ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

V.N. Baier, V.M. Katkov, V.M. Strakhovenko

SIMPLE MODEL OF KINETIC PHENOMENA FOR DESCRIPTION OF RADIATION IN SINGLE CRYSTALS

ПРЕПРИНТ 90-87

НОВОСИБИРСК

SIMPLE MODEL OF KINETIC PHENOMENA FOR DESCRIPTION OF RADIATION IN SINGLE CRYSTALS

V.N.Baier, V.M.Katkov, and V.M.Strakhovenko

ABSTRACT

A model of kinetic phenomena in single crystals considering both radiative effects and multiple scattering of the electrons is proposed. The model allows one to describe the radiation of photons by high-energy electrons starting with crystal thicknesses, where a hard peak appears in energy loss spectra of aligned electrons.

The radiation of photons by high-energy electrons in aligned single crystals attracts a continual attention. This is owing to the fact, that at energies of the projectiles available now the uniqe opportunity to investigate the QED in intense external field is permitted just by crystals. Aligned single crystals are besides the most effective radiators, since corresponding radiation lengths become much shorter than in an amorphous case. An additional interest in the process was stimulated by the observation of a hard peak in electron energy loss distribution [1]. An origin of this peak is due to multiple photon emission by an electron traversing a crystal of sufficient thickness and to actual addition of photon energies by the detector. This explanation was given in Ref. 2 and then confirmed by re-analysis of the experimental data [3]. It should be emphasized that although soft photons are mainly emitted, the radiation under conditions of Ref. 1 represent a rare example of the QED process where the quantum effects induced by an external field are of the order of unity. Remind that the magnitude of the quantum effects is determined by the parameter $\chi_s = \gamma E(a_s)/E_0$ ($E(a_s)$ is the electric field of the axis at screening radius a_s, $E_0 = m^2/e = 1.32 \cdot 10^{16}$ V/cm, $\gamma = \varepsilon/m$, $\varepsilon(m)$ is the energy (mass) of an electron) which also is of the order of unity: $\chi_s \simeq 0.8$ under conditions mentioned. The radiation of a photon by an electron side by side with the electron-positron pair creation by a photon is a component of the specific cascade [4] which is generated over length one or two order of magnitude shorter than in an appropriate amorphous medium. A detailed investigation of these cascades started in Ref. [5]

(theory) and Ref. 6 (experiment) is a necessary step towards a high energy electron and photon detector [7] based on single crystals.

The radiation characteristics contain indirect information about kinetics of electrons traversing a crystal. This kinetics is mainly determined by the multiple scattering and radiation processes. The magnitude of multiple scattering and radiation energy losses can be characterized by energy dependent dechanneling length $\ell_d(\varepsilon)$ and radiation length $L_{rad}(\varepsilon)$. At some energy ε_{cr} [8] these two lengths become equal (e.g. for Ge, $\varepsilon_{cr} \simeq$ 40-60 GeV depending on axis and temperature). At $\varepsilon < \varepsilon_{cr}$ the multiple scattering dominates in the kinetics, but at $\varepsilon > \varepsilon_{cr}$ a part of the radiation becomes more and more important. In turn, a correct description of the radiation is possible only with an adequate allowance for the kinetics. This is especially important for crystals of intermediate thickness, where the radiation from particles initially captured into a channel is very essential [2]. According to Ref. 2 there is a group of electrons, which while traversing a crystal are staying all the time close to the axis. In this region of distances $\underline{\varrho}$ from the axis both multiple scattering and the radiation (including bremsstrahlung) are enhanced by a factor about 10 as compared to above-barrier values. Multiple scattering results on the average in increase of transverse energy $\varepsilon_\perp$ in contrast with the radiation action decreasing on the average $\varepsilon_\perp$. These processes together cause a strong intermixing of electrons belonging to this group (first one in terms of Ref. 2) over $\varepsilon_\perp$, leading to the uniform distribution over transverse coordinate $\underline{\varrho}$ for

them. Therefore the approach of the type used in Ref. 9 , where this kinetics was completely ignored can not be adequate*). Kinetic equations with allowance for radiation and scattering may be solved by using numerical simulation [10], [11]. The most advanced calculation of this type was carried out in Ref. 12 under conditions of Ref. 1 .

In the present paper a simple model, based on the approach developed in Ref. 2 is stated, which permits one in particular to predict the energy loss spectra of electrons traversing a crystal of intermediate (and larger) thickness. For the sake of succession, we use an axially symmetric approximation $\mathcal{U}(x)$ of a crystal potential in the region per one axis with an area $S_0 = \pi r_0^2 = (nd)^{-1}$, where n is a density of atoms in a crystal, d is a mean distance between them along the axis. A generalization onto arbitrary potential $\mathcal{U}(\underline{\varrho})$ is straightforward. For electrons

$$\mathcal{U}(x) = V_0 \ln\left[(1 + 1/\eta)/(1 + 1/(x+\eta))\right], \quad (1)$$

where $x = \underline{\varrho}^2/a_s^2$; V_0, a_s, η are fitting parameters (see e.g. Table I in Ref. 14).

For given $\varepsilon_\perp$ the scattering magnitude may be characterized by the parameter $\ell_d(\varepsilon_\perp)$:

$$\ell_d(\varepsilon_\perp) = \ell_d(\varepsilon)\left[\vartheta(\varepsilon_\perp - \mathcal{U}_0) + \vartheta(\mathcal{U}_0 - \varepsilon_\perp)(x+\eta)/(x_0+\eta)\right] \quad (2)$$

Here $\ell_d(\varepsilon) = (\alpha/2\pi)\mathcal{U}_0 \varepsilon L_{rad}/m^2$ is the conventional dechanneling length, $\mathcal{U}_0 \equiv \mathcal{U}(x_0)$ is the potential well depth, $x_0 = r_0^2/a_s^2$ is the boundary value of x , L_{rad} is

*) Very recently the conventional point of view was accepted by these authors (see e.g. [13]).

the radiation length in a corresponding amorphous medium. The dependence of $\varkappa$ in eq.(2) on $\varepsilon_\perp$ is defined by the equation $\mathcal{U}(x) = \varepsilon_\perp$. Relation (2) denotes that for channeld ($\varepsilon_\perp < \mathcal{U}_0$) particles the length $\ell_d(\varepsilon_\perp)$ decreases practically ($\eta \ll 1$) in proportion to the area $S(\varepsilon_\perp)$ accessible for motion: $\ell_d \propto S(\varepsilon_\perp)/S_0$. The similar behaviour shows the radiation length in the continious potential $L_{ch}(\varepsilon_\perp) = \varepsilon / I(\varepsilon, \varepsilon_\perp)$, where $I(\varepsilon, \varepsilon_\perp)$ is the radiation intensity from the electron with total energy ε and transverse energy $\varepsilon_\perp$:

$$L_{ch}(\varepsilon_\perp) = L_{ch}(\varepsilon)\left[\vartheta(\varepsilon_\perp - \mathcal{U}_0) + \vartheta(\mathcal{U}_0 - \varepsilon_\perp)\frac{2x+1}{2x_0+1}\right] \quad (3)$$

here $L_{ch}(\varepsilon)$ is the radiation length for above-barrier ($\varepsilon_\perp > \mathcal{U}_0$) particles.

Let us remind main points of the approach of Ref. 2 . The particles having initial values of $\varepsilon_\perp$ smaller than some boundary value $\varepsilon_\perp^b$ belong to the first group (I), all the others, including above-barrier ones belong to the second group (II). The radiation from electrons of the second group is described by the simulation of a cascade under above-barrier motion conditions, when within the constant field approximation the probability (and the intensity, see eq.(3)) of photon emission is independent of $\varepsilon_\perp$. In view of a strong intermixing in the transverse phase space of particles belonging to the first group, a photon emission from them is described by the simulation of a cascade at some intermediate value of $\varepsilon_\perp = \varepsilon_\perp^{ef} < \varepsilon_\perp^b$. The simulation is carried out with the probability corresponding to this value $\varepsilon_\perp = \varepsilon_\perp^{ef}$ for all particles of the first group. Then radiation characte-

ristics (energy loss spectra, photon multiplicity, etc) are presented as a sum of the contributions from two groups multiplied by the fraction of particles in every one of them:

$$F = a F_I + (1-a) F_{II} \quad (4)$$

The constant field approximation is valid for incident angles ϑ_0 smaller than $\vartheta_V = V_0/m$. Then orientation dependence of the yield F in this interval of ϑ_0 is entirely due to that of the fraction a of electrons belonding to the first group, since the parameters $\varepsilon_\perp^b$ and $\varepsilon_\perp^{ef}$ depend only on initial electron energy ε_0 , the type and the thickness of the crystal.

Generally speaking the parameters $\varepsilon_\perp^b$ and $\varepsilon_\perp^{ef}$ were used in Ref. 2 as fitting ones. Now we can propose analytical expressions for them basing on carefully investigated time dependence of ε and $\varepsilon_\perp$ mean values described by Eq.(2) in Ref. 2 , the allowance for arising widths of distributions, and numerous simulations developed. Let us formulate the prescription.

1) The parameter $\varepsilon_\perp^{ef}$ is given by the solution of the following equation:

$$\exp\left[L/L_{ch}(\varepsilon_\perp^{ef})\right] = \ell_d(\varepsilon_\perp^{ef})/L \,, \quad (5)$$

where L is the crystal thickness, the functions $\ell_d(\varepsilon_\perp)$, $L_{ch}(\varepsilon_\perp)$ are defined by Eqs. (2), (3).

2) The equation on the parameter $\varepsilon_\perp^b$ reads

$$\frac{L_{ch}(\varepsilon_\perp^b)}{L}\left\{\exp\left[L/L_{ch}(\varepsilon_\perp^b)\right] - 1\right\} = 1 + \frac{\ell_d(\varepsilon_\perp^b)}{2L_{ch}(\varepsilon_\perp^b)}\left(\frac{V_0}{\mathcal{U}_0}\right)^2 . \quad (6)$$

It is important, that only above-barrier values of the radiation $L_{ch}(\varepsilon_o)$ and dechanneling $\ell_d(\varepsilon_o)$ lengths at initial electron energy ε_o are sufficient for calculation of the parameters $\varepsilon_\perp^{ef}$ and $\varepsilon_\perp^{b}$. The meaning of eq.(5) is in selfconsistent determination of the $\varepsilon_\perp$ domain boundary, below of which the transverse phase space distribution gets uniform for given crystal thickness. A detailed analysis of this problem was performed in Ref. 8 without allowance for the radiation action. Let us suppose, that some initial electron state is narrow over $\varepsilon_\perp$. While particles are traversing a crystal the width of the state is increasing because of both scattering and radiation effects. Relation (6) determines a boundary of the $\varepsilon_\perp$ domain, from which particles are captured into the first group, just because of the mentioned spread of states.

For fixed ε_o a peak like that observed in Ref. 1 appears in the electron energy loss distribution for crystall thickness L larger than L_o defined by the equation $\varkappa_{ef}(L_o)=1$ ($U(\varkappa_{ef})=\varepsilon_\perp^{ef}$, see eq.(5)). Note, that the thikness L_o under conditions of Ref. 1 estimated in Ref. 2 by using similar arguments was then confirmed experimentally [15]. For $L < L_o$ a separation of all particles into two groups becomes insufficient (see a footnote in Ref. 2). Indeed, for very small thicknesses $L \ll L_o$ no intermixing occurs and one has to use continuons initial distribution over $\varepsilon_\perp$ corresponding in our terms to the infinite number of groups. Thus, the proposed model is valid for $L > L_o$. Beginning with the thickness L_1, defined by the equation $\varkappa_{ef}(L_1) = \varkappa_o$, all particles belong to the second group.

Let us compare energy loss spectra calculated by means of this model to experimental data [16] on radiaton from 150 GeV electrons traversing aligned Ge (axis $\langle 110 \rangle$, L = 0.205 mm) and Si ($\langle 110 \rangle$, L = 0.5 mm) crystals. The approximate potential (1) was used. The results are shown in Figs.1-2. The values of the parameters $\varkappa_{ef}$, $\varkappa_b$ were obtained from Eqs.(5), (6) being $\varkappa_{ef}$ = 2.39 and $\varkappa_b$ = 5.08 for Ge, $\varkappa_{ef}$ = 2.75 and $\varkappa_b$ = 4.53 for Si. The fractions of particles captured into the first group (the quantuty a in eq. (4) are found to be respectively 0.29, 0.193, 0.124, 0.082 for incident angles 0-9, 9-15, 15-21, 21-25 μrad in Ge and 0.246, 0.156, 0.107 for incident angles 0-9, 9-13, 13-17 μrad in Si. The peak disappears at some incident angle of narrow (over ϑ_o) beam. This occurs for a≠0, at ϑ_o connected to relative magnitude of derivatives of two groups contributions with respect to y ($y = \Delta\varepsilon/\varepsilon_o$, $\Delta\varepsilon$ is the energy loss) near the peak position. The values of parameters $\varkappa_{ef}$ obtained for Ge and Si correspond to transverse energy $\varepsilon_\perp^{ef}$ = 0.88 U_o in contrast with a statement of Ref. 16 , that only low-lying states, give rise to the peak.

The influence of the potential choice on theoretical description of QED - processes in crystals is of evident interest. We have analyzed this problem for cold (T = 100 K) Ge $\langle 110 \rangle$, i.e. under conditions of Refs. 1 , 15 . In Fig. 3 is shown the energy dependence of inverse radiation length $L_{rad}^{-1} = I(\varepsilon)/\varepsilon$ calculated for various potentials of separate atom: (a) Doyle-Turner, (b) Moliere, and (c) Moliere with additional axially symmetric approximation in the form of (1). Let us stress that in all three cases the field not of the separate axis, but of the whole crystal is taken into account.

Near the axis the potential is axially symmetric with a good accuracy. Therefore at moderate energies, when just this region of most strong fields $E(\underline{\varrho})$ contributes mainly into radiation, the description of processes by using the potential (1) is rather accurate. This applies also to the radiation from particles of the first group staying all the time close to the axis. The local value of the quantum parameter $\chi(\underline{\varrho})$ increases with energy, extending the region of contributed distances $\underline{\varrho}$. Althongh the mentioned symmetry of the potential holds for $\langle 111 \rangle$ axis practically in the whole area per one axis, it is not the case for $\langle 110 \rangle$ axis. As a result the difference between curves (b) and (c) in Fig.3 characterizing the accuracy of axially symmetric approximation (1) increases with energy. Let us note also the systematic difference between curves (a) and (b) in Fig.3, owing to use of different atomic potentials. Our analysis shows, that calculations using Doyle-Turner potential agree with all experimental data obtained in cold Ge $\langle 110 \rangle$ better than for other approximations. So, the QED processes may test the potential at different $\underline{\varrho}(x)$ depending on particle energy. In particular, the creation of e^+e^- pair by a photon in near-threshold region is very sensitive to the potential for $x \lesssim 1$.

In Fig.4 are shown the calculated energy loss spectrum in Ge (T = 100 K, $\langle 110 \rangle$ axis) crystal for L = 0.185 mm and experimental data [15]. The Doyle-Turner potential without additional approximation was used. By using the potential (1) the region of $y < 0.5$ rises about 20%. The main contribution to this region is given by particles of the second group. The scale of the alteration approximately corresponds to the

difference between curves (a) and (c) in Fig.3. The dependence of the type presented in Fig.3 for Si and Ge at room temperature will be investigated elsewhere. However, it is clear from above consideration, that the use of the Doyle-Turner potential may result mainly in some lowering of theoretical curves in Figs.1-2 in the region of $y < 0.5$.

The agreement between theoretical and experimental data may be always improved by letting ε_1^{ef} and ε_1^{b} be fitting parameters. The loss of prediction ability of the model is a price to pay for. In this way one can obtain, in particular quite satisfactory description of the data [15] for Ge crystal with the thickness L = 0.07 mm, which is somewhat smaller than the boundary value L_0 of the model.

In the Table are listed for some crystals thicknesses L_0 and L_1 determining an interval, where the kinetics of initially channeld particles is important, causing an appearance of the peak in energy loss spectra. We expect the most pronounced peak at crystal thickness $L \approx 2L_0$. The phenomenon exists for $\varepsilon > \varepsilon_{cr}$, which values are tabulated as well. In addition the energy should be sufficiently high providing essentially quantum character of a radiation process determined by the parameter χ_s . In tungsten the peak may be observed at relatively low energy $\varepsilon \simeq \varepsilon_{cr} = 30$ GeV, where all necessary conditions are fulfilled.

In conclusion it can be stated that the presented model gives not only qualitative, but also quite satisfactory quantitative description of the phenomena. The model permits one to find regions of thicknesses and energies where the peak appears and to predict the shape of spectra without any fitting parameters. It allows one to estimate a physical situation, in particular while planning new experiments.

References

1. A.Belkacem, G.Bologna, M.Chevallier et al., Phys. Lett. B 177 (1986) 211.
2. V.N.Baier, V.M.Katkov and V.M.Strakhovenko, Phys. Lett. A 132 (1988) 211.
3. A.Belkacem, G.Bologna, M.Chevallier et al., Phys. Lett. B 206 (1988) 561.
4. V.N.Baier, V.M.Katkov and V.M.Strakhovenko. Sov. Phys. Dokl. 30 (1985) 474.
5. V.N.Baier V.M.Katkov and V.M.Strakhovenko. Nucl. Instrum. Methods B 27 (1987) 360.
6. J.Bak, D.Barbenis, T.Brodbeck et al., Phys. Lett. B 213 (1988) 242.
7. V.N.Baier, V.M.Katkov and V.M.Strakhovenko. Nucl. Instrum. Methods A 250 (1986) 514.
8. V.N.Baier, V.M.Katkov and V.M.Strokhovenko. Preprint INP 83-70 (Novosibirsk, 1983) Sov. Phys. Dokl. 29 (1984) 306.
9. Yu.V.Kononets and V.A.Ryabov. Sov. Phys. JETP Lett. 48 (1988) 333 .
10. X.Artru. Phys. Lett. A 128 (1988) 302.
11. V.V.Tichomirov Nucl. Instrum. Methods B36 (1989) 282.
12. X.Artru Nucl.Instrum. Methods. B48 (1990) 278.
13. Yu.V.Kononets and V.A.Ryabov. Nucl.Instrum. Methods B48 (1990) 269.
14. V.N.Baier, V.M.Katkov and V.M.Strakhovenko. Nucl.Instrum. Methods B16 (1986) 5.
15. X.Artru, A.Belkacem, G.Bologna et al., Report at 13 International Conference "Atomic Collisions in Solids". Aarhus, Denmark, August 1989.

16. R.Medenwaldt, S.P.Møller, A.Sørensen et al., Phys. Rev. Lett. 63 (1989) 2827.

T A B L E

Crystal, axis	$\mathcal{E}_{cr}$, GeV	$\chi_s(\mathcal{E}_{cr})$	$\mathcal{E}_o = \mathcal{E}_{cr}$		$\mathcal{E}_o$ = 150 GeV		$\mathcal{E}_o$ = 250 GeV		$\mathcal{E}_o$ = 500 GeV	
			L_o	L_1	L_o	L_1	L_o	L_1	L_o	L_1
C, ⟨111⟩	48	0.06	0.31	1.5	0.46	2.1	0.5	2.3	0.58	2.6
Si, ⟨110⟩	53	0.17	0.14	1.9	0.21	2.7	0.23	2.9	0.27	3.3
Ge, ⟨110⟩	60	0.29	0.06	0.9	0.1	1.3	0.11	1.4	0.14	1.7
W, ⟨111⟩	30	0.86	0.007	0.25	0.016	0.5	0.019	0.6	0.025	0.76

All quantities were calculated at room temperature, thicknesses L_o, L_1 are given in mm.

Figure Captions

Fig.1. Energy loss spectra for 150 GeV electrons traversing a 0.205 mm Ge ⟨110⟩ crystal at indicated incident angles, $y = \Delta\mathcal{E}/\mathcal{E}_o$. Experimental data from Ref. 16 .

Fig.2. As fig.1, but for a 0,5 mm Si ⟨110⟩ crystal.

Fig.3. Inverse radiation length depending on electron energy for different potentials (see text).

Fig.4. As fig.1, but for a 0.185 mm, Ge ⟨110⟩ crystal, T = 100 K. Experimental data from Ref. 15 .

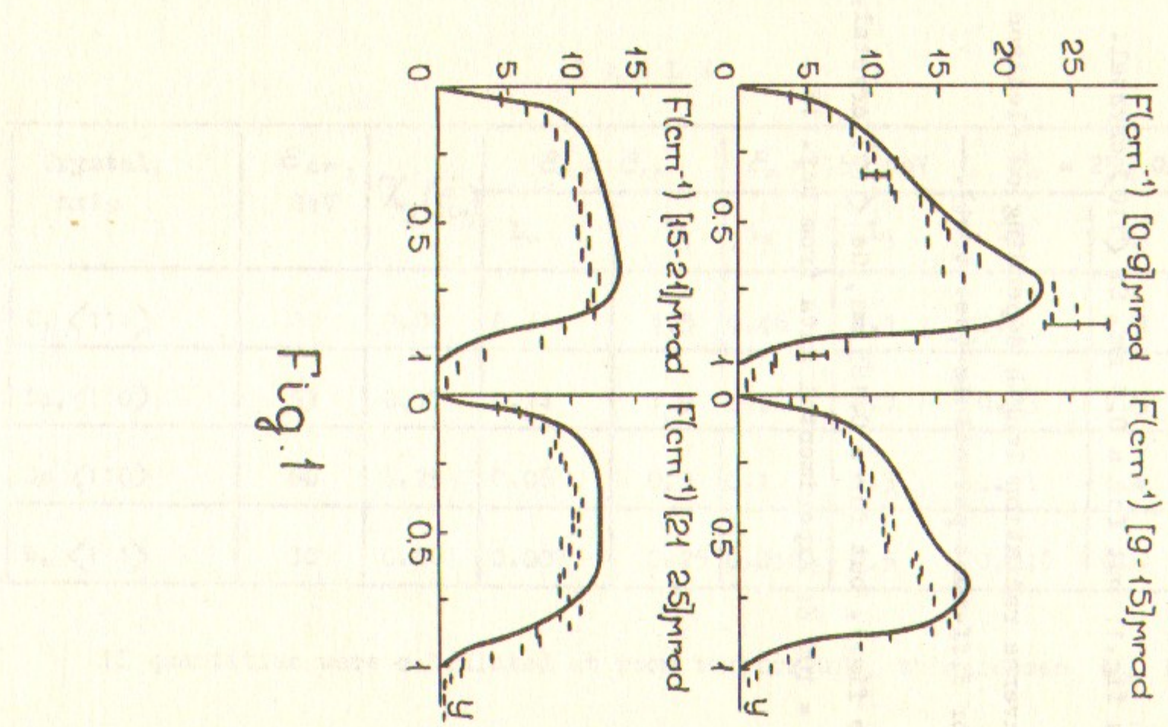
F(cm⁻¹) [0-9]mrad
F(cm⁻¹) [9-15]mrad
F(cm⁻¹) [15-21]mrad
F(cm⁻¹) [21 25]mrad
y

Fig. 1

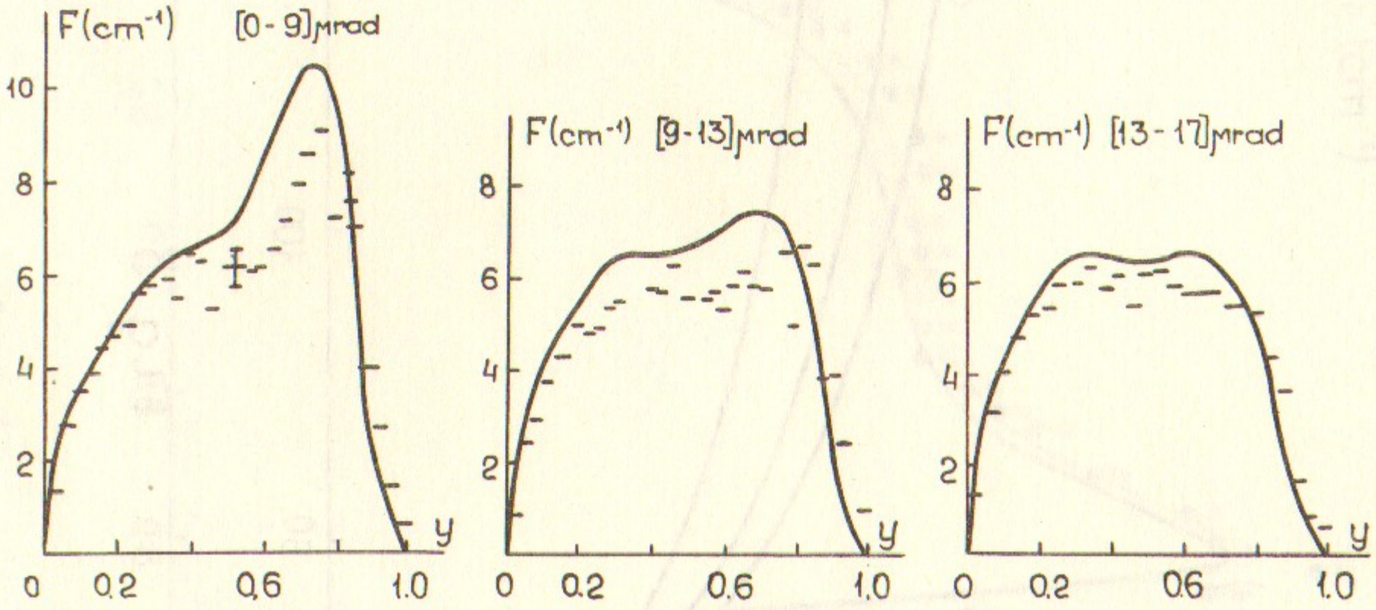
F(cm⁻¹) [0-9]mrad
F(cm⁻¹) [9-13]mrad
F(cm⁻¹) [13-17]mrad
y

Fig. 2

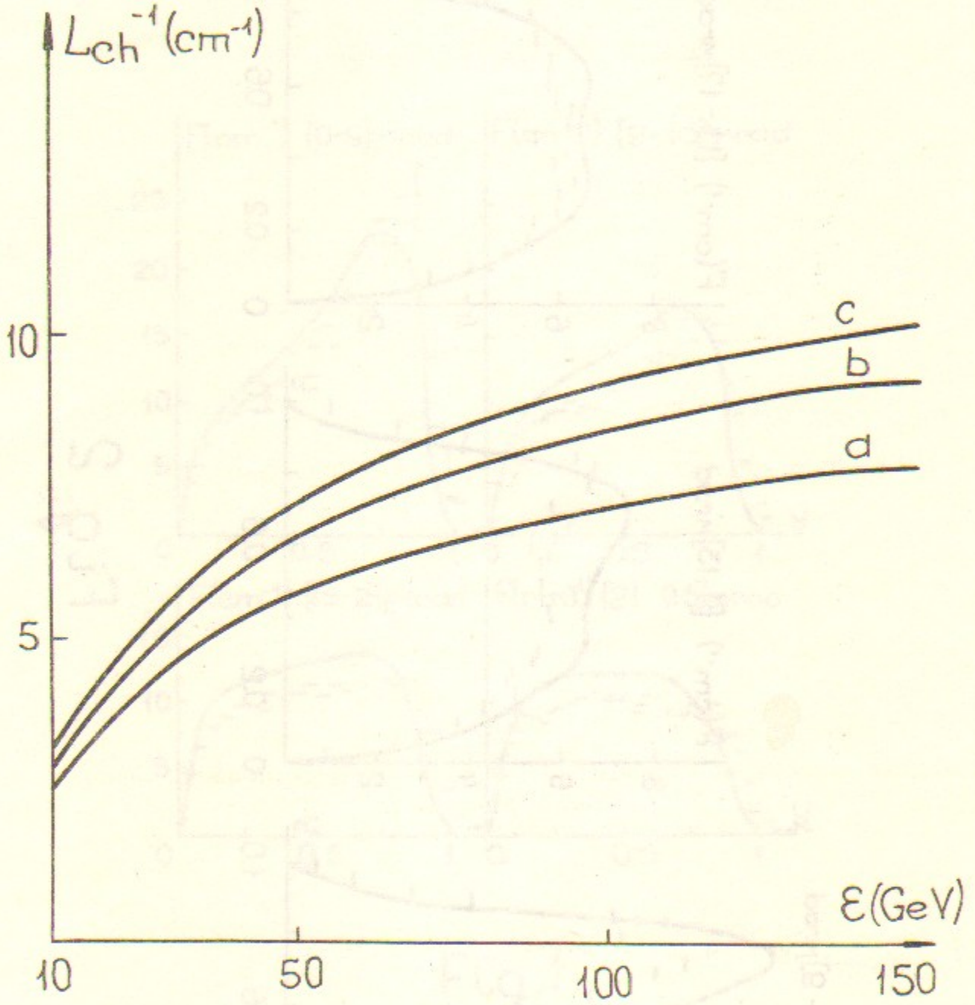
$L_{ch}^{-1}(cm^{-1})$
10
5
c
b
a
E(GeV)
10
50
100
150

Fig. 3

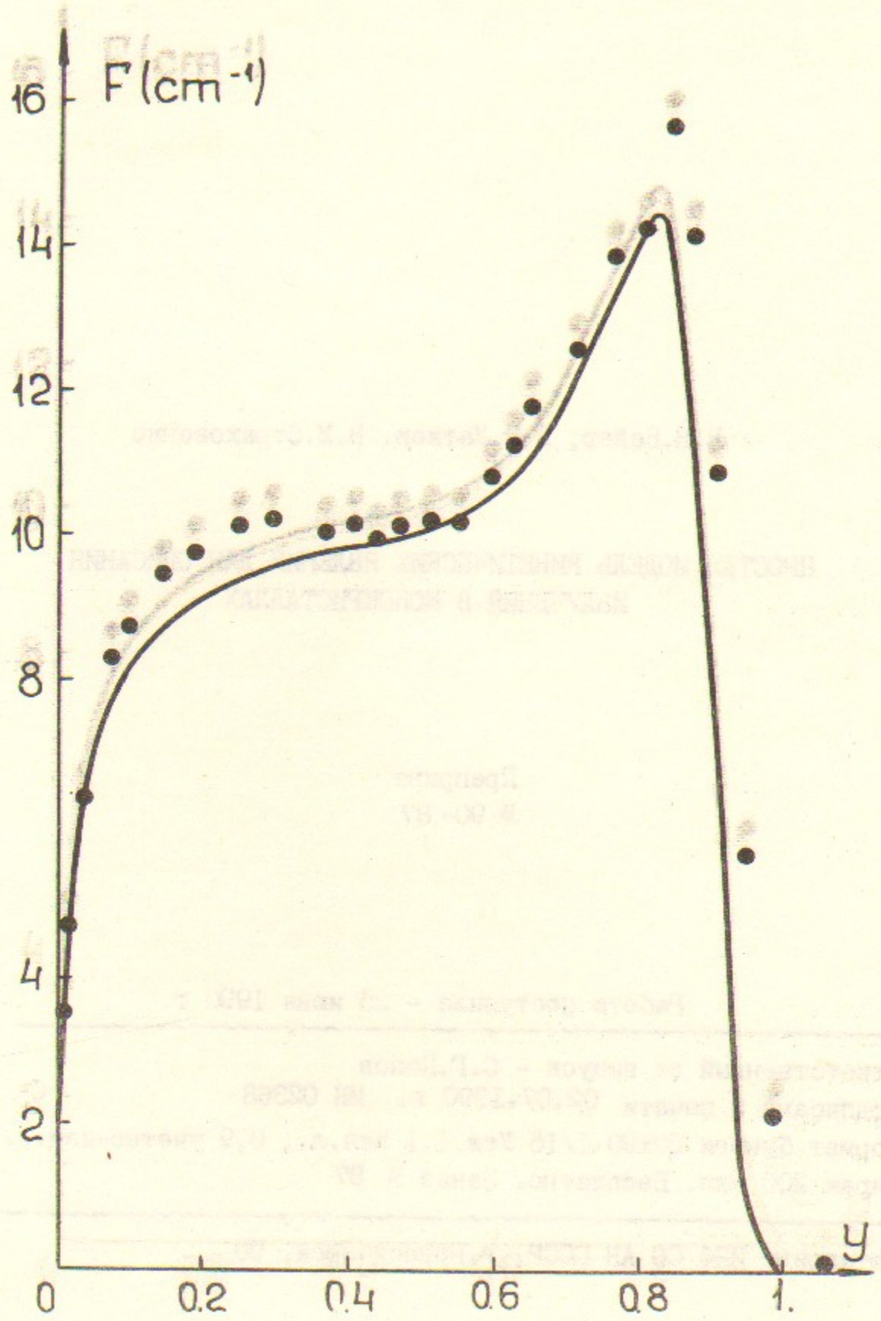
F(cm⁻¹)
16
14
12
10
8
4
2
y
0
0.2
0.4
0.6
0.8
1.

Fig. 4

В.Н.Байер, В.М.Катков, В.М.Страховенко

ПРОСТАЯ МОДЕЛЬ КИНЕТИЧЕСКИХ ЯВЛЕНИЙ ДЛЯ ОПИСАНИЯ ИЗЛУЧЕНИЯ В МОНОКРИСТАЛЛАХ

Препринт
№ 90-87

Работа поступила - 25 июня 1990 г.

Ответственный за выпуск - С.Г.Попов
Подписано к печати 02.07.1990 г. МН 02368
Формат бумаги 60x90 1/16 Усл.1,1 печ.л., 0,9 учетно-изд.л.
Тираж 200 экз. Бесплатно. Заказ № 87

Ротапринт ИЯФ СО АН СССР, г.Новосибирск, 90