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CONTACT PROBLEM

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НОВОСИБИРСК

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1. The problem of classical mechanics in its canonical version is to find the trajectories of bodies moving under external forces. The trajectories are thought of as lines of a certain class in Euclidean (Riemannian) three-dimensional space and parametrized with a segment of the number axis (time).

The scheme is related to an experiment via the standard procedure: space-time points are fixed using rods and clocks (the reference frame). Measurement results naturally depend not only on the motion itself but on the reference frame construction as well, whose suitability is checked up using some standard reference trajectories (see, for instance, the detailed discussion in [1]).

To substantiate this procedure they often refer to a typical experiment which, in its turn, greatly relies on the successes of technology. An experiment, however, is not free of a'priori ideas* and the estimation of the results of technology considerably depends on its aims.

To break the vicious circle one must consider the applications of mechanics and ask: what for should I find trajectories? Moreover, what for should I formulate the concept of space-time? To distinguish points?

If the trajectories of two bodies A and B intersect there may occur an interaction unique for the given problem which will be called a "contact". The nature of the interaction may be

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* According to Einstein, to measure the velocity of light one needs a theoretical concept of velocity.

different, but if the trajectories do not intersect, there is no contact. The problem is to find out whether the given bodies A and B will come into contact or not. The task, thus, is to control, if it is desirable, the occurrence of the contact choosing from the available set of influences $\mathcal{F}$ a certain $F \in \mathcal{F}$, so that each time a whole class of contact problems (CP) is solved, each with its own F. The result of the solution may be used then to advise somebody the variant of action.

The contact of A and B (denoted hereafter as $\{AB\}$ is unique only in that it is final for the given CP. There may be other interactions A and B or of any of them with other bodies, some of them are used to define $\mathcal{F}$ itself. What happens when the contact occurs cannot be found whithin the limits of the given problem, it should be known before the CP is formulated. Thus, the bodies may move under electric attraction and the final contact that is in question (collision) is also an electromagnetic (interatomic) interaction, it is only collision that is considered as the unique contact mentioned above.

Thus the introduced concept is, in fact, of informational meaning: there is the contact or there is not. A contact must be pointwise since it is analogous to the information bit thus differing from the main canonical version concept of "event", i.e. "an instant flash of a small lamp". The latter presumes some previously stated metric structure for its very definition.

A CP solution usually implies not a contact simply but a contact at a definite moment. This is meaningful if the CP is not completed - either the bodies A and/or B may experience some changes and the final contact depends on the phase of these internal processes, or the CP, when solved, is to be included in a more general problem, mechanical or some other. The situation

may be simulated introducing "multiple contact" that implies more than two bodies, that is $A, B, C\ldots - \{ABC\ldots\}$.

All the other concepts of mechanics, such as trajectories, space, time and so on are introduced only for a CP solution. The very concept of "body" is also used in a CP only.

The canonical version is supposed to use a reference frame for a CP solution, i.e. a three-dimensional lattice formed by rods with clocks at the knots. There has always been a question if the construction of the reference frame, which is only an intermediate stage to compare the given motion to some standard one, introduces into the solution of the problem anything of its own, as a real physical object. This would restrict the generality of situations that could be analysed in mechanics. The question could be answered inambiguously by comparing one motion to another directly, without any reference frame.

Let there be besides $A, B, C\ldots$ dealt with in the given CP some other bodies used as reference ones. In the simplest case, using for the time being the concepts of the canonical version, they may be taken as bodies that move uniformly and rectilinearly with arbitrary velocities in all points of space and moments of time.

The concept of contact covers reference bodies as well, i.e. their mutual contacts and those with $A, B, C\ldots$. Unlike the final contact that may result even in the bodies destruction, these measuring ones must not (in classical mechanics) even disturb the trajectories, i.e. the removal of any of them leaves others unchanged. But just like the final contact these measuring ones are to be defined in the informational way: for any pair of bodies it may be always said if there is such a contact or there is not.

The term "body" is used here in the meaning of a "material point", which implies a little body. However, as it often happens, the concept of "zero size" may be used without introducing the general concept of size (e.g. "zero mesure", zero dimension", etc.). In this case it is supposed that the occurence of a mesuring contact is the only information needed to predict the final contact and that the mesurement does not disturb the motion.

Reference bodies are empirically distinguishable one from another. They may be thought of as provided with numbers. Their totality forms a set R. Hereafter they will be denoted as X's with indexes. Bodies $A, B, C...$ come into measuring contacts with a subset of R. There are also measuring contacts $\{Ax\}$, $x \in R$, that correspond to the final contact $\{AB\}$.

A CP may be formulated like this: given measuring contacts $A, B...$ with a subset of R one must find out whether the final contact will take place. Naturally, the information presented is supposed to be minimal; exclusion of any measuring contact makes the result ambiguous.

Since the CP is useful because of the possibility to change the final contact occurrence, its solution is a form of prediction: the final contact follows the measuring ones. In other words, there must be asymmetry in the set of contacts, i.e. the order (before/after) is necessary.

Let there exist contacts $\{X_1 X_2\}$ and $\{X_1 X_3\}$. If R has been chosen properly $\{X_1 X_2\}$ will come either before or after $\{X_1 X_3\}$, or they will be simultaneous giving the double contact $\{X_1 X_2 X_3\}$. No order will be introduced if all the bodies participating in the contacts are different (e.g $\{X_1 X_2\}$ and $\{X_3 X_4\}$).

Such an order has but a weak relation to the usual concept of time. Of course, one may learn it using a clock attached to the body X_1 but the only thing it should do is to show the proper direction of time. It would be enough to mark X_1 when the first contact takes place. Then, if X_3 comes into contact with marked X_1 this will be considered a later contact. The experiment is supposed to determine only the occurrence of the bodies' mutual contacts and their order. All the other concepts introduced for the solution of a CP are to be expressed via these terms.

The set R with its contact scheme (i.e. with a rule that states what bodies come into contact and in what order) forms the so-called space of the given CP. In contrast to the canonical version it is in no way supposed to be given in advance. On the contrary, one should construct the set R in a way proper for the given CP.

The concept of the universal space is raised by the problem to find R so complete that it is sufficient to construct the solution of any CP and at the same time has no elements unnecessary for the solution.

The contact scheme unduces the standard geometrical structure in R because it specifies some subsets and their relations. The structure is examined below. Section 2 is mainly supposed to visualize the constructions. Here the Euclidean space geometry is accepted unconditionally. Reference bodies trajectories are taken as uniform and rectilinear and the scheme of the bodies' contacts is made up by the intersections of straight lines in Euclidean space. Basic geometric objects and their relations are defined by the contact scheme only. Such constructions are called

"natural". Section 3 discusses what rules of mechanics are natural. To substantiate a proposition in mechanics is to establish its naturalness, by definition.

Contact scheme in its internal terms (and the corresponding R geometry) that is not based on any a'priori model is discussed in the section 4. The section considers the conditions that the contact scheme is to satisfy so that any CP could be solved. A proper set of reference bodies is found and so the use of three-dimensional space and one-dimensional time (homeomorphic to a segment of the number axis) is substantiated. The subsets of $E^3 \times T$ are established that are good enough to represent the bodies' trajectories. The internal definition of the subsets is also presented.

2. Let there be a pair of bodies (A and B) moving in a Euclidean space. Let then a reference body X ($\exists \{AX\}$ start at some moment from A and reach B ($\{AX\} \prec \{BX\}$). All the bodies in R that leave A simultaneously and reach B do not necessarily arrive there at the same time. For any pair of such bodies X_1 and X_2 there must be an order relation: $\{BX_1\} \prec \{BX_2\}$ or $\{BX_2\} \prec \{BX_1\}$ or there exists $\{BX_1X_2\}$. In this special case there is no need to divide distance by time in order to introduce the concept of "velocity" and to regulate the magnitudes of velocities: the order relation is just a property of a certain subset of R formulated in terms that express the occurrence and order of the contacts. The proposition that the bodies move along straight lines in $E^3 \times T$ is not significant in this case. In every such situation one can establish an order, but generally X_1 and X_2 do not always move the same way. Nevertheless, it will be always said that X_1 is faster than X_2 if $\{BX_1\} \prec \{BX_2\}$. This definition is convenient especially if there exist the fastest body: no matter what the trajectories may be, starting from A simultaneously with other bodies it reaches B sooner than others (absolute extremum). The bodies for R are chosen by the experimenter. For a CP to be always solved in R the set must include the fastest body for each of the special cases. Indeed, communication between A and B through other bodies is necessary already for the determination of the initial state, without which the CP does not exist. One must be sure that with any influence from $\mathcal{F}$ upon, say, A the contact $\{AB\}$ that is in question will not occure before the measuring contacts that determine the initial state. On the other hand, the maximal speed cannot be infinite:

$\{Ax\}$ and $\{Bx\}$ cannot be simultaneous since then, according to the contact ordering rule, there must also be $\{AB\}$ which in fact is only to be found. The fastest bodies will be hereafter called photons, though not due to their electromagnetic nature. They have their own definition in R as top velocity bodies.

At the first glance, it seems that Nature need not take into account the way an observer chooses bodies for R and may provide bodies faster than photons. But then the observer will not take them as bodies since the concept of body is used only for a CP solving.

For example, let it be not A that comes into contact with B but some A_1 which is "very much like" A (the so-called "phase" speed that may be infinite). A and A_1 are never considered to be the same body though for the final contact results the difference between the same body but at another point and a quite another body may be insignificant. In situations with such phase velocity it is impossible to affect the final contact $\{AB\}$ by accelerating A and so they are of no interst for mechanics. Thus top velocity is at the basis of the very idea of moving bodies since one implies the same body at all phases of the motion. In the canonical version this is as if disguised. Using a reference frame one seems to be able to obtain an instant photography of the whole space, the top velocity being introduced as an independent experimental result that may turn out to be wrong. The general examination of the CP shows that this experiment only reveals the contents of the basic concepts.

The special role of photons in R is used to specify geometrical objects and quantities unequivocally reproduced in the experiment. As a limiting element the photon is defined in a unique way and its contacts overtake all the others. Photons will be

denoted by V's with indexes.

The main part of many constructions is a double contact concept of the kind of $\{X_1X_2X_3\}$. Let a pair of photons $V_{12}^{(1)}$ and $V_{13}^{(1)}$ leave X_1 simultaneously and go toward X_2 and X_3 correspondingly ($\{X_1V_{12}^{(1)}V_{13}^{(1)}\} \prec \{X_1X_2X_3\}$). When the photons reach X_2 and X_3 there start backward to X_1 the "reflected" photons $V_{12}^{(2)}$ and $V_{13}^{(2)}$ forming new contacts $\{X_1V_{12}^{(2)}\} \succ \{X_2V_{12}^{(1)}V_{12}^{(2)}\} \succ \{X_1V_{12}^{(1)}\}$ and $\{X_1V_{13}^{(2)}\} \succ \{X_3V_{13}^{(1)}V_{13}^{(2)}\} \succ \{X_1V_{13}^{(1)}\}$. Such photon oscillations go on up to $\{X_1X_2X_3\}$. Let there be n_{12} oscillations between X_1 and X_2 that started at some arbitrary moment. The quantity n_{12} corresponds to another quantity n_{13} the number of oscillations between X_1 and X_3 so that $\{X_1V_{12}^{(n_{12})}\} \prec \{X_1V_{13}^{(n_{13})}\}$ and $\{X_1V_{13}^{(n_{13}+1)}\} \succ \{X_1V_{12}^{(n_{12})}\}$. The quantity $X_1\gamma_{X_2X_3} = \lim_{n_{12}\to\infty} n_{12}/n_{13}$ is introduced, where the limit corresponds to approaching $\{X_1X_2X_3\}$. Using the canonical version concepts one can easily calculate:

$$X_1\gamma_{X_2X_3} = \lim_{n_{12}\to\infty} n_{12}/n_{13} = \frac{\ln\frac{1-\beta_3}{1+\beta_3}}{\ln\frac{1-\beta_2}{1+\beta_2}}\ ;\quad \beta_a = \sqrt{\beta_{a\nu}^2}\ ; \quad (1)$$

$$a = 1, 2, 3;\quad \mu = 1, 2, 3$$

where $\beta_{a\mu}$ denote $v_{a\mu}/c$, c is light velocity and $v_{2\mu}$ and $v_{3\mu}$ are the components of the velocities of the bodies X_2 and X_3 in the reference frame where X_1 is at rest: $\beta_1 = 0$.

The limit has been defined correctly since it does not depend either on the beginning of the oscillation count, nor on the precise time position of the (n_{12})-th photon arrival bet-

ween the (n_{i3})-th and the $(n_{i3}+1)$-th ones.

If the trajectories are not uniform and rectilinear, but their coordinates can be presented through left-differentiable time functions, then the instant velocity values in the space-time point corresponding to the contact $\{X_1 X_2 X_3\}$ are to be used in (1).

Formula (1) can be presented in the four-dimensional form, which is convenient if one needs to obtain $X_2 \gamma_{X_1 X_3}$ besides $X_1 \gamma_{X_2 X_3}$ (then there is no need to calculate them in different reference frames):

$$X_1 \gamma_{X_2 X_3} = \frac{\ln \dfrac{u_{1i} u_{3i} - \sqrt{(u_{1i} u_{3i})^2 - 1}}{u_{1i} u_{3i} + \sqrt{(u_{1i} u_{3i})^2 - 1}}}{\ln \dfrac{u_{1i} u_{2i} - \sqrt{(u_{1i} u_{2i})^2 - 1}}{u_{1i} u_{2i} + \sqrt{(u_{1i} u_{2i})^2 - 1}}} \qquad (2)$$

where u_{ai} denote the four-dimensional velocities of X_a.

The formulas obtained correspond to the case when the photon oscillation process precede $\{X_1 X_2 X_3\}$. If one is interested in the trajectories intervals that follow the common contact, it is necessary to continue them to the area preceding the contact, to find γ there and to use this value for the whole vicinity of $\{X_1 X_2 X_3\}$. The same value of γ can be obtained (but for uniform rectilinear trajectories only) if one starts counting oscillation number at a moment after $\{X_1 X_2 X_3\}$ and goes on up to $t = \infty$. Oscillation numbers are infinite in both cases though in the first case the duration of the process is finite.

Formula (1) presents the correspondence between γ values, obtained by counting oscillation numbers, and β as measured

by means of rods and clocks. But in the first case determination of γ is given in terms expressing only the occurrence and order of the contacts. In other words, it is a natural procedure. It is meaningful even when there are no metric relations at all being a direct experimental procedure.

In the general situation, however, both the possibility of evaluating numbers n and the existence of limits are not given a'priori, but must be guaranteed by the proper choice of the contact scheme in R. In the canonical version the limits exist due to the smoothness of the trajectories that are used in standard constructions.

Hence, if it is desirable to use this construction (and it is a basic one), one must choose R so that some definite conditions hold for the mutual contacts scheme (see section 4). In other words, one is to construct a space with sufficient resolution and continuity in order to solve the CP.

Given $X_1 \gamma_{X_2 X_3}$ the quantity $X_2 \gamma_{X_1 X_3}$ may be of different values corresponding in the reference frame where X_1 is at rest to different directions of the velocity of X_2 with X_3 given. Among these values in the canonical version there must be minimum and maximum (perhaps, infinite) ones. They are realized when X_1, X_2 and X_3 belong to a straight line in E^3, X_2 and X_3 being at the same or at opposite sides as regards X_1. If now, leaving aside any interpretation in $E^3 \times T$, one imagines a contact scheme with the same extremum properties, there arises a possibility to introduce a linear (vector) space $K(X_0|X_0')$ associated with any contact $\{X_0 X_0'\}$ in natural terms. It contains all X's taking part in $\{X X_0 X_0'\}$. Each pair $X_1, X_2 \in K$ corresponds to $X_\Sigma = X_1 + X_2$, $X_\Sigma \in K$ according to the "parallelogram rule": 1) $X_1 \gamma_{X_0 X_\Sigma} = X_2 \gamma_{X_\Sigma X_0}$ and 2) among all X'_Σ

satisfying 1), one takes X_Σ with the minimum value of $x_0\gamma_{x_\Sigma x'_\Sigma}$.

The element $\theta X \in K$ is defined for each $X \in K$ and a real number θ : 1) $x_0\gamma_{x(\theta x)}=\theta$ and 2) among all $(\theta x)'$ satisfying 1), one takes θX with the minimum value of $x\gamma_{x_0(\theta x)'}$. The element $(-X)$ opposite to X is defined by $x_0\gamma_{(-x)x}=1$ and $x\gamma_{(-x)x_0}$ is minimum.

The double contact concept can be generalized for the important case of two pairs of simple contacts connected by a photon. If for four bodies X_1, X_2, X_3 and X_4 there exist $\{X_1X_2\}$ and $\{X_3X_4\}$, and for some V there exists $\{X_1X_2V\}\prec\{X_3X_4V\}$ one may send a photon V' from X_1 to X_3 starting the counting of oscillation number n_{12} at the same moment and n_{34} when V' reaches X_3 and so define the quantity $x_1x_3\gamma_{x_2x_4}$ as the limit for n_{12}/n_{34} when the contacts $\{X_1X_2\}$ and $\{X_3X_4\}$ are approached.

The multiple contact concept is the basis for specifying various classes of subsets in R, primarily the "spheres". For the given $\{X_0X'_0\}$ sphere $S(X_0|X'_0)$ with the centre $X_0|X'_0$ consists of all elements coming into contact $\{X_0X'_0S(X_0|X'_0)\}$ with the centre, $x_0\gamma_{x_1x_2}$ being equal 1 for each pair $X_1, X_2 \in S(X_0|X'_0)$.

According to the canonic version concepts $S(X_0|X'_0)$ consists of all bodies moving with equal velocities toward X_0 from all directions in the reference frame where X_0 is at rest at the moment of the contact. The sphere "draws up" to $\{X_0X'_0\}$ and then, after the contact, "expands" from it.

Mutual positions of different spheres with the common centre $X_0|X'_0$ is characterized by γ numbers. The number $x_0\gamma_{x_1x_2}$ $(X_1 \in S_1, X_2 \in S_2)$ is determined for a pair $S_1(X_0|X'_0)$ and $S_2(X_0|X'_0)$. The set Λ consisting of all spheres with the

centre $X_0|X'_0$ can be defined in a unique way if to use the contact scheme that is defined by straight lines intersections in $E^3\times T$. This set can be parametrized with the number $x_0\gamma_{x_1x_\alpha}$ if a "unit" sphere S_1 is marked in R. The set $K(X_0|X'_0)=\bigcup_\alpha S_\alpha$ contains all elements of all spheres making up Λ.

The extremum relation used above to introduce the algebraic structure into K may also be used to uniformly subdivide S. For this one takes n elements of the sphere $X_1 \dots X_i \dots X_n$ and looks for maximum $x_e\gamma_{x_mx}=\tilde{\gamma}(X_i)$ $(e\neq m;\ e, m=1\dots n)$ among all the values of e and m. The set (X_i) is said to be distributed uniformly in the sphere if $\tilde{\gamma}(X_i)$ is minimum, i.e. for any other n elements $(\bar{X}_i)$: $\tilde{\gamma}(X_i)\leq\tilde{\gamma}(\bar{X}_i)$.

A sequence of subdivisions of spheres $S_\alpha \in \Lambda$ as $n\to\infty$ together with the usual partition of the parameter $x\gamma_{x_1x_2}$ variance interval is used for the natural definition of the integration in K (section 3).

The concept of bodies with equal velocity vectors though positioned differently is significant in the canonical version. The analogous definition can be obtained in natural terms. Consider some X, a pair of photons V_1 and V_2 $(\{XV_1\}\prec\{V_1V_2\}\prec\{XV_2\})$ and a pair of auxiliary elements X_1 and X_2 $(\{V_1V_2\}\prec\{X_1V_2\}\prec\{XV_2\}, \{XV_1\}\prec\{X_2V_1\}\prec\{V_1V_2\})$. The element $\bar{X}$ is said to be "parallel" to X $(\bar{X}\parallel X)$ if $\exists\{\bar{X}V_1X_2\}\prec\{\bar{X}V_2X_1\}$ and $x_1x_2\gamma_{\bar{x}x}=1$.

Using this construction, one has to consider sufficiently long time intervals in order to construct directly $\bar{X}$ that is far from X in the canonical sense. It is possible to avoid this inconvenience using sequences of the type $X_1\parallel X, X_2\parallel X_1 \dots X_n\parallel\bar{X}$ and choosing X_i close to X_{i+1}.

The similar construction fits as the basis of another method to introduce linear spaces $K(X_0|X'_0)$.

Straight lines $\ell(X1X')\,(\exists\{XX'\})$ analogous to those in E^3 are introduced as sets of elements of the type θX. Similar to spheres, the lines move with arbitrary transverse velocities (in the canonical interpretation).

If the bodies $A, B, C...$ move "freely", i.e. like reference bodies, then $\{ABC...\}$ can be predicted using the intersection scheme in R known from the previous construction (it is assumed that A moves like X or is kinematically equal to X, if for any X' the presence or absence of $\{XX'\}$ brings about the same as for $\{AX'\}$).

The influence of $F \in \mathcal{F}$ on A (F is supposed not to influence R) changes its motion so, that A is no longer kinematically equivalent to any reference body. One codes $\mathcal{F}$ corresponding to any F some contact scheme of the type $\{AM\}$ $M \subset R$. Having in mind canonical models, it is worthwhile to define "chains" in R made up of the parts of reference trajectories and constituting a sequence $\{X_0X_1\} \prec \{X_1X_2\} \prec ....$ To every chain there corresponds a body trajectory kinematically equivalent to X_K of the K-th link.

The differential of a curve in $E^3 \times T$ is naturally associated with the set of uniform and rectilinear trajectories. The characteristic distinction between a straight line and a curve is natural, provided, as it is assumed, the basic set of lines called "straight" was stated in advance and provided with a usual intersection scheme. Each pair of lines intersects at most in a single point, whereas each pair of points on a curve belongs to a straight line crossing the curve. That is why there exists the natural straight line approximation for the A trajectory structure "at a point". One chooses a sequence $(X_K)\ (K=0,1,...;\ \exists\{A(X_K)\};$ $\exists\{AX_K\} \prec \{AX_{K+1}\} \prec \{A(X_K)\}$ and constructs the associated

sequence $A\gamma_{X_0X_K}$, the oscillation number count for any K starting after the first contact $\{AX_K\}$. The limit $A\gamma_{X_0X_K} \to 0$ determines the tangent element to the A trajectory for the $\{AX_0\}$ contact.

The natural construction of an integral curve, i.e. finding the limit of the chain sequences, is based on using oscillation number as an argument. Let photons oscillate between bodies A_1, A_2, A_3 (that move in E^3) starting at some moment. If $\eta_{A_1A_3} < \eta_{A_1A_2}$ it is considered that at this interval A_2 is closer to A_1 than A_3. When $\eta_{A_1A_2} \to 0$ (still being much more than unity, if the trajectories are close) the concept of closeness becomes local. At the trajectories intersection points η numbers are infinite, and one is to examine the vicinities changing the starting and final moments of the photon counting. A_2 trajectory is said to be closer to A_1 one than A_3 one, if it is closer everywhere.

The smallness of two chains at a given interval may be compared by counting the number of links for the given number of mutual oscillations. At any interval the smaller chain has more links, their number being much more than unity.

Chains, as a scheme, are sufficient to represent trajectories, as there always exists a chain connecting an arbitrary initial link with any contact chosen beforehand, the latter coming after the corresponding photon contact.

Any subset of R that is the limit of a chain sequence is, by definition, the trajectory of a body in mechanics. The usefulness of such a definition will be discussed in section 4. But it is clear already now that the very possibility to ascribe a definite trajectory to a body pressupposes that the measuring contacts do not influence the motion.

3.One more set of bodies $\mathcal{P}$ (probe bodies) is to be introduced in order to code $\mathcal{F}$ in natural terms. Unlike X's bodies $\rho \in \mathcal{P}$ react to external influences. In this respect they are similar to bodies $A, B\ldots$ whose contact is discussed in the problem. On the other hand, while $A, B\ldots$ are given, the set $\mathcal{P}$ just like R is to be chosen specially for the CP solution. Recommendations for the choice of a proper $\mathcal{P}$ suitable for solving a wide enough class of CP's is the subject of dynamics (i.e. classification of influences), whereas a proper choice of R is the subject of geometry.

Like $A, B\ldots$ elements of $\mathcal{P}$ come into measuring contacts with R, the whole information about the set $\mathcal{F}$ is coded by transition from one contact to another. The probe bodies may be thought of as noncontacting, as the contacts of the type $\{X\rho\}$ $(X \in R, \rho \in \mathcal{P})$ are known and may be used to compare the motion of A with that of some subset of $\mathcal{P}$.

The trajectories $A, B\ldots$ must be known fully as it is they that predict the final contact of these very bodies. Contrary to this, it is only the local behaviour that is essential for $\rho \in \mathcal{P}$ since one may compare the motion of, say, A with various ρ's: ρ's pass A from one to another as in "relay race".

It is useful to distinguish the zero influence in $\mathcal{F}$ with respect to $\mathcal{P}$. It is characterized by the kinematic equivalence of R and $\mathcal{P}$ that is violated if $F \neq 0$. By a special empirical procedure with the help of $\mathcal{P}$ each nonzero F determines in R elementary links. For an arbitrary $\{\rho X\}$ an element X_ρ tangent to ρ at $\{\rho X\}$ is found. After this contact ρ and X_ρ trajectories diverge under the influence of F. Then $X' \left(\exists \{X' X_\rho\} \succ \{\rho X_\rho\},\ \exists \{\rho X'\} \succ \{X' X_\rho\} \right)$ is

chosen. The set $\Gamma(X')$ containing all such elements having a common contact $\{\rho \Gamma(X')\}$ includes X'_ρ $(\rho \gamma_{X' X'_\rho} = \min[\Gamma(X')])$. The transition $X_\rho \to X'_\rho$ locally corresponds to ρ at $(\{\rho X_\rho\}, \{\rho X'_\rho\})$.

Since the set R is supposed not to contain unnecessary elements, each $X \in R$ may turn out to be kinematically equivalent to A in some CP. Hence, for every $\{X_\rho X'_\rho\}$ $(X_\rho, X'_\rho \in R, \exists \{X_\rho X'_\rho\})$ there must be an element in $\mathcal{P}$ tangent to X_ρ and X'_ρ. It would seem that all transitions of the kind $X_\rho \to X'_\rho$ should be given in order to code $\mathcal{F}$. This would be inconvenient as then one would have to include in $\mathcal{P}$ all the bodies "the same as A", i.e. dynamically equivalent to A *. It turns out, however, that it is possible in natural terms to reduce $\mathcal{P}$ by one dimension.

Locally A and ρ trajectories are compared by means of element X_ρ kinematically equivalent to the given A and the chosen ρ: X_ρ is tangent to A and ρ at $\{X_\rho \rho A\}$ (the denotation $\{X_\rho \rho A\}$ is meaningful since the contact A with ρ is indirectly fixed through X_ρ, though it is not defined by itself). Let there be found for this case the element X'_ρ : transition $X_\rho \to X'_\rho$ locally corresponds to ρ. One has to find the transition $\dot{X}_\rho \to X'_A$, i.e. the influence of F upon A from the known influence of F upon ρ. This may be done in natural terms not only when ρ does not differ from A, but also when there is a subset in R containing X'_ρ, whose elements may be distinguished naturally. According to sec.2 there is such subset in R - the straight line, i.e.

* It is enough to imagine that to predict the trajectory of a very heavy body one would have to use similar heavy probe bodies everywhere. This would considerably narrow the application of mechanics.

one-parameter subgroup of $\theta X'_P$ elements from $K(X'_P|X_P)$.

The choice of X'_A could be explained otherwise as follows. The given transition $X_P \to X'_P$ is supplemented with an auxiliary one X' : $\exists\{X_P X'\} \prec \{X_P X'_P\}$, $\exists\{X'_P X'\} \prec \{X_P X'_P\}$. Then X'_A is found: $\exists\{X'_A X_P X'_P\}$, $\exists\{X'_A X'\} \prec \{X_P X'_P\}$, $X_P \gamma_{X'_A X'_P} = \theta$, where θ is a given number. The process of finding X'_A that determines the A motion locally is a natural construction equivalent to Newton's second law. Indeed, the law enables one to find A acceleration using the given force and mass. The force, in its turn, can be found from the acceleration of probe bodies. which mostly differ in their masses from A .

In fact, to find A acceleration one must fix P versus A mass ratio. According to Newton's law, P and A accelerations must not differ by "more" than a scalar parameter, i.e. they must have the same direction.

The natural representation explains the real reason of this habitual fact - only such situatiohs can be expressed in terms of a contact scheme.

Besides, $\mathcal{P}$ obtains an important property of uniqueness. Only one element of $\mathcal{P}$ is needed at each space-time point and at each velocity: if $X_P = X_{P'}$, then $X'_P = X'_{P'}$.

From the view point adopted here, these considerations are to be taken as the foundation on Newton't method: one motion is compared to another directly, since the procedure can be expressed in natural terms.

To find the limit, as chains become smaller, one deals with velocity increments that are small as compared to C . Then it follows from formula (1): $x_P \gamma_{X'_A X'_P} \approx \frac{v_P}{v_A}$ where v_P and v_A denote absolute values of P and A velocities. Thus the ratio of the masses is the inverse ratio of the velocities, i.e. it

is the momentum increments that are equal for A and P .

Left differentiableness that is a necessary condition for the previous construction, must be considered as a requirement that is to be fulfilled with the components of the scheme ($\mathcal{P}$, $\mathcal{F}$, R and also $A, B\ldots$) in order to use it in mechanics. The narrowing of the class of situations fit for analysis is compensated by the efficiency of investigation.

The next step in this direction will be the selection of problems that allow to present influences at some basis in R .

Choosing three elements $(X_1 X_2 X_3) \equiv (X_a)$ $(a=1,2,3; \exists\{(X_a)\})$ not belonging to a straight line, one may fix any element X $(\exists\{X(X_a)\}$ with its decomposition coefficients in the basis (X_a).

The numbers $x_a \gamma_{X X_B}$ may be taken as the coefficients (the numbers $x_a \gamma_{X_B X_C}$ are known as (X_a) has already been chosen).

An influence F can be presented by means of a basis if $\mathcal{F}$ locally conserves the ratios of oscillation numbers, i.e. if $x'_a \gamma_{X'_B X'_C} = x_a \gamma_{X_B X_C}$, (X_a) being chosen arbitrary. The set of forces satisfying this condition in the canonical version may be found from formula (2): $x_1 \gamma_{X_2 X_3}$ do not vary if scalars $u_{1i} u_{2i}$ and $u_{1i} u_{3i}$ remain unchanged. This is the only case possible in a homogeneous R (there being no explicit dependence on the contact "number").

Hence, the influence of F must be such that in the first order with respect to independent variable it would not change the scalar products of four-dimensional velocities of the bodies having a common contact.

It is Lorentz force that satisfies this requirement. Let u_{1i} and u_{2i} be the initial velocities in a link, u'_{1i} and u'_{2i}

- the final. Then

$$u'_{1i}u'_{2i} \approx (u_{1i} + \frac{e_1}{m_1c^2}F_{iK}u_{1K}\,dS_1)(u_{2i} + \frac{e_2}{m_2c^2}F_{iK}u_{2K}\,dS_2) \approx$$

$$\approx u_{1i}u_{2i} + \frac{e_1}{m_1c^2}F_{iK}u_{1K}u_{2i}\,dS_1 + \frac{e_2}{m_2c^2}F_{iK}u_{2K}u_{1i}\,dS_2$$

The term linear in dS will equal zero if $\frac{e_1}{m_1} = \frac{e_2}{m_2} = \frac{e}{m}$ $dS_1 = dS_2 = dS$ as in this case after changing the sum indexes in the second term one obtains

$$u'_{1i}u'_{2i} \approx u_{1i}u_{2i} + \frac{e}{mc^2}\,u_{1K}u_{2i}(F_{iK}+F_{Ki})dS = u_{1i}u_{2i}$$

due to the antisymmetry of F_{iK} for the Lorentz force. But it is in the natural representation of $X \to X'$ transitions that all dS_i are equal, because for the three (X_a) the beginning of counting photon oscillation numbers to measure the influence result is automatically determined in a symmetric way. Elements (X_a) belong to a sphere (perhaps, not to a single one). It is reasonable to begin the oscillation numbers counting at the time when a photon starts from the sphere centre that is fixed in R as the sphere has been already chosen. In any sphere radial velocities of its elements are equal in the reference *frame* where its centre is at rest. Hence, the corresponding dS_i are equal too. Thus, Lorentz force corresponds to the natural procedure of finding the influence result at any link of a chain. First, the basic three (X_a) is chosen. Second, a sphere containing the basis is chosen, and the initial photon from its centre coming into contact with A at the end of the preceding link is found. Third, the elements (X'_a) as tangent to (P_a) are found (as there exists $\{(X_a)(X'_a)\}$, the three (X'_a) can be fixed by numbers

γ in the (X_a) basis too; the conservation of oscillation number ratios at $X_a \to X'_a$ transitions guarantees that if elements chosen as (P_a) have equal e/m, then F is Lorentz force). Fourth, X_A tangent to A before the common contact $\{A(X_a)(X'_a)\}$ is found and its decomposition coefficients γ in the basis (X_a) are determined; the element X' having the same γ's in the basis (X'_a) corresponds to the element P with the same e/m as the basic ones. Fifth, the element X'_A tangent to A at the next link is found as described above using the given $\theta = (e/m)_A/(e/m)_P$ by Newton's construction.

When comparing X'_a decomposition in the basis (X_a) with the canonical version, one must take into account that in the scalar $u'_{ai}u_{ai}$ in formula (2), determining $X_a\gamma X_\beta X'_a$ one must find the terms up to the second order with respect to dS, which is not necessary in $u_{ai}u_{\beta i}$ $(a \neq \beta)$. After taking the root and expanding the logarithm as a Taylor series these turn out to be the first-order terms, since u_a are perpendicular to du_a/dS:

$$u'_{ai}u_{ai} \approx (u_{ai} + \frac{du_{ai}}{dS}dS + \frac{1}{2}\frac{d^2u_{ai}}{dS^2}dS^2)u_{ai} =$$

$$= -1 + \frac{1}{2}u_{ai}\frac{d^2u_{ai}}{dS^2}dS^2 = -1 + \frac{1}{2}\left(\frac{e}{mc^2}\right)^2 F_{mn}F_{nq}u_{am}u_{aq}\,dS^2;$$

$$\ln\frac{u'_{ai}u_{ai} + \sqrt{(u'_{ai}u_{ai})^2 - 1}}{u'_{ai}u_{ai} - \sqrt{(u'_{ai}u_{ai})^2 - 1}} \approx \frac{e}{mc^2}\sqrt{F_{mn}F_{nq}u_{am}u_{aq}}\,dS$$

In the reference frame where X_a is at rest, $u_{ai} = (0,0,0,i)$, and hence in the last formula under the root sign $-F_{4n}F_{n4} = E^2$ where E is the absolute value of the electric field strenth

in this frame. Of course, this may be directly obtained from formula (1) which is, however, inconvenient for determining numbers $x_a \gamma x_\beta x'_a$.

If $\beta \ll 1$, the Lorentz force properties that make it representative in the contact scheme are obvious in three-dimensional notations. The electric field adds equally to the velocities of P_a and being equivalent to the local change of the reference frame does not change the oscillation numbers. The magnetic field does not change the scalar product in the first order in dt due to the mutual compensation of the terms with the scalar triple products: $\bar{V}_1(\bar{V}_2 \times \bar{H}) + \bar{V}_2(\bar{V}_1 \times \bar{H}) = 0$.

It is useful to replace canonical relations by the natural ones, primarily, to substantiate mechanics, i.e. to prove that its main concepts do not depend on intermediate objects - rods and clocks. One the other hand, oscillation number counting provides a definite empirical result even if the basic trajectories of bodies from R are not uniform and rectilinear, though the contact scheme may be the same.

Local correlation between A and P established in a certain way does not necessarily mean a CP solution in its original sense, even though this correlation may be known all along the A trajectory and the solution may be found by integration. Indeed, the solution that is learned only at the end of the motion does not predict the final contact: by the moment the solution is obtained the occurrence of the contact is evident by itself. This is not useful for the main purpose - to influence the final contact occurrence by the proper choice of F.

Hence, one needs to further restrict the set $\mathcal{F}$ that is to be analyzed. Besides codifying in R the set must make it possible to restore every $F \in \mathcal{F}$ from the initial datas; known initially only for some subsets $\mathcal{P}' \subset \mathcal{P}$, F must be extended to others by some regular procedure. Investigation of such extensions is the subject of the "field theory".

Correlations of A and P's as well as different P's have been established up to now only for the case of their common contact. Now the analysis should be extended to a more general situation.

As mentioned in the section 2, simple natural representation assumes not only multiple contacts, but a set of contacts as well, connected by photons. In this case the starts and the finishes of oscillation counting can be coordinated as well.

Let a pair of contacts be marked - $\{XX_1\} < \{XX_2\}$. In the set $K(X|X_1)$ every body $\tilde{X}$ may be communicated to $\{XX_2\}$ with a photon: as X is not a photon there exists $\tilde{\nu}$ $(\{X\tilde{X}X_1\} < \{\tilde{\nu}\tilde{X}\} < \{\tilde{\nu}XX_2\}$. In particular, the set of photons that start from X simultaneously with $\{XX_1\}$ posesses this property too. This set forms the limit sphere in $\Lambda(X|X_1)$ denoted as $S^+(X|X_1)$, corresponding to the light cone of the future with respect to the contact $\{XX_1\}$. Analogously, there exists for $\{XX_2\}$ the sphere $S^-(X|X_2)$, corresponding to the light cone of the past, that has contacts with $S^+(X|X_1)$.

The set of elements $\bar{X}$ $(\bar{X} \| X; \exists \{\bar{X} S^-(X|X_2) S^+(X|X_1)\})$ is initial for the extension of F. Given the $\bar{X} \to \bar{X}'$ transitions, one can find that of $X \to X'$: Each $\bar{X} \to \bar{X}'$ must influence $X \to X'$, if F can be extended through the final intervals. On the other hand, transitions inside the sphere do not influence each other; $\bar{X}_1 \to \bar{X}'_1$ cannot influence $\bar{X}_2 \to \bar{X}'_2$ since they both influence $X \to X'$

and the existence of an "extra" way contradicts to the limit character of photons.

Hence, the only possibility in this case is to define $X \to X'$ using a simple arithmetic average for some quantities that are given at the sphere. Thus, it is necessary to define some sort of integration on the sphere $S^-(X|X_2)$. This requires the iniform subdivision of the sphere. However, this cannot be done directly, since at the light cone γ numbers are meaningless. So, a limit process is to be defined. A sphere $S_1(X|\tilde{X})$ from the set $\Lambda(X|\tilde{X})$ is chosen and a sequence $S_1 \dots S_n \dots, S_n \in \Lambda(X|\tilde{X})$ is constructed so that ${}_X\gamma_{X_n X_1} \to 0$ as $n \to \infty$. The sphere S_n may be represented by m elements being located uniformly on it (see sec.2). If, for example, to choose $m = n^\delta$ (δ be positive and large enough), then the subdivisions become smaller as $S_n \to S^-$ inspite of the increasing of the "radius". This construction will be taken as the integral subdivision sequence on the sphere $S^-(X|X_2)$. The limit will be used for the integral; it is determined in a unique way in $E^3 \times T$ as $\{XX_1\}$ and $\{XX_2\}$ are fixed.

The simplest class of influences $\mathcal{F}_0$, whose elements can be extended by integration, contains such F that $X \to X'$ transition is the arithmetic average of $\bar{X}_n^{(q)} \to \bar{X}_n^{(q)\prime}$ transitions, where $\bar{X}_n^{(q)}$ is the q^{th} element from the n^{th} sphere subdivision. According to sec. 2 there is the vector space $K(X|X_2)$ at $\{XX_2\}$. Hence, the element $X'_n = \frac{1}{m}\sum_{q=1}^{m} X_n^{(q)\prime}$; $(X_n^{(q)\prime} \| \bar{X}_n^{(q)\prime})$ may be determined. The limit of the sums as $n \to \infty$ presents the element X'.

The extension procedure for $F \in \mathcal{F}_0$ expresses Huygens' principle directly.

The next, more complicated class $\mathcal{F}_1$ can be constructed if to take into account, besides $\bar{X} \to \bar{X}'$ transitions, their "derivatives". Space differentiation cannot be represented naturally by itself. Indeed, let a pair of transitions $\bar{X}_0 \to \bar{X}_0'$ and $\bar{X}_1 \to \bar{X}_1'$ ($\bar{X}_0 \| \bar{X}_1$) be communicated with a photon. It seems, it is possible to define some sort of closeness of $\bar{X}_0$ to $\bar{X}_1$ along the photon trajectory by the oscillation number n_{01} when starting the count at some $\{\bar{X}_0 \bar{X}_0''\} \prec \{\bar{X}_0 \bar{X}_0'\}$ and finishing at $\{\bar{X}_0 \bar{X}_0'\}$. The sequence $\bar{X}_1, \dots \bar{X}_K \dots$ ($\bar{X}_K \| \bar{X}_0$) with the initial photon ν' ($\exists\{\bar{X}_0 \bar{X}_0'' \nu'\}$; $\exists\{X_K \nu'\}$ and $\{\bar{X}_0 \bar{X}_0'' \nu'\} \prec \{\bar{X}_K \nu'\} \prec \{\bar{X}_{K-1} \nu'\}$) corresponds to the sequence of the increasing numbers n_{0K}. But this procedure does not yet define any natural object like derivative (say, $\lim_{K\to\infty} n_{0K}(\bar{X}_K' - \bar{X}_0')$) as n_{0K} depends on the initial moment of the count too. However, it is possible to define in a unique way "integral of differential", and this is the only needed to find $X \to X'$ transition. The sequence $(\bar{X}_K)$ is coordinated with a partition of the photon ν trajectory ($\{\nu \bar{X}_0 \bar{X}_0'\} \prec \{\nu X X'\}$: the $(\bar{X}_{ot})$ sequence is found ($\exists\{\bar{X}_{ot}\nu\}, \bar{X}_{01} = \bar{X}_{01}$ $\bar{X}_{ot} \| X, \{\bar{X}_{ot}\nu\} \prec \{\bar{X}_{o(t+1)}\nu\} \prec \{XX'\nu\}$). Beginning with the contact $\{\nu''\bar{X}_{ot}\}$ ($\exists\{\nu'\nu''\bar{X}_0\}, \exists\{\nu''\bar{X}_{ot}\}, \{\nu''\bar{X}_0\} \prec \{\nu''\bar{X}_{ot}\}$) and up to $\{\nu\bar{X}_{ot}\}$ the numbers $n_{t(t+1)}$ are counted; for each value n_{0K} the sequence $(\bar{X}_{ot})$ must be chosen in such a way that there would be $n_{0K} = n_{t(t+1)}$ for all t. Then some definite number $\tau(K)$ of the elements in the $(\bar{X}_{ct})$ sequence corresponds to any $\bar{X}_K$. The contribution to the integral sum determining $X \to X'$ and corresponding to $\bar{X}$ increment along the ν' photon trajectory is $\lim_{K\to\infty} \tau(K)(\bar{X}_K' - \bar{X}_0')$.

Thus, it is necessary to make the set of operations in order to find $X \to X'$ transition. First, to construct $\bar{X}'_{K0}$ ($\exists\{\bar{X}_0 \bar{X}_0' \bar{X}'_{K0}\}$, $\bar{X}'_{K0} \| \bar{X}'_K$). Second, to determine the difference $\Delta\bar{X}'_K = \bar{X}'_{K0} - \bar{X}_0'$ in the linear space $K(\bar{X}_0|\bar{X}_0')$. Third, to determine at $\{XX'\}$ the element parallel to $\Delta\bar{X}'_K$ and to multiply it by $\tau(K)$ in

$K(X|X_2)=K(X|X')$. Fourth, to find the limit as $K\to\infty$. Fifth, to average arithmetically on the sphere, as $(\bar{X}_o)$ is an element from the set of the sphere $S^-(X|X_2)$ partitions (it would be written in full $\bar{X}^{(q)}_{on}(q=1..m)$, and the double limit as $K\to\infty$ and $n\to\infty$ would be used). Sixth, to add the result to a solution from $\mathcal{F}_o$ ($\mathcal{F}_o\subset\mathcal{F}_1$).

It is easy to see the connection of the $\mathcal{F}_1$ class with the solutions of the Cauchy problem in the canonical version for the homogeneous wave equation. The $\mathcal{F}_o$ class corresponds to the solutions with the initial time derivative equal zero.

It is known that it is a V' photon from the $S^+(X|X_1)$ sphere (not from $S^-(X|X_2)$ containing V) that must be used for the solutions would exist in reality. That is because the differentials along V are not independent one from another. The $\bar{X}_o\to\bar{X}'_o$ transitions influence all the intermediate $\bar{X}_{ot}\to\bar{X}'_{ot}$ ones as they determine $X\to X'$. Differentials along V' are called usually the leading out the light cone with the vertex at $\{XX_2\}$ ones.

The $\mathcal{F}_2$ class corresponds to the nonhomogeneous wave equation. It is constructed like $\mathcal{F}_1$, but instead of $\tau(K)\Delta\bar{X}'_K$ one forms $\sum_{t=1}^{\tau}\Delta_t\bar{X}'_K$ that sums the leading out differencies at the subdivision point of the V trajectory. The same result may be obtained if to express $\mathcal{F}_2$ as the integrals on the $\mathcal{F}_1$ subsets, i.e. to use the differencies $\Delta_t\bar{X}'_K-\Delta_{t+1}\bar{X}'_K$ and the sums $\sum_{t=1}^{\tau}t(\Delta_t\bar{X}'_K-\Delta_{t+1}\bar{X}'_K)$. In the canonical form of the wave equation it is just these differencies that are equals the right hand side function.

Thus, it is Kirchoff's and Poisson's solutions that are basic in the natural representation, not the wave equation itself. These solutions are meaningful as the natural constructions even if the

wave equation cannot be written at all: the necessary derivatives do not exist. Natural constructions correspond to so-called distributions (or generalized functions). Hence, these are the real objects that are examined in a CP.

The elements from $\mathcal{F}_2\supset\mathcal{F}_1\supset\mathcal{F}_o$ are known as "fields". The $\mathcal{F}_1$ class (the "free" fields) consists of F's that are fit for CP solutions. This is wrong for $\mathcal{F}_2$: the information is required up to the very $\{XX'\}$ in order to construct the extension of $F\in\mathcal{F}_2$. But the very concept of field has been introduced just to exclude such information. So, the effective realization of $\mathcal{F}_2$ class is possible only if the information is presented in the initial datas by itself.

For example, the A motion under a force depending on another body motion, that allows the natural extension as the initial contacts are given, will be a $\mathcal{F}_2$ element that is not a $\mathcal{F}_1$ element. The forcing body motion must be known beforehand or constructed recursively together with the solution. It is only meaningful that all the information is presented naturally.

In distinction from elements of $\mathcal{P}$ these bodies trajectories are to be known totally, not only locally. On the other hand, it is sufficient often to use only some general properties of the trajectories (especially, if there is a lot of such "field" bodies). Thus, it may be useful not to distinguish bodies with equal "charges" and so to form the concept of "current" when admitting the field bodies transpositions (in addition to their motion law) as an operation that does not change F.

In essence, the $\mathcal{F}_2$ class corresponds to the problem: to find some body trajectories given some others ones. It is usual to call the latters the "field generating sources". The natural representation of F is equivalent at the case to the wave

equation as solved by means of Green function. As distributions are used, the initial data may be added to the source function in the form of delta-function and its time derivative.

The concept of source in the form of the set of moving charged bodies, as applicated to the Lorentz force, turns out to be the only possible and in the canonical version results directly in the special splitting of the wave equation - the Maxwell equations. The divergency $\frac{\partial F_{ik}}{\partial x_k}$ yields the functions j_i fulfiling the continuity equation $\frac{\partial j_i}{\partial x_i} = 0$, due to F_{ik} antisymmetry. This is the consequence of the Lorentz force naturalness, as it has been shown above. In this context, the so-called Maxwell system second pair $\frac{\partial F_{ik}}{\partial x_k} = \frac{4\pi}{c} j_i$ is not connected as yet with the motion of any body set, since F_{ik} has been defined originally only by the influence on probe bodies, but not by any description as how to generate fields. But the continuity equation allows to describe a source like the flux of bodies, just the same as those in a CP, as one means them nonvanishing in the process of their motion. Otherwise, a CP would become pointless, because by its very meaning there is no doubt that all the time the same body moves that has started.

Thus, the Lorentz force naturalness shows a way to realize the $\mathcal{F}_2$ class. Any $F \in \mathcal{F}_2$ can be composed from a free part (a field from $\mathcal{F}_1$) and a part that is due to the field particles.

In the same way as the divergency equation, the nonhomogeneous wave equation for F_{ik} ($\frac{\partial^2 F_{ik}}{\partial x_e^2} = \Pi_{ik}$) looks to be only a notation with the antisymmetric Π_{ik} for the sums of the F_{ik} second derivatives. But if now to account only charged bodies to be

field sources, then Π_{ik} must be functions of j_i (more precisely, of their derivatives, i.e. of F_{ik} second derivatives). The only possible antisymmetric combination of the type is $\frac{\partial j_k}{\partial x_i} - \frac{\partial j_i}{\partial x_k}$ that must be, hence, equal Π_{ik}. This is a physically meaningful proposition: Only charged bodies can be the sources of fields. This is always true since the continuity equation holds for the fluxes of nonvanishing bodies. This guarantees, in its turn, the natural representation of a CP as a whole.

The connection between the wave equation and the Maxwell second pair results in the Maxwell first pair: $\frac{\partial F_{ik}}{\partial x_e} + \frac{\partial F_{ke}}{\partial x_i} + \frac{\partial F_{ei}}{\partial x_k} = 0$. Indeed, differentiating this equation in respect to x_e summing over e and presenting the third term by means of j_i from the second Maxwell pair, one obtains: $\Pi_{ik} = \frac{\partial j_k}{\partial x_i} - \frac{\partial j_i}{\partial x_k}$.

For the approach adopted here, the presented consideration must be taken as the substantiation of the theory of electricity by the possibility of its natural representation. The effectiveness of a CP solution forces them to perform experiments in such a way that electromagnetic phenomena are revealed and distinguished.

Gravitation cannot be represented in the same way as Lorentz force. It is impossible to construct a set $\mathcal{P}$ different from R and to measure F by the mutual divergence of P and X since the accelerations of bodies do not depend on their masses.

Gravitation but then influences geometry, i.e. the structure of a set R. The influence may be presented in different ways, for example, by means of the number $\chi_1 \chi_1' \gamma_{\chi_2 \chi_2'}$ that was defined in the section 2 for a pair of contacts $\{\chi_1 \chi_2\}$ and $\{\chi_1' \chi_2'\}$ connected with the photon ν. In a Euclidean (metrically) space the equation $\chi_1 \chi_1' \gamma_{\chi_2 \chi_2'} = 1$ holds if $\chi_1 \parallel \chi_1'$, $\chi_2 \parallel \chi_2'$ and the

trajectories of bodies of R are uniform and rectilinear ones. The "noneuclideanness" can be represented in natural terms as the violation of this equation.

One can express the violation locally only in respect to a fixed element X_0 in R. For any contact $\{X_0\bar{X}_0\}$, a photon V $(\exists\{VX_0\bar{X}_0\})$ and a contact $\{X_1\bar{X}_1\}$ $(\{VX_1\bar{X}_1\}>\{VX_0\bar{X}_0\}$, $X_1\|X_0, \bar{X}_1\|\bar{X}_0)$ the oscillation number n^e_{01} is determined when starting the count from an arbitrary moment $\{X_0X_e\}$ and counting up to $\{X_0\bar{X}_0\}$. A next pair $(X_1'X_1')$ $(\{VX_1\bar{X}_1\}<\{VX_1'\bar{X}_1'\}<\{VX_0\bar{X}_0\}$, $X_1'\|X_1, \bar{X}_1'\|\bar{X}_1)$ is chosen so that $n^e_{X_1X_1'}/n^e_{X_1X_0}>1$. A sequence $(X_1'\bar{X}_1')\dots(X_1^{(K)}\bar{X}_1^{(K)})\dots$ is constructed under the condition $n^e_{X_1X_1^{(K)}}/n^e_{X_0X_1}\to\infty$ as $K\to\infty$. The quantity $Ve=$ $=\lim_{K\to\infty} n^e_{X_1X_1^{(K)}}/n^e_{X_0X_1}(X_1\bar{X}_1\gamma_{X_1^{(K)}\bar{X}_1^{(K)}}-1)$ is calculated. Then the sequence (X_e) is chosen: $\{X_0X_e\}<\{X_0X_{e+1}\}<\{X_0\bar{X}_0\}$, $n^e_{X_1X_1^{(K)}}/n^{e+1}_{X_1X_1^{(K)}}>1$. The quantity $V=\lim_{e\to\infty} Ve$ may be taken to measure the violation of euclideanness. The analogous construction must be done for the inverse order of the initial contacts $(\{VX_1\bar{X}_1\}<\{VX_0\bar{X}_0\})$. It is essential that the value of V depends, generally speaking, on the choice of X_0.

Another way to measure the noneuclideanness is to choose a three $X_1\|X_2\|X_3$ and to examine the function d^2n_{12}/dn_{13}^2 that equals zero for an Euclidean space.

The natural representation of the so widely used in the canonical considerations concept as extended three-dimensional bodies is analogous, in some sense, to gravitation. Just as gravitation disturbs the reference bodies trajectories, the extended objects may be thought of as an obstacle that disturbs their contact scheme. Otherwise, if the object T is transparent for reference bodies there exist complex continuum - type subsets of contacts in R between T and X''.

4. The contact scheme that is defined by intersections of straight lines in $E^3\times T$ seems to be very specific. On the other hand, all the investigation complex, including R, must be chosen basing only on the aim of the problem that is to predict the final contact occurence. Hence, there arises the problem to define a contact scheme in the internal terms only, i.e. without any reference to a model like $E^3\times T$. Of course, the model may appear as the result of a construction. It may turn out that there are no objects in Nature that are suitable as the elements of R. But the analysis purpose is to establish the necessity: what sort of elementary objects has an experimenter to look for and how to use them for a CP solution.

It is useful at first to consider some more general problem than CP in mechanics.

Let some system A be able to occupy different states $q's$ from a set Q (the way to distinguish and to mark states is of no importance here, though it is presumed to be made somehow, for example, by colour). Let it exist, further, among these states some state q_1 (the "final" one), and the aim is to put A to this state.

If there is no way, according to the rules stated beforehand, to reach q_1 then there is no transition problem at all: a person wanting to use the solution is "helpless". If it is possible to transit simply A to q_1 from any state, then it is superfluous to solve the problem: a person wanting to reach q_1 is "omnipotent". The problem takes place if there exists a way to reach q_1 though not directly but putting at first A to some intermediate state $q_{1/2}$.

The state q_1 must be descripted externally as the need to reach it must be shown before the problem is formulated. All the

other elements of the set Q (including q_1 but now being regarded as an element of Q, i.e. with its internal definition in respect to the problem) are to be chosen in order to solve the problem. Hence, it may occure that it is possible to reach q_1 through intermediate states if Q has been chosen succesfully and not to reach q_1 if the choice of Q was wrong.

An initial state will be denoted as q_0. Then there exist the transitions $q_0 \to q_{1/2}$ and $q_{1/2} \to q_1$, their order corresponding to the very problem: to reach just q_1. Hence, there arises the so-called causal relation. Consequence that is something important by itself has an external description, while the cause, on the contrary, must be defined internally and is important only since it implies the consequence. Hence, causality implies asymmetric relation - order.

Let now the state $q_{1/2}$ be not such yet that the transition $q_{1/2} \to q_1$ is known directly, but, nevertheless, it is "easier" somehow to find $q_{1/2} \to q_1$ than $q_0 \to q_1$. Then one may look for $q_{3/4}$ that realises else easier transitions $q_{1/2} \to q_{3/4}$ and $q_{3/4} \to q_1$. In the same way, there may exist $q_{1/4}$, the transitions $q_{1/4} \to q_{1/2}$ and $q_0 \to q_{1/4}$ be intermediate for $q_0 \to q_{1/2}$.

Continuing the process, one looks for a set $\ell'(q_0, q_1) \subset Q$, the transitions along ℓ' lead from q_0 to q_1 without fail. This set must be infinite. Indeed, if it would be finite then it would contain the last element $q_{(K)}$, the transition $q_{(K)} \to q_1$ be guaranteed. But in this case there is no reason to distinguish $q_{(K)}$ from q_1 in Q: this would be superfluous for the transition problem solving. As identifying q_1 to $q_{(K)}$, the same situation is obtained in respect to $q_{(K-1)}$ and so on.

As ℓ' is infinite, there may exist false sequences that do not lead to any limit element initial for further transitions. Consequently, $\ell'(q_0, q_1)$ must be completed up to $\ell(q_0, q_1)$ by the limits of all such sequences. A proper space Q by its technical realization must include all the needful subsets of the type $\ell(q_0, q_1)$ - the "paths".

The procedure used to obtain sets ℓ is such that the indexes marking the elements of ℓ form a set similar to the real number axis segment, i.e. ℓ's are "perfect". The distinction of the "rational" elements that are used for the initial construction from the "irrational" ones, which are the limits of the sequences of rational elements is nothing but nominal. There is no difference in their realization as devices for fixing elements in Q. Some other choice of elements for the initial construction would replace rational elements by irrational ones and vice versa. Change of numeration conserving order (similar mapping) would not change ℓ as the subset of Q. In applications, there are examined often several $\ell \subset Q$ simultaneously. This is necessary, for example, when the solutions for variants of influences F are compared, or the field bodies trajectories are fixed (as in the section 3). However, the order has been defined for different ℓ's independently and cannot be transfered, generally speaking, to Q as a whole. The index of $q \in \ell$ is not its own one (as a "number that is written" on q). It is only an external notation ascribed to q as the set $\ell \ni q$ is constructed; if $q \in \ell_1$ and $q \in \ell_2$, there may be different indexes of q in ℓ_1 and ℓ_2. The set Q that includes different ℓ's is not ordered.

It may be possible to construct a space having its own partial order (clocks at various points) and to use similar mappings in

order to select a linear ordering in every ℓ as induced from Q. This is the way adopted in the canonical version. But this is not convenient for the initial consideration that cannot use some ready technical realization. On the contrary, one has to find what a construction of Q may be useful to solve the externally formulated problem.

As a matter of fact, if to introduce some partial order in Q from the very beginning, if would forbid some trajectories (ordered incompatibly to Q). This results in that not for each pair (q_0, q_1) there exists $\ell \ni (q_0, q_1)$. Hence, one has to control the choice of an element, for instance, $q_{1/2}$ according to the order in Q stated a'priori, i.e. $q_{1/2}$ cannot be any. But then the partial order in Q turns to be some basic hypothesis that must be checked in experiment. The last must be performed according to some rules. These would be external as regards to the problem and hence unacceptable for the approach adopted here, requiring that all the constructions (including Q itself) must originate from the very statement of the problem only.

Hence, it would be better to keep, at the stage, the property of Q to communicate each pair of elements with at least one path. Then one obtains free indexes for any ℓ. This is useful for the subsequent inclusion of the partial problem in a whole one by the proper fixing of indexes.

There arise the complications, but then, as regards to the practical solutions for ℓ. The different parts of a path, say, the parts $q_0 \to q_{1/2}$ and $q_{1/2} \to q_1$ are constructed independently. The whole ℓ is obtained as the parts are unificated at $q_{1/2}$. It cannot be excluded, then, that common elements may arise and so a selfintersection of ℓ would take place. At the case, order in ℓ would be broken. For some pair (q_i, q_k) there occur both $q_i < q_k$

and $q_k < q_i$. The simplest situation of the kind may be presented as a single circle. The elements $q_{1/4}$ and $q_{3/4}$ coincide in the sequence $q_0 \to q_{1/4} \to q_{1/2} \to q_{3/4} \to q_1$. Denoting $q_{1/4} = q_{3/4} = \bar{q}$ one can see that there arise the partial paths $q_0 \to \bar{q} \to q_{1/2}$ and $q_{1/2} \to \bar{q} \to q_1$ forming together the path with broken order. Order may be restored if to introduce inertia, i.e. if to suggest that in order to find an element in a sequence from the motion law it is necessary to know not only the preceeding element but also the pre-preceeding one. Then for the example presented above it is $q_{1/2}$ (not q_1) that follows $q_0 \to \bar{q}$ just as it is q_1 (not $q_{1/2}$) that follows $q_{1/2} \to \bar{q}$. This is the informational meaning of the concept of inertia (not in mechanics only). Inertia is used to restore the proper order in circled paths, i.e. the order that corresponds to the initial one.

It may occure in some more complicated cases that the selfintersection is multiple. There arises the question as to how complex a selfintersection scheme might be for it is possible yet to restore the initial order of a path (i.e. the path round similarity class) from the path very picture. If so, it is possible to divide, according to this order, the selfintersection (branchy) points and so to find the full solution with the order fixing indexes.

The choice of the proper branch at a branchy point q should be determined by inertia if there is some q' that belongs, together with q to a linearly ordered part of ℓ - without branchy points. The set of the branches at any point must be not more than countable (and, moreover, descrete) for the parts could be rounded one after another.

For the same reason, the set of the branchy points can be not more than countable too. There cannot exist, further, any branchy

point sequence that converges to a branchy point (the term "converges" means here that there is no linear ordered part containing the points of the sequence in the limit point branch). Any such sequence would converge either to a point with only one exiting branch or to the final point q_1.

All the variety of ℓ's is bounded by the necessity to state the unique round of any path, the order coincides for the linear ordered parts with their own.

The space Q itself is meaningful only as a structure that contains all kinds of paths and does not contain elements unnecessary for a path construction. This is the basic remark for determining of the geometrical structure of Q as a whole, if the structure of paths is given.

Each path is provided with the topology connected with the path order structure. If to choose axioms that are convenient for applications (though, perhaps, not looking extremely abstract) then so usual concepts of topology as neighbourhoods or open sets do not seem to be the basic ones. They may originate in the concept of paths: Any point of an open set cannot be reached along a path missing all the other points of the set.

The topology may be then introduced into Q as follows. There specified some system L of subsets Q - the path system. Each path bears the internal topology that is defined beforehand. A set $O \subseteq Q$ is called open if it turns for any $\ell \in L$ that $O \cap \ell$ is open in ℓ [2].

The internal topology of ℓ's themselves may be also derived from the point accessibility along ℓ. The last is based, in its turn, on the order structure. In the natural topology of an ordered set the basis consists of open intervals $(q' q'') : q' < q < q''$.

The path structure presented above requires that every point belongs to at least one neighbourhood $V(q)$ that is composited of the intervals of linearly ordered parts of the ℓ so that q is the only branchy point in $V(q)$. Hence, one can reach q along ℓ in no other way than passing these intervals points. Open sets in ℓ are all the neighborhood unions.

Provided with such a topology ℓ turns out to be an one-dimensional metrizable space with a countable base. It is known [3] that such a space can be embedded topologically into a three-dimensional Euclidean space. But it is just the topological embedding that is in question since the intersection of paths is the only essential problem when the subsequent passing of states is examined.

Hence, a space Q whose only function is to contain all kinds of paths as subsets (i.e. to include sufficient "stock" of states), may be in all cases realized as a space gomeomorphic to Euclidean E^3 at each point. The simple meaning of this statement is that each curve can be embedded in E^3, though one cannot, for example, leave a circle along a continious curve in E^2 without an intersection with its circumference. Thus, E^2 is not "free" enough. It "forces" upon intersections. But in a universal space it is only a process itself that determines whether there is a real intersection or there is not.

Returning to mechanics, as it is, one has to find a way to fix states in order to predict the trajectory of a body comparing its motion to the known trajectories of some auxiliary bodies. These may be always represented by curves in E^3. Any procedure of trajectory measurement must fix completely the trajectory of A via the scheme of its contacts with reference bodies.

This is possible if the set R would be of sufficient discernment, that is for each pair $\{x'A\} \prec \{x''A\}$ there exists $X: \{x'A\} \prec \{XA\} \prec \{x''A\}$. This must be true, in particular, for the bodies from the set R itself (one may take $\bar{X} \in R$ instead of A). Then any part of A trajectory would be coded just as a part (not as a point $\{\bar{x}A\}$) even if the part is common for $\bar{X}$ and A trajectories, because of the contacts of $\bar{X}$ and A with other $X \in R$.

In general, the elements X' and X'' may come into more than a single contact or to be out of contact at all. One has to deal, in some cases, with the sequence of elements coming into a common contact. This takes place, for example, when the local correspondence $X \leftrightarrow P$ must be stated. In order to find the limit of such a sequence in a unique way, it is necessary to mean just the same common contact when considering the next term of the sequence after a preceding one. In order to make this possible, the set R must be chosen in such a way that for any pair (X', X'') the mutual contacts $\{X'X''\}$ would be distributed descretely, that is for any $\{X'X''\}$ the next contact $\{X'X''\}' \succ \{X'X''\}$ exists (there is no contact $\{X'X''\}''$ such that $\{X'X''\}' \succ \{X'X''\}'' \succ \{X'X''\}$).

These conditions are satisfied if to present R by $E^3 \times T$ with the system of uniform and rectilinear trajectories or with some other line system topologically equivalent to this.

It is the multiple contact of the trajectory of a body with some X that corresponds to this trajectory selfintersection. This may be, in particular, even the intersection point of a pair of straight lines being defined as in the section 2. Due to inertia that is necessary to represent selfintersecting routes, it is only left-differentiable curves that are retained for the analysis.

The set of uniform and rectilinear trajectories forms a basis sufficient to present any such curve. Thus, this set fits as a device for motion coding.

To predict $\{AB\}$ with laws of motion, it is necessary to introduce some topological structure into the space, as the contact would be predicted if it is found from the laws that A tends to B in the sense of the topology (this is the reason to introduce irrational points as the trajectories must be perfect ordered sets in order to make it sure that there is in fact $\{AB\}$ if the trajectory of A tends to that of B.

It is required for bodies from $\mathcal{P}$ to be massive in order to use inertia for the decoding of routes in practice, massive $\rho \in \mathcal{P}$ guarantee the differentiableness of trajectories in measurements. The smaller the links of chains are used the less masses are required for P. In the limiting case it is only necessary that the masses would be finite.

5. As compared with the canonical version, the approach adopted here is based in some less rate on hypotheses and in some more on axioms. The difference is in that it must be checked in experiment whether the hypotheses are true or they are not true. But the answer depends on the solution of the problem as to how to perform experiments. As to axioms, someone who intends to use the theory may be asked whether the axioms are good enough for his aim. Therefore, it is important to formulate axioms in such a way that their relation to practice would be clear.

Some hypotheses remain in the natural constructions too. Bodies must exist. They must not "resolve" and vanish in the course of motion unless the final contact is reached. Their possible disintegration may be taken into account only if the concept of composite bodies can be introduced.

Measuring contacts must not destroy bodies and even influence their trajectories (in classical mechanics).*

All this is assumed in the canonical version also. But, moreover, there must exist rods and clocks that may be transported from one place to another without the distortion of measurement. This is to be checked up as examining some specific motions of bodies ("free" motions) with the use of rods and clocks. So arise, for example, Lorentz transformations.

An analysis of mechanics without using reference frames reveals the essential features of its geometry from some more general viewpoint. The concept of space-time as of some devices or instruments for the examination of motion that are not given from the very beginning together with the body in question, was

* The question in quantum mechanics is not the same as in classical one. Only probability distributions are to be found. Hence, some disturbance of trajectories may be allowed though this must be minimum in some sense [4].

used long ago.

Einstein and Poincaré constructed space basing on the properties of hard bodies. Some body was extended by means of puting others to it. It is necessary to use the original concept of hardness or changelessness to construct space. The Feynman's construction of clocks with the photon oscillations between two mirrors is based on the possibility to retain somehow distances. Just the opposite, the Born's definition of hardness requires the use of clocks only, distances are measured by photon oscillations.*

It is assumed in all these constructions that the main geometrical properties of space-time are known, and it is only the minimum means for their realization that is in question.

Einstein, however, thought about the dependence of topology on interactions, just like he postulated that metric depends on them. This is evident from the remark in his "The meaning of relativity": "That Euclidean geometry, from this point of view, affirms something more than the mere deductions derived logically from definitions may be seen from the following simple consideration.

Between n points of space there are $\frac{n(n-1)}{2}$ distances, $S_{\mu\nu}$; between these and the $3n$ coordinates we have the relations

$$S^2_{\mu\nu} = (X_{1(\mu)} - X_{1(\nu)})^2 + (X_{2(\mu)} - X_{2(\nu)})^2 + \dots$$

From these $\frac{n(n-1)}{2}$ equations the $3n$ coordinates may be eliminated, and from this elimination at least $\frac{n(n-1)}{2} - 3n$ equations in the $S_{\mu\nu}$ will result. Since the $S_{\mu\nu}$ are measurable quantities, and by definition are independent of each other,

* The review of a lot of papers in which the problem is examined in different ways, see in the books [1,5].

these relations between the $S_{\mu\nu}$ are not necessary a'priori".

According to this remark, the relation of the primary measurement procedure to the dimension of the space seems paradoxical as the dependence of this relation on real physical forces is not explained explicitly: bodies (i.e. material points) do not interact necessarily.

The paradox is explained at once if to remember that forces themselves show only via their influence upon motion. In other words, they are to be coded via the same coordinates. Hence, it is necessary to construct a reference frame before the very concept of interaction could be introduced and so there must be some independent explanation of the reference frame construction. This can be found if to regard space-time as an auxiliary device for the description of trajectories and so to prepare it in accordance with the problem to be solved.

It is difficult to understand the structure of any instrument if to know its material embodiment only. It may be understood why the form of a ruler is just such as it is among all the forms of all the kinds of objects only if to know the way it is used. It is so habitual to use space-time that it seems to be given from the very beginning and, therefore, the procedure of its preparation or choice for a CP solution is not realized explicitely.

From this point of view a physical theory is a language that is needed to ask Nature. Any answer is perceived just as an answer only if it is formulated in the terms of the question. Thus, the general meaning of the answer (the set of its variants) is contained in the very question. Just this aspect of mechanics is at the basis of the approach stating the necessity of the scheme of mechanics.

But then there arises some more questions. Why are CP's so usual? Why are experiments performed in such a way as they are? The consideration of these questions in details seems to be out of the framework of this paper. However, one may take into account the efficiency of a CP solution. Guarantees may be obtained when the problem can be reduced to a CP. A hare, when meeting a wolf, will not be eaten surely (the wolf, maybe, is not hungry), but it would be safer for the hare not to meet a wolf at all. This leads to the tendency to try to reduce any problem, first of all, to a contact problem.

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