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ABSTRACT

A process of e^+e^- pair production in electron-positron linear colliders is considered. Various mechanisms of the pair creation are analysed: photon radiation by an electron in the field of the oncoming beam and subsequent pair production by this photon in the same field (cascade process), a direct electroproduction of the pair in the field through virtual photon; a mixed production, when the photon is radiated in the field (coherently) and then it produces the pair in a collision with an individual particle of the oncoming beam (incoherently) and vice versa. A modification of the incoherent pair production cross section in a presence of the strong field is considered. A dependence of the pair production process on a longitudinal beam size which is particularly important when it is comparable with pair formation length and on transverse beam sizes, the latter owing to a change of the minimal momentum transfer, is taken into account as well.

1. INTRODUCTION

In linear colliders the probability of photon emission by a particle during collision time turns out to be of order of unity and the density of the accompanying photon beams becomes comparable to that of the primary charged beams. The magnetic bremsstrahlung radiation mechanism dominates, and its characteristics are determined by the value of the quantum parameter $\chi(t)$, dependent on the strength of the oncoming beam field at the moment t (constant field limit)*)

$$\chi^2 = -\frac{e^2}{m^6}(F^{\mu\nu}p_\nu)^2, \quad \chi = \frac{\gamma F}{H_0}, \tag{1.1}$$

where $p^\nu(\varepsilon, \vec{p})$ is a particle four-momentum, $F^{\mu\nu}$ is an external electromagnetic field tensor, $\gamma = \varepsilon/m$, $\vec{F} = \vec{E}_\perp + \vec{v}\times\vec{H}$, $\vec{E}$ and $\vec{H}$ are electric and magnetic fields in the laboratory-system, $\vec{E}_\perp = \vec{E} - \vec{v}(\vec{v}\vec{E})$, $\vec{v}$ is the particle velocity and $H_0 = m^2/e = 4.41\cdot 10^{13}$ Oe. At $\chi \gtrsim 1$ the mean photon energy $\bar{\omega} \simeq \varepsilon\chi/(2(1+2\chi))$ is comparable with that of an electron. The radiation energy losses in linear colliders have been considered in Refs [1—7].

At $\varkappa = \omega F/mH_0 \gtrsim 1$ the emitted photons with high probability convert into electron-positron pairs in the field of the oncoming

*) In this paper, we employ units $\hbar = c = 1$ and $e^2 = \alpha = 1/137$, metrics $ab = a^\mu b_\mu = a^0 b^0 - \vec{a}\vec{b}$.

beam. The radiation and the pair production in external fields, as well as the influence of these fields on incoherent processes at particles collisions, have been studied in detail recently [8—14]. Note that in Ref. [8] the production of a e^+e^- pair by the photon of a definite energy has been considered with the allowance for the beam field inhomogeneity and end effects.

The present paper deals with e^+e^- pair production at beam-beam collisions in linear colliders. Some aspects of this problem have been solved in authors paper [15], where both e^+e^- and heavy- particle pair production have been considered. The pairs can be produced by the photons, emitted at beam-beam collisions. In case of $\chi \gg 1$ one should also take into account the direct electroproduction process ($e \to 3e$), owing to virtual intermediate photons. The creation of pairs by charged particles and secondary photons also takes place in incoherent processes, being essentially modified in the presence of the strong field. The finiteness of transverse beam sizes resulting in an increase of the minimal momentum transfer also decreases corresponding cross sections. At $\chi < 1$ the incoherent processes dominate and the e^+e^- pair production by a magnetic bremsstrahlung photon on the particles of the oncoming beam and by two virtual photons are most important.

The arising pairs can serve as unavoidable source of the background. Thereby it seems very important not only to know the total number of produced pairs, but also their energy spectrum, since the latter affects the angular distribution of emitted particles owing to the rapid increase of the particle deflection angle in the field of the oncoming beam along decrease of its energy.

2. PHOTON RADIATION AND PAIR PRODUCTION PROBABILITIES

Let us consider the photon radiation and subsequent pair production by this photon in the field of the oncoming beam. The photon formation length in an external field is determined by the expression (see, e. g. [9])

$$l_c(\chi, u) = \lambda\!\!\!^{-}_c \frac{H_0}{F} \left(1+\frac{\chi}{u}\right)^{1/3} = \frac{\lambda\!\!\!^{-}_c \gamma}{\chi} \left(1+\frac{\chi}{u}\right)^{1/3}, \tag{2.1}$$

where $\lambda\!\!\!^{-}_c = \hbar/mc$ is the electron Compton wave length, $u = \omega/\varepsilon'$, $\varepsilon' = \varepsilon - \omega$. The field of the oncoming beam changes very slightly

along the formation length l_c, if the condition $l_c \ll \sigma_z$ (σ_z is the longitudinal beam size) is satisfied, providing a high accuracy of the well-known magnetic bremsstrahlung approximation, for which the photon radiation probability per unit time has the form [10]

$$\frac{dw_\gamma}{dt} \equiv dW_\gamma(t) = \frac{\alpha \Phi_\gamma (1+\vec{\lambda}\vec{\xi})}{2\sqrt{3}\,\pi\gamma^2}\, d\omega;$$

$$\Phi_\gamma = \left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right) K_{2/3}(z) - \int_z^\infty K_{1/3}(y)\, dy + \frac{\omega}{\varepsilon} (\vec{\zeta}\vec{h})\, K_{1/3}(z),$$

$$z = 2u/3\chi, \tag{2.2}$$

where ω is a frequency, $\vec{\lambda}$ are Stokes parameters of radiated photons for the following choice of axes: $\vec{e}_1 = \vec{v} \times \vec{h}$, $\vec{h} = \vec{F}^*/F$, $\vec{e}_2 = \vec{h}$, $\vec{F}^* = (e/|e|)\ (\vec{H}_\perp + \vec{E} \times \vec{v})$, e is the particle charge, $\vec{\zeta}$ is the spin vector of the initial electron. The vector $\vec{\xi}$ determines mean photon polarization and is given by the following expression

$$\xi_1 = (\omega (\vec{\zeta}\vec{v}\vec{h})/\varepsilon' \Phi_\gamma)\, K_{1/3}(z),$$

$$\xi_2 = ((\vec{\zeta}\vec{v})/\Phi_\gamma) \left[\left(\frac{\varepsilon}{\varepsilon'} - \frac{\varepsilon'}{\varepsilon}\right) K_{2/3}(z) - \frac{\omega}{\varepsilon} \int_z^\infty K_{1/3}(y)\, dy\right],$$

$$\xi_3 = \frac{1}{\Phi_\gamma} \left[K_{2/3}(z) + \frac{\omega}{\varepsilon'} (\vec{\zeta}\vec{h})\, K_{1/3}(z)\right]. \tag{2.3}$$

Here $K_\nu(z)$ are the MacDonald functions, which are exponentially small at $z \gg 1$ while at $z \to 0$ they grow as $z^{-\nu}$:

$$K_\nu(z) = \sqrt{\pi/2z}\; e^{-z} \quad (z \gg 1),$$

$$K_\nu(z) = \frac{\Gamma(\nu)}{2} \left(\frac{2}{z}\right)^\nu \quad (z \ll 1). \tag{2.4}$$

Using asymptotic expression (2.4) we obtain in the frequency range $u = \frac{\omega}{\varepsilon'} \ll \chi$, important for further analysis, the following expression

$$\Phi_\gamma = \Gamma(2/3)\, (3\chi/u)^{2/3} \left(1 + \frac{\omega^2}{2\varepsilon\varepsilon'}\right);$$

$$\xi_1 = 0, \quad \xi_2 = \frac{\varepsilon^2 - \varepsilon'^2}{\varepsilon^2 + \varepsilon'^2} (\vec{\zeta}\vec{v}), \quad \xi_3 = \frac{\varepsilon\varepsilon'}{\varepsilon^2 + \varepsilon'^2}. \tag{2.5}$$

The probability of the pair production by a photon in the external field can be obtained from formulae (2.2), (2.3) with the following substitutions $\varepsilon\to-\varepsilon$, $\omega\to-\omega$, $\vec{\zeta}\to-\vec{\zeta}$, $\lambda_2\to-\lambda_2$, $\lambda_{1,3}\to\lambda_{1,3}(\vec{e}\to\vec{e}^{\,*})$, $\omega^2 d\omega\to-\varepsilon^2 d\varepsilon$ (see, e. g., Ref. [10]). Performing these substitutions we obtain:

$$\frac{dw_e}{dt}\equiv dW_e(t)=\frac{\alpha m^2\Phi_e(1+\vec{\lambda}\vec{\Sigma})}{2\sqrt{3}\,\pi\omega^2}\,d\varepsilon\,;$$

$$\Phi_e=\left(\frac{\varepsilon}{\varepsilon'}+\frac{\varepsilon'}{\varepsilon}\right)K_{2/3}(y)+\int_y^\infty K_{1/3}(x)\,dx+\frac{\omega}{\varepsilon}(\vec{\zeta}\vec{h})\,K_{1/3}(y)\,,$$

$$y=\frac{2\omega^2}{3\varepsilon\varepsilon'\varkappa},\qquad \Sigma_1=-\omega(\vec{\zeta}\vec{v}\vec{h})\,K_{1/3}(y)/(\varepsilon'\Phi_e)\,,$$

$$\Sigma_2=\frac{(\vec{\zeta}\vec{v})}{\Phi_e}\left[\left(\frac{\varepsilon}{\varepsilon'}-\frac{\varepsilon'}{\varepsilon}\right)K_{2/3}(y)+\frac{\omega}{\varepsilon}\int_y^\infty K_{1/3}(x)\,dx\right],$$

$$\Sigma_3=-\frac{1}{\Phi_e}\left[K_{2/3}(y)+\frac{\omega}{\varepsilon'}(\zeta\vec{h})\,K_{1/3}(y)\right]. \tag{2.6}$$

Here ε is the energy of the produced electron, $\varepsilon'=\omega-\varepsilon$ is the energy of the positron, $\vec{\zeta}$ is the electron spin vector, $\vec{\lambda}$ are Stokes parameters of the initial photon. Summing up over final electron states in formula (2.6) (some terms have been integrated by parts) yields:

$$W_e=\frac{2\alpha m^2}{3\sqrt{3}\,\pi\omega}\int_0^1 dx\left[\frac{1-3\lambda_3}{2}+\frac{1}{x(1-x)}\right]K_{2/3}\left(\frac{2}{3\varkappa x(1-x)}\right). \tag{2.7}$$

Using the presented above probabilities as the kernels of the corresponding kinetic equation, one can calculate the characteristics of the cascade generated by the initial particle. However in the case when the number of secondary charged particles is small one can use the method of successive approximations. Allowing for the smallness of the relative energy loss of the particle in the colliders the following expression for the spectrum of produced pairs (over total pair energy $\omega=\varepsilon+\varepsilon'$) is obtained:

$$\frac{dw_r}{d\omega}=\int_{-\infty}^{\infty}\frac{dW_\gamma(t)}{d\omega}\,w_e(\omega,t)\,dt=$$

$$=\int_{-\infty}^{\infty}dt\,\frac{dW_\gamma(\varepsilon,t)}{d\omega}\left[1-\exp\left(-\int_t^\infty W_e(\omega,t')\,dt'\right)\right]. \tag{2.8}$$

The expression (2.8) is valid, if $w_r\ll1$. When the probability w_e is small, one can perform expansion of the exponent in (2.8). In what follows unpolarized initial particles are considered and the shape of the beams during the collision is assumed to be stable, thus, the electron and the radiated photon move at a certain distance $\vec{\rho}$ from the axis of the oncoming beam. In this case the probability (2.8) takes the form:

$$\frac{dw_r(\vec{\rho})}{d\omega}=\frac{1}{2}\,w_e(\omega,\xi_3,\vec{\rho})\,\frac{dn_\gamma(\vec{\rho},\omega)}{d\omega}\,, \tag{2.9}$$

where $n_\gamma(\vec{\rho})$ is the total number of photons, radiated by the particle. In deriving formula (2.9) the use was made of the oncoming beam longitudinal symmetry and of the summing rule

$$\frac{1}{2}\sum_\lambda(1+\vec{\xi}\vec{\lambda})(1+\vec{\lambda}\vec{\Sigma})=(1+\vec{\xi}\vec{\Sigma})\to(1+\xi_3\Sigma_3)\,.$$

To get the total number of the produced pairs moving along one beam, expressions of the type (2.9) should be averaged over the transverse coordinate distribution $n_\perp(\vec{\rho})$ and multiplied by the total number of particles in this beam N.

3. COHERENT CASCADE PROBABILITY

The calculation of the probability of coherent cascade (going through a real intermediate photon) at $\chi\ll1$ is especially simple. Here one can use for corresponding probabilities their asymptotic expressions (see Eq. (2.4) at $z\gg1$), further performing the integration by Laplace method. Thus, for nonpolarized electrons, using Eqs (2.2) and (2.3) we have:

$$\Phi_\gamma=\frac{1}{2}\sqrt{\frac{3\pi\chi}{u}}\left(1+\frac{u^2}{1+u}\right)\exp\left(-\frac{2u}{3\chi}\right);$$

$$\xi_1=\xi_2=0, \quad \xi_3=\frac{1+u}{1+u+u^2}. \tag{3.1}$$

Using the explicit form $\chi(t)=\chi_0(\vec{\rho})\exp(-2t^2/\sigma_z^2)$ we obtain

$$dn_\gamma(\vec{\rho},\omega)=\frac{\sqrt{3}\,\alpha m^2\chi_0\sigma_z}{4\varepsilon}\exp\left(-\frac{2u}{3\chi_0}\right)\frac{(1+u+u^2)\,du}{u(1+u)^3}. \tag{3.2}$$

Thereby for the pair production probability we get:

$$w_e=\frac{9\sqrt{\pi}}{64\sqrt{2}}\frac{\alpha m^2\sigma_z}{\omega}\left(1-\frac{\xi_3}{3}\right)\varkappa_0^{3/2}\exp\left(-\frac{8}{3\varkappa_0}\right). \tag{3.3}$$

Taking into account that $\varkappa=u\chi/(1+u)$ and integrating in Eq. (2.9) over u by Laplace method (here $u_0=2$, $\xi_3=3/7$, we obtain:

$$w_r(\rho)=\frac{\sqrt{3}\,\pi}{512}\frac{\alpha^2m^4\sigma_z^2}{\varepsilon^2}\chi_0^3\exp\left(-\frac{16}{3\chi_0}\right). \tag{3.4}$$

Note, that in this case the energy of the initial particles is equally shared among three final particles. The probability (3.4) should be averaged over the particle distribution in the transverse plane. For round beams the corresponding Laplace integration with a density

$$n_\perp(\vec{\rho})=\exp(-\rho^2/2\sigma_\perp^2)/(2\pi\sigma_\perp^2)$$

yields the following result:

$$\bar{w}_r^{rd}=c_r^{rd}\alpha^2\left(\frac{\sigma_z}{\gamma\lambda_c}\right)^2\chi_m^{7/2}\exp\left(\frac{-16}{3\chi_m}\right), \tag{3.5}$$

where

$$c_r^{rd}=\frac{3(\pi f_0)^{3/2}}{2^{10}\sqrt{2}\,|f_0''|}\simeq 6.7\cdot10^{-3},$$

χ_m is the maximum value of χ. The use is made of functions and quantities

$$f_0=f(x_0), \quad f'(x_0)=0, \quad x=\rho/\sigma_\perp,$$

$$f(x)=(1-e^{-x^2/2})/x, \quad x_0=1.585, \quad f_0=0.451;$$

$$\chi_0^{rd}(\rho)=\chi_m^{rd}\frac{f(x)}{f_0}, \quad \chi_m^{rd}=0.720\alpha N\gamma\frac{\lambda_c^2}{\sigma_\perp\sigma_z}. \tag{3.6}$$

Let us perform analogous calculations for flat $(\sigma_y\ll\sigma_x)$ beams. In this case

$$\chi_0^{fl}(\vec{\rho})=\chi_m^{fl}\exp\left(-\frac{x^2}{2\sigma_x^2}\right)\Phi\left(\sqrt{\frac{y^2}{2\sigma_y^2}}\right), \quad \chi_m^{fl}=2N\gamma\alpha\frac{\lambda_c^2}{\sigma_x\sigma_z},$$

where Φ is the error function, and averaging of Eq. (3.4) yields:

$$\bar{w}_r^{fl}=c_r^{fl}\alpha^2\left(\frac{\sigma_z}{\gamma\lambda_c}\right)^2\chi_m^{9/2}\exp\left(-\frac{16}{3\chi_m}\right),$$

$$c_r^{fl}=\frac{9\pi}{2^{15}}\simeq 8.63\cdot10^{-4}. \tag{3.7}$$

Note that expression (3.4) is very sensitive to the variation of χ_0, which in its turn depends on a probable change in the beams' shape during collision. This circumstance should be taken into account while averaging over $\vec{\rho}$.

When $\chi_m\gg1$ (supercollider) the contribution to the process under consideration, as will be shown below, is given with a logarithmic accuracy by soft photons $(\omega\ll\varepsilon)$. Neglecting the terms ω/ε, with the help of Eqs (2.5), (2.2) one obtains:

$$dn_\gamma(\vec{\rho},\omega)=\frac{\alpha\sigma_z\,d\omega}{2\sqrt{\pi}\,\gamma^2}\Gamma\left(\frac{2}{3}\right)\left(\frac{3\chi_0}{u}\right)^{2/3}; \quad \xi_{1,2}=0, \quad \xi_3=\frac{1}{2}. \tag{3.8}$$

The minimal value of frequency for which this formula is valid is determined by the condition $l_c(\omega_1)=\sigma_z$ (see Eq. (2.1)). Substituting asymptotic expression (2.4) into Eq. (2.7) and integrating over x one obtains for the probability of pair production by a polarized photon at $\varkappa=\chi\omega/\varepsilon\gg1$

$$W_e=\frac{3^{7/6}\Gamma^3(2/3)}{14\pi\Gamma(1/3)}\frac{\alpha m^2}{\omega}\varkappa^{2/3}(5-\xi_3),$$

$$w_e=\frac{3^{5/3}\Gamma^3(2/3)}{28\sqrt{\pi}\,\Gamma(1/3)}\frac{\alpha m\sigma_z}{\omega\lambda_c}\varkappa_0^{2/3}(5-\xi_3). \tag{3.9}$$

The product of probabilities (3.8) and (3.9) behaves as $d\omega/\omega$ at $\omega_m<\omega<\varepsilon$, where $\omega_m=\max(\omega_1,\omega_2)$. The value ω_1 was determined above, while ω_2 is obtained from the condition $\varkappa(\omega_2)=\chi\omega_2/\varepsilon=1$, since the probability of pair production in an external field at $\varkappa\ll1$

is exponentially small. Substituting Eqs (3.8) and (3.9) in Eq. (2.9) one obtains with a logarithmic accuracy for the total probability of the process at $\chi_0 \gg 1$ (Setting $\omega_m = \omega_2 = \varepsilon/\chi_0 = mH_0/F(\rho)$)

$$w_r(\rho) = 0.0884 \frac{\alpha^2 m^4}{\varepsilon^2} \left(\int\limits_{-\infty}^{\infty} \chi^{2/3}(t)\, dt \right)^2 \ln \chi_0 = 0.208 \left(\frac{\alpha \sigma_z}{\gamma \lambda\!\!\!^{-}_c} \right)^2 \chi_0^{4/3} \ln \chi_0 . \qquad (3.10)$$

Since the relative energy loss of the particle in the case under consideration has the form (Eq. (2.5) should be substituted in Eq. (2.2) and integrated with the factor ω)

$$\frac{\Delta \varepsilon}{\varepsilon} = 0.371 \frac{\alpha m^2}{\varepsilon} \int\limits_{-\infty}^{\infty} \chi^{2/3}(t)\, dt = 0.569\, \chi_0^{2/3} \frac{\alpha \sigma_z}{\gamma \lambda\!\!\!^{-}_c} . \qquad (3.11)$$

Eq. (3.10) can be presented in the form of

$$w_r = 0.642 \left(\frac{\Delta \varepsilon}{\varepsilon} \right)^2 \ln \chi_0 . \qquad (3.12)$$

In calculation within the power accuracy the quantity in Eqs (3.10), (3.12) is substituted by $\ln \chi_0 - 3.254$.

In the general case of arbitrary values of χ, using Eqs (2.9), (2.2) and (2.6) one obtains the following expression for the spectral probability of pair production owing to a real intermediate photon

$$\frac{dw_r}{dE} = \frac{1}{6} \left(\frac{\alpha \sigma_z}{\pi \gamma \lambda\!\!\!^{-}_c} \right)^2 \int\limits_{-\infty}^{\infty} dt_1\, dt_2 \int\limits_{E}^{\varepsilon} \frac{d\omega}{\omega^2} [\Phi_\gamma(\vec{\zeta}=0, t_1) \times$$

$$\times \Phi_e(\vec{\zeta}=0, t_2) - K_{2/3}(z(t_1))\, K_{2/3}(y(t_2))] , \qquad (3.13)$$

where E is the energy of one of the created particles. The function $\Phi_\gamma(z)$ if defined by Eq. (2.2), $z(t) = 2\omega/(3\varepsilon' \chi(t))$, while $\Phi_e(y)$ is defined in Eq. (2.6) with the following substitutions $\varepsilon \to E$, $\varepsilon' \to E'$, then $y(t) = 2\omega^2/3\varkappa(t) EE'$, $E' = \omega - E$. Let us stress, that Eq. (3.13) is not reduced to the product of probabilities for inpolarized particles. The additional term $-K_{2/3}(z) K_{2/3}(y)$ appears owing to the fact, that radiated photons are polarized, and in turn, the probability of the pair production depends on this polarization. Fig. 1 shows the spectra for various values of χ_0 ($x = E/\varepsilon$) normalized to unity. The

position of the spectral maximum is well described by the relation $\chi_{max} \simeq (3 + \chi_0)^{-1}$.

The results of the calculation of the total probability w_r averaged over the transverse Gaussian particle distribution can be presented for any value of χ_m (to within 5%) in the form of simple analytical expressions, which were derived on the basis of the asymptotic behaviour of the probabilities at small and large values of χ_m

$$\bar{w}_r^{fl} = \frac{0.369\, \chi_m^{7/2} \beta^2}{(1+\chi_m)^{3/2}} \frac{(25+\chi_m)^{2/3}}{(25+4\chi_m)^{4/3}} \ln\left(1 + \frac{\chi_m}{50}\right) \exp\left(-\frac{16}{3\chi_m}\right),$$

$$\bar{w}_r^{rd} = \frac{1.446\, \chi_m^{5/2} \beta^2}{(7+3\chi_m)^2} (10 + \chi_m)^{5/6} \ln\left(1 + \frac{\chi_m}{30}\right) \exp\left(-\frac{16}{3\chi_m}\right),$$

$$\beta = \alpha \sigma_z / (\gamma \lambda\!\!\!^{-}_c) . \qquad (3.14)$$

The quantity χ_m is determined in Eqs (3.5) and (3.7). At $\chi_m \ll 1$ the probability (3.14) is negligibly small, nevertheless it increases very rapidly with the parameter χ_m growing and at $\chi_m > 1$ the contribution of the coherent cascade becomes dominant. From Eq. (3.14) it also follows that the asymptotic expressions at $\chi_m \gg 1$ are valid only for rather large values of $\chi_m \gg 100$.

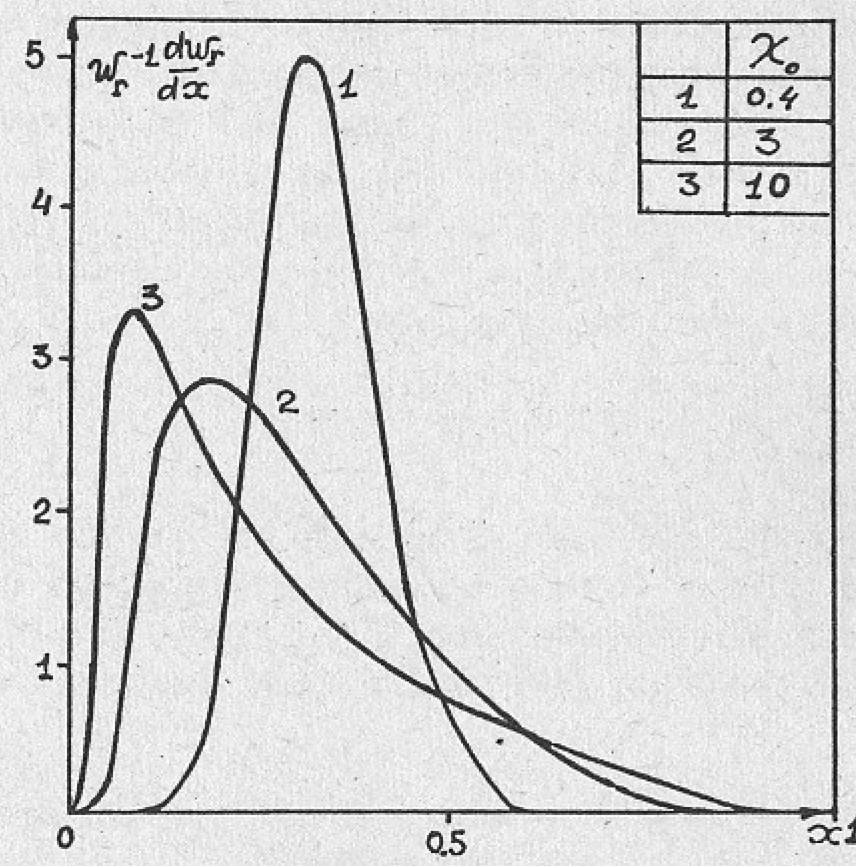

Fig. 1. Spectrum of the produced electrons in the coherent cascade ($x = E/\varepsilon$).

4. DIRECT ELECTROPRODUCTION OF ELECTRON-POSITRON PAIR

At $\chi \gg 1$ the contribution of the virtual intermediate photon to the considered process should also be taken into account. This contribution can be calculated with a logarithmic accuracy by the method of equivalent photons

$$\frac{dW_v}{d\omega} = n(\omega)\, W_e(\omega), \tag{4.1}$$

where for soft photons ($\omega \ll \varepsilon$)

$$n(\omega) = \frac{2\alpha}{\pi\omega} \ln \frac{\Delta}{q_m};$$

$$\Delta = m(1+\varkappa)^{1/3}, \qquad q_m = m\frac{\omega}{\varepsilon}\left(1+\frac{\varepsilon\chi}{\omega}\right)^{1/3} + \frac{1}{\sigma_y} \equiv q_F + q_\sigma. \tag{4.2}$$

Here $W_e(\omega)$ is the probability of the corresponding unpolarized photoprocess, Δ is the upper boundary of the transverse momentum transfer $k_\perp$ for which the condition $W_e(\omega, k_\perp) \simeq W_e(\omega, 0)$ is fulfilled, q_m determines the lower boundary of the momentum transfer, σ_y is the smallest transverse beam size. The value of Δ in Eq. (4.2) is dependent on characteristic relative angle of outgoing electron and positron in the pair photoproduction process ($\vartheta_e \sim (m/\omega)(1+\varkappa)^{1/3}$). The dependence of the quantity q_m on χ and on transverse dimension was found in Refs [11] and [9]. As will be shown below, the main contribution to the integral over ω in Eq. (4.1) is given by large (numerically) values of $\varkappa$, therefore the spectrum of equivalent photons (4.2) takes the form ($\chi \gg 1$)

$$n(\omega) = \frac{2\alpha}{3\pi\omega} \ln \frac{\varepsilon}{\omega}. \tag{4.3}$$

In this form the spectrum is independent of the external field magnitude and differes from the standard expression by the factor 1/3 only. Setting in expression for $W_e(\omega)$ (2.7) $\lambda_3 = 0$, and substituting it and the spectrum (4.3) in Eq. (4.1), one obtains:

$$dW_v = \frac{4\alpha^2 m^2 \chi}{9\sqrt{3}\,\pi^2 \varepsilon} \frac{d\varkappa}{\varkappa^2} \ln\frac{\chi}{\varkappa} \int_0^1 dx \left(\frac{1}{2} + \frac{1}{x(1-x)}\right) K_{2/3}(y), \tag{4.4}$$

$$y = \frac{2}{3\varkappa x(1-x)}.$$

The integral over $\varkappa$ in Eq. (4.4) can be calculated going over to the variable $1/\varkappa$ and using known integrals of K_ν, then the integration over x is easily performed. As a result, we obtain

$$W_v = \frac{13\alpha^2 m^2 \chi}{18\sqrt{3}\,\pi\varepsilon} (\ln\chi - C_1),$$

$$C_1 = C + \ln 4\sqrt{3} + 77/39 = 4.487\ldots, \qquad C = 0.577\ldots \tag{4.5}$$

Representation on the function $\ln\chi - C_1$ in Eq. (4.5) in the form of $\ln(\chi/\bar{\varkappa})$, where $\bar{\varkappa} \simeq 89$, confirms the fact that the contribution to the integral over x in Eq. (4.4) is given by large values of $\varkappa$ and justifies the use of the spectrum (4.3). The direct electroproduction of a pair in a constant field was considered in Refs [12, 13]. At $\chi \gg 1$ the probability has the form of Eq. (4.5), if C_1 is substituted by C_2, where, according to Ref. [13]

$$C_2 = C + \ln 2\sqrt{3} + 142/39 = 5.461\ldots \tag{4.6}$$

Hence the use of the spectrum (4.3) practically reproduces the correct result. The coincidence will be complete, if one put

$$n(\omega) = \frac{2\alpha}{3\pi\omega}\left(\ln\frac{2\varepsilon}{\omega} - \frac{5}{3}\right). \tag{4.7}$$

Let us assume, that the parameters of the both colliding beams are the same. Then, taking into accoint Eq. (4.6) ($C_1 \to C_2$) after integrating over t and averagins over $\vec{\rho}$ Eq. (4.5) we obtain for the total number of the pairs moving in one direction the following expressions:

$$N_v^{rd} = 0.097\,\alpha^3 N^2 \frac{\lambda\!\!\!^{-}_c}{\sigma_\perp} \ln\left(1+\frac{\chi_m}{460}\right),$$

$$N_v^{fl} = 0.12\,\alpha^3 N^2 \frac{\lambda\!\!\!^{-}_c}{\sigma_x} \ln\left(1+\frac{\chi_m}{820}\right). \tag{4.8}$$

In the range of validity of expression (4.8) ($\chi_m \gtrsim 10^3$) the ratio of the contributions of virtual and real intermediate photons (see Eq. (3.14)) to the considered process has the form (for round beams)

$$\delta^{rd}=\frac{N_v^{rd}}{N_r^{rd}}=0.84\,\frac{\gamma\lambda\!\!\!^{-}_c}{\sigma_z\chi_m^{1/3}}\,\frac{\ln(1+\chi_m/460)}{\ln(1+\chi_m/30)}. \qquad (4.9)$$

The ratio (4.9) can be presented in the form of $\delta\simeq l_c(\chi_m,\ u\sim 1/\chi_m)/\sigma_z$ (for l_c see Eq. (2.1)), valid at any value of χ_m. At $\chi_m\lesssim 1$ the ratio l_c/σ_z is small, providing the applicability of the constant field limit. For example, for the collider projects with an energy below 1 TeV this ratio is $l_c/\sigma_z<10^{-2}$. Thus, the contribution of the direct electroproduction can be neglected at $\chi_m\lesssim 1$.

For supercollider parameters

$$\varepsilon=5\ \text{TeV},\quad \sigma_\perp=5\cdot10^{-8}\ \text{cm},\quad \sigma_z=4\cdot10^{-5}\ \text{cm},\quad N=1.2\cdot10^{8} \qquad (4.10)$$

one obtains:

$$\chi_m=4600,\quad N_r\simeq 4.4\cdot10^{6},\quad \delta\simeq 0.22, \qquad (4.11)$$

so that the contribution of the direct electroproduction $(e\to 3e)$ becomes noticeable.

It should be noted, that in heavy particles pair production (e. g. $e\to e+\mu^-\mu^+$) the virtual photon contribution to the coherent photoproductions dominates in the near-threshold region. In fact, in this region the probability of a direct electroproduction of a pair of heavy particles in the constant field has the form [12] (for definiteness, spinor particles are considered)

$$W_v(\mu)=\frac{\alpha^2 m^2\chi^{5/2}}{192\cdot 3^{1/4}\sqrt{\pi}\,\varepsilon}\left(\frac{m}{\mu}\right)^5\exp\left\{-\frac{4\sqrt{3}}{\chi}\left(\frac{\mu}{m}\right)^2\right\}, \qquad (4.12)$$

where μ is muon mass.

At the same time, the probability of $\mu^+\mu^-$ pair production by a real photon with an energy $\omega=\varepsilon$ contains an exponential factor $\exp(-8/3\chi(\mu))=\exp(-(8/3\chi)(\mu/m)^3)$, thus, the cascade process in the near-threshold region $\chi\sim(\mu/m)^2$ can be neglected if $\mu\gg m$.

5. PAIR PRODUCTION OWING TO MIXED MECHANISM

There is also mixed mechanism of pair production, e. g. when photon is radiated in the bremsstrahlung process, i. e. incoherently, and the pair is produced by this photon in the field of oncoming beam. Bremsstrahlung cross-section in constant external field was obtained with relativistic accuracy in Ref. [14]. With regard for smallness of the transverse beam sizes this cross-section can be

presented with logarithmic accuracy:

$$\frac{d\sigma_\gamma}{d\omega}=\frac{4\alpha^3}{15m^2\omega}f_\gamma(z)\ln\left[m\sigma_y\left(1+\frac{\chi}{u}\right)^{1/3}\right], \qquad (5.1)$$

where

$$f_\gamma(z)=\frac{\omega^2}{\varepsilon^2}\left(z^4\Upsilon-3z^2\Upsilon'-z^3\right)+$$

$$+\left(1+\frac{(\varepsilon-\omega)^2}{\varepsilon^2}\right)\left(z^4\Upsilon+3z\Upsilon-5z^2\Upsilon'-z^3\right);$$

$$z=(u/\chi)^{2/3},$$

$$\Upsilon=\Upsilon(z)=\int\limits_0^\infty d\tau\,\sin\left(z\tau+\frac{\tau^3}{3}\right). \qquad (5.2)$$

Here $\Upsilon(z)$ is the Hardy function (see Ref. [16], §6.4, 10.2)

$$\Upsilon(z\gg1)=\frac{1}{z}+\ldots,\qquad \Upsilon(0)=\frac{3^{1/3}}{6}\Gamma\left(\frac{1}{3}\right). \qquad (5.3)$$

In case of $\chi\gg1$ $(z\ll1)$ using formula (5.3) we obtain

$$\frac{d\sigma_\gamma}{d\omega}=\frac{2}{5}\,\frac{\alpha^3\Gamma(1/3)}{m^2\omega}\left(\frac{u}{3\chi}\right)^{2/3}\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)\ln\left[\frac{\sigma_y}{\lambda\!\!\!^{-}_c}\left(\frac{\chi}{u}\right)^{1/3}\right]. \qquad (5.4)$$

For soft photons $\omega\ll\varepsilon$ this cross-section and the propability of pair creation by the photon in an external field (3.9) behave as $\omega^{-1/3}$, so that the main contribution to the probability of considered mixed cascade is given by the range $\omega\sim\varepsilon$ $(\varkappa\gg1)$. This justifies the use of Eq. (3.9). From Eq. (5.4) one obtains for the number of photons which have been radiated by the particle up to time t

$$\frac{dw_\gamma^{inc}(t)}{d\omega}=\frac{\sqrt{3}\,\alpha^3N}{5m^2\omega}\Gamma\left(\frac{1}{3}\right)\left(\frac{u}{3\chi_0}\right)^{2/3}\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)\times$$

$$\times\ln\left[\frac{\sigma_y}{\lambda\!\!\!^{-}_c}\left(\frac{\chi_0}{u}\right)^{1/3}\right]n_\perp(\bar\rho)\left(1+\Phi\left(\sqrt{\frac{2}{3}}\frac{t}{\sigma_z}\right)\right), \qquad (5.5)$$

here Φ is the error function, Gaussean distribution of the particles density over z was used. Using Eq. (3.9) for the photoprocess $(\xi_3=0)$ we obtain for the cascade probability owing to given mixed mechanism

$$w_p^{(1)} = \frac{\sqrt{3\pi}\,\pi}{5\varepsilon}\alpha^4\sigma_z N n_\perp(\vec\rho)\ln\left(\frac{\sigma_y}{\lambda\!\!\!^{-}_c}\chi_0^{1/3}\right). \qquad (5.6)$$

Averaging this probability over transverse coordinate with additional factor $n_\perp(\rho)$, we finally obtain for round beams

$$w_{p(rd)}^{(1)} = \frac{\sqrt{3\pi}}{20}\,\frac{\alpha^4 N\sigma_z \lambda\!\!\!^{-}_c}{\gamma\sigma_\perp^2}\ln\left(\frac{\sigma_\perp}{\lambda\!\!\!^{-}_c}\chi_m^{1/3}\right). \qquad (5.7)$$

In another mixed mechanism a photon is radiated in magnetic bremsstrahlung way, and a pair is produced at the interaction of this photon with individual particles of oncoming beam, i. e. at potential fluctuations. The cross-section of the latter process can be obtained from Eq. (5.1) by the substitutions $\omega^2 d\omega \to -\varepsilon^2 d\varepsilon$, $\omega \to -\omega$, $\varepsilon \to -\varepsilon$. As a result, we have

$$\frac{d\sigma_p}{dx} = \frac{4\alpha^3}{15m^2} f_p(z)\ln\left[\frac{\sigma_y}{\lambda\!\!\!^{-}_c}\left(1+\varkappa x(1-x)\right)^{1/3}\right], \quad x = \frac{\varepsilon}{\omega}, \qquad (5.8)$$

where

$$f_p(z) = z^4 - 3z^2\Upsilon' - z^3 + [x^2 + (1-x)^2](z^4\Upsilon + 3z\Upsilon - 5z^2\Upsilon' - z^3),$$
$$z = (\varkappa x(1-x))^{-2/3}. \qquad (5.9)$$

Since a probability of the magnetic bremsstrahlung behaves as $\varkappa^{-2/3}$ and a cross-section (5.8) behaves as $(1+\varkappa)^{-2/3}$, the contribution to the cascade probability is given by $\varkappa \sim 1$, and we cannot use an asymptotic expansion for the function $f_p(z)$ in Eq. (5.8). But with the help of the integral representation (5.2) for the function $\Upsilon(z)$, going over from the variable $\varkappa$ to the variable z, we can take first integral over z, and then over τ and x. As a result, we get

$$w_p^{(2)}(\vec\rho) = c_p^{(2)}\frac{\alpha^4}{\varepsilon}N n_\perp(\vec\rho)\,\sigma_z\chi_0^{1/3}\ln\frac{\sigma_y}{\lambda\!\!\!^{-}_c},$$

$$c_p^{(2)} = \frac{47\pi^{3/2}3^{4/3}\Gamma(2/3)}{35\cdot 2^{5/6}\Gamma(1/3)} = 9.178\ldots \qquad (5.10)$$

Averaging the probability (5.10) over transverse coordinates with the factor $n_\perp(\rho)$ we obtain (for round beams)

$$\bar w_{p(rd)}^{(2)} = 0.64 N\alpha^4\frac{\sigma_z\lambda\!\!\!^{-}_c}{\gamma\sigma_\perp^2}\chi_m^{1/3}\ln\frac{\sigma_\perp}{\lambda\!\!\!^{-}_c}. \qquad (5.11)$$

Comparing the probability (5.11) with the probability of another mixed cascade (5.7) we have at $\chi_m \gg 1$

$$\frac{\bar w_{p(rd)}^{(2)}}{\bar w_{p(rd)}^{(1)}} = \frac{4.2\chi_m^{1/3}\ln(\sigma_\perp/\lambda\!\!\!^{-}_c)}{\ln\left(\frac{\sigma_\perp}{\lambda\!\!\!^{-}_c}\chi_m^{1/3}\right)} \gg 1. \qquad (5.12)$$

In turn, the probability $w_p^{(2)}$ is negligibly small at $\chi_m \gg 1$ comparing to the coherent cascade case:

$$\frac{\bar w_{p(rd)}^{(2)}}{\bar w_r^{rd}} = \frac{5.5\,\alpha\lambda\!\!\!^{-}_c\ln(\sigma_\perp/\lambda\!\!\!^{-}_c)}{\sigma_\perp \ln\chi_m} \ll 1. \qquad (5.13)$$

The mixed cascade can compete with the coherent process considered in Section 3 only if the latter is exponentially suppressed, i. e. at $\chi_m < 1$. In this case the parameter $\varkappa$ of the radiated photon is sufficiently small and the cross-section (5.8) is weakly dependent on the photon frequency

$$\sigma_p = \frac{28}{9}\alpha^3\lambda\!\!\!^{-2}_c\ln\frac{\sigma_y}{\lambda\!\!\!^{-}_c}\left(1+\frac{396}{1225}\varkappa^2\right). \qquad (5.14)$$

Using Eqs (2.2) and (2.5) one can show that $\bar\omega^2/\varepsilon^2 < 1/6$. If the term $\frac{396\varkappa^2}{1225} < \frac{\chi^2}{20}$ being neglected, the pair production probability is factorized and the effective cross-section of the pair electroproduction can be introduced

$$\sigma_{ef}(\vec\rho) = \frac{1}{2}n_\gamma(\vec\rho)\,\sigma_p, \qquad (5.15)$$

where $n_\gamma(\vec\rho)$ is the total number of photons emitted by one electron at given impact parameter during collision. For this quantity the simple analytical expression is obtained which is valid at all values of $\chi_0(\vec\rho)$ to within 1%:

$$n_\gamma(\vec\rho) = \frac{1.81\,\chi_0(\alpha\sigma_z/\gamma\lambda\!\!\!^{-}_c)}{[1+1.5(1+\chi_0)\ln(1+3\chi_0)+0.3\chi_0^2]^{1/6}}. \qquad (5.16)$$

Note that in all linear colliders projects the value n_γ is of the order of unity. For estimate we use an expression for radiation probability which is valid at $\chi \ll 1$. At $\chi \ll 1$, using Eqs (5.14) — (5.16) we have

$$w_p^{(2)}(\vec\rho) = \frac{35}{9}\sqrt{\frac{\pi}{6}}\,\frac{\alpha^4}{\gamma}N\sigma_z\lambda\!\!\!^{-}_c\, n_\perp(\rho)\,\chi_0\ln\frac{\sigma_y}{\lambda\!\!\!^{-}_c}. \qquad (5.17)$$

Calculating an analogous asymptotics of the probability $w_p^{(1)}(\vec\rho)$ we obtain for the ratio.

$$w_p^{(1)}(\vec{\rho})/w_p^{(2)}(\vec{\rho})=4.7\cdot10^{-2}\chi_0^{3/2}\exp(-8/3\chi_0)\,.$$

Thus at all values of the parameter χ (see also Eq. (5.12)) the mixed cascade probability is determined by the incoherent photoprocess. Averaging Eq. (5.17) over $\vec{\rho}$ with the weight $n_\perp(\vec{\rho})$ we obtain

$$\bar{w}^{(2)}_{p(rd)}=c^{(2)}_{p(rd)}\,N\alpha^4\chi_m\frac{\sigma_z\lambda_c}{\gamma\sigma_\perp^2}\ln\frac{\sigma_\perp}{\lambda_c},$$

$$c^{(2)}_{p(rd)}=\frac{35}{18f_0}\left(\frac{1}{\sqrt{6}}-\frac{1}{3}\right)=0.161... \qquad (5.18)$$

For flat beams we obtain correspondingly

$$\bar{w}^{(2)}_{p(fl)}=c^{(2)}_{p(fl)}\,N\alpha^4\chi_m\frac{\sigma_z\lambda_c}{\gamma\sigma_x\sigma_y}\ln\frac{\sigma_y}{\lambda_c},$$

$$c^{(2)}_{p(fl)}=\frac{35}{54\pi^{3/2}}\operatorname{arc\,tg}\frac{1}{\sqrt{2}}=0.072... \qquad (5.19)$$

Comparing the probabilities (5.18), (5.19) to Eqs (3.5) — (3.7) and using the expressions for χ_m, we see that the mixed cascade is dominant at $\chi_m<\chi_c$

$$\chi_c=16\left(3\ln\left(\frac{\sigma_y}{\lambda_c\ln(\sigma_y/\lambda_c)}\right)\right)^{-1}. \qquad (5.20)$$

Let us consider the spectral distribution of charged particles, produced in the mixed cascade under the condition of factorization in form of Eq. (5.15). For the spectral photoproduction cross-section which has the form

$$\frac{d\sigma_e}{dx}=\sigma_0\varphi(x)\,,\qquad \sigma_0=\frac{4\alpha^3}{m^2}\ln\frac{\sigma_y}{\lambda_c},$$

$$\varphi(x)=1-\frac{4}{3}x(1-x)\,,\qquad x=\frac{E}{\omega} \qquad (5.21)$$

and the photon spectrum $n_\gamma(y)$ $(y=\omega/\varepsilon)$, the resulting distribution of the effective cross-section over the variable $z=E/\varepsilon$ (E is the energy of one of the produced particles) may be represented as

$$\frac{d\sigma_{ef}}{dz}=\frac{\sigma_0}{2}\int\limits_z^1\frac{dx}{x}\varphi(x)\,n_\gamma(z/x)\,. \qquad (5.22)$$

For soft particles, which are deflected at relatively large angles giving the main contribution to the background, one can use asymptotic expressions for the photon spectrum (2.2), (2.5) and extend the integration in Eq. (5.22) to 0. As a result ($z\ll\chi_0$, $z\chi_0\ll1$) we obtain

$$\frac{d\sigma_{ef}}{dz}=\frac{6\,\Gamma(2/3)}{5\sqrt{\pi}}\frac{\alpha^3\sigma_z}{\varepsilon}\left(\frac{3\chi_0}{z}\right)^{2/3}\ln\frac{\sigma_y}{\lambda_c}. \qquad (5.23)$$

6. INCOHERENT ELECTROPRODUCTION OF THE PAIRS

Let us discuss incoherent electroproduction, when both intermediate photons are virtual using the method of equivalent photons (see Eqs (4.1), (4.2)). As follows from Eq. (5.8), the probability of the corresponding photoprocess at $\varkappa\gg1$ decreases as $\varkappa^{-2/3}$, providing the dominant contribution of photons with the frequencies, restricted by the condition $\varkappa(\omega)=\omega\chi/\varepsilon\lesssim1$, so that $\omega_{\max}=\varepsilon/(1+\chi)$. For the frequencies $\omega\ll\omega_{\max}$ the pair production probability by two photons, as well as upper boundary of the momentum transfer Δ, determined just by this process are independent of the external field. Only the spectrum of equivalent photons changes in this range of ω with the lower boundary of frequencies being determined by the condition $\omega\omega'=m^2$ as in the absence of the field, yielding $\omega_{\min}=m^2(1+\chi)/\varepsilon$. The lower boundary of the momentum transfer $q_m(\omega)$ in Eq. (4.2) essentially depends on ω. For example, at $\omega=\omega_{\min}$ it is defined by the beam transverse sizes $q_\sigma=\sigma_y^{-1}(\sigma_y\lesssim\sigma_x)$, while at sufficiently large ω it is connected with the frequency ω and the external field magnitude

$$q_F=m\frac{\omega}{\varepsilon}\left(1+\frac{\varepsilon\chi}{\omega}\right)^{1/3}.$$

Let us introduce the parameter η, characterizing the ratio of effective momentum transfers q_F and q_σ at $\omega=\omega_{\min}$

$$\eta=\frac{q_F(\omega_{\min})}{q_\sigma}=\frac{\sigma_y\chi^{1/3}(1+\chi)^{2/3}}{\lambda_c\gamma^{4/3}}. \qquad (6.1)$$

Then the boundary of frequencies where the influence of transverse beam sizes on the equivalent photon spectrum begins is determined by the relation

$$\omega_\sigma = \omega_{min}\eta^{-3/2}. \qquad (6.2)$$

Using the expression for the cross-section of the pair photoproduction

$$\sigma_p = \frac{28\alpha^3}{9m^2}\ln\frac{m}{q_m(\omega)} \qquad (6.3)$$

and Eq. (4.2) with $\Delta = m$, we obtain in the main logarithmic approximation for the cross-section of the pair electroproduction the following result

$$\sigma(2e\to 4e) = \frac{56\alpha^4}{9\pi m^2}\int_{\omega_{min}}^{\omega_{max}}\frac{d\omega}{\omega}\ln\frac{m}{q_m(\omega)}\ln\frac{m}{q_m(\omega')}, \qquad (6.4)$$

where $q_m(\omega)$ is determined in Eq. (4.2), $\omega' = m^2/\omega$. Integrating in Eq. (6.4) we obtain with adopted accuracy

$$\sigma(2e\to 4e) = \frac{28}{3\pi}\frac{\alpha^4}{m^2}\left[L_1 L_2^2 - L_2 L_3^2 + \varphi\left(\left(\frac{m}{\omega_\sigma}\right)^{4/3}\right)\right];$$

$$L_1 = \frac{2}{3}\ln\frac{\omega_{max}}{\omega_{min}}, \quad L_2 = \ln\frac{\sigma_y}{\lambda_c}, \quad L_3 = \frac{2}{3}\ln\frac{\omega_{max}}{\omega_\sigma},$$

$$\varphi(z) = \int_0^\infty\frac{dx}{x}\ln(1+x)\ln\left(1+\frac{z}{x}\right). \qquad (6.5)$$

The term $\varphi(z)$ should be retained in Eq. (6.5) only at $z \gg 1$, when $\varphi(z) \simeq \ln^3(z/6)$. At $\chi_m \leqslant 1$ ratio of the cross-section (6.5) to the standard Landau—Lifshitz one σ_{LL} obtained in Ref. [17], to within 10% can be represented in the form of:

$$\frac{\sigma(2e\to 4e)}{\sigma_{LL}} \simeq \frac{\ln(\sigma_y/\lambda_c)}{\ln\gamma^2}. \qquad (6.6)$$

Let us consider the electron-positron pair production by two real photons. The cross-section of this process for linearly-polarized photons has the form (see, e. g. [18])

$$\sigma_{\gamma\gamma}(y) = \frac{\pi\alpha^2}{m^2 y}\left[f_0(y) + \frac{1}{2}\xi^{(1)}\xi^{(2)}f_1(y)\right];$$

$$f_0(y) = \left(2 + \frac{2}{y} - \frac{1}{y^2}\right)\ln\left(\sqrt{y}+\sqrt{y-1}\right) - \sqrt{1-\frac{1}{y}}\left(1+\frac{1}{y}\right),$$

$$f_1(y) = \frac{1}{y}\sqrt{1-\frac{1}{y}} - \frac{1}{y^2}\ln\left(\sqrt{y}+\sqrt{y-1}\right), \quad y = \frac{\omega_1\omega_2}{m^2}. \qquad (6.7)$$

Since the cross-section (6.7) is maximal at $y \sim 1$, the contribution to the pair creation with logarithmic accuracy is given by soft photons $\omega \ll \varepsilon$, the spectral distribution of which is described by Eq. (3.8). The validity of the latter is restricted from below by the frequencies, violating the condition $l_c < \sigma_z$ (see Eq. (2.1))

$$\omega_0 = \left(\frac{2\gamma\lambda_c}{\sigma_z}\right)^3\frac{4\varepsilon}{\chi_0^2}, \qquad \omega > \omega_0. \qquad (6.8)$$

Eq. (6.7) is valid as long as the parameter $\varkappa(\omega) = \varkappa(\omega_1 + \omega_2)$ is small, otherwise the invariant mass of the created pair $W \sim m\varkappa^{1/3}$ becomes large comparing with m and the cross-section decreases as $y^{-1} \sim \varkappa^{-2/3}$. It yields $\omega_m = \chi_0\varepsilon/(1+\chi_0)^2$, and the restriction (6.8) turns out to be essential for the process under consideration, since usually $\omega_0\omega_m > m^2$. Note that for $\omega_m \ll m$ the probability of the considered process is exponentially suppressed. Provided that $\omega_0 \ll m$ the integral over $y = x_1x_2$ of the cross-section (6.7) can be taken in the range $1 \leqslant y \leqslant \infty$, then

$$\Sigma_0 \propto \int_1^\infty\frac{dy}{y^{5/3}}f_0(y) = \frac{36\sqrt{\pi}\,\Gamma(2/3)}{5\,\Gamma(1/6)},$$

$$\Sigma_1 \propto \int_1^\infty\frac{dy\,f_1(y)}{y^{5/3}} = \frac{27\sqrt{\pi}\,\Gamma(2/3)}{91\,\Gamma(1/6)}. \qquad (6.9)$$

Using Eqs (3.8), (6.7) — (6.9) one can find the effective cross-section of the pair electroproduction by two real intermediate photons. The polarization effects can be neglected in this case ($\Sigma_1/\Sigma_0 \sim 5\cdot 10^{-3}$)

$$\Sigma(\vec{\rho}) = 3.08\,\alpha^4(\chi_0/\gamma^2)^{4/3}\sigma_z^2\ln\left(\frac{m}{\omega_0}\right). \qquad (6.10)$$

In the opposite case $\omega_0 \gg m$ this cross-section has the form:

$$\Sigma(\vec{\rho}) = A\alpha^4(\chi_0 m/\gamma^2\omega_0)^{4/3}\sigma_z^2\ln(\omega_0/m). \qquad (6.11)$$

Using the constant field approximation for the description of the photon radiation with the frequency $\omega \sim \omega_0$ one, gets in Eq. (6.11)

$A \simeq 2.23$. But since the frequency $\omega = \omega_0$ determines the validity boundary of this approximation, the expression (6.11) is of qualitative character.

Comparing the cross sections (6.10) (6.5) and (5.15) in the case of linear colliders with the energy of 0.5—1 TeV, we obtain that the relative contribution of the mixed cascade Eq. (5.15) and the virtual process Eq. (6.5) are of the same order, while the contribution of the two-photon real process is by one or two orders of magnitude smaller. But it can determine the pair production of heavy particles (see Ref. [15]). For the supercollider project the virtual process turns out to be the main incoherent process, and the relative contribution of the mixed cascade is by the order of magnitude smaller.

7. BOUNDARY PHOTON CONTRIBUTION TO THE PAIR PRODUCTION

In beam-beam collision side by side with magnetic bremsstrahlung photons the boundary photons are radiated [6]. They are formed mainly outside of the oncoming beam at the length $l_\omega = 2\gamma\lambda\!\!\!^{-}_c/u \equiv 2l_0/u \gg \sigma_z$ and the radiation itself (break down of coherence) occurs inside the region occupied by the field, at its boundary. The spectral distribution of these photons under the condition $l_0/\sigma_z \gg u \gg \omega_0/\varepsilon$ was obtained in Ref. [7]

$$dw_i = \frac{\alpha\, d\omega}{2\pi\omega}\left\{\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)\left[\ln\left(\frac{l_\omega}{\sigma_z}\sqrt{\ln\frac{\omega}{\omega_0}}\right) - 1.714\right] - \frac{2\varepsilon'}{\varepsilon}\right\}. \qquad (7.1)$$

At $\omega \ll \omega_0$ only the boundary photons contribution survives and the angle-like trajectory approximation is applicable. In this case the photon distribution is described by the formula (see, e. g. [10])

$$dw_i(Q) = \frac{\alpha\, d\omega}{\pi\omega}\left\{\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)\ln\frac{Q}{m} - \frac{\varepsilon'}{\varepsilon}\right\}, \qquad (7.2)$$

where Q is the total momentum transfer from the field $(Q/m \sim \chi_0\sigma_z/\gamma\lambda\!\!\!^{-}_c \gg 1)$. Two expressions (7.1) and (7.2) coincide with logarithmic accuracy at $\omega \sim \omega_0$. It should be noted, that the spectrum (7.2) differs from the equivalent photon spectrum by the argument of logaritnm only. That is why at certain conditions the boundary photon contribution to the pair production can be comparable with that of virtual one.

For example, let us consider the e^+e^- pair production by the boundary photon in the field of oncoming beam. The calculation is completely similar to that performed in Sec. 4.

Using Eqs (7.1), (2.7) and (2.9) (without the factor 1/2 in the latter) and integrating over ω and t we obtain for the probability of the process at fixed $\vec{\rho}$

$$w_i^\rho(\chi_0) = \frac{23}{24}\frac{\alpha^2\sigma_z\chi_0}{\sqrt{6\pi}\,\gamma\lambda\!\!\!^{-}_c}\ln\left(1+\frac{\xi_0}{1150}\right), \quad \xi_0 \geqslant 10^3,$$

$$\xi_0 = \frac{\gamma\lambda\!\!\!^{-}_c}{\sigma_z}\chi_0\sqrt{\ln q}, \quad q \simeq \chi_0\left(\frac{\sigma_z}{\gamma\lambda\!\!\!^{-}_c}\right)^3, \quad q \gg 1. \qquad (7.3)$$

Here the fact that the polarization degree of soft boundary photons is $\xi_3 = 1/2$ was taken into account. Averaging Eq. (7.3) over the transverse distribution of particles we have for the round beams

$$w^p_{i(rd)}(\chi_m) = 0.18\frac{\alpha^2\sigma_z}{\gamma\lambda\!\!\!^{-}_c}\chi_m\ln\left(1+\frac{\xi_m}{1360}\right), \quad \xi_m = \xi(\chi_m). \qquad (7.4)$$

The ratio of the boundary photon contribution into e^+e^- pair production to the corresponding contribution of the virtual photon (see Eq. (4.8)) for round beams is

$$\delta_i^{rd} = \frac{N_i^{rd}}{N_v^{rd}} = \frac{69}{52}\,\frac{\ln\left(1+\frac{\sigma_m}{1360}\right)}{\ln\left(1+\frac{\chi_m}{460}\right)}. \qquad (7.5)$$

For supercollider parameters $\xi_m \simeq 7.5\cdot 10^4$, $q_m = 5$, $\delta_i \simeq 2.2$. Taking into accound Eqs (4.9) — (4.11) we obtain for the total contribution to the coherent pair production the following expression

$$N_p = N_r + N_v + N_i = N_r[\delta(1+\delta_i)+1] \simeq 7.5\cdot 10^6 = \frac{N}{16}. \qquad (7.6)$$

At $q \ll 1$, the magnetic bremsstrahlung type radiation for the frequencies giving the main contribution to the pair production is suppressed **because** the interaction length $\sim\sigma_z$ is much less than the formation length. In this case the boundary photon spectrum is given by Eq. (7.2) and the corresponding pair production probability with logarithmic accuracy has the form

$$w_i^p(\chi_0) = \frac{23}{12}\,\frac{\alpha^2\sigma_z\chi_0}{\sqrt{6\pi}\,\gamma\lambda\!\!\!^{-}_c}\ln\frac{\chi_0\sigma_z}{\gamma\lambda\!\!\!^{-}_c}; \qquad \bar{w}^{p(cr)} = 0.36\,\frac{\alpha^2\sigma_z\chi_m}{\gamma\lambda\!\!\!^{-}_c}\ln\frac{\chi_m\sigma_z}{\gamma\lambda\!\!\!^{-}_c}. \qquad (7.7)$$

8. CONCLUSION

Relying on the analysis performed above, let us sum up the role of the basic mechanisms of e^+e^- pair production depending on the energy of particles in colliders. At relatively low energies $\varepsilon \leqslant 200$ GeV the quantum parameter $\chi_m \simeq 2\alpha N\gamma^2\lambda\!\!\!^{-}_c{}^2/(\sigma_x+\sigma_y)\sigma_z$ is sufficiently small $\chi_m \leqslant 10^{-2}$. In this case soft photons having energy $\bar{\omega} \simeq \varepsilon\chi/2$, are emitted in the field of the oncoming beam. Their radiation is described by the classical theory and the coherent pair production probability by these photons is exponentially small. If the formation length $l_c \sim \gamma\lambda\!\!\!^{-}_c/\chi$ is small comparing to the beam length σ_z ($\sigma_z/l_c \sim Nr_e/(\sigma_x+\sigma_y)$, where $r_e = e^2/m = 2.82\cdot10^{-13}$ cm is the classical electron radius) then the constant field limit is applicable for the description of the radiation. At $\chi \ll 1$ the incoherent processes are the main mechanisms of pair production at beam-beam collision and the magnetic bremsstrahlung photons can also be involved in them. The number of these photons per one primary particle is $n_\gamma \sim \alpha\sigma_z/l_c$, and their density can exceed that of virtual (equivalent) photons in some part of the spectrum. The estimates of electromagnetic background processes withih the framework of classical radiation theory were given in Ref. [19].

With the growth of particle energy the parameter χ increases. For the energies $\varepsilon \sim 1$ TeV its value becomes comparable to unity $\chi_m \sim 1$, then the classical radiation theory is invalid (already at $\chi = 0.1$ this theory overestimates 1.5 times the radiation intensity, see, e. g. [20].

In the region $1 \leqslant \chi \leqslant 15$ the radiation intensity to within 10% is described by the simple formula $I \simeq 2\alpha m^2\chi/15$ [21], and the mean photon energy is $\bar{\omega} \simeq \varepsilon\chi/(2+5\chi)$. The e^+e^- pair production by the emitted photon in the field of the oncoming beam becomes possible. The threshold value of the parameter χ for this process χ_c is determined by Eq. (5.20). It should be kept in mind that in the region $\chi \sim \chi_c$ the variation of χ e. g. 2 times can result in several order of magnitude change of the process probability. Such variations of the parameter χ can be caused by the change in the beam shape during collision. For this very reason all the estimates of the pair production probability, neglecting the change of the beam shape, in the

region $\chi_m \simeq \chi_c$ should be taken carefully. At $\chi > \chi_c$ the coherent cascade probability increases rapidly and at $\chi_m > 1$ this pair production mechanism becomes dominant. The photon formation length at the energy $\omega \sim \varepsilon$ is for $\chi \sim 1$ of the same order of magnitude as the pair formation length $l_c \sim \gamma\lambda\!\!\!^{-}_c$ and for the applicability of the constant field limit the condition $\gamma\lambda\!\!\!^{-}_c \ll \sigma_z$ should be satisfied.

The growth of the radiation intensity depending on the particle energy at $\chi \geqslant 10^2$ is slowing down $I \simeq 0.37\alpha m^2\chi^{2/3}$, and a photon carries on the average approximately a quarter of this energy $\bar{\omega} \simeq \varepsilon/4$. At large values of χ side by side with the coherent cascade the direct electroproduction of the e^+e^--pair becomes noticeable, and the energy fraction of this pair mainly is small $\omega \sim \varepsilon/\chi_m$ This property essentially increases outgoing angles of the produced particles after collision e. g., for the supercollider parameters $v_\perp \sim 1/30$. One should bear in mind that the soft pair formation length can become comparable with the length of the oncoming beam, even if for the main process the condition $l_c(\omega \sim \varepsilon) \ll \sigma_z$ is fulfilled

$$\frac{l_c}{\sigma_z}\left(\omega \sim \frac{\varepsilon}{\chi}\right) \sim \frac{l_c(\omega \sim \varepsilon)}{\sigma_z}\chi^{1/3} \sim \frac{\gamma\lambda\!\!\!^{-}_c}{\sigma_z\chi^{1/3}}. \qquad (8.1)$$

Thus for the supercollider project the ratio (8.1) is of the order of unity, that shows a limited validity of the constant field approximation in this region of ω. But the accuracy of the obtained expressions is completely sufficient for estimates in this case as well.

The results of the calculation of the pair production probabilities caused by the different mechanisms for the existing collider projects are presented in the Table 1. The collider parameters are conventional enough, showing a tendency to vary in time. However, the table shows obviously the scale of the effects discussed above and the role of the specific mechanism an different values of the parameter χ.

Table 1

Characteristics of the Pair Production Process

Project	σ_x (cm)	σ_y (cm)	σ_z (cm)	N	χ_m	n_γ	Number of the coherent pairs per beam: Coherent cascade	Landau–Lifshits mechanism	Mixed cascade
INP(0.5)	$3\cdot10^{-4}$	$4\cdot10^{-6}$	$7.5\cdot10^{-2}$	$2\cdot10^{11}$	0.189	1.64	$8.3\cdot10^{-6}$	$1.8\cdot10^{4}$	$4.4\cdot10^{4}$
INP(1.0)	$3\cdot10^{-4}$	$2\cdot10^{-6}$	$7.5\cdot10^{-2}$	$2\cdot10^{11}$	0.378	1.57	49	$3.6\cdot10^{4}$	$8.2\cdot10^{4}$
KEK(0.5)	$4.3\cdot10^{-5}$	—	$6\cdot10^{-2}$	$4.8\cdot10^{10}$	0.143	2.25	$2.3\cdot10^{-9}$	750	$2.5\cdot10^{3}$
CERN(1.0)	$6.5\cdot10^{-6}$	—	$3\cdot10^{-2}$	$5.35\cdot10^{9}$	0.421	1.51	34	390	$8.0\cdot10^{2}$
SLAC(0.5)	$4\cdot10^{-5}$	$3\cdot10^{-7}$	$4\cdot10^{-3}$	$1.8\cdot10^{10}$	2.394	0.85	$5.5\cdot10^{6}$	$9.6\cdot10^{3}$	$2\cdot10^{4}$
SUPER(5)	$5\cdot10^{-8}$	—	$4\cdot10^{-5}$	$1.2\cdot10^{8}$	4600	0.41	$4.4\cdot10^{6}$	$1.1\cdot10^{3}$	$1.2\cdot10^{2}$

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Electron-Positron Pair Production in Linear Colliders

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Рождение e^+e^- пар в линейных коллайдерах

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