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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ СО АН СССР

E.A. Kuraev, T.V. Kukhto, A. Schiller

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НОВОСИБИРСК

Contribution of Fermionic Loops to the Muon Anomalous Magnetic Moment

E.A. Kuraev, T.V. Kukhto)*, *A. Schiller***)*

Institute of Nuclear Physics
630090, Novosibirsk, USSR

ABSTRACT

The contribution of diagrams with fermion three vertex loops in lowest (fourth) order of electroweak perturbation theory to the muon anomalous magnetic moment has been calculated. It reaches $-(17.1/\sin^2 2\theta_W)\cdot 10^{-11}$ (without quarks in the fermion loop) and has to be taken into account for experiments with accuracy $\sim 10^{-10}$. One and two loop contributions of possible scalars and pseudoscalars to the magnetic moment are analyzed. Upper limits on their coupling constants are found for light and heavy particles. Their contribution to the reaction $e^+e^-\to 2\gamma$ is presented.

*) Byelorussian State University, Minsk, USSR.

**) Karl-Marx-Universität, Leipzig, DDR.

For a long time the anomalous magnetic moment of the muon a_μ is of nondiminishing interest for theoreticians and experimentalists. Besides of electromagnetic interactions a_μ is sensitive also to strong and weak interactions due to its not too small mass scale. Presently, the calculated electromagnetic, strong and weak contributions to a_μ allow to define its value properly up to 9 digits. A lot of efforts has been made to extend the accuracy to 10 digits. The contribution of electromagnetic interactions, completed up to 8th order of perturbation theory [1] gives the following result:

$$a_\mu(\mathrm{QED}) = 0.5\frac{\alpha}{\pi} - 0.766\left(\frac{\alpha}{\pi}\right)^2 + 24.1\left(\frac{\alpha}{\pi}\right)^3 + 122\left(\frac{\alpha}{\pi}\right)^4 + a_\mu(\tau) =$$
$$= 1165848108\,(42)(28)\cdot 10^{-12}. \qquad (1)$$

The hadronic [2] and weak [3] contributions are estimated as

$$a_\mu(\mathrm{had}) = 6940\,(142)\cdot 10^{-11},$$
$$a_\mu(\mathrm{weak}) = 195\,(1)\cdot 10^{-11}. \qquad (2)$$

For a_μ(weak) a not too small Higgs mass ($m_H \gtrsim 5$ GeV) is assumed, the error does not contain also higher order corrections. The difference between the theoretical value

$$a_\mu(\mathrm{theor}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{had}) + a_\mu(\mathrm{weak}) \qquad (3)$$

and the most precise experimental measurement [4] vanishes within experimental and theoretical errors

$$a_\mu(\exp) - a_\mu(\text{theor}) = 35\,(85)\cdot 10^{-10} \qquad (4)$$

showing no signal of new physics.

The situation for a_μ may change for an experimental accuracy of order $\sim 10^{-10}$. Such an accuracy requires the inclusion of two loop corrections to a_μ(weak) arising in particular from fermionic anomalies. Possible «exotic» particles can also contribute to a_μ in one and two loop approximation.

In the present work we calculate the correction to the weak part of the muon magnetic moment arising from fermionic anomalies (two loop approximation). We present the corresponding contributions for scalars and pseudoscalars known already in one loop. Comparing their values to the improved a_μ (weak) and the relation (4) upper limits on the coupling constants are found for light and heavy particles.

We represent the vertex function of the muon V^μ in terms of the Dirac $f(k^2)$ and Pauli $\varphi(k^2)$ form factors:

$$V^\mu(p_2, p_1, k) = \bar u(p_2)\,\Gamma^\mu(k)\,u(p_1)\,,$$

$$\Gamma^\mu(k) = \gamma^\mu f(k^2) + \frac{1}{4\mu}\,(\hat k\gamma^\mu - \gamma^\mu \hat k)\,\varphi(k^2)\,, \qquad (5)$$

where $p_1^2 = p_2^2 = \mu^2$, $\hat k = k_\mu \gamma^\mu$, $k = p_2 - p_1$, μ is the muon mass. The value of the Pauli form factor at $k^2=0$ defines its anomalous magnetic moment

$$\varphi(0) = a = \frac{1}{2}\,(g-2) = a^{(2)} + a^{(4)} + \dots \qquad (6)$$

In the lowest order of perturbation theory the contribution of electromagnetic interactions has been calculated by Schwinger already in 1952:

$$a_\gamma^{(2)} = \frac{\alpha}{2\pi}. \qquad (7)$$

This value corresponds to an one loop diagram, with a virtual photon (Fig. 1,*a*). The one loop diagrams of standard electroweak theory with Z^0- and W-boson are shown in Fig. 1,*b*,*c* resulting in the contributions to the muon magnetic moment

$$a_W^{(2)} = \frac{G_F\mu^2}{8\pi^2\sqrt{2}}\,\frac{10}{3} = 389\cdot 10^{-11},$$

$$a_Z^{(2)} = -\frac{G_F\mu^2}{8\pi^2\sqrt{2}}\,\frac{1}{3}\,(5-(1-4\sin^2\theta_W)^2) = -194\cdot 10^{-11} \qquad (8)$$

with θ_W the Weisberg angle. To obtain the numbers of eq. (8). we have used $G_F = 1.1664\cdot 10^{-5}\,\text{GeV}^2$ and $\sin^2\theta_W = 0.226$.

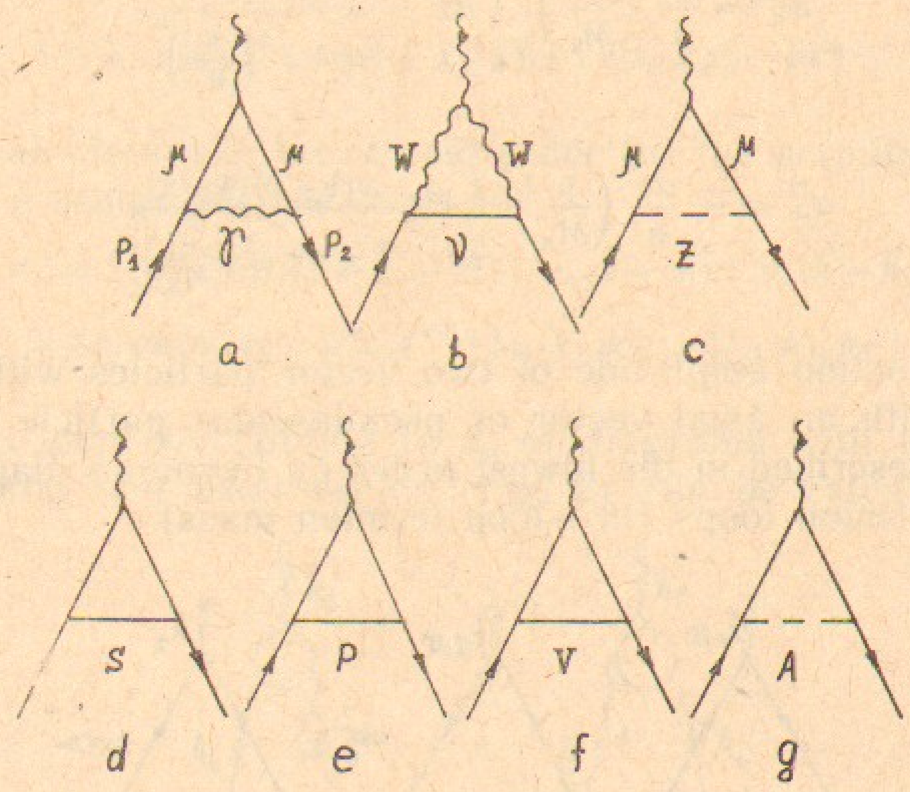


Fig. 1. One loop vertex diagrams.

Assuming the following Lagrangian interaction parts of scalar [4, 5], pseudoscalar [6], massive vector [5, 7] and axial vector (Fig. 1,*c*,*d*,*e*,*f*) particles to fermions

$$g_S S(x)\,\bar\psi(x)\,\psi(x)\,, \quad i g_P P(x)\,\bar\psi(x)\,\gamma_5\,\psi(x)\,, \quad g_V \bar\psi(x)\,\hat V(x)\,\psi(x)\,,$$

$$g_A \bar\psi(x)\,\hat A(x)\,\gamma_5\,\psi(x)\,, \quad \alpha_i = \frac{g_i^2}{4\pi}, \quad i = S, P, V, A \qquad (9)$$

we present here, for completeness, the corresponding one loop results (M_i — particle masses)

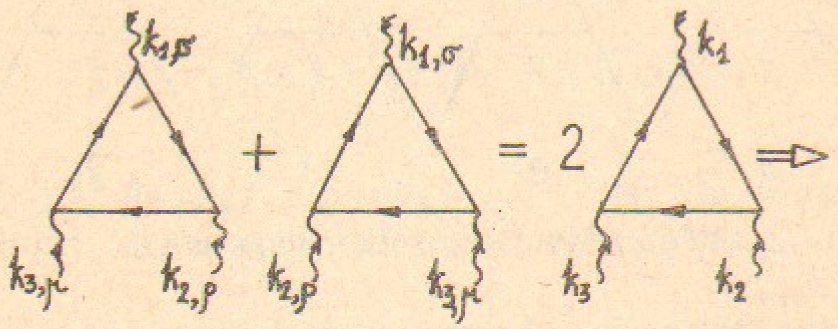
$$a_S^{(2)} = \frac{\alpha_S}{2\pi}\left(\frac{\mu}{M_S}\right)^2 \int_0^1 dx\,\frac{x^2(2-x)}{1-x+x^2\left(\frac{\mu}{M_S}\right)^2} = \alpha_S A_S^{(2)},$$

$$a_P^{(2)} = -\frac{\alpha_P}{2\pi}\left(\frac{\mu}{M_P}\right)^2 \int_0^1 dx \frac{x^3}{1-x+x^2\left(\frac{\mu}{M_P}\right)^2} \equiv -\alpha_P A_P^{(2)},$$

$$a_V^{(2)} = \frac{\alpha_V}{\pi}\left(\frac{\mu}{M_V}\right)^2 \int_0^1 dx \frac{x^2(1-x)}{1-x+x^2\left(\frac{\mu}{M_V}\right)^2},$$

$$a_A^{(2)} = -\frac{\alpha_A}{\pi}\left(\frac{\mu}{M_A}\right)^2 \int_0^1 dx \frac{x(1-x)(4-x)}{1-x+x^2\left(\frac{\mu}{M_A}\right)^2}. \qquad (10)$$

The interaction amplitude of two vector particles with momenta k_1 and k_2 with an axial vector or pseudoscalar particle of momentum k_3 are described in the lowest order by Feynman diagrams with triangular fermion loops (m — loop fermion mass)



leading to the definition of tensors

$$R_{\sigma\rho\mu}(k_1,k_2,k_3) = 2\int d^4k\, \mathrm{Sp}\{(\hat{k}+\hat{k}_1-m)^{-1}\gamma_\sigma(\hat{k}-m)^{-1}\gamma_\rho(\hat{k}-\hat{k}_2-m)^{-1}\gamma_\mu\gamma_5\},$$

$$R_{\sigma\rho}(k_1,k_2,k_3) = 2\int d^4k\, \mathrm{Sp}\{(\hat{k}+\hat{k}_1-m)^{-1}\gamma_\sigma(\hat{k}-m)^{-1}\gamma_\rho(\hat{k}-\hat{k}_2-m)^{-1}\gamma_5\},$$

$$k_1+k_2+k_3=0. \qquad (11)$$

$R_{\sigma\rho\mu}$ and $R_{\sigma\rho}$ have been calculated by Adler and Rosenberg [8]:

$$R_{\sigma\rho\mu}(k_1,k_2,k_3) = A_1(k_1,k_2)\,k_1^\tau \varepsilon_{\tau\sigma\rho\mu} + A_3(k_1,k_2)\,k_{1\rho}k_1^\xi k_2^\tau \varepsilon_{\xi\tau\sigma\mu} +$$
$$+ A_4(k_1,k_2)\,k_{2\rho}k_1^\xi k_2^\tau \varepsilon_{\xi\tau\sigma\mu} + (k_1 \leftrightarrow k_2,\ \sigma \leftrightarrow \rho),$$

$$R_{\sigma\rho}(k_1,k_2,k_3) = B(k_1,k_2)\,k_1^\xi k_2^\eta \varepsilon_{\xi\eta\sigma\rho} \qquad (12)$$

with

$$A_1(k_1,k_2) = k_1k_2A_3(k_1,k_2) + k_2^2A_4(k_1,k_2),$$

$$A_3(k_1,k_2) = -16\pi^2 I_{11}(k_1,k_2),$$

$$A_4(k_1,k_2) = 16\pi^2[I_{20}(k_1,k_2) - I_{10}(k_1,k_2)],$$

$$B(k_1,k_2) = 8m\pi^2 I_{00}(k_1,k_2) \qquad (13)$$

and

$$I_{st}(k_1,k_2) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2\, x_1^t x_2^s \cdot$$
$$[x_1(1-x_1)k_1^2 + x_2(1-x_2)k_2^2 + 2k_1k_2x_1x_2 - m^2]^{-1}. \qquad (14)$$

A similar calculation in the lowest order for the amplitude with two vector and a scalar particle leads to

$$S_{\sigma\rho}(k_1,k_2,k_3) = 2\int d^4k\, \mathrm{Sp}\{(\hat{k}+\hat{k}_1-m)^{-1}\gamma_\sigma(\hat{k}-m)^{-1}\gamma_\rho(\hat{k}-\hat{k}_2-m)^{-1}\} =$$
$$= 8\pi^2 mi\,(k_1^\rho k_2^\sigma - k_1k_2 g^{\rho\sigma})\,[I_{00}(k_1,k_2) - 4I_{11}(k_1,k_2)]. \qquad (15)$$

The two loop contribution to a_μ of diagrams with fermion loop, virtual photon and axial vector (Fig. 2) can be calculated as follows.

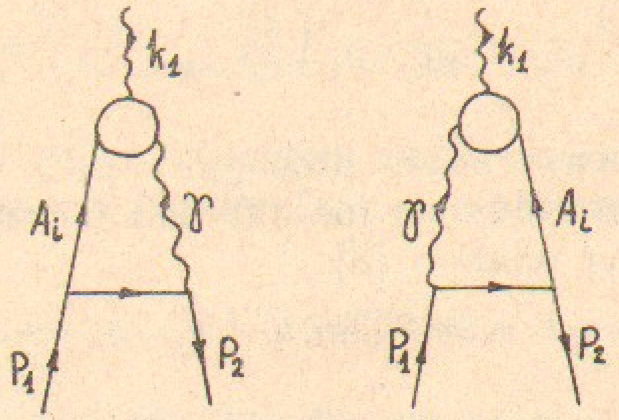


Fig. 2. Two loop diagrams with three vertex fermion loops.

The corresponding part in the vertex function V^μ is proportional to

$$(-1)\int d^4k \frac{R_{\sigma\rho\mu_1}(k_1,k,-k_1-k)}{k^2(k^2-M_A^2)}\left(g_{\mu\mu_1} - \frac{k_\mu k_{\mu_1}}{M_A^2}\right),$$

$$\bar{u}(p_2)\left[\frac{\gamma^\rho(\hat{p}+\hat{k}+\mu)\gamma_\mu\gamma_5}{(p+k)^2-\mu^2} + \frac{\gamma_\mu\gamma_5(\hat{p}-\hat{k}+\mu)\gamma^\rho}{(p-k)^2-\mu^2}\right]u(p_1), \qquad (16)$$

where the factor (-1) comes from the fermion loop of $R_{\sigma\rho\mu}$. To calculate $\varphi(k_1^2)$ at $k_1^2=0$ it is sufficient to consider the limit of small k_1: $|k_1| \ll |k|$. In this limit the tensor $R_{\sigma\rho\mu}$ simplifies to the expression

$$R_{\sigma\rho\mu}(k_1, k, -k) = 4\pi \int_0^1 \frac{dx}{k^2 - m_*^2} \{ -2k_1^\tau \varepsilon_{\tau\xi\sigma\mu} (k_\rho k_\xi - k^2 g_{\rho\xi}) +$$

$$+ (k_1 k g_{\tau\sigma} - k_\sigma k_{1\tau}) k^\lambda \varepsilon_{\lambda\tau\rho\mu} \}, \quad m_*^2 = \frac{m^2}{x(1-x)} \qquad (17)$$

To integrate over d^4k we parametrize the denominators as follows:

$$\frac{1}{(k^2+2pk)} \frac{1}{k^2(k^2-M_A^2)(k^2-m_*^2)} = \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \frac{6x_3^2 x_2}{[(k+p(1-x_2))^2 - M^2]^4} \qquad (18)$$

with

$$M^2 = \mu^2(1-x_3)^2 + M^2 x_1 x_2 x_3 + m_*^2 x_3 (1-x_2) \qquad (19)$$

and perform the integrations with the formulae

$$\int \frac{d^4k}{i\pi^2} \frac{1}{(k^2-M^2)^4} = \frac{1}{6M^4}, \qquad \int \frac{d^4k}{i\pi^2} \frac{k_\mu k_\nu}{(k^2-M^2)^4} = -\frac{g_{\mu\nu}}{12M^2}.$$

We note that the heavy boson propagator part $\sim k_\mu k_\nu / M_A^2$ in (16) leads to nonzero contributions for different fermions in the fermion loop due to the Adler relation [8]

$$-k^{\mu_1} R_{\sigma\rho\mu_1}(k_1, k, -k_1-k) = 2m R_{\sigma\rho}(k_1, k, -k_1-k) + 8\pi^2 k_1^\xi (k+k_1)^\tau \cdot \varepsilon_{\xi\tau\sigma\rho}.$$

However, the first part in the r.h.s. leads to a numerically small contribution to $a_\mu \sim (m\mu/M_A^2)^2$, $M_A \gg \mu$, m. Taking into account all leptons and quarks in the fermion loop the total sum of all contributions vanishes. This fact is known in the standard model as cancellation of anomalies. Therefore, for a heavy axial vector we neglect the part $\sim k_\mu / k_\mu / M_A^2$ of the boson propagator.

Using the properties of the Dirac matrices

$$\mathrm{Sp}(\gamma_\tau \gamma_\xi \gamma_\sigma \gamma_\mu \gamma_5) = 4i\, \varepsilon_{\tau\xi\sigma\mu}, \qquad \varepsilon_{0123} = 1;$$

$$2\gamma^\mu \varepsilon_{\mu\sigma\tau\rho} = i\gamma_5 (\gamma_\sigma \gamma_\tau \gamma_\rho - \gamma_\rho \gamma_\tau \gamma_\sigma)$$

and selecting the structure according to the Pauli form factor (5) we obtain

$$a_A^{(4)} = -\frac{\alpha}{\pi} \frac{\alpha_A}{\pi} \left(\frac{\mu}{M_A}\right)^2 \int_0^1 dx \int_0^1 dx_2 \int_0^1 dx_3 x_3 [(10-6x_3) L_A - 2(1-x_3) Q], \qquad (20)$$

$$L_i = \ln\left(1 + \frac{M_i^2 x_2 x_3}{\mu^2 (1-x_3)^2} Q\right), \quad i = A, V, S, P,$$

$$Q = \left(1 + \frac{m^2}{\mu^2} \frac{x_3(1-x_2)}{x(1-x)(1-x_3)^2}\right)^{-1}. \qquad (21)$$

For an electron in the fermion loop we have $L_A \approx \ln \frac{M_A^2}{\mu^2} + \ln \frac{x_2 x_3}{(1-x_3)^2}$ and $Q=1$ and the integration in (20) can be carried out completely

$$a_A^{(4)} = -2 \frac{\alpha}{\pi} \frac{\alpha_A}{\pi} \left(\frac{\mu}{M_A}\right)^2 \left(3 \ln \frac{M_A}{\mu} + \frac{5}{4}\right). \qquad (22)$$

Using

$$\alpha_A = \frac{\alpha}{4 \sin^2(2\theta_W)} \approx \frac{\alpha}{3}, \quad M_A = M_Z = 91.2 \text{ GeV}, \qquad (23)$$

we obtain the two loop contribution to a_μ(weak) for an electron in the fermion loop

$$a_Z^{(4)}|_{m=m_e} = -10.4 \cdot 10^{-11}. \qquad (24)$$

The contribution of muon, τ-lepton and t-quark (mass 100 GeV, without color and quark charge $Q_t = e$), obtained numerically, are given as follows:

$$(-)(8.12, 4.26, 0.221) \cdot 10^{-11}. \qquad (25)$$

For heavy scalar and pseudoscalar particles we obtain similarly

$$a_S^{(4)} = -2 \frac{\alpha_s}{\pi} R \left(\frac{\mu}{M_S}\right)^2 \int_0^1 dx \frac{1-2x(1-x)}{x} \int_0^1 dx_2 \int_0^1 dx_3 x_3 L_S = -\alpha_S A_S^{(4)}, \qquad (26)$$

$$a_P^{(4)} = 2 \frac{\alpha_P}{\pi} R \left(\frac{\mu}{M_P}\right)^2 \int_0^1 \frac{dx}{x} \int_0^1 dx_2 \int_0^1 dx_3 x_3 (L_P - Q) = \alpha_P A_P^{(4)}. \qquad (27)$$

An extra factor

$$R = \frac{\alpha}{\pi} \frac{m}{\mu} \qquad (28)$$

compared to the one loop contribution (10) appears in eqs (26), (27). For loop fermion masses much larger than the muon mass (e. g. τ, t) the factor R is not small and one and two loop results are expected to be of the same order.

In the limit $M_i \gg \mu$ and $m \gg \mu$ eqs (26), (27) reduce to

$$A_S^{(4)}\Big|_{\substack{M_S \gg \mu \\ m \gg \mu}} = \frac{1}{\pi} R \left(\frac{\mu}{M_S}\right)^2 \int_0^1 dz \frac{1+z^2}{1-z^2-\frac{4m^2}{M_S^2}} \ln\left(\frac{M_S^2}{4m^2}(1-z^2)\right),$$

$$A_P^{(4)}\Big|_{\substack{M_P \gg \mu \\ m \gg \mu}} = \frac{2}{\pi} R \left(\frac{\mu}{M_P}\right)^2 \int_0^1 dz \frac{1}{1-z^2-\frac{4m^2}{M_P^2}} \ln\left(\frac{M_P^2}{4m^2}(1-z^2)\right). \qquad (29)$$

Tables 1 and 2 present numerical results on $A_{S,P}^{(2)}$ and the ratio $A_{S,P}^{(4)}/A_{S,P}^{(2)}$ for several masses $M_{S,P}$ and loop fermion masses m. We notice that for not too small masses m and $M_{S,P}$ the two loop contribution are not neglectable compared to the one loop ones. Supposing all uncertainty in (4) arises from contribution of heavy «exotic» particles we obtain upper limits for their coupling to fermions:

$$|\alpha_{S,P}| < 2\cdot 10^{-3}, \quad M_{S,P} = 100 \text{ GeV};$$

$$|\alpha_{S,P}| < 2\cdot 10^{-5}, \quad M_{S,P} = 10 \text{ GeV}. \qquad (30)$$

Now we consider the case of light (zero mass) scalar and pseudoscalar (axion) particles. The one loop contributions are (see (10))

$$A_P^{(2)}\big|_{M_P=0} = \frac{1}{4\pi}, \quad A_S^{(2)}\big|_{M_S=0} = \frac{3}{4\pi}. \qquad (31)$$

In the considered two loop approximation the largest contributions arise from fermion loops with electrons and muons

$$A_P^{(4)}\Big|_{\substack{M_P=0 \\ m=m_e}} = \frac{\alpha}{3}, \quad A_P^{(4)}\Big|_{\substack{M_P=0 \\ m=\mu}} = 1.43\,\frac{\alpha}{\pi^2},$$

$$A_S^{(4)}\Big|_{\substack{M_S=0 \\ m=m_e}} = \frac{\alpha}{2}, \quad A_S^{(4)}\Big|_{\substack{M_S=0 \\ m=\mu}} = 1.29\,\frac{\alpha}{\pi^2}. \qquad (32)$$

For large loop fermion masses we obtain the expressions

$$A_P^{(4)}\big|_{m \gg \mu \gg M_P} = \frac{\alpha}{\pi^2}\frac{\mu}{m}\left(\ln\frac{m}{\mu} + \frac{5}{4}\right),$$

$$A_S^{(4)}\big|_{m \gg \mu \gg M_S} = \frac{2}{3}\frac{\alpha}{\pi^2}\frac{\mu}{m}\left(\ln\frac{m}{\mu} + \frac{11}{6}\right). \qquad (33)$$

Table 1

One Loop $A_S^{(2)}$ and Ratio of Two to One Loop Contribution $A_S^{(4)}/A_S^{(2)}$ for Several Loop Fermion Masses m and Masses of the Scalar Particle M_S

M_S (GeV)	$A_S^{(2)}$	$A_S^{(4)}/A_S^{(2)}$					
		m_e	μ	m_τ	5 GeV	80 GeV	100 GeV
0.0	0.239	$1.43\cdot10^{-2}$	$3.99\cdot10^{-3}$	$5.70\cdot10^{-3}$	$2.48\cdot10^{-4}$	$2.31\cdot10^{-5}$	$1.90\cdot10^{-5}$
10	$1.41\cdot10^{-4}$	$3.98\cdot10^{-4}$	$2.07\cdot10^{-2}$	$5.79\cdot10^{-2}$	$5.54\cdot10^{-2}$	$1.46\cdot10^{-2}$	$1.25\cdot10^{-2}$
20	$4.14\cdot10^{-5}$	$4.04\cdot10^{-4}$	$2.37\cdot10^{-2}$	$8.75\cdot10^{-2}$	0.100	$3.92\cdot10^{-2}$	$3.41\cdot10^{-2}$
30	$2.00\cdot10^{-5}$	$4.09\cdot10^{-4}$	$2.55\cdot10^{-2}$	0.107	0.135	$6.85\cdot10^{-2}$	$6.03\cdot10^{-2}$
40	$1.19\cdot10^{-5}$	$4.12\cdot10^{-4}$	$2.68\cdot10^{-2}$	0.122	0.164	0.101	$8.95\cdot10^{-2}$
50	$7.93\cdot10^{-6}$	$4.16\cdot10^{-4}$	$2.78\cdot10^{-2}$	0.135	0.188	0.134	0.121
60	$5.68\cdot10^{-6}$	$4.18\cdot10^{-4}$	$2.86\cdot10^{-2}$	0.145	0.209	0.169	0.153
70	$4.29\cdot10^{-6}$	$4.20\cdot10^{-4}$	$2.93\cdot10^{-2}$	0.154	0.227	0.205	0.187
80	$3.36\cdot10^{-6}$	$4.22\cdot10^{-4}$	$2.99\cdot10^{-2}$	0.162	0.244	0.241	0.221
90	$2.70\cdot10^{-6}$	$4.24\cdot10^{-4}$	$3.04\cdot10^{-2}$	0.168	0.259	0.277	0.256
100	$2.23\cdot10^{-6}$	$4.26\cdot10^{-4}$	$3.09\cdot10^{-2}$	0.175	0.273	0.313	0.290

Table 2

$A_p^{(2)}$ and $A_p^{(4)}/A_p^{(2)}$ Contributions for the Pseudoscalar Particle

M_P (GeV)	$A_P^{(2)}$	$A_P^{(4)}/A_P^{(2)}$					
		m_e	μ	m_τ	5 GeV	80 GeV	100 GeV
0.0	$7.96\cdot10^{-2}$	$3.00\cdot10^{-2}$	$1.33\cdot10^{-2}$	$2.24\cdot10^{-3}$	$1.00\cdot10^{-3}$	$9.67\cdot10^{-5}$	$7.96\cdot10^{-5}$
10	$1.29\cdot10^{-4}$	$4.38\cdot10^{-4}$	$2.67\cdot10^{-4}$	$8.42\cdot10^{-2}$	$8.38\cdot10^{-2}$	$2.33\cdot10^{-2}$	$2.00\cdot10^{-2}$
20	$3.84\cdot10^{-5}$	$4.42\cdot10^{-4}$	$2.97\cdot10^{-2}$	0.122	0.146	$6.12\cdot10^{-2}$	$5.34\cdot10^{-2}$
30	$1.87\cdot10^{-5}$	$4.46\cdot10^{-4}$	$3.15\cdot10^{-2}$	0.146	0.192	0.106	$9.34\cdot10^{-2}$
40	$1.12\cdot10^{-5}$	$4.49\cdot10^{-4}$	$3.28\cdot10^{-2}$	0.164	0.229	0.154	0.138
50	$7.45\cdot10^{-6}$	$4.52\cdot10^{-4}$	$3.38\cdot10^{-2}$	0.179	0.260	0.205	0.184
60	$5.36\cdot10^{-6}$	$4.54\cdot10^{-4}$	$3.46\cdot10^{-2}$	0.191	0.286	0.256	0.233
70	$4.05\cdot10^{-6}$	$4.56\cdot10^{-4}$	$3.59\cdot10^{-2}$	0.201	0.309	0.308	0.283
80	$3.17\cdot10^{-6}$	$4.58\cdot10^{-4}$	$3.59\cdot10^{-2}$	0.210	0.329	0.361	0.333
90	$2.56\cdot10^{-6}$	$4.60\cdot10^{-4}$	$3.64\cdot10^{-2}$	0.217	0.347	0.413	0.383
100	$2.11\cdot10^{-6}$	$4.61\cdot10^{-4}$	$3.69\cdot10^{-2}$	0.224	0.364	0.465	0.434

We conclude that for light scalar and pseudoscalar particles the two loop diagrams with fermion loops are completely neglectable compared to the one loop result. The coupling constants, estimated from (4), are

$$|\alpha_P| < 5\cdot10^{-8}, \quad |\alpha_S| < 10^{-8}, \quad M_S = M_P = 0. \qquad (34)$$

Additionally we remark the following: some restrictions on mass, width and coupling constants to fermions of heavy pseudoscalar and scalar particles can be obtained studying the deviation of QED predictions in the two photon production of high energy e^+e^--annihilation. The source of deviation for the considered interactions (9) arises only from fermionic anomalies. We obtain the single photon angular distribution

$$\left(\frac{d\sigma}{d\cos\theta}\right)^{e^+e^-\to2\gamma} = \frac{\pi\alpha^2}{2E^2}\,\frac{1+\cos^2\theta}{1-\cos^2\theta} + \frac{\alpha^2\alpha_S^2\left|\sum_f m_f\left(\left(\frac{1}{2}\rho_f^2-\frac{\pi^2}{2}-2\right)-i\pi\rho_f\right)\right|^2}{2\pi\left((4E^2-M_S^2)^2+M_S^2\Gamma_S^2\right)} +$$

$$+\frac{\alpha^2\alpha_P^2\left|\sum_f m_f\left(\left(\frac{1}{2}\rho_f^2-\frac{\pi^2}{2}\right)-i\pi\rho_f\right)\right|^2}{2\pi\left((4E^2-M_P^2)^2+M_P^2\Gamma_P^2\right)}, \quad 1-\cos^2\theta\sim1. \qquad (35)$$

Here θ denotes the angle of the photon in the cms of e^+e^--beam, $2E$ — the total energy. The first part in (35) corresponds to the lowest order QED contribution $e^+e^-\to2\gamma$. The remaining two parts consider the e^+e^--annihilation into intermediate scalar and pseudoscalar particles followed by a two photon decay via the triangular anomaly in the lowest order. m_f denotes the loop fermion mass, $\rho_f = \ln(4E^2/m_f^2)$. In eq. (35) we have neglected interference terms of order $O(m_e/E)$ compared to one.

We summarize our results. The contributions of fermionic anomalies in the lowest order of perturbative theory to the weak part of the muon magnetic moment have been calculated. For three leptons in the fermion loop we obtain (see (23) — (25))

$$-23\cdot10^{-11} \approx -17.1/\sin^2 2\theta_W\cdot10^{-11}. \qquad (36)$$

We expect for light quarks in the fermion loop a result of the same order. Important two loop electroweak corrections to a_μ can also arise from diagrams with electroweak gauge bosons which have to be calculated in the future.

Using the uncertainty in a_μ we have found restrictions on coupling constants to fermions of possible heavy and light pseudoscalar and scalar particles (see (30), (34)).

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E.A. Kuraev, T.V. Kuchto, A. Schiller

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Э.А. Кураев, Т.В. Кухто, А. Шиллер

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