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THE DEVICE FOR BUNCH SELFFOCUSSING

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ABSTRACT

The new device for damping the longitudinal single bunch instability in storage rings is proposed. This simple device is the dielectric canal insert of definite length in vacuum chamber. The structure of wake fields, induced by intensive bunch in such a canal is that, that backward action on bunch particles not only preserves but also decreases bunch length, i. e. leads to bunch selffocussing. The conditions under which this phenomenon reveals itself and can be applied to electron-positron factories are considered.

1. INTRODUCTION

The growth of a bunch longitudinal phase volume with the increasing of the number of particles—that is sc called bunch lengthening—was observed at different high energy storage rings. The reason for it is the bunch particles interaction with inhomogeneous vacuum chamber. The description of bunch lengthening effect based on wide-band impedance model was considered by many authors. Nevertheless the proposal how to avoid this effect was only given in the report [1] and for the first time. There, in particular the steady state distribution was shown to be selffocussed and stable, if the wake potential is a step-like function. In this case the bunch is compressed without energy widening when one increases the number of particles in it.

According to [1] step-like function $W(z)$ is the function, which Fourier transform-impedance $Z(k)$

$$Z(k)=\int_0^{\infty} W(z)\,e^{-ikz}\,dz$$

satisfies the following requirements:

$$\left|Z\left(\frac{1}{\sigma}\right)-Z_{step}\left(\frac{1}{\sigma}\right)\right|\leqslant\left|Z_{step}\left(\frac{1}{\sigma}\right)\right|, \qquad (1)$$

$$s\geqslant z_0,\quad \sigma_T,$$

where

$$Z_{step}(k) = W_0/ik$$

is the Fourier transform of the exact step-function

$$W_{step}(z) = W_0\,\Theta(z)\,, \quad \Theta(z) = \begin{cases} 1 & \text{for} \quad z>0 \\ 0 & \text{for} \quad z<0 \end{cases}$$

and s is the effective step-like function length above which the difference between $W(z>s)$ and $W_{step}(z)$ is not relatively small; z_0 is the coherent bunch displacement due to energy loss; σ is r.m.s. bunch length; σ_T is r.m.s. bunch length at zero current. In this definitions impedance is dimensionless: one unit of dimensionless impedance is equal to 30 Ohms.

What real vacuum chamber structure can present wake potential of step-like type? It may be noticed that step-likeness means finite energy loss of a point charge travelling in some kind of electrodynamic structure. This is occured in a dielectric canal (Fig. 1), where maximum frequency of Cherenkov radiation is limited by inner radius a. So it may be supposed that dielectric canal is characterized by wake potential of required type.

2. WAKE POTENTIAL OF DIELECTRIC CANAL

Let us derive the fields induced by a point charge travelling along the axis of cylindrically symmetric dielectric canal, presented in Fig. 1. The outer surface of the canal is covered by ideal metal. We assume that the bunch velocity is equal to that of light.

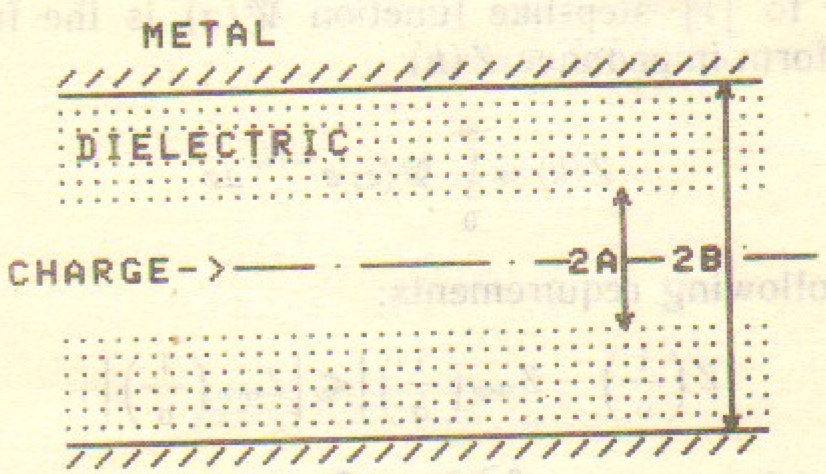

Fig. 1. Dielectric canal.

Substitute

$$E_\varphi = H_r = H_z = 0\,; \quad c=1\,; \quad \frac{\partial}{\partial t} = -\frac{\partial}{\partial z}\,;$$

$$\rho(\vec{r},t) = \rho(\vec{r}_\perp)\,\delta(z-t)\,; \quad \vec{j} = \vec{n}_z\,\rho(\vec{r},t)$$

into Maxwell equations one obtain a set of equations for nonzero component of electromagnetic field E_r, E_z, $H_\varphi = H$ in vacuum part of canal

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = \frac{\partial H}{\partial z}\,,$$

$$\frac{\partial H}{\partial z} = \frac{\partial E_r}{\partial z}\,, \qquad r<a\,,$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rH) = -\frac{\partial E_z}{\partial z} + 4\pi\rho$$

and in dielectric tube

$$E_r = \frac{H}{\varepsilon}\,,$$

$$\frac{\partial E_z}{\partial r} = \left(\frac{1}{\varepsilon} - 1\right)\frac{\partial H}{\partial z}\,, \qquad a<r<b\,,$$

$$\frac{1}{r}\frac{d}{dr}(rH) = -\varepsilon\,\frac{\partial E_z}{\partial r}\,.$$

Hence, in vacuum:

$$\frac{\partial E_z}{\partial r} = 0\,, \qquad E_z = E(z)\,,$$

$$E_r = H = -\frac{r}{2}\frac{dE}{dz} + \frac{2\delta(z)}{r}\,.$$

and in dielectric:

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} - (\varepsilon - 1)\frac{\partial^2 E_z}{\partial z^2} = 0\,. \tag{2}$$

On the boundary between vacuum and dielectric (for $r=a$) electromagnetic field components E_z and H have to satisfy the condition of continuity

$$E_z(a,z) = E(z)\,,$$

$$\left.\frac{\partial E_z}{\partial r}\right|_{r=a+0} = -\frac{\varepsilon-1}{\varepsilon}\frac{d}{dz}\left(-\frac{r}{2}\frac{dE}{dz}+\frac{2\delta(z)}{r}\right). \tag{3}$$

Next step for solving (2) together with (3) is to carry Fourier transformation, taking into account the principle of causality $(E(z>0)=0)$;

$$E_k^{(r)} = \int_{-\infty}^{0} E(r,z)\, e^{-ikz}\, dz, \tag{4}$$

one can find

$$E_k(r) = C_k\, \mathcal{Z}_0\left(kr\sqrt{\varepsilon-1}\right),$$

where $\mathcal{Z}_0(x)$ is the solution of Bessel equation.

The requirement of boundary condition on metal surface $(E_k(b)=0)$ gives the following expression for $\mathcal{Z}_0(\varkappa r)$:

$$\mathcal{Z}_0(\varkappa r) = H_0^{(1)}(\varkappa r)\, H_0^{(2)}(\varkappa b) - H_0^{(2)}(\varkappa r)\, H_0^{(1)}(\varkappa b),$$

$$\varkappa = k\sqrt{\varepsilon-1},$$

where $H_0^{(1)}(x)$, $H_0^{(2)}(x)$ are Hankel functions [2].

Constant C and Fourier component of E_k in vacuum can be evaluated by using boundary condition (3):

$$E_k = \frac{2i\sqrt{\varepsilon-1}}{a\varepsilon}\,\frac{1}{\frac{\mathcal{Z}_1}{\mathcal{Z}_0}-\frac{\varkappa a}{2\varepsilon}}, \tag{5}$$

where

$$\mathcal{Z}_0 = \mathcal{Z}_0(\varkappa a), \quad \mathcal{Z}_1 = -\frac{d\mathcal{Z}_0(\varkappa a)}{d(\varkappa a)}.$$

Functions $\mathcal{Z}_0$ and $\mathcal{Z}_1$ can be described by Neiman (N_0, N_1) and Bessel (J_0, J_1) functions, so that

$$\frac{\mathcal{Z}_1}{\mathcal{Z}_0} = \frac{N_1(\varkappa a)\, J_0(\varkappa b) - J_1(\varkappa a)\, N_0(\varkappa b)}{N_0(\varkappa a)\, J_0(\varkappa b) - J_0(\varkappa a)\, N_0(\varkappa b)}. \tag{6}$$

For $\varkappa a \gg 1$ the asymptotic expression for (6) is given by

$$\frac{\mathcal{Z}_1}{\mathcal{Z}_0} = \operatorname{ctg}(\varkappa(b-a)). \tag{7}$$

It is clear that $E_k(r)$ for $r>a$ is a regular function in upper half-plane of complex k. It means that the causality condition is satisfied.

The field $E(-z)$ is calculated by reverse Fourier trasformation of (5):

$$E(-z) = \int E_k\, e^{-ikz}\frac{dk}{2\pi}$$

The contour of integration is lying above all singularities of E_k, in the particular above the cut, connected with $N_{0,1}$ multisignificance. It is suitable to make the cut along the negative imaginary half-axis. When moving the contour of integration to lower half-plane it catches the cut and also poles, lying on the real axis, where

$$\frac{\mathcal{Z}_1}{\mathcal{Z}_0} = \frac{\varkappa a}{2\varepsilon}.$$

Thus the contour is transformed to that presented in Fig. 2. The dominant part of the integral over the cut is in the region

$$|\varkappa a| \leqslant 1.$$

Outside this region the difference between integrated function at opposite edges of the cut is exponentially small. For $|z| < a\sqrt{\varepsilon-1}$ the integral is independent of z. Taking into account Bessel function properties one can show that inside mentioned above region the difference of contributed values of integral over the left and right

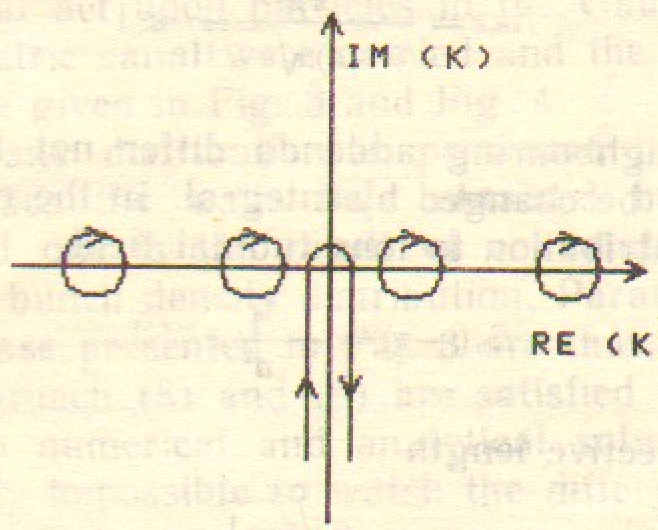

Fig. 2. Contour of integration.

parts of contour is of the same order as the value itself, so the estimation of integral over the cut is

$$E_c \approx \frac{2\sqrt{\varepsilon-1}}{a\varepsilon}\,\frac{1}{a\sqrt{\varepsilon-1}} \simeq \frac{2}{a^2\varepsilon}.$$

For $|z| > a\sqrt{\varepsilon-1}$ the integral decreases when z is increasing.

Next step is to evaluate the pole contribution to the total integral. The number of poles is infinite and all of them are simple and located on the real axis. The poles that give dominated contribution into integral are concentrated in the region

$$\frac{\varkappa a}{2\varepsilon} \leqslant 1$$

Supposing that

$$2\varepsilon \gg 1 \tag{8}$$

one can use asymptotic expression for Bessel functions (7). When the dielectric layer is thick enough,

$$b-a \gg \frac{\pi a}{2\varepsilon}, \tag{9}$$

the derivative of the denominator in the pole is

$$\frac{d}{d\varkappa}\left(\operatorname{ctg}\varkappa(b-a) - \frac{\varkappa a}{2\varepsilon}\right) = -(b-a)\left(1+\left(\frac{\varkappa a}{2\varepsilon}\right)^2\right).$$

And if

$$\Delta kz = \frac{\pi z}{(b-a)\sqrt{\varepsilon-1}} \ll 1$$

the phases of neighbouring addenda differ not too much and the sum of series can be changed by integral. In the result one can evaluate the pole contribution to longitudinal field

$$E_p(-z) = -\frac{4}{a^2}\, e^{-z/s}, \tag{10}$$

where s is the effective length

$$s = \frac{a\sqrt{\varepsilon-1}}{2\varepsilon}. \tag{11}$$

Under the condition (9) E_p is antiperiodical function:

$$E_p\left(-z-(b-a)\sqrt{\varepsilon-1}\right) = -E_p(-z).$$

Finally the total wake potential for a charge travelling in a dielectric canal is

$$W(z) = -E(-z)\,L = \frac{4L}{a^2}\left(e^{-z/s} - \frac{f}{2\varepsilon}\right) \tag{12}$$

for

$$0 \leqslant z \leqslant a\sqrt{\varepsilon-1}, \quad (b-a)\sqrt{\varepsilon-1}, \tag{13}$$

where L is the canal length, f is the constant. The value of f obtained by numerical calculation is

$$f = 0.37.$$

As it could be seen the wake potential of dielectric canal $W(k)$ is a step-like function. Its impedance $Z(k)$ under the condition $ks \gg 1$ is pure capacitive:

$$Z(k) = \frac{4L}{ik\,a^2}. \tag{14}$$

It is clear from (11) that the optimum value of dielectric constant is $\varepsilon=2$, in this case the step length achieves its maximum

$$s_{max} = a/4.$$

The problem of wake field calculation was also treated by numerical method for Maxwell equations. Calculations for longitudinal wake fields that act upon particles in the Gaussian bunch travelling along dielectric canal were carried and the results for some particular cases are given in Fig. 3 and Fig. 4.

The bunch density distribution is presented by dashed curve, solid curve describes the wake field obtained by the numerical method and dotted one is the result of convolution of the wake potential (12) and bunch density distribution. Parameters of dielectric canal for the case presented in Fig. 3 are that, that the conditions of analytic approach (8) and (9) are satisfied well enough. The agreement between numerical and analytical solutions is so good that it is practically impossible to watch the difference. It's interesting to note the good agreement also takes place in the case (Fig. 4) when the conditions (8) and (9) are not satisfied.

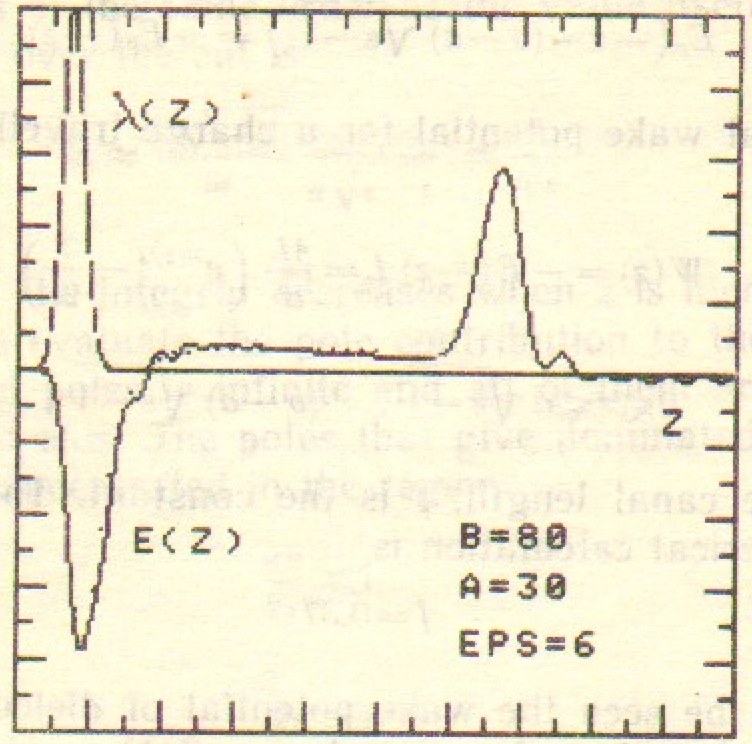

Fig. 3. Wake field of Gaussian bunch in dielectric canal (thick dielectric layer).

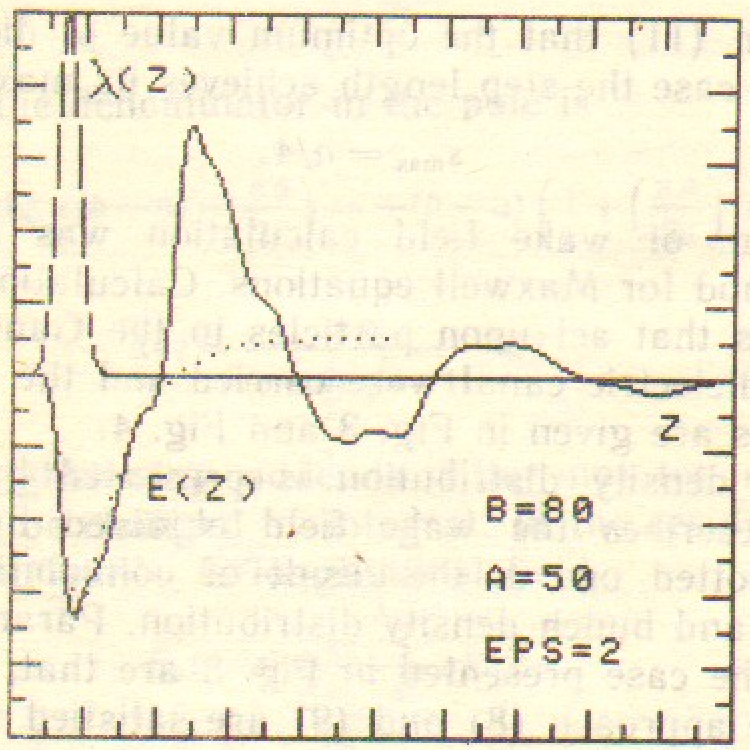

Fig. 4. Wake field of Gaussian bunch in dielectric canal (thin dielectric layer).

3. BUNCH STEADY STATE FORMED BY DIELECTRIC CANAL

According to (1) the main condition for bunch selffocussing is that, that wide-band impedance of dielectric canal must be higher than parasitic one due to inhomogeneity of vacuum chamber. Using the derived estimation (14) this condition takes the following form:

$$\frac{4L\sigma_T^2}{a^2R} \geqslant \left|\frac{Z}{n}\right|_{par}, \qquad (15)$$

where R is the average radius of a storage ring, $|Z/n|_{par}$ is parasitic impedance.

Also it is needed the coherent displacement z_0 to be not much higher then wake step length s. In order to get exact quantitative expression for this condition the bunch steady state equation (Haissinski equation [3]) for wake potential of dielectric canal (12) was solved numerically. One of the results for proper wake potential is shown in Fig. 5, where bunch density distribution is presented. The analytical solution for step function [1] is shown in this figure by dotted curve and the density distribution at low current is demonstrated by dashed curve.

When the coherent displacement z is increasing the bunch shape is changing: new local condensations of particles arise and grow, as it is shown in Fig. 6.

Due to this shape reconstruction the bunch r.m.s. length σ is increasing and the selffocussing effect is transformed to bunch lengthening. Fig. 7 demonstrates r.m.s. bunch length dependence vs its coherent displacement z_0 for different values of wake step length s. All values are measured in units of natural bunch length σ_T. As it can be seen from the Fig. 7 the minimum bunch length is achieved at

$$z_0 \approx 3s\,.$$

Thus the second condition for parameter z_0 of wake step-like function is

$$z_0 \lesssim 3s\,. \qquad (16)$$

4. REQUIERED PARAMETERS

The coherent displacement z_0 is determined by external RF rigidity $\varkappa_{RF}$:

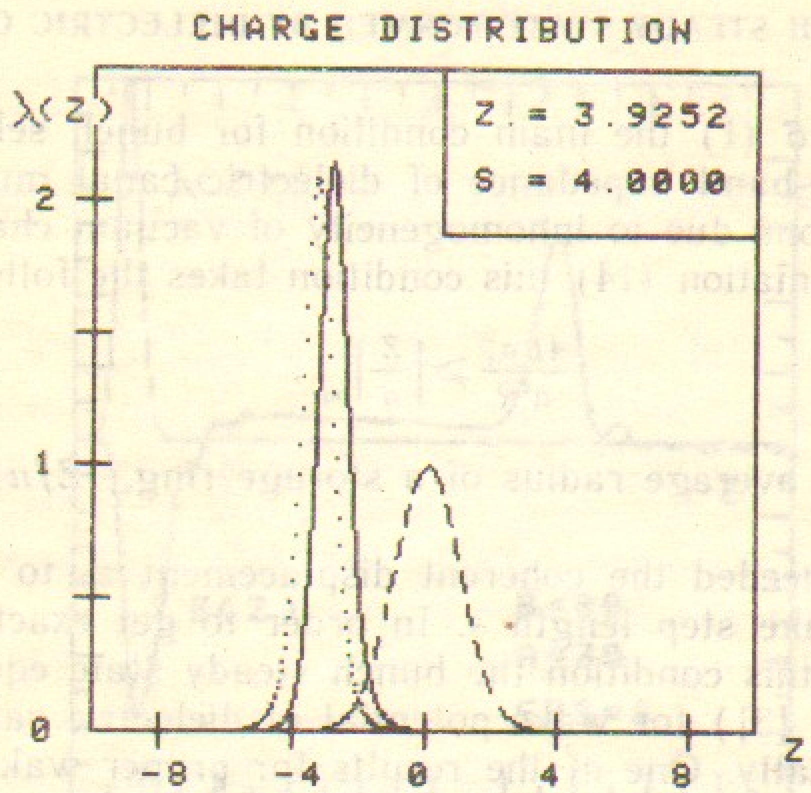

Fig. 5. Steady state charge distribution (selffocussing).

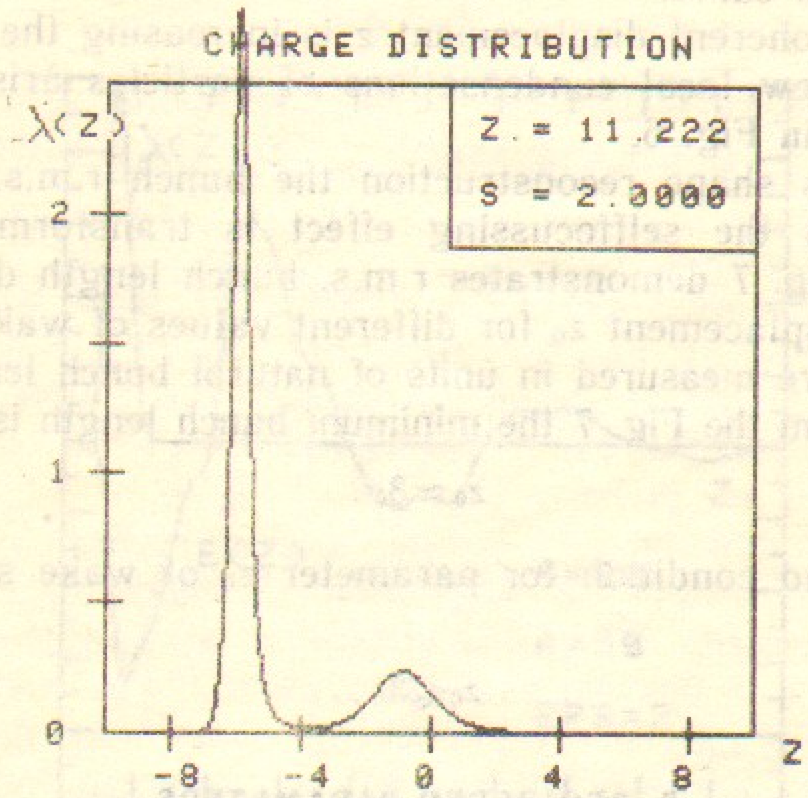

Fig. 6. Steady state charge distribution (lengthening).

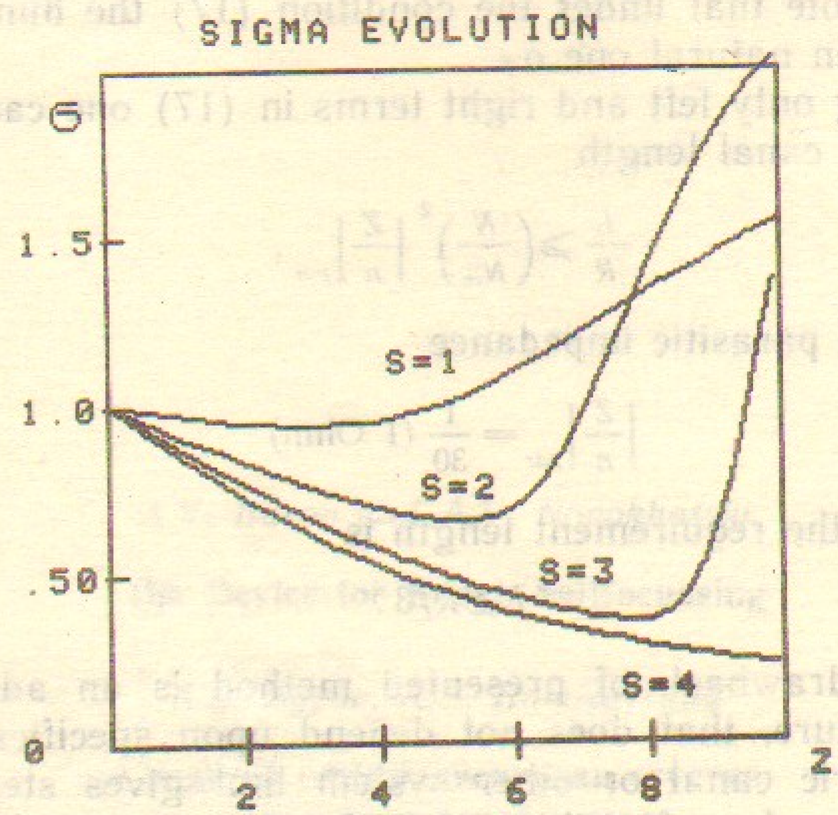

Fig. 7. Bunch length vs coherent displacement.

$$z_0 = \frac{2LNe^2}{\varkappa_{RF}\, a^2}$$

and in its turn the rigidity is depended upon RF harmonic number q_{RF} and voltage amplitude U_{RF}

$$\varkappa_{RF} = \frac{q_{RF}\, e U_{RF}}{R}.$$

Conditions (15), (16) are also can be presented in the following way:

$$\left(\frac{N}{N_{th}}A\right)^{1/3} \leqslant \frac{a}{2\sigma_T} \leqslant A^{1/2}, \qquad (17)$$

where

$$A = \frac{L}{R}\left|\frac{Z}{n}\right|^{-1}_{par}$$

and N_{th} is the threshold of bunch lengthening due to parasitic impedance:

$$N_{th} = \frac{\sqrt{2\pi}\,\varkappa_{RF}\,\sigma_T^3}{e^2\,|Z/n|_{par}\,R}.$$

It is wise to note that under the condition (17) the bunch length σ will be less then natural one σ_T.

Considering only left and right terms in (17) one can obtain the requirement on canal length

$$\frac{L}{R} \geqslant \left(\frac{N}{N_{th}}\right)^2 \left|\frac{Z}{n}\right|_{par}.$$

For example, if parasitic impedance

$$\left|\frac{Z}{n}\right|_{par} = \frac{1}{30} (1\ \text{Ohm})$$

and $N/N_{th}=3$ the requirement length is

$$L \geqslant R/3.$$

The main drawback of presented method is an additional RF power expenditure, that does not depend upon specification of the device (dielectric canal or other system that gives step-like wake potential). The value of additional average power needed for compensation the energy loss due to coherent radiation of M bunches with N particles in each is

$$P = \frac{\varkappa_{RF} z_0 NMc}{2\pi R} = \frac{(Ne)^2 MLc}{\pi R a^2} \geqslant \frac{(Ne)^2 Mc}{4\pi\sigma_T^2} \left|\frac{Z}{n}\right|_{par}.$$

To get numerical data one can evaluate the additional power for B-factory (one of possible projects, INP). If one proposes to use $M=2\cdot 150$ bunches with $N=6\cdot 10^{10}$ particles in each and r.m.s. bunch length $\sigma_T=0.75$ cm, and if the vacuum chamber will have parasitic impedance $|Z/n|_{par}=0.3$ Ohm, then additional RF power would be of order $P=1$ MW. The required RF power seems to be high enough, however it permits to avoid bunch lengthening and approach to very high luminosity of the electron-positron factory.

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