ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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PARTON'S PICTURE OF THE ELECTROWEAK PROCESSES IN HIGH ENERGY e^+e^- COLLISIONS

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Parton's Picture of the Electroweak Processes in High Energy e^+e^- Collisions

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ABSTRACT

Regarding the initial e^+, e^- as a sources of partons (e^+, e^-, γ), we obtain the distributions on invariant mass of system Φ produced in processes $e^+e^- \to e^{+\prime}e^{-\prime}\Phi$, $e^{+\prime}\gamma\Phi$, $\gamma\gamma\Phi$ with detection of $e^{+\prime}e^{-\prime}$, $e^{+\prime}\gamma$, $\gamma\gamma$ emitted near the beam's directions. Radiative corrections are taken into account in principle to all orders of perturbation theory (in leading logarithmical approximation). Polarized and unpolarized cases are considered. Estimations of number of $W^\pm$, W^+W^-, Z, produced are given and they are of order $10^4 \div 10^5$ for annual luminosity integral $\int L dt \simeq 10^4\,\text{pb}^{-1}$.

Additional hard photon spectra in narrow resonance production is obtained for the cases of large and small emission angles (in c.m. system). The «reversed radiative tail» in photon spectra may be a test of a resonance. We discuss also the tagging conditions of charged particles.

The next generation of e^+e^- colliders with energies above the current energy of LEP, i.e. up to $\sqrt{S} \geqslant 200$ GeV will be aimed at learning both the interaction of heavy vector bosons $W^\pm$, Z, Z' and the search for the bound states of top quark. Expecting numbers of events per year at a luminosity $L \sim 10^{33}\,\text{cм}^{-2}\text{s}^{-1}$ are about 10^5 for $W^\pm$, W^+W^-, $Z^0\gamma$, Z^0Z^0 production and of order 10^3 for the processes with an additional photon production. An account of radiative corrections (r.c.) seems to be relevant especially in the case of the narrow resonance production where higher orders of perturbation theory (p.t) are essential.

In this paper we propose a simple method with sufficient accuracy for a calculation of cross sections of the above mentioned processes with an account of r.c.: we use the parton approach. Distributions of partons (e^+, e^-, γ) in the initial e^- or e^+ are determined by Altarelly—Parisi—Lipatov (APL) equations [1]. We put them as well as their approximate solutions for polarized and unpolarized case in Appendix A.

We may consider the initial particles (e^+, e^-) as a sources of partons and write down the cross section of creation of some system Φ in e^+e^- collision in form of Drell—Yan process [2]:

$$\sigma^{e^+e^-}_{\lambda_+\lambda_-}(S) = \sum_{A,\lambda_A} \sum_{B,\lambda_B} \int dx_1 \int dx_2\, D^{A,\lambda_A}_{e^-,\lambda_-}(x_1, S)\, D^{B,\lambda_B}_{e^+,\lambda_+}(x_2, S)\, \hat{\sigma}^{\lambda_A\lambda_B}_{AB\to\Phi}(Sx_1x_2)\,. \quad (1)$$

Where the structure function $D^{A,\lambda_A}_{e,\lambda}(x, S)$ describe the distribution of partons of the kind A with the helicity λ_A, energy fraction x

and momentum square q^2 up to S into e with helicity λ; $\hat{\sigma}^{\lambda_A, \lambda_B}_{AB \to \Phi}(S_1)$ is the cross section of partonic subprocess.

We believe that using the different tagging conditions of scattered particles at small (to beams directions) angles $e^{\pm\prime}$, γ one may extract the different $\hat{\sigma}$. One may use (1) for estimation of number of events. Choosing $A=B=\gamma$ one may write down (1) in unpolarized case in form

$$\sigma(S) = \beta^2 \int_{S_{th}}^{S} \frac{dS_1}{S_1} \hat{\sigma}^{\gamma\gamma \to \Phi}(S_1)\, \varphi_{\gamma\gamma}\left(\frac{S_1}{S}\right), \quad \beta = \frac{2\alpha}{\pi}(L-1), \quad L = \ln \frac{S}{m_e^2}, \quad S = 4\varepsilon^2,$$

$$\varphi_{\gamma\gamma}(x) = (2+x)^2 \ln \frac{1}{x} - 2(3+x)(1-x), \qquad (2)$$

obtained first by Brodski—Kinoshita—Terazawa [3]. It's known that the expression (2) have a low accuracy ($\sim 20\%$) [4]. The reason is in the fact that the contribution to (2) go from the region $S_1 \sim S_{th} = \left(\sum_{\Phi} m\right)^2$ where the renormalization group (used to obtain φ) don't work.

Much better accuracy $\sim 1\%$ will have the distribution on invariant mass square of produced system

$$\frac{\partial \sigma^{e^+e^- \to e^{+\prime} e^{-\prime} \Phi}(S_1, S)}{\partial S_1} = \frac{\beta_q^2}{S_1} \varphi_{\gamma\gamma}\left(\frac{S_1}{S}\right) \hat{\sigma}^{\gamma\gamma \to \Phi}(S_1), \quad S_1 \gg m^2, \quad \beta_q = \frac{2\alpha}{\pi}(L_q - 1),$$

$$L_q = \ln |q^2/m_e^2|, \quad q_1^2 \approx q_2^2 \approx q^2 = (p_1 - p_1')^2 = (p_2 - p_2')^2, \quad m_e^2 \ll |q_{1,2}^2| \ll S. \ (3)$$

This distribution may be measured in experiment with tagging of both electron and positron scattered at small angles to beam axes with four-momentums p_1', p_2' respectivelly. One may measure the cross sections of subprocesses $\gamma e^{\pm} \to \Phi$, $e\bar{e} \to \Phi$ when tagging energy of photons, emitted close to beam directions (say by means of calorimeter—type detectors).

This subprocesses have the important role in investigation of standard model (SM) of electroweak interactions.

In section 1 we obtain the cross sections of producing the system Φ in γe, $\gamma\gamma$, $e\bar{e}$ subprocesses in form (3) for polarized and unpolarized cases. In section 2 we consider the similar processes with an additional hard photon in a final state. For such an experiment when only emitting angle and the energy fraction of photon are detected in narrow resonance production we obtain the compact

formulas of type (1) for the cases of small and large photon's emission angles. The photon fraction's distribution have an characteristic «reversal radiative tail» which, we believe, may be a test for resonance. For $e^+e^- \to Z^0\gamma$ processes our result agree with the result of lower order calculation of r.c. which was performed in [5]. In section 3 we estimate roughly the annual number of events of production of systems with heavy vector bosons and discuss the tagging conditions of charged particles.

1. The distribution on invariant mass square of Φ may be expressed in terms of convolution

$$f * g = g * f = \int_x^1 \frac{dy}{y} g(y)\, f\left(\frac{x}{y}\right).$$

For unpolarized case we may write down

$$\frac{\partial \sigma^{e\bar{e} \to (\gamma\gamma)\Phi}}{\partial S_1} = \frac{1}{S} \hat{\sigma}^{e\bar{e} \to \Phi}(S_1)\, |1 - \Pi(S_1)|^{-2} \Phi_{e\bar{e}}\left(\frac{S_1}{S}, \beta\right), \qquad (4a)$$

$$\frac{\partial \sigma^{e\bar{e} \to (e\gamma)\Phi}}{\partial S_1} = \frac{1}{S_1} \hat{\sigma}^{\bar{e}\gamma \to \Phi}(S_1)\, \Phi_{\gamma e}\left(\frac{S_1}{S}, \beta\right), \qquad (4b)$$

$$\frac{\partial \sigma^{e\bar{e} \to (e\bar{e})\Phi}}{\partial S_1} = \frac{1}{S_1} \hat{\sigma}^{\gamma\gamma \to \Phi}(S_1)\, \Phi_{\gamma\gamma}\left(\frac{S_1}{S}, \beta_q\right). \qquad (4c)$$

Notation (ij) in left hand side (4) means that the i, j particles are to be tagged, $\Pi(S_1) = \frac{\alpha}{\pi}\left(\frac{1}{3} \ln \frac{S_1}{m_e^2} - \frac{5}{9}\right)$, is the known factor of polarization of vacuum. The functions $\Phi_{e\bar{e}}$, $\Phi_{\gamma e}$, $\Phi_{\gamma\gamma}$ are the conversions $D*D$, $x \cdot D*G$ and $x \cdot G*G$ respectively; D, G—structure functions [6, 7], discribing the probability to find out an electron and photon in initial unpolarized electron. Using the expressions quoted in Appendix A, we obtain (only terms of lowest orders on β are put down):

$$\Phi_{e\bar{e}}(x, \beta) = \beta \left[\left(1 + \frac{3}{4}\beta\right)(1-x)^{\beta - 1} - \frac{1}{2}(1+x)\right], \qquad (5a)$$

$$\Phi_{\gamma e}(x, \beta) = \frac{\beta}{2}(1 + (1-x)^2)\left(1 + \frac{3}{8}\beta\right)(1-x)^{\beta/2} +$$

$$+ \frac{1}{8}\beta^2 [x(2-x) \ln x - (1-x)(3-2x)], \qquad (5b)$$

$$\Phi_{\gamma\gamma}(x,\beta_q)=\beta_q^2\,\varphi_{\gamma\gamma}(x)+\frac{1}{8}\beta_q^3\Big[4(2+x)^2\int\limits_x^1\frac{dy}{y}\ln(1-y)+x(4+x)\ln^2 x-$$
$$-x(12+5x)\ln x-(1-x)(24+8x)\ln(1-x)+(1-x)(10+8x)\Big]. \quad (5c)$$

Let us consider now a case when the initial leptons age completely longitudinally polarized see also [7,a]. For the case when electron and positron are tagged in a case of the same (positive) helicities of initial particles one measure the linear combination of $\hat\sigma^{++}_{\gamma\gamma\to\Phi}$ and $\hat\sigma^{-+}_{\gamma\gamma\to\Phi}$:

$$\frac{\partial\sigma_{++}}{\partial S_1}=\frac{\beta_q^2}{S_1}\Big[\varphi_1\Big(\frac{S_1}{S}\Big)\hat\sigma^{+-}_{\gamma\gamma}(S_1)+\varphi_2\Big(\frac{S_1}{S}\Big)\hat\sigma^{++}_{\gamma\gamma}(S_1)\Big]; \quad (6)$$

another combination may be measured in the case of opposite helicities of initial e^+, e^-:

$$\frac{\partial\sigma_{+-}}{\partial S_1}=\frac{\beta_q^2}{S_1}\Big[\varphi_2\Big(\frac{S_1}{S}\Big)\hat\sigma^{+-}_{\gamma\gamma}(S_1)+\varphi_1\Big(\frac{S_1}{S}\Big)\hat\sigma^{++}_{\gamma\gamma}(S_1)\Big], \quad (7)$$

where

$$\varphi_1(x)=\frac{1}{8}\Big[\ln\frac{1}{x}-\frac{1}{2}(1-x)(3-x)\Big],$$

$$\varphi_2(x)=\frac{1}{16}\Big[(2+4x+x^2)\ln\frac{1}{x}-3(1-x^2)\Big]. \quad (8)$$

When the tagged particles are small angle scattered electron and the photon beam in opposite direction, one obtain

$$\frac{\partial\sigma_{++}}{\partial S_1}=\frac{1}{S_1}\Big[\psi_1\Big(\frac{S_1}{S},\beta\Big)\hat\sigma^{++}_{e^+\gamma}(S_1)+\psi_2\Big(\frac{S_1}{S},\beta\Big)\hat\sigma^{+-}_{e^+\gamma}(S_1)\Big], \quad (9)$$

$$\frac{\partial\sigma_{-+}}{\partial S_1}=\frac{1}{S_1}\Big[\psi_2\Big(\frac{S_1}{S},\beta\Big)\hat\sigma^{++}_{e^+\gamma}(S_1)+\psi_1\Big(\frac{S_1}{S},\beta\Big)\hat\sigma^{+-}_{e^+\gamma}(S_1)\Big], \quad (10)$$

where

$$\psi_1(x,\beta)=\frac{\beta}{4}x^2+\frac{3}{64}\beta^2(2x+x^2+4\ln(1-x)),$$

$$\psi_2(x,\beta)=\frac{\beta}{4}(1-x)^2+\frac{3}{32}\beta^2\Big((x^2-2x)\ln\frac{1}{x}+$$
$$+2(1-x)^2\ln(1-x)+x(1-x)\Big). \quad (11)$$

When both tagged particles are photons one has ($\sigma^{++}_{e^+e^-}\ll\sigma^{+-}_{e^+e^-}\neq\sigma^{-+}_{e^+e^-}$):

$$\frac{\partial\sigma^{++}}{\partial S_1}=\frac{\beta^2}{48}\frac{1}{S_1}\Big(1-\frac{S_1}{S}\Big)^3|1-\Pi(S_1)|^{-2}[\hat\sigma^{+-}_{e^+e^-}(S_1)+\hat\sigma^{-+}_{e^+e^-}(S_1)], \quad (12)$$

$$\frac{\partial\sigma^{+-}}{\partial S_1}=\frac{1}{S}\Phi_{ee}\Big(\frac{S_1}{S},\beta\Big)\hat\sigma^{+-}_{e^+e^-}(S_1)\,|1-\Pi(S_1)|^{-2}. \quad (13)$$

2. Consider now the inclusive process on photon cros ssection

$$e^+(p_+)+e^-(p_-)\to\gamma(k)+\Phi. \quad (14)$$

The cross section of this process in the quasireal electron approximation [8] can be written as

$$d\sigma=dW_{p_-}(k)\,d\sigma_0(p_--k,p_+)+dW_{p_+}(k)\,d\sigma_0(p_-,p_+-k), \quad (15)$$

where $dW_p(k)$ is the probability of radiation a photon with momentum K by a lepton with momentum p:

$$dW_p(k)=\frac{\alpha}{4\pi^2}\Big(\frac{\varepsilon_p^2+(\varepsilon_p-\omega)^2}{\omega\cdot\varepsilon_p(kp)}-\frac{\varepsilon_p-\omega}{\varepsilon_p}\cdot\frac{m^2}{(kp)^2}\Big)\frac{d^3k}{\omega}, \quad (16)$$

$d\sigma_0$ — the cross section of process without photon. The first term in r.h.s. (15) discribe the process of emission of photon along initial electron momentum, second — emission along the positron. The accuracy of this approximation is insufficient in the case of narrow resonance production. The structure function method [6] modify (15), providing the accuracy $\sim 1\%$.

For definiteness consider the case when photon is emitted along the direction of the initial electron. The simple parton-like formulas may be obtained in the cases of large angle emission $\theta=\widehat{\vec p_-,\vec k}\sim 1$ and the emission in the forward direction $\frac{m_e}{\varepsilon}\ll\theta\ll 1$. In the first case the electron after the «point of large angle photon emission» go far off mass shell. In this state the collinear singularities does not appear, as well as the evolution up to «point of hard partonic process». So we have in this case:

$$\frac{d^3\sigma^{e\bar e\to\gamma\Phi}}{d\Omega\,dZ}=\int\limits_Z^1 dx\,D(x,\beta)\frac{dW}{d\Omega\,dZ}(x,Z)\int\limits_0^1 dy\,D(y,\beta)\,\hat\sigma_0^{e\bar e\to\Phi}(Sy(x-Z)),$$

$$\frac{dW}{d\Omega\, dZ}(x, Z) = \frac{\alpha}{4\pi^2} \frac{\left(1+\left(1-\frac{Z}{x}\right)^2\right)}{x(1-\cos\theta)}, \tag{17}$$

where εZ, $d\Omega$ — the photon energy in c.m.s. and it's emission solid angle. For the resonance production

$$\sigma_0^{e\bar{e}\to\Phi}(S) = \frac{12\pi B_{ee}(\Gamma/M^2)^2}{\left(\frac{S}{M^2}-1\right)^2+(\Gamma/M)^2}; \tag{18}$$

rearranging the order of integration in (17), one may obtain:

$$\frac{d^3\sigma^{e^+e^-\to\gamma\Phi}}{d\Omega\, dZ} = \frac{\sigma}{2\sin^2\left(\frac{\theta}{2}\right)} \cdot F(Z), \quad \sigma = \frac{3\alpha\Gamma_{ee}}{\pi M^3} = 0{,}3\ \text{pb}\ (Z^0),$$

$$F(Z) = \frac{\delta}{Z} \int\limits_0^{1-Z} dt\, B\left(\frac{t}{\mu}\right) F(t, Z), \quad B(\rho) = [(1-\rho)^2+\delta^2]^{-1},$$

$$\delta = \frac{\Gamma}{M}, \quad \mu = \frac{M^2}{S}, \tag{19}$$

M, Γ, Γ_{ee} are the mass of resonance, its total and electron-positron widths;

$$F(t, Z) = \int\limits_y^{1-Z} \frac{dy}{y}\left(1+\left(\frac{y}{y+Z}\right)^2\right) D(y+Z, \beta)\, D\left(\frac{t}{y}, \beta\right) \simeq$$

$$\simeq \frac{\beta}{2}\left(1+\frac{3}{4}\beta\right)(1-Z-t)^{\beta-1}\left[t^{-\beta/2}\left(1+\left(\frac{t}{t+Z}\right)^2\right)+\right.$$

$$\left. + (1-Z)^{-\beta/2}(1+(1-Z)^2)\right] -$$

$$-\frac{\beta}{4}\left[(1-Z+t)\left(1+\frac{1}{(1-Z)^2}\right)+(1+Z+t)\left(1+\left(\frac{t}{t+Z}\right)^2\right)\right]. \tag{20}$$

For the case of emission in forward direction it's necessary also to take into account the evolution «between the points of hard photon emission and point of resonance production». The cross section has the form

$$\frac{d\sigma^{e^+e^-\to\gamma\Phi}}{d\Omega\, dZ} = \frac{2\sigma}{\theta^2}\Phi(Z),$$

$$\Phi(Z) = \frac{\delta}{Z}\int\limits_Z^1 dx\, D(x, \beta)\left(1+\left(1-\frac{Z}{x}\right)^2\right)\int\limits_0^1 d\lambda\, D(\lambda, \beta)\int\limits_0^1 dt\, D(t, \beta)\, \times$$

$$\times B\left(\frac{t\lambda(x-Z)}{\mu}\right) = \frac{\delta}{Z}\int\limits_0^{1-Z} dt\, B\left(\frac{t}{\mu}\right)\Phi(t, Z), \tag{21}$$

$$\Phi(t, Z) = \frac{\beta}{2}\left(1+\frac{9}{8}\beta\right)(1-Z-t)^{\frac{3\beta}{2}-1} \times$$

$$\times\left[t^{-\beta}\left(1+\frac{t^2}{(t+Z)^2}\right)+2(1-Z)^{-\beta}(1+(1-Z)^2)\right] -$$

$$-\frac{\beta}{2}\left[(1-Z+t)\left(1+\frac{1}{(1-Z)^2}\right)+\frac{1}{2}(1+Z+t)\left(1+\frac{t^2}{(t+Z)^2}\right)\right]. \tag{22}$$

The function's $F(Z)$, $\Phi(Z)$ are drawn in Fig. 1. for the parameters $\delta = 2.74 \cdot 10^{-2}$, $\mu = 0.208$, $\beta = 0.129$, corresponding to $e^+e^- \to Z^0\gamma$ process at $\sqrt{S} = 200$ GeV. This functions have an characteristic reversed radiative tail with maximum value at $Z = Z_0 = 1-\mu$ and sharp decrease at $Z > Z_0$. We believe that inclusive photon spectra may be useful to detect such a resonance as toponium.

For the resolution $\Delta\omega \ll \omega$ of the photon's energy $\omega = \varepsilon Z$ small enough to allow only the emission of virtual and soft additional photons with the total energy don't exceed $\Delta\omega$, one obtains

$$\frac{d\sigma}{d\Omega\, dZ} = \frac{d\sigma_0}{d\Omega\, dZ} R;$$

$$R = \int\limits_{1-\frac{\Delta\omega}{\varepsilon}}^1 dx/x^2 \int\limits_{\left(1-\frac{\Delta\omega}{\varepsilon}\right)/x}^1 dt/t\left(1-\frac{Z}{x}\right)^{-1} D(x)\, D(t)\, D\left(\frac{\mu}{tx\left(1-\frac{Z}{x}\right)}\right) P_e^\gamma\left(\frac{Z}{x}\right), \tag{23}$$

where $d\sigma_0/d\Omega dZ$ — the cross section of (13) in Born approximation.

Lowest orders expansion on β of R is

$$R = 1 - \left(\frac{\alpha L}{\pi}\right)\ln\frac{S}{4\Delta\omega^2} + \frac{1}{2}\left(\frac{\alpha L}{\pi}\right)^2 \ln^2\left(\frac{S}{4\Delta\omega^2}\right), \tag{24}$$

agreess with the result obtained in [5].

3. To estimate roughly number of events one may use expressi-

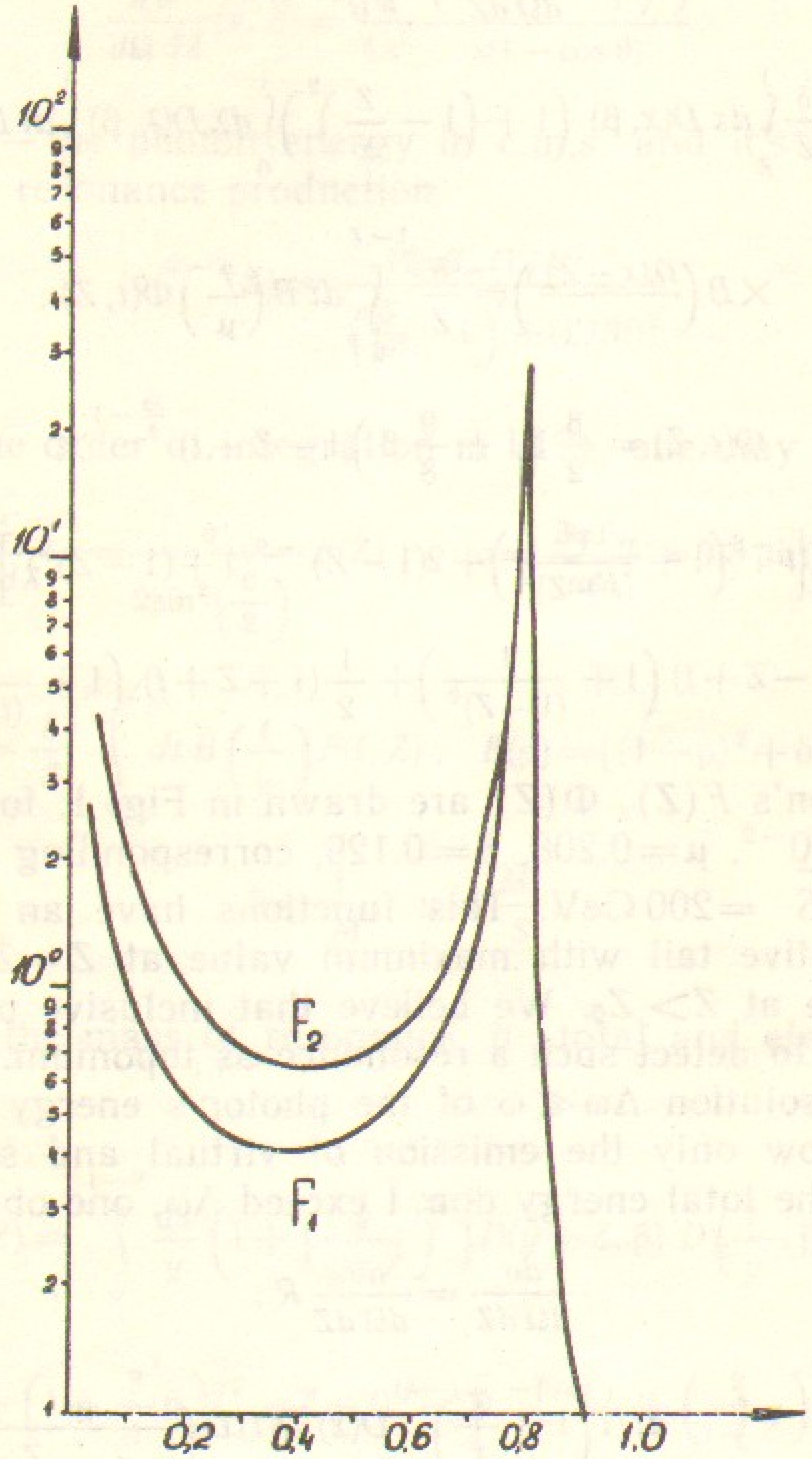


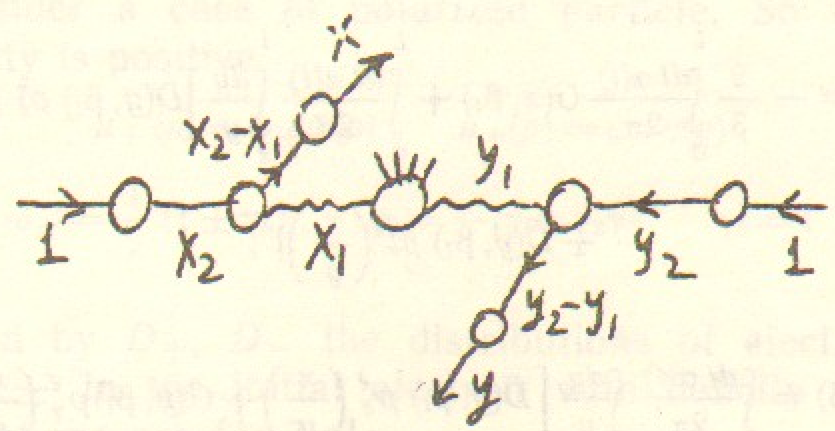
Fig. 1. Functions $F=F_1$ and $\Phi=F_2$ (19), (21) for $\delta=2{,}74\cdot10^{-2}$, $\mu=0{,}208$; $\beta=0{,}129$.

ons of a type (2) (we use the subprocesses cross sections from [9], see Appendix B). We'll take one's bearings to the year integrated luminosity $\int Ldt=10^{40}\,\text{cm}^{-2}=10^4\text{pb}^{-1}$. One obtains $N_{W^-}\approx5\cdot10^4$ for setting up an experiment when photon and scattered positron are detected using (4b) and $\hat\sigma^{\gamma e^-\to W^-\gamma}(S)\approx20$ pb at $\sqrt{S}=200$ GeV.

For the production of Z^0 in the subprocess $\gamma e\to Z^0\gamma$, one obtains $NZ_0\approx10^5$. Using the phenomenological expression for cross section of hadron production in $\gamma\gamma$-collisions $\sigma^{\gamma\gamma\to h}(S)=(255+300/S^{1/2}$ (GW)) nb, one obtains $N_h\sim10^7$. The number of W^+W^--pairs is of order $2.5\cdot10^4$. The inclusive photon cross section $e^+e^-\to Z^0\gamma$ for $\theta\sim\pi/4$ emitting angle at peak is of order 80 pb and practically don't depend on energy in region $\sqrt{S}\geqslant200\ GeV$.

So the numbers of events per one year are comparatively large [10] and the taking into account of r.c. is essential.

Due to bremsstrahlung process the energies of leptons, produced in hard subprocess are exceed ones measured by detectors. It's necessary to take into account the «evolution» of charged leptons lines «from point of hard process to the detection point». In the case when two charged particles are detected the distribution on $\tilde S$, the «invariant mass square of produced system» $\tilde S=Sxy$, εx, εy — the measured lepton's energies (which in fact do not coincide with the real one $S_1=Sx_1y_1$), will be modified according to the following parton' s scenario [7,b]



$$F(S_1,S)\equiv\frac{\partial\sigma(S_1,S)}{\partial S_1}\Rightarrow\bar F(S,x,y)\,,$$

$$\bar F(S,x,y)=\int dx_2\,D(x_2,\beta_q)\,G\left(\frac{x_1}{x_2},\beta_q\right)D\left(\frac{x}{x_2-x_1},\beta_q\right)\frac{dx_1}{x_2(x_2-x_1)}F(Sx_1y_1)\times$$

$$\times\int dy_2\,D(y_2,\beta_q)\,G\left(\frac{y_1}{y_2},\beta_q\right)D\left(\frac{y}{y_2-y_1},\beta_q\right)\frac{dy_1}{y_2(y_2-y_1)}, \qquad (25)$$

D—functions describe the bremsstrahlung process by initial and final charged particles.

Note that instead of using the complicated expressions of type (25) one may apply the Monte-Carlo simulations program to describe the density of the probability of the emission of photon with the energy fraction x in the process of electron scattering with the momentum transferred q:

$$\frac{dp_q(x)}{dx}=\frac{\beta_q}{2}\left(\left(1+\frac{3}{8}\beta_q\right)(1-x)^{\frac{\beta_q}{2}-1}-\frac{1}{2}(1+x)\right),\quad \beta_q=\frac{2\alpha}{\pi}\left(\ln\left|\frac{q^2}{m_e^2}\right|-1\right),$$

$$q^2=(p_1-p_1')^2,\quad m_e^2\ll|q^2|\ll S. \qquad (26)$$

We are grateful to V.S. Panin, Z.K. Silagadze and V.S. Fadin for discussions and critical comments.

Appendix A

The Altarelli—Parisi—Lipatov equations for the distributions $D\equiv D_e^e$, $G\equiv D_e^\gamma$, $\bar{D}\equiv D_e^{\bar{e}}$ which presented densities of numbers correspondently electrons, photons and positrons and in for unpolarized electron are:

$$D(x,\beta)=\delta(1-x)+\int_0^L\frac{dt\,\alpha(t)}{2\pi}\int_x^1\frac{dy}{g}\left[D(y,\beta_t)\,p_e^e\left(\frac{x}{y}\right)+G(y,\beta_t)\,p_\gamma^e\left(\frac{x}{y}\right)\right];$$

$$G(x,\beta)=-\frac{2}{3}\int_0^L\frac{dt\,\alpha(t)}{2\pi}G(x,\beta_t)+\int_0^L\frac{dt\,\alpha(t)}{2\pi}\int_x^1\frac{dy}{y}\left[D(y,\beta_t)\,p_e^\gamma\left(\frac{x}{y}\right)+\right.$$

$$\left.+\bar{D}(y,\beta_t)\,p_{\bar{e}}^\gamma\left(\frac{x}{y}\right)\right];$$

$$\bar{D}(x,\beta)=\int_0^L\frac{dt\,\alpha(t)}{2\pi}\int_x^1\frac{dy}{y}\left[\bar{D}(y,\beta_t)\,p_{\bar{e}}^{\bar{e}}\left(\frac{x}{y}\right)+G(y,\beta_t)\,p_\gamma^{\bar{e}}\left(\frac{x}{y}\right)\right], \qquad (A1)$$

where

$$p_e^e(Z)=p_{\bar{e}}^{\bar{e}}(Z)=\frac{1+Z^2}{1-Z}-\delta(1-Z)\int_0^1\frac{dx\,(1+x^2)}{1-x};\quad \beta=\frac{2\alpha}{\pi}(L-1);\quad L=\ln\frac{S}{m_e^2},$$

$$p_\gamma^e(Z)=p_\gamma^{\bar{e}}(Z)=Z^2+(1-Z)^2;\quad p_e^\gamma(Z)=P_{\bar{e}}^\gamma(Z)=\frac{1}{Z}(1+(1-Z)^2), \qquad (A2)$$

$$\beta_t=\frac{2\alpha}{\pi}(t-1)$$

$\alpha(t)=\alpha\cdot\left(1-\frac{\alpha t}{3\pi}\right)^{-1}$—the «running» coupling constant, $\alpha=\frac{1}{137}$.

For the real case when $\sqrt{S}\approx 200$ GeV the value $\beta\approx 0.129$ is small, $\beta\ll 1$. We'll take one's bearings to the accuracy of the order of $\sim 1\%$ estimated the cross sections. Therefore one finds enough the solution of the eq. (A1) using an iteration method and the known solution for the nonsinglet contribution in the D (the singlet contribution is of the order $\sim\beta^2$):

$$D(x,\beta)=\frac{\beta}{2}\left(\left(1+\frac{3}{8}\beta\right)(1-x)^{\frac{\beta}{2}-1}-\frac{1}{2}(1+x)\right), \qquad (A3)$$

Substitute (A3) in (A1) and obtain

$$\bar{D}(x,\beta)=\frac{1}{32}\beta^2\left[-4x(1+x)\ln\frac{1}{x}+\frac{1}{3}(1-x)(4+7x+4x^2)\right];$$

$$G(x,\beta)=\frac{1}{4x}\beta(1+(1-x)^2)+\frac{1}{64}\beta^2\left[(3+4\ln(1-x))\frac{(1+(1-x)^2)}{x}+\right.$$

$$\left.+2(2-x)\ln x+\frac{2}{x}(1-x)(2x-3)\right]. \qquad (A4)$$

Let's consider a case of polarized particle. So let the initial electron helicity is positive.

$$u_+(p)=\omega_+\bar{u}(p),\quad u_+(p)=\omega_-v(p),$$

$$\omega_\pm=\frac{1}{2}(1\pm\gamma_5),\quad \gamma_5=-i\gamma_0\gamma_1\gamma_2\gamma_3,\quad \gamma_5^2=1. \qquad (A5)$$

If it's denoted by D_+, D_- the distributions of electrons, photons with helicity $\pm$ in the initial electron with helicity $+$ then their evolution equations are [1; 7,a]:

$$D_+(x,\beta)=\delta(1-x)+\int_0^L\frac{dt\,\alpha(t)}{2\pi}\int_x^1\frac{dy}{y}\left[D_+(y,\beta_t)\,P_{e+}^{e+}\left(\frac{x}{y}\right)+\right.$$

$$\left.+G_+(y,\beta_t)\,p_{\gamma+}^{e+}\left(\frac{x}{y}\right)+G_-(y,\beta_t)\,p_{\gamma-}^{e+}\left(\frac{x}{y}\right)\right],$$

$$D_-(x,\beta)=\int_0^L \frac{dt\,\alpha(t)}{2\pi}\int_x^L \frac{dy}{y}\Big[D_-(y,\beta_t)\,p_{e^-}^{e^-}\left(\frac{x}{y}\right)+$$

$$+G_+(y,\beta_t)\,p_{\gamma+}^{e^-}\left(\frac{x}{y}\right)+G_-(y,\beta_t)\,p_{\gamma-}^{e^-}\left(\frac{x}{y}\right)\Big],$$

$$G_+(x,\beta)=-\frac{2}{3}\int_0^L \frac{dt\,\alpha(t)}{2\pi}G_+(x,\beta_t)+$$

$$+\int_0^L \frac{dt\,\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}\Big[D_+(y,\beta_t)\,p_{e+}^{\gamma+}\left(\frac{x}{y}\right)+D_-(y,\beta_t)\,P_{e^-}^{\gamma+}\left(\frac{x}{y}\right)\Big], \quad \text{(A6)}$$

$$G_-(x,\beta)=-\frac{2}{3}\int_L^0 \frac{dt\,\alpha(t)}{2\pi}G_-(x,\beta_t)+$$

$$+\int_0^L \frac{dt\,\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}\Big[D_+(y,\beta_t)\,p_{e+}^{\gamma-}\left(\frac{x}{y}\right)+D_-(y,\beta_t)\,P_{e^-}^{\gamma-}\left(\frac{x}{y}\right)\Big],$$

$$D_+ + D_- = D\,,\quad G_+ + G_- = G\,,\quad p_{e+}^{e+}=p_{e-}^{e-}=p_e^e;$$

$$p_{e+}^{\gamma+}(Z)=\frac{1}{Z};\quad p_{e+}^{\gamma-}(Z)=\frac{(1-Z)^2}{Z};\quad p_{\gamma+}^{e+}(Z)=Z^2;\quad p_{\gamma+}^{e-}(Z)=(1-Z)^2,$$

where the distributions $\bar{D}_\pm$ are neglected because of ones are of the order of β^2.

Approximate solutions of (A6) are

$$D_+(x,\beta)=D(x,\beta)\,,$$

$$G_+(x,\beta)=\frac{\beta}{4x}+\frac{\beta^2}{64x}(3+4\ln(1-x)-(1-x)(3+x))\,,$$

$$G_-(x,\beta)=\frac{\beta}{4x}(1-x)^2+\frac{\beta^2}{32x}\Big[(x^2-2x)\ln\frac{1}{x}+2(1-x)^2\ln(1-x)+x(1-x)\Big],$$

$$D_-(x,\beta)=\frac{\beta^2}{48x}(1-x)^3. \quad \text{(A7)}$$

Appendix B

To estimate the annual numbers of produced particles $N^i=\sigma^i\int Ldt$ cited above we use $\int Ldt=10^{33}\cdot 10^7=10^{40}\text{cm}^2=10^4\text{pb}^{-1}$ and simplified version of equations similar to (2). We also use the cross sections of some subprocesses from [9] (see referencies in this article), $\sin^2\theta_W=0.25$.

$$\sigma^{e^+e^-\to(\gamma\gamma)W^+W^-}(S)=\int_1^\rho dx\,\varphi_{e\bar{e}}\left(\frac{x}{\rho},\beta\right)\cdot\hat{\sigma}^{e\bar{e}\to WW}(x)\,, \quad \text{(B1)}$$

$$\hat{\sigma}^{e\bar{e}\to WW}(x)=\sigma_0\frac{v}{2x}\Big[\left(1+\frac{1}{2x}+\frac{1}{8x^2}\right)l-\frac{5}{4}+\frac{1}{2}(3x-1)^{-1}\Big(\frac{8x+1}{8x^2}l-\frac{1}{3}x-$$

$$-\frac{5}{3}-\frac{1}{4x}\Big)+\frac{v^2}{24}(3x-1)^{-2}(4x^2-20x+3)\Big]\,,\quad \sigma_0=51\text{ pb}\,,$$

$$x=\frac{S_1}{4M_W^2};\quad \rho=\frac{S}{4M_W^2};\quad v=\sqrt{1-\frac{1}{x}};\quad l=\frac{1}{v}\ln\frac{1+v}{1-v}. \quad \text{(B2)}$$

For W^+W^- production in $\gamma\gamma\to W^+W^-$ subprocess, we obtain

$$\sigma^{e^+e^-\to(e^+e^-)W^+W^-}(S)=\beta^2\int_1^\rho\frac{dx}{x}\varphi_{\gamma\gamma}\left(\frac{x}{\rho}\right)\hat{\sigma}^{\gamma\gamma\to W^+W^-}(x),\quad \rho=\frac{S}{4M_W^2}, \quad \text{(B3)}$$

$$\hat{\sigma}^{\gamma\gamma\to W^+W^-}(x)=\tilde{\sigma}_0\cdot v\left[1+\frac{3}{16x}+\frac{3}{16x^2}-\frac{3}{16}\left(1-\frac{1}{2x}\right)l\right],\quad \tilde{\sigma}_0=86\text{ pb}\,. \quad \text{(B4)}$$

For W^- production in subprocess $e^-\gamma\to W^-\nu$, we obtain

$$\sigma^{e^+e^-\to(e^+\gamma)W^-\nu}(S)=\frac{1}{2}\beta\int_1^{\rho_1}\frac{dx}{x}\eta\left(\frac{x}{\rho_1}\right)\hat{\sigma}^{\gamma e\to W^-\nu}(x);\qquad \eta(Z)=1+(1-Z)^2,$$

$$\rho_1=\frac{S}{M_W^2},\qquad \sigma_0=47\text{ pb}\,. \quad \text{(B5)}$$

$$\hat{\sigma}^{\gamma e\to W^-\nu}(x)=\sigma_0\Big[\left(1-\frac{1}{x}\right)\left(1+\frac{5}{4x}+\frac{7}{4x^2}\right)-\frac{1}{x}\left(2+\frac{1}{x}+\frac{1}{x^2}\right)\ln x\Big]. \quad \text{(B6)}$$

$$\sigma_0=47\text{pb}$$

For Z_0 production in subprocess $\gamma e\to Z^0 e$ we obtain

$$\sigma^{e+e\to(e+\gamma)Z^0e}(S)=\frac{\beta}{2}\int_1^{\rho_1}\frac{dx}{x}\eta\left(\frac{x}{\rho_1}\right)\hat{\sigma}^{\gamma e\to Z^0e}(x), \quad \rho_1=S/m^2 \qquad (B.7)$$

$$\hat{\sigma}^{\gamma e\to Z^0e}(x)=\sigma_0\frac{1}{x}\left[\left(1-\frac{2}{x}+\frac{2}{x^2}\right)l+\frac{1}{2}\left(1-\frac{1}{x}\right)\left(1+\frac{7}{x}\right)\right]; \qquad (B8)$$

$$l=24+\ln\frac{(x-1)^2}{x}; \qquad \sigma_0\approx 6\text{pb}.$$

REFERENCES

1. *L.N. Lipatov.* Yad. Fif. 1974, v.20, p.181;
   G. Altarelly, G. Parisi. Nucl. Phys., B126 (1977) p.298.
2. *S.D. Drell, T.M. Yan.* Phys. Rev. Lett., 1970, v.25, p.316.
3. *S. Brodsky et. al.* Phys. Lett., 25 (1970) p.972.
4. *V.M. Budnev et. al.* Phys. Rep. C., 1975, v.15, p.183.
5. *F.A. Berends et. al.* Phys. Lett., B177, (1986) p.191.
6. *E.A. Kuraev, V.S. Fadin.* Yad. Fiz., 1985, v.41, p.753.
7. QED Structure Functions, Ann Arbor, MI, 1989, Conference Preceedings N 201 Particles and Fields, ser.39:
   a) V.S. Fadin's lesson;
   b) L. Trendature and O. Nicrosini's lesson.
8. *V.N. Baier et. al.* Nucl. Phys. B65 (1973) p.381.
   R. Dalitz, D.R. Yennia. Phys. Rev., 105 (1957) 1598;
   R. Curtis. Phys. Rev., 104 (1956) 211.
9. *I.F. Ginzburg and V.G. Serbo.* Nucl. Phys., 1983, B228, p.285.
10. *V. Barger et. al.* Phys. Rev., N 1, D39 (1990) 146.

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