma^{i+j}, \quad (26)$$

and the characteristic equation (20) is

$$(\sigma-\gamma) \Sigma a_{ij} \sigma^{i+j} = 0. \quad (27)$$

The solution $\sigma = \gamma$ with nonzero flux I exists automatically, if the sum in (27) is nonzero for $\sigma = \gamma$. Other solutions of (27) are those of $I = 0$. Note the special choice of a_{ij} to have the only solution $\sigma = \gamma$:

$$\partial n/\partial t = a\,(v_k/k)[\gamma n^2 - n\,(k\, \partial n/\partial k)]. \quad (28)$$

$$I = -(a/2)\,(v_k^3 k^{-3})\, n^2.$$

The stationary solution of (25)

$$n(k) = n_0 k^{\gamma}, \quad v_k = v_0 k^{1-(2\gamma/3)} \quad (29)$$

means for the spectral density E(k) of kinetic energy

$$E(k) = \langle |v(k)|^2 \rangle k^2 \propto k_k^2 \int n(q)(v_q q^{-3})^2\, \partial q \propto k^{-1-\gamma/3}. \quad (30)$$

The spacing $L(R)$ is $L \propto (kn(k))^{-1/3}$, so the parameter Π (see (4)) is

$$\Pi = R/L(R) \propto k^{-\delta}, \quad \delta = 2-\gamma. \quad (31)$$

The case $\delta = 0$ gives the value of $\gamma = 2$ and the Kolmogorov-Obukhov low $E(k) \propto k^{-5/3}$. The condition $\Pi \ll 1$ may still be fitted for this value of δ if Π is small numerically. Another possibility is a week dependence like $\Pi^{-1} \propto \ln(k)$.

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