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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

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CONTINUOUS TIME REGGE GRAVITY IN THE TETRAD-CONNECTION VARIABLES

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Continuous Time Regge Gravity
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ABSTRACT

The continuous time limit of the tetrad-connection formulation of Regge calculus is found. The shift and lapse functions take on their values loosely. In the continuous space limit the action obtained is reduced to the continuous general relativity action.

This paper continues the preceding author's work [1] which develops the approach to the Regge calculus based on the discrete moving frame formalism by Bander [2]. It is shown in [1] that using the discrete analogs of the tetrad and connection introduced in [2] the action can be formulated in terms of independent variables of this type. Excluding connections via the equations of motion leads to the Regge action as function of link lengths only. (The problem of uniqueness of the solution of these equations was considered in [1] and will not be addressed here.)

Now we construct the $3+1$ continuous time Regge calculus in the tetrad representation [1]. The continuous time formalism was studied in a number of works [3—10], see also [11]. We follow the idea of [4, 9]: this formalism should be the limit of the 4-dimensional Regge calculus when the distance between the spacelike leaves ε tends to zero. The difference between these works and the present one is both the formal (using the new variables simplifying the action) and the essential one—analogs of the shift and lapse functions are freely chosen. This is achieved by assuming the angle defects on the spacelike bones being nonzero at $\varepsilon\to 0$. This does not contradict to finiteness of the action [9] since the closely located at $\varepsilon\to 0$ such defects cancel each other. (This possibility naturally arises in the tetrad-connection variables.)

Regge spacetime we consider is that of simplest periodic structure [12]. Apart from the degrees of freedom which turn to the tetrad in the continuous spacetime limit there are extra degrees of freedom (some defects) inessential in this limit but making the analysis quite difficult. Freezing them we get the simplest Regge minisuperspace system [13] being able to approximate any continuous spacetime. To simplify geometrical interpretation we assume the metric being positively defined although the results can be easily modified for the pseudo-Riemannian manifold. Passing to the continuous time limit means that the orders in ε are ascribed to the set of quantities in a consistent way. For example, we can imagine a sequence of Regge manifolds with an arbitrarily small ε inscribed in a fixed smooth 4-surface in an Euclidean space of sufficiently high dimension [14]. From now on the terms «timelike» and «spacelike» will be referred to the further specified k-simplices (links at $k=1$) of measure (length at $k=1$) $O(\varepsilon)$ and $O(1)$, respectively.

Let us give notations concerning the Regge manifold [12] and the formalism [2, 1]. Topologically, the Regge manifold periodic cell is a 4-cube divided into 24 4-simplices sharing the hyperbody diagonal. Let the indices $\mu, \nu, \lambda, ... = 1, 2, 3, 4$ label the cube edges emerging from a vertex O, the edge 4 being the timelike one. The T_μ, $T_\mu^{-1} = \bar{T}_\mu$ are operators of the translations to the neighbouring vertices in the direction μ. The 1-simplices σ_1 (links) will be labelled by multiindices $M, N, P, ...$, the unordered sequences of (different) indices: $M = (\mu\nu...\lambda)$. By definition, link M connects O and $T_M O = T_\mu T_\nu ... T_\lambda O$. The k-simplex σ^k labelled by ordered sequence of multiindexes $[M_1 M_2 ... M_k]$ is spanned by the links M_1, $(M_1 M_2), ..., (M_1 M_2 ... M_k)$. It is spacelike if $M_1, M_2, ..., M_k = 4$ and timelike if it has the form $[...4...]$. If confusion can not appear, the round and square brackets will be omitted. There are the following σ^k, $1 \leqslant k \leqslant 4$, at the given vertex O: (i) 15 links μ, $\mu\nu$, $\mu\nu\lambda$, 1234; (ii) 50 2-simplices (bones) $\mu\nu$, $(\mu\nu)\lambda$, $\mu(\nu\lambda)$, $\mu(\nu\lambda\rho)$, $(\mu\nu)(\lambda\rho)$ $(\mu\nu\lambda)\rho$; (iii) 60 3-simplices $\mu\nu\lambda$, $(\mu\nu)\lambda\rho =: d\lambda\rho$, $\mu(\nu\lambda)\rho =: \mu d\rho$, $\mu\nu(\lambda\rho) =: \mu\nu d$ (d means «diagonal»); (iv) 24 4-simplices $\mu\nu\lambda\rho$. The 2-simplices $\mu(\nu\lambda\rho)$, $(\mu\nu)(\lambda\rho)$, $(\mu\nu\lambda)\rho$ and 3-simplices $...d...$ meeting at the diagonal 1234 will be called the internal ones: they are contained in the interior of the 4-cube. In the same sence all the 4-simplices are internal simplices. The 3-dimensional indexes $\alpha, \beta, \gamma, ... = 1, 2, 3$ and multiindexes $A, B, C, ...$ refer to the spacelike leaves which are themselves Regge manifolds of the type [12]. The $[A_1 A_2 ... A_{k-1}]$ means both the $(k-1)$-simplex σ^{k-1} and the k-prism with the bases σ^{k-1} and $T_4 \sigma^{k-1}$.

Basic quantities are the link vectors l^a_M, bone bivectors V^{ab}_{MN} and curvatures R^{ab}_{MN}, connections $\{MNP\}^{ab}$ (denoted as $\Omega(\sigma^3)$ in [1]) on the 3-simplices. Euclidean vector indexes $a, b, c, ...$ are related to the

local frames associated with the separate 4-simplices σ; this relation for a quantity $Q^{ab...c}$ will be denoted by vertical bar: $Q^{ab...c}_{|\sigma}$. The $\{\}$-matrixes $\{MNP\}$ are $O(4)$-rotations relating the local frames of any two 4-simplices sharing the (oriented) 3-face $[MNP]$. To each link M the 4-simplex $\sigma(M)$ is assigned so that $l^a_M := l^a_{M|\sigma(M)}$ is considered as independent variable. The bivector V^{ab}_{MN} can be constructed in three distinct ways of three pairs of the triangle MN edge vectors; we choose

$$V^{ab}_{MN} := V^{ab}_{MN|\sigma} = (l^a_{M|\sigma} l^b_{MN|\sigma} - l^b_{M|\sigma} l^a_{MN|\sigma})/2 =: [l_{M|\sigma}, l_{MN|\sigma}]^{ab} \qquad (1)$$

for some $\sigma = \sigma([MN])$. In general $l^a_{M|\sigma}$, $l^a_{MN|\sigma}$ are functions of l^a_M, l^a_{MN} and $\{\}$, and so V is. Here $\{\}$ are still not independent variables but rather a particular set $\{\}(l)$ of the Regge manifold connections [1]. Let $\sigma(M)$, $\sigma([MN])$ be attributed to the same vertex O as M, $[MN]$ are. Then only internal connections $\{...d...\}(l)$ are required to define V. These ones follow from the equations

$$R_M(\{...d...\}(l)) = \exp(\varphi_M {}^*V_M) \qquad ({}^*V^{ab} := \varepsilon^{abcd} V^{cd}/2), \qquad (2)$$

$$(l_{MN|\sigma} - l_{M|\sigma})^2 = T_M l^2_N, \qquad (3)$$

where the subscript M at R, V, φ means MN at $N = (1234 \backslash M)$ (an internal bone), φ_M are the parameters. Due to the Bianchi identity [15, 12] for the hyperbody diagonal the 14 internal curvatures are parametrized by 13 matrixes $\{...d...\}$; let the other $\{...d...\}(l)$'s be trivial. This means that the local frame is maximally extended on the whole 4-cube with the cuts across the internal bones. A choice of the cuts and of $\sigma(M)$ is shown in Fig. 1. This picture results from projection of the 4-cube simplices on the 3-plane orthogonal to l_{1234} by intersecting it with a 2-sphere centered at O. This scheme leads to the following R-dependent bivectors:

$$V_{(14)2} = [R_{(124)} R_{(12)} R_1 l_{14}, l_{142}], \quad V_{4(12)} = [R_{(124)} R_{(12)} R_1 R_{(14)} l_4, l_{142}],$$
$$V_{1(42)} = [l_1, \bar{R}_{(12)} l_{142}],$$
$$V_{4(23)} = [R_{(234)} R_{(23)} R_2, R_{(24)} R_{(124)} R_{(12)} R_1 R_{(14)} l_4, l_{243}],$$
$$V_{(24)3} = [R_{(234)} R_{(23)} R_2 l_{24}, l_{243}], \quad V_{42} = [R_{(124)} R_{(12)} R_1 R_{(14)} l_4, l_{42}], \qquad (4)$$
$$V_{2(43)} = [l_2, \bar{R}_{(23)} l_{243}], \quad V_{43} = [R_{(234)} R_{(23)} R_2 R_{(24)} R_{(124)} R_{(12)} R_1 R_{(14)} l_4, l_{43}],$$
$$V_{(34)1} = [R_{(314)} R_{(31)} R_{(123)} R_3 l_{34}, l_{341}], \quad V_{31} = [l_3, \bar{R}_{(123)} l_{31}],$$
$$V_{3(41)} = [l_3, \bar{R}_{(123)} R_{(31)} l_{341}].$$

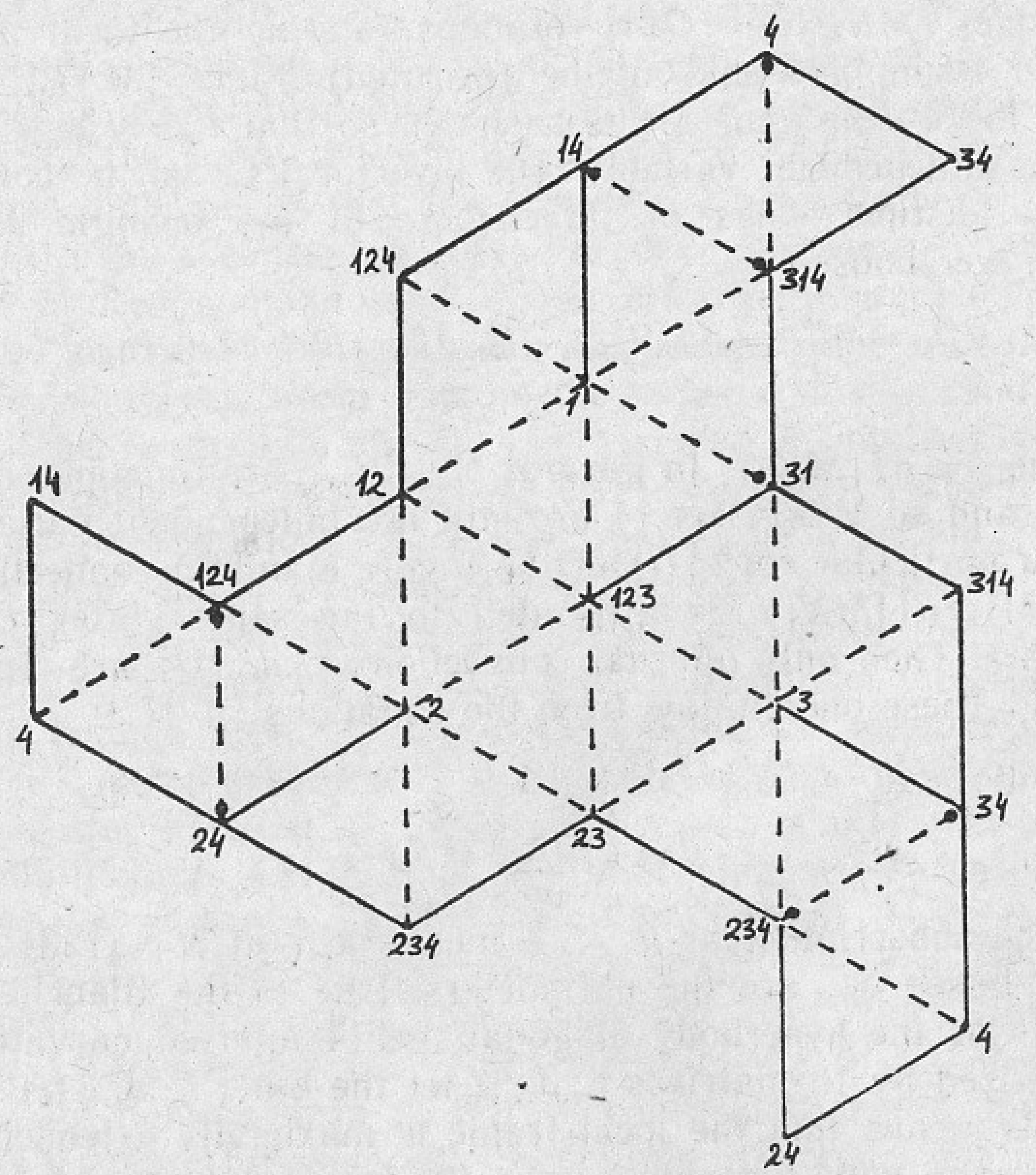

Fig. 1. The scheme of cuts in the 4-cube. The internal k-simplices are shown as the $(k-2)$-simplices on the 2-sphere. In particular, the 3-simplices on which the connection is nontrivial are shown by solid line, the other ones are by dashed one. The bivectors $[l_M, l_{1234}]$ are shown as vertices, the numerals meaning M. The bolded point indicates an edge of the cut on which l^a_M is defined.

All other bivectors take the simple form

$$V_{MN}=[l_M, l_{MN}]\,. \tag{5}$$

To define a set of independent tetrad variables let us examine the eqs (2), (3) using the above parametrization of $\{...d...\}$ in terms of $R_M(l)$. Eqs (2) are already in the desired form with exception of that the Bianchi identity should be also taken into account,

$$R_4 R_{(314)} R_{(31)} R_{(123)} R_3 R_{(34)} R_{(234)} R_{(23)} R_2 R_{(24)} R_{(124)} R_{(12)} R_1 R_{(14)}=1 \tag{6}$$

(in fact, $R=R(\{\,\})$ is a general solution of the Bianchi identities). Eq. (6) is equivalent to three scalar equations (in the 3-plane orthogonal to l_{1234}). Therefore the 11 of 14 parameters φ_M are independent ones. The 11 eqs (3) for MN appearing in (4) form a minimal set required to express these φ's in terms of the link vectors (thereby (4) is a minimal set of R-dependent bivectors). Given the tetrad l^a_μ and 5 of 11 parameters φ_M at each vertex the remaining 44 components l^a_M and six φ_M can be found from 50 eqs (3). We can find that generally φ_A, $\varphi_{(A4)}=O(1)$ at $\varepsilon\to0$, $A\neq(123)$. It is consistent with the reasoning of [9, 11] which concludes from vanishing the spacelike defects* at $\varepsilon\to0$ and from the Bianchi identities for the link $(A4)$ that the planes of $[A4]$ and $[4A]$ coincide thus imposing a constraint on the tetrad. We have succeeded in finding an explicit form of the Lagrangian for the particular values of 5 parameters φ. Namely, at $\varepsilon\to0$ we denote

$$\varepsilon N^a := l^a_4\,,\quad \varepsilon N^a_A := l_{A4}-l_A\,,\quad \tilde\varphi_{(123)} := \varphi_{(123)}\,\varepsilon\,,\quad \tilde\varphi_4 := \varphi_4\varepsilon\,,\quad \varphi'_A := \varphi_A+\varphi_{(A4)} \tag{7}$$

with finite N, N_A, $\tilde\varphi_{(123)}$, $\tilde\varphi_4$. Using φ'_A instead of $\varphi_{(A4)}$ we put

$$\tilde\varphi_{(123)}, \varphi'_A=0\,,\qquad A=1,2,(12),(23)\,. \tag{8}$$

Then (6) gives $\tilde\varphi_4=\varphi'_3=\varphi'_{(31)}=0$. Since $R_A l_{A4}=l_A+O(\varepsilon)$ all the bivectors are, accurate to the leading order in ε, independent on R. The 6 remaining parameters φ_A, $A\neq(123)$ enter (and can be found from) only the 6 equations (3) at $MN=(\alpha4)\beta$, $\alpha(4\beta)$, $\alpha(4\beta)$, $\alpha\beta=12, 23, 31$ (see (4)) which therefore can be omitted. Eqs (3) present the bilinear system of constraints on l^a_M,

$$(l_M-l_{MN})^2=T_M l^2_N,\quad MN\neq(\alpha4)\beta\,,\quad \alpha(4\beta)\,,\quad \alpha\beta=12,23,31\,. \tag{9}$$

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*) The defect $\alpha_{\sigma^2}=\varphi_{\sigma^2}\mu_{\sigma^2}$ where α_{σ^2} is the area of σ^2.

In the continuous time notations this reads

$$(l_{AB}-l_A)\, N_{AB}=T_A l_B N_B \quad (AB\neq 12,23,31)\,, \quad (l_{AB}-l_A)^2=T_A l_B^2,$$
$$(l_{AB}-l_A)\,(N_{AB}-N_A)=T_A \dot{l}_B l_B \quad (AB\neq 12,23,31)\,, \quad N_A^2=T_A N^2\,, \tag{10}$$
$$l_A(N_A-\dot{N})=\dot{l}_A l_A.$$

This system is uniquely solvable for l^a_A, N^a_A in terms of l^a_α, N^a as it was checked for small variations about flat spacetime where l^a_α, $N^a=\mathrm{const}$,

$$l^a_A=\sum_{\alpha\in A} l^a_\alpha\,, \quad N^a_A=N^a.$$

Return to the finite ε case. Regge action in the tetrad representation takes the form

$$S(l,\{\,\})=-\sum_n\sum_{MN}\mu_{MN}\sin^{-1}\mathrm{tr}[{}^*U_{MN}R_{MN}(\{\,\})/2] \quad (U_{MN}=V_{MN}/\mu_{MN})\,, \tag{11}$$

where $n=(n^1, n^2, n^3, n^4)$ label the vertices, μ_{MN} is the area of $[MN]$. The curvature matrix $R_{MN}(\{\,\})$ is the product of $\{\sigma^3\}$, $\sigma^3\supset[MN]$, ordered along the loop which encloses $[MN]$. Assuming a definite orientation of faces write R_{MN} in some definite 4-simplices σ in the following form:

$$\begin{aligned}
R_{41}&=\overline{\{413\}}(\bar{T}_2\overline{\{241\}})(\bar{T}_{23}\overline{\{d41\}})(\bar{T}_3\{341\})\{412\}\{41d\}\,,\\
R_{4(23)}&=\overline{\{4d1\}}\,\overline{\{423\}}\,(\bar{T}_1\{14d\})\{432\}\,,\\
R_{23}&=\overline{\{23d\}}\,\overline{\{231\}}(\bar{T}_4\overline{\{423\}})\,(\bar{T}_{14}\{d23\}(\bar{T}_1\{123\})\{234\}\,,\\
R_{2(43)}&=\overline{\{2d1\}}\,\overline{\{234\}}\,(\bar{T}_1\{12d\})\{243\}\,,\\
R_{(24)3}&=\overline{\{d31\}}\,\overline{\{243\}}\,(\bar{T}_1\{1d3\})\,\{423\}\,,\\
R_{1(32)}&=\overline{\{132\}}\,(\bar{T}_4\overline{\{41d\}})\{123\}\{1d4\}\,,\\
R_{1(432)}&=\overline{\{1d4\}}\,\{12d\}\,\{1d3\}\{14d\}\overline{\{1d2\}}\,\overline{\{13d\}}\,,\\
R_{(14)(32)}&=\overline{\{14d\}}\{d23\}\{41d\}\overline{\{d32\}}\,,\\
&\ldots\text{(cyclic permutations of } 1,2,3)\ldots,\\
R_{4(123)}&=\{4d3\}\overline{\{42d\}}\{4d1\}\overline{\{43d\}}\,\{4d2\}\overline{\{41d\}}\,.
\end{aligned} \tag{12}$$

The rest of R-matrices can be obtained by index **group** permutations: if $R_{(\mu\ldots\nu)(\lambda\ldots\rho)}=\Pi T_{(\ldots)}\{\ldots\mu\ldots\nu\lambda\ldots\rho\ldots\}$ then $\bar{R}_{(\lambda\ldots\rho)(\mu\ldots\nu)}=\Pi T_{(\ldots)}\{\ldots\lambda\ldots\rho\mu\ldots\nu\ldots\}$. The action presents the sum of contributions from the groups of (closely located at $\varepsilon\to0$) timelike $A4$, $4A$ and spacelike AB, $A(4B)$, $(A4)B$ bones which will be called contributions of the 2-prisms A and 3-prisms AB, respectively.

Next consider the limit $\varepsilon\to0$ in the connection sector of the theory. If $M, N, P\neq4$ then

$$\{MNP\}=1+\varepsilon f_{MNP}\,, \quad \bar{f}_{MNP}=-f_{MNP}=O(1)\,, \tag{13}$$

which is an analog of the continuum connection for the parallel vector transport at a distance $O(\varepsilon)$ (orthogonally to the spacelike leaf of the foliation). Let $M, N\neq4$. The R_{MN} is shown above to admit the general form $\exp(O(1))$; the finite contribution is given here by the connections $\{\ldots4\ldots\}$. For the limiting Lagrangian being finite the sum of $O(1)$ contributions to the action of the 3-prisms attributed to the vertices of any spacelike leaf,

$$\sum_{\bar{n}}\sum_{AB}\mu_{AB}\sum_{MN}\sin^{-1}({}^*U^{ab}_{AB}\langle MN\rangle^{ab}/2)\,, \tag{14}$$

should vanish where $\bar{n}=(n^1, n^2, n^3)$; $MN=AB$, $A(4B)$, $(A4)B$; $\langle AB\rangle:=\overline{\{AB4\}}\{4AB\}$, $\langle A(4B)\rangle:=\overline{\{A4B\}}\{AB4\}$, $\langle(A4)B\rangle:=\overline{\{4AB\}}\{A4B\}$. The $\langle MN\rangle$ is an $O(1)$ part of R_{MN}. In fact, finiteness of the Lagrangian follows from the equations of motion. Indeed, $O(\varepsilon)$-fluctuations of $\{\,\}$ in (14), $\delta\{MNP\}=\varepsilon\{MNP\}\omega_{MNP}$, $\bar{\omega}_{MNP}=-\omega_{MNP}=O(1)$, generally result in the finite terms in the action. Equations of motion for ω^{ab}_{MNP} take the form of the following constraints:

$$\Gamma_{MNP}V_{AB}+V_{AB}\bar{\Gamma}_{MNP}=V_{AB}\,\mathrm{tr}\,\Gamma_{MNP}\,, \quad MNP=4AB\,,\ A4B\,,\ AB4\,, \tag{15}$$

where

$$\Gamma_{4AB}=\langle AB\rangle/\cos\alpha_{AB}-\langle(A4)\,B\rangle/\cos\alpha_{(A4)\,B}\,,\ldots,(2\sin\alpha:={}^*U^{ab}R^{ab})\,.$$

Solving (15) with taking into account the identity $\langle AB\rangle\langle(A4)B\rangle\langle A(B4)\rangle=1$ leads in any given 3-prism AB to

$$\langle MN\rangle=\exp(\varphi_{MN}{}^*V_{AB}+{}^*\varphi_{MN}V_{AB})\,,$$
$$\sum_{MN}\varphi_{MN}=\sum_{MN}{}^*\varphi_{MN}=0\,, \quad \alpha_{MN}=\varphi_{MN}\mu_{MN} \tag{16}$$

thereby making (14) vanish. Thus, independent 3-prism connection variables are a matrix $\{AB\}$ (let $\{AB\}:=\{4AB\}$) and four of the parameters $\varphi_{MN}, {}^*\varphi_{MN}$ (we choose $MN=AB$, $(A4)B$). Unlike this situation the analogous quantities φ appearing above in the tetrad sector serve simply to parametrize the link vectors.

Let us proceed to computing the Lagrangian. Now we take into account the $O(\varepsilon)$-corrections $dR_{MN}=\langle AB\rangle r_{MN}dt$ $(dt:=\varepsilon,\ \bar{r}_{MN}=-r_{MN})$, $d\mu_{MN}$, dU_{MN} modifying the 3-prism action (14):

$$\mathscr{L}_{AB}dt=-\Sigma(\mu+d\mu)\sin^{-1}\mathrm{tr}[{}^*(U+dU)\langle\ \rangle(1+rdt)/2] \qquad (17)$$

(the summation variable MN being suppressed). Taking into account (16) and the identities $\mathrm{tr}(dU{}^*\langle\ \rangle)=0$, $\mathrm{tr}({}^*U\langle\ \rangle r)=\mathrm{tr}({}^*Ur)\cos\alpha$ we obtain

$$2\mathscr{L}_{AB}dt=\Sigma\,\varphi d\mu^2-\mathrm{tr}({}^*V\Sigma\, r)\,dt, \qquad (18)$$

$$\Sigma\,\varphi d\mu^2=\varphi_{(A4)B}(\mu^2_{(A4)B}-\mu^2_{A(4B)})+\varphi_{AB}(\mu^2_{AB}-\mu^2_{A(4B)})\,.$$

The $(V+dV)$'s depend on $\varphi_A(l)$ in the order $O(\varepsilon)$ in the 3-prisms 12, 23, 31 (see (4)). Therefore to define the area differences the following formula is useful:

$$4\mu^2_{MN}=\mu^2(l^2_M, l^2_{MN}, T_M l^2_N)\,,$$

$$\mu^2(s_1,s_2,s_3)=2(s_1s_2+s_2s_3+s_3s_1)-s_1^2-s_2^2-s_3^2\,. \qquad (19)$$

In the other 3-prisms sufficient is the formula

$$4\mu^2_{MN}=2[\,l_M, l_{MN}]^2=l^2_M l^2_{MN}-(l_M l_{MN})^2\,. \qquad (20)$$

The matrices r linearly depend on the time derivatives $\{\dot{A}B\}=(1-\bar{T}_4)\ \{AB\}\varepsilon^{-1}$ and on the matrices f (see (13)) summed over the 4-prisms $\alpha\beta\gamma$:

$$h_{\alpha\beta\gamma}=f_{\alpha\beta\gamma}+f_{d\beta\gamma}+f_{\alpha d\gamma}+f_{\alpha\beta d}\,. \qquad (21)$$

The curvature on the timelike bone $A4$ or $4A$ is the product of $\{...4...\}$'s and thus it is parametrized by φ_{MN}, ${}^*\varphi_{MN}$ and $\{AB\}$.

The resulting Lagrangian takes the form

$$L=\sum(\mathscr{L}_{\{\}}+\mathscr{L}_h+\mathscr{L}_\varphi+\mathscr{L}_N)\,, \qquad (22)$$

$$2\mathscr{L}_{\{\}}=-l^a_3 l^b_{23}\,{}^*(\overline{\{32\}}\,\{\dot{3}2\})^{ab}-l^a_{23} l^b_2\,{}^*(\overline{\{23\}}\,\{\dot{2}3\})^{ab}-$$

$$-l^a_{123} l^b_1\,{}^*(\overline{\{1d\}}\,\{1\dot{d}\})^{ab}-l^a_{23} l^b_{123}\,{}^*(\overline{\{d1\}}\,\{d\dot{1}\})^{ab}+\ldots, \qquad (23)$$

$$2\mathscr{L}_h=[\,l^a_3 l^b_{23}+l^a_{23} l^b_{123}+l^a_{123} l^b_3-T_3(\{21\}\,l_2)^a(\{21\}\,l_{21})^b]\,{}^*h^{ab}_{321}+$$

$$+[\,l^a_{12} l^b_1+(\{d3\}\,l_{123})^a(\{d3\}\,l_{12})^b+(\{1d\}\,l_1)^a(\{1d\}\,l_{123})^b-$$

$$-T_1(\{23\}\,l_{23})^a(\{23\}\,l_2)^b]\,{}^*h^{ab}_{123}+\ldots, \qquad (24)$$

$$2\mathscr{L}_\varphi=(\varphi'_{32}N^a_3\partial^a_3-\varphi_{32}N^a_{32}\partial^a_{32})\,\mu^2_{32}+(\varphi'_{1d}N^a_1\partial^a_1-\varphi_{1d}N^a_{123}\partial^a_{123})\,\mu^2_{1d}+$$

$$+(\varphi'_{d1}N^a_{32}\partial^a_{32}-\varphi_{d1}N^a_{123}\partial^a_{123})\,\mu^2_{d1}-\varphi_{23}[(T_2 l^a_3 N^a_3)(T_2\partial^2_3)+l^a_{23}N^a_{23}\partial^2_{23}]\,\mu^2_{23}+$$

$$+\varphi'_{23}\{l^a_2 N^a_2\partial^2_2+[T_2 l^a_3(\dot{l}_3-N_3)^a](T_2\partial^2_3)\}\,\mu^2_{23}+\ldots, \qquad (25)$$

where $\partial^a_A:=\partial/\partial l^a_A$, $T_A\partial^2_B:=2\partial/\partial(T_A l^2_B)$,

$$\mathscr{L}_N=\mu_{41}\sin^{-1}\frac{1}{2}U^{ab}_{41}\,{}^*[\overline{\{13\}}\,(\bar{T}_2\overline{\langle 21\rangle'}\,\overline{\{21\}})(\bar{T}_{23}\overline{\langle d1\rangle'}\,\overline{\{d1\}})\times$$

$$\times(\bar{T}_3\{31\}\overline{\langle 31\rangle'})\{12\}\{1d\}]^{ab}+$$

$$+\mu_{14}\sin^{-1}\frac{1}{2}\bar{U}^{ab}_{14}\,{}^*[\overline{\langle 13\rangle'}\,\overline{\{13\}}\,(\bar{T}_2\langle 21\rangle\overline{\{21\}})(\bar{T}_{23}\langle d1\rangle\overline{\{d1\}})\times$$

$$\times(\bar{T}_3\{31\}\langle 31\rangle)\{12\}\overline{\langle 12\rangle'}\{1d\}\overline{\langle 1d\rangle'}]^{ab}+$$

$$+\mu_{4(23)}\sin^{-1}\frac{1}{2}U^{ab}_{4(23)}\,{}^*[\overline{\{d1\}}\,\overline{\{23\}}\,(\bar{T}_1\{1d\}\overline{\langle 1d\rangle'})\{32\}]^{ab}+$$

$$+\mu_{(23)4}\sin^{-1}\frac{1}{2}\bar{U}^{ab}_{(23)4}\times$$

$$\times{}^*[\overline{\langle d1\rangle'}\,\overline{\{d1\}}\,\overline{\langle 23\rangle}\,\overline{\{23\}}\,(\bar{T}_1\{1d\}\langle 1d\rangle)\{32\}\overline{\langle 32\rangle}]^{ab}+\ldots+$$

$$+\mu_{4(123)}\sin^{-1}\frac{1}{2}U^{ab}_{4(123)}\,{}^*(\{d3\}\overline{\{2d\}}\,\{d1\}\,\overline{\{3d\}}\,\{d2\}\overline{\{1d\}})^{ab}+$$

$$+\mu_{(123)4}\sin^{-1}\frac{1}{2}\bar{U}^{ab}_{(123)4}\,{}^*(\{d3\}\overline{\langle d3\rangle}\,\overline{\langle 2d\rangle}\,\overline{\{2d\}}\,\{d1\}\overline{\langle d1\rangle}\times$$

$$\times\overline{\langle 3d\rangle}\,\overline{\{3d\}}\,\{d2\}\overline{\langle d2\rangle}\,\overline{\langle 1d\rangle}\,\overline{\{1d\}})^{ab}. \qquad (26)$$

Here $\langle AB\rangle':=\langle(A4)B\rangle$, $\varphi'_{AB}:=\varphi_{(A4)B}$; $\langle MN\rangle$ is given by (16); $U_{4A}=[N,l_A]\ ([N,l_A]^2/2)^{-1/2}$, $\bar{U}_{A4}=[N_A,l_A]\ ([N_A,l_A]^2/2)^{-1/2}$, for μ_{AB} see (19), (20); d means «diagonal» (in three dimensions): $1d:=1(23)$ and so on; dots mean the two cyclic permutations of 1, 2, 3 in the preceding terms. The field variables are l^a_A, N^a, N^a_A parametrized by N^a, l^a_α via the constraints (10), $\{AB\}^{ab}=(\{AB\}^{-1})^{ba}$, $h^{ab}_{\alpha\beta\gamma}=-h^{ba}_{\alpha\beta\gamma}$ and φ_{MN}, ${}^*\varphi_{MN}$ at $MN=AB$, $(A4)B$. The symmetry of the formalism w.r.t. the odd permutations of indices 1, 2, 3 is broken spontaneously while choosing an edge of a 4-cube cut to define a link vector on it (see Fig. 1).

The equation of motion for a connection $\{MNP\}$ expresses (modulo extra conditions of the type $RV\bar{R}=V$) vanishing the bivector sum over 2-faces of the 3-simplex MNP [1]. Now the $h_{\alpha\beta\gamma}$ is a multiplier of the constraint which presents such property for the spacelike 3-simplex $\alpha\beta\gamma$. The equation for $\{AB\}$ proves to require vanishing the bivector sum for the 3-prism AB under extra conditions of the type $R_{MN}V_{MN}\bar{R}_{MN}=V_{MN}$, $MN=A4$, $4A$. Under the same conditions the equations for ${}^*\varphi$ claim the two spacelike 2-faces of the timelike 3-simplex being in the same 3-plane and φ equations relate the area difference between these faces to the area of projection onto them of the two timelike faces of this 3-simplex.

Finally, the continuous Einshtein—Hilbert action can be reproduced if the spacelike link length scale a tends to zero. In this limit the different objects have the following orders in a ($\{AB\}=:\exp\omega_{AB}$):

$$N, N_A, h_{\alpha\beta\gamma}, \varphi, {}^*\varphi\sim 1; \quad l_A, \quad \omega_{AB}\sim a; \quad \Delta_A:=T_A-l\sim a. \tag{27}$$

In these orders

$$N_A=N, \quad h_{\alpha\beta\gamma}=h_{\beta\alpha\gamma}=h_{\alpha\gamma\beta}=:h, \quad \omega_{AB}=\omega_{BA},$$
$$\dot{l}_A=\sum_{\alpha\in A} l_\alpha, \quad \Delta_A=\sum_{\alpha\in A}\Delta_\alpha. \tag{28}$$

The terms $O(a^3)$ should be retained in L. Within this accuracy φ, ${}^*\varphi$ do not enter L and ω_{AB} can be grouped in ω_α given by $\omega_1=\omega_{23}+\omega_{2d}-\omega_{3d}$ and cyclic permutations of 1, 2, 3 in it. So we get

$$S=\int dt\sum_{\bar{n}} \pi^{\gamma ab}\,{}^*\dot{\omega}_\gamma^{ab}+\varepsilon^{\alpha\beta\gamma}N^a l_\alpha^b\,{}^*(\Delta_\beta\omega_\gamma+\omega_\beta\omega_\gamma)^{ab}+$$
$$+(\Delta_\alpha\pi^\alpha+[\omega_\alpha,\pi^\alpha])^{ab}\,{}^*h^{ab}\left(\pi^{\gamma ab}:=\frac{1}{2}\varepsilon^{\alpha\beta\gamma}l_\alpha^a l_\beta^b\right). \tag{29}$$

Redenote $e_\alpha^a:=l_\alpha^a a^{-1}$, $e_4^a:=N^a$, $\omega_4^{ab}:=h^{ab}$ and introduce the world coordinates x^α coinciding with $n^\alpha a$ at the vertices $\bar{n}$. Eq. (29) takes the form

$$2S=\frac{1}{2}\int d^4x\varepsilon_{abcd}\,\varepsilon^{\mu\nu\lambda\rho}e_\mu^a e_\nu^b[\partial_\lambda+\omega_\lambda, \partial_\rho+\omega_\rho]^{ab}+O(a), \tag{30}$$

which is just the Einshtein—Hilbert action in the variables e, ω [16].

The Lagrangian (22) supplemented with the system of constraints (10) is a starting point for constructing the Hamiltonian formalism and canonical quantization of Regge spacetime, which we expect to consider in a separate publication. Unlike it's continuum limit the discrete space action depends on the analogs of lapse and shift functions $N^a=(\bar{N}, N^4)$ nonlinearly. Another interesting feature of the formalism is the presence of the tetrad velocities $\dot{l}_\alpha$ in both the Lagrangian (25) and constraints (10).

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REFERENCES

1. *Khatsymovsky V.* 1989 Tetrad and Self-Dual Formulation of Regge Calculus. Novosibirsk, Preprint INP 89-69, 1989; Class. Quantum Grav. to be published.
2. *Bander M.* Phys. Rev. Lett., 1986, v.57, p.1825;
Bander M. Phys. Rev., 1987, v.D36, p.2297;
Bander M. Phys. Rev., 1988, v.D38, p.1056.
3. *Lund F. and Regge T.* Unpublished.
4. *Collins P.A. and Williams R.M.* Phys. Rev., 1973, v.D7, p.965;
Collins P.A. and Williams R.M. Phys. Rev., 1974, v.D10, p.3537.
5. *Williams R.M.* Class. Quantum Grav., 1986, v.3, p.853.
6. *Friedman J.L. and Jack I.* J. Math. Phys., 1986, v.27, p.2973.
7. *Piran T. and Williams R.M.* Phys. Rev., 1986, v.D33, p.1622.
8. *Porter J.* Class. Quantum Grav., 1987, v.4, p.375; p.391; p.651.
9. *Brewin L.* Class. Quantum Grav., 1987, v.4, p.899;
Brewin L. Class. Quantum Grav., 1988, v.5, p.839.
10. *Tuckey P.A. and Williams R.M.* Class. Quantum Grav., 1988, v.5, p.155.
11. *Tuckey P.A.* Independent Variables in 3+1 Regge Calculus Cambridge. Preprint DAMTP 87-32, 1987.
12. *Rocek M. and Williams R.M.* Phys. Let., 1981, v.104B, p.31;
Rocek M. and Williams R.M. Z. Phys., 1984, v.C21, p.371.
13. *Hartle J.B.* J. Math. Phys., 1985, v.26, p.804.
14. *Feinberg G., Friedberg R. Lee T.D. and Ren M.C.* Nucl. Phys., 1984, v.B245, p.343.
15. *Regge T.* Nuovo Cim., 1961, v.19, p.558.
16. *Schwinger J.* Phys. Rev., 1963, v.130, p.1253.

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