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INFLUENCE OF A COULOMB-LIKE INTERACTION ON RADIATION IN NONRELATIVISTIC PAIR PRODUCTION

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НОВОСИБИРСК

INFLUENCE OF A COULOMB-LIKE INTERACTION ON RADIATION IN NONRELATIVISTIC PAIR PRODUCTION

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A b s t r a c t

In nonrelativistic pair production, the matrix element of the radiation is represented as a product of the matrix element of pair production and the nonrelativistic radiation amplitude, with taking a Coulomb type interaction into account. The interaction is shown to exert considerable effect on the integral and spectral characteristics of the radiation. The cases of photon and gluon emission in lepton and quark pair production are considered in detail.

1. Introduction

One of the basic purposes of modern accelerator physics is to discover and study new particles. Besides the t-quark, which was predicted long before its observation, the discovery of other heavy quarks and leptons is also possible.

In the process of heavy pair production, the range of energies at which the heavy particles are produced as nonrelativistic ones enlarges with increasing their masses. Both in Quantum Electrodynamics (QED) and in Quantum Chromodynamics (QCD), it is convenient to split the photons and gluons in this range into 'Coulomb' and transverse ones (i.e. to use Coulomb gauge) since their interaction with nonrelativistic particles is different: the effective coupling is $\frac{\alpha}{v}\left(\frac{\alpha_s}{v}\right)$ in the case of Coulomb photons (gluons) and $\alpha v^2 (\alpha_s v^2)$ in the case of transverse ones (v is the relative velocity of the particles).

At rather low v , the Coulomb-like interaction should be taken into account in all the orders of perturbation theory. In QED, this problem was considered as long ago as the 1930s /1/. Our approach is close to that reported in /2/, where allowance was made for the Coulomb-like final-state interaction for the case of e^+e^- pair production. The results of Ref./2/ are easily applicable to the case of $Q\bar{Q}$ pair production (see Ref./3/).

The present work deals with the influence of the Coulomb-like interaction on the emission of a transverse gluon (photon) during the nonrelativistic quark (lepton) pair production. For known flavours of leptons and quarks, this problem is of rather theoretical than practical interest. For leptons, this is the case since either the region where the effects of Coulomb interaction are significant ($\frac{\alpha}{v} \sim 1$) is too narrow or the lifetime of leptons is too short so that the radiation their decay products rather than the radiation of their own is observable. For quarks, similar circumstances are of minor importance; the nonperturbative and large-distance contributions (where α_s is not small) gain importance here. However, as the quark masses increase, these contributions lose their influence, and the Coulomb-like interaction becomes determining /3-5/.

Gluon emission is the basic mechanism responsible for energy losses of quarks and multiple hadron production in hard processes (see, e.g., Ref./6/) and the problem in question therefore acquires great practical interest.

In section 2 the matrix element (ME) of the process under consideration is shown to be represented as a product of the hard-process ME M_0 and the nonrelativistic radiation amplitude, with the Coulomb-like interaction taken into account. In section 3 we calculate the nonrelativistic radiation amplitude, and the spectrum and the energy losses caused by photon emission are analysed in section 4. At last, section 5 is concerned with gluon emission. The results obtained are discussed in the Conclusion.

2. Factorization of the interaction and radiation by nonrelativistic final-state pair.

As shown in /2/ (see also Ref./7/), the production amplitude for a pair of charged particles with low relative velocities is as follows if we take into consideration the Coulomb final-state interaction:

$$M = M_0 \cdot \left(\psi_{\vec{p}}^{(-)}(0)\right)^* . \qquad (1)$$

Here M_0 is the corresponding amplitude without regard for the Coulomb interaction, $\vec{p}$ - the particle momentum in the c.m.s., $\psi_{\vec{p}}^{(-)}(\vec{r}) = \langle \vec{r} | \vec{p}, out \rangle$ the wave function of the 'out'-state of relative pair motion, i.e. the solution of the Schrodinger equation for a particle with the mass $\frac{m}{2}$ (m is the mass of the particle being produced) in Coulomb field, which contains incoming spherical waves and outgoins planar waves with momentum $\vec{p}$ (normalized to $((2\pi)^3 \delta(\vec{p}-\vec{p}')$). For further generalization let us represent the factor in (1) which incorporates the final-state interaction as follows:

$$\left(\psi_{\vec{p}}^{(-)}(0)\right)^* = \langle \vec{p} | \, \mathcal{U}(\infty, 0) | 0 \rangle \qquad (2)$$

Here $|\vec{p}\rangle$ is the state with momentum $\vec{p}$, $\langle\vec{p}|\vec{r}\rangle = e^{-i\vec{p}\vec{r}}$, and $\mathcal{U}(t_2,t_1)$ is the time translation operator in the Dirac representation for a particle in the field with potential energy $V(\vec{r}) = -\frac{\alpha}{r}$:

$$\mathcal{U}(t_2,t_1) = T\exp\left(-i\int_{t_1}^{t_2} V(\vec{r},t)\,dt\right), \tag{3}$$

$$V(\vec{r},t) = e^{iH_0 t}\,V(\vec{r})\,e^{-iH_0 t}, \tag{4}$$

$$H_0 = -\frac{\Delta}{m}. \tag{5}$$

The meaning of formulae (1) and (2) is evident. Pair production lasts $\lesssim \frac{1}{m}$; the distance is not larger than $\frac{1}{m}$ either. For the Coulomb interaction, essential are much longer distances, $\ell \sim \frac{1}{|\vec{p}|} = \frac{2}{mv} \gg \frac{1}{m}$, and time spans, $\tau \sim \frac{\ell}{v} \gg \frac{1}{m}$. In view of this, the pair may be assumed to produce at the point $\vec{r} = 0$ for $t = 0$ with the probability amplitude M_0. After that, the interaction starts to influence which determines the amplitude of the probability to detect the pair in the state $|\vec{p}\rangle$.

It is clear that together with the Coulomb interaction, the interaction with transverse photons, which leads to their emission, may be considered as well. For the latter, also significant are large (as compared to $1/m$) distances and time spans $\tau_e \sim \frac{1}{\omega} \gg \frac{1}{m}$, $\ell_e \sim \tau v \gg \frac{1}{m}$, at frequencies $\omega \lesssim E = \frac{\vec{p}^2}{m}$. Therefore, the amplitudes of the processes followed by photon emission by a pair of produced particles are of the form

$$M_e = \langle f|\mathcal{U}_e(\infty,0)|0\rangle \cdot M_0 \tag{6}$$

where $|f\rangle$ is the product of the vector of the state $|\vec{p}\rangle$ of the produced pair and the vectors of the states of emitted photons, whereas the time translation operator $\mathcal{U}_e$ is given by formulae (3)-(5) with the substitutions $V(\vec{r}) \to V_e(\vec{r},t)$, where

$$V_e(\vec{r},t) = V(\vec{r}) - \frac{2e}{m}\vec{A}(t)\hat{\vec{p}} + \frac{e^2}{m}\vec{A}^2(t). \tag{7}$$

In the above formula, $\hat{\vec{p}} = -i\vec{\nabla}$, $\vec{A}(t)$ is the double-quantized transverse vector-potential of an electromagnetic field in the interaction representation. We have neglected its dependence on $\vec{r}$ because the photon wavelength is large as compared to the sizes of the emitting system (dipole nature of radiation).

In the zero order with respect to the interaction involving transverse photons, formula (6) is transformed into (1). In the first order, we get the radiation amplitude for a photon with momentum $\vec{k}$ and helicity λ by the produced pair:

$$M^{\lambda}(\vec{p},\vec{k}) = \frac{2e}{m}\langle\vec{p},out|\vec{e}^{*\lambda}(\vec{k})\hat{\vec{p}}(H-E_0-i0)^{-1}|0\rangle M_0, \tag{8}$$

Here $E_0 = \frac{\vec{p}^2}{m}+\omega$ is the energy of the pair before emission and H is the Hamiltonian incorporating the Coulomb interaction,

$$H = H_0 + V(\vec{r}). \tag{9}$$

Formula (8) can be derived by a direct summation of the main contributions given by the diagrams of perturbation theory, as it was done in Ref./7/. This formula reduces the problem to a quantum-mechanical one.

Using formula (8) and the relations

$$\hat{\vec{p}} = \frac{im}{2}[H-E_0,\vec{r}], \quad \vec{r}|0\rangle = 0, \tag{10}$$

we may represent the matrix element $M^{\lambda}(\vec{p},\vec{k})$ as follows:

$$M^{\lambda}(\vec{p},\vec{k}) = A^{\lambda}(\vec{p},\vec{k}) \cdot M_o \quad , \tag{11}$$

where

$$A^{\lambda}(\vec{p},\vec{k}) = -i\omega e \langle \vec{p}, out | \overset{*\lambda}{\vec{e}}(\vec{k}) \, \vec{r} \, (H - E_o - i0)^{-1} | 0 \rangle . \tag{12}$$

Besides, it is convenient to decompose the state $|\vec{p}, out\rangle$ into states $|E, L, L_z\rangle$ with definite values of the angular momentum L of the pair and its projection onto the direction of photon escape L_z . Since the pair produces at $\vec{r} = 0$, the final states with $L = 1$ and $L_z = -\lambda$ are only possible. Therefore, we obtain from (12)

$$A^{\lambda}(\vec{p},\vec{k}) = -\frac{2\pi\omega}{p} e^{i\delta_1} Y_{1,-\lambda}\left(\frac{\vec{p}}{p}\right) \cdot \\ \cdot \langle E, 1, -\lambda | \overset{*\lambda}{\vec{e}}(\vec{k}) e \cdot \vec{r} \, [H - E_o - i0]^{-1} | 0 \rangle , \tag{13}$$

where δ_1 is the scattering phase in the state with $L = 1$. We use a conventional normalization of states:

$$\langle E, L, L_z | E', L, L_z \rangle = 2\pi \delta(p - p') \quad , \tag{14}$$

where $p = \sqrt{mE}$.

For the cross section of the photon emission with momentum $\vec{k}$ and helicity λ we obtain from (11) and (1)

$$d\sigma_{\gamma}^{\lambda} = d\sigma_o \left| \frac{A^{\lambda}(\vec{p},\vec{k})}{\psi^{(-)}_{\vec{p}_o}(0)} \right|^2 \frac{p}{p_o} \frac{d^3\vec{k}}{(2\pi)^3 2\omega} \quad , \tag{15}$$

where $d\sigma_o$ is the cross section of the production of the pair with invariant mass $2m + E_o$, with the final-state interaction

taken into account; the factor p/p_o appears in (15) since the cross section $d\sigma_o$ is taken at the invariant mass of the radiationless process.

Formulae (11)-(15) admit a simple generalization to the case of gluon emission at $Q\bar{Q}$ pair production. In this case, the operator of electric dipole moment $e \cdot \vec{r}$ should be replaced by the operator of colour dipole moment:

$$e \cdot \vec{r} \longrightarrow g \vec{r} \left(\frac{t^a - \bar{t}^a}{2} \right) \quad , \tag{16}$$

where $t^a(\bar{t}^a)$ are the generators of coolour group SU(3) for quarks (antiquarks) and "a" is the colour index of the emitted gluon. Now the Hamiltonian H is the matrix in colour space:

$$H = -\frac{\Delta}{m} + \frac{\alpha_s}{r} t^a \bar{t}^a \quad , \tag{17}$$

where $\alpha_s = g^2/4\pi$ is the coupling constant in QCD. The colour states of the $Q\bar{Q}$ pair is convenient to choose as the eigenstates of the operator $(t^a + \bar{t}^a)^2$. In the basis of these states, the H is diagonalized; note that

$$H^{(T)} = -\frac{\Delta}{m} + \frac{\alpha_s}{r} \left(\frac{T}{2} - \frac{4}{3} \right) \quad , \tag{18}$$

where T is the eigenvalues of $(t^a + \bar{t}^a)^2$, which may be equal to 0 or 3. The zero value of T corresponds to the singlet representation, while $T = 3$ to the octet one. Within the representation, the states are distinguished by a colour index "a". The colour wave functions $\chi(T, a)$ are of the form

$$\chi^{\alpha}_{\bar{\alpha}}(0,0) = \frac{\delta^{\alpha}_{\bar{\alpha}}}{\sqrt{3}} \quad , \quad \chi^{\alpha}_{\bar{\alpha}}(3, a) = \sqrt{2} \, (t^a)^{\alpha}_{\bar{\alpha}} \quad , \tag{19}$$

where $\alpha(\bar{\alpha})$ are the colour indices of a quark (antiquark).

With the said above taken into consideration, for gluon emission at $Q\bar{Q}$ production the elementary algebra yields, instead of (11),

$$M^{\lambda a}_{T_f a_f}(\vec{p},\vec{k}) = A^{\lambda}_{T_f,T_i}(\vec{p},\vec{k})\cdot M_0^{T_i a_i}\cdot C^{T_f T_i}_{a_f a_i a}\,, \tag{20}$$

where $T_i, a_i\ (T_f, a_f)$ are the colour quantum numbers of the $Q\bar{Q}$ pair before (after) the emission, $C^{T_f T_i}_{a_f a_i a}$ are the group coefficients,

$$C^{00}_{a00} = 0\,, \quad C^{03}_{0a_i a} = \frac{\delta_{a_i a}}{\sqrt{6}}\,,$$

$$C^{30}_{a_f 0 a} = \frac{\delta_{a_f a}}{\sqrt{6}}\,, \quad C^{33}_{a_f a_i a} = \frac{1}{2} d_{a_f a_i a}\,, \tag{21}$$

and the nonrelativistic amplitude $A^{\lambda}_{T_f,T_i}$ is equal to

$$A^{\lambda}_{T_f,T_i}(\vec{p},\vec{k}) = -\frac{2\pi\omega}{p} e^{i\delta_1^{(T_f)}} Y_{1,-\lambda}\left(\frac{\vec{p}}{p}\right)\langle E, T_f, 1, -\lambda|\overset{*}{\vec{e}}{}^{\lambda}(\vec{k})\, g\vec{r}(H^{(T_i)}-E_0-i0)^{-1}|0\rangle. \tag{22}$$

For the cross sections, summed over the colour states of gluon and quarks, we obtain from (20) and (21) (cf.(15)), using the relation $d_{abi}d_{abj} = \frac{5}{3}\delta_{ij}$,

$$d\sigma_g^{\lambda} = \left\{\frac{4}{3} d\sigma_0^{(0)}\left|\frac{A^{\lambda}_{3,0}}{\psi^{(-)}_{\vec{p}_0,0}(0)}\right|^2 + d\sigma_0^{(3)}\left[\frac{1}{6}\left|\frac{A^{\lambda}_{0,3}}{\psi^{(-)}_{\vec{p}_0,3}(0)}\right|^2 + \frac{5}{12}\left|\frac{A^{\lambda}_{3,3}}{\psi^{(-)}_{\vec{p}_0,3}(0)}\right|^2\right]\right\}\frac{p}{p_0}\,\frac{d^3\vec{k}}{(2\pi)^3 2\omega}\,, \tag{23}$$

where $d\sigma_0^{(T)}$ is the cross section of the $Q\bar{Q}$ pair production in singlet ($T = 0$) or octet ($T = 3$) state with invariant mass $2m+E_0$, with the final state interaction taken into account, and $\psi^{(-)}_{\vec{p},T}(\vec{r})$ is the corresponding wave function.

3. Calculation of the nonrelativistic amplitude

Using the expansion in terms of the complete set of states $|\vec{r}\rangle$, we may rewrite formula (22) as follows:

$$A^{\lambda}_{T_f,T_i}(\vec{p},\vec{k}) = -\frac{2\pi g\omega}{p}\cdot e^{i\delta_1^{(T_f)}} Y_{1,-\lambda}\left(\frac{\vec{p}}{p}\right)\cdot$$

$$\cdot\int d^3\vec{r}\,\langle\vec{r}|[H^{(T_i)}-E_0-i0]^{-1}|0\rangle\cdot\overset{*}{\vec{e}}{}^{\lambda}(\vec{k})\cdot\vec{r}\,Y^{*}_{1,-\lambda}(\theta,\varphi)R^{(T_f)}_{p1}(r) \tag{24}$$

where $R^{(T_f)}_{p1}(r)$ is the radial wave function for the Hamiltonian (18). With the polarization vectors $\vec{e}^{\lambda}(\vec{k}) = -\frac{i\lambda}{\sqrt{2}}(e_x + i\lambda e_y)$ and spherical functions $Y_{1,\lambda} = -i\lambda\sqrt{\frac{3}{8\pi}}\sin\theta\cdot e^{i\lambda\varphi} = \vec{e}^{\lambda}\vec{n}\sqrt{\frac{3}{4\pi}}$ chosen in a conventional way and after the integration over the angles we get

$$A^{\lambda}_{T_f,T_i}(\vec{p},\vec{k}) = -\frac{2\pi g\omega}{p^2} e^{i\delta_1^{(T_f)}}\overset{*}{\vec{e}}{}^{\lambda}(\vec{k})\vec{p}\int_0^{\infty} r^3 dr\,\langle r|(H^{(T_i)}-E_0-i0)^{-1}|0\rangle R^{(T_f)}_{p1}(r)\,. \tag{25}$$

For the Green function in (25), we use the integral representation in the Meixner form /8/ (see also Ref./4/):

$$\langle\vec{r}|\left(H^{(T)}-\frac{\vec{p}_0^{\,2}}{m}-i0\right)^{-1}|0\rangle = -\frac{imp_0}{2\pi} e^{ip_0 r}\int_0^{\infty} dt\left(\frac{1+t}{t}\right)^{\frac{im\alpha^{(T)}}{2p_0}} e^{2ip_0 rt}\,, \tag{26}$$

here

$$\vec{p}_0^{\,2} = mE_0\,, \quad \alpha^{(T)} = \alpha_s\left(\frac{4}{3}-\frac{T}{2}\right). \tag{27}$$

For the radial part of the wave function we take the known representation (Ref./9/)

$$R^{(T)}_{p1}(r) = C^{(T)}_{p1}\frac{(2pr)^{-2}}{2\pi}\oint_C d\tau\, e^{2ipr\tau}\left(\tau+\frac{1}{2}\right)^{\frac{im\alpha^{(T)}}{2p}-2}\left(\tau-\frac{1}{2}\right)^{-\frac{im\alpha^{(T)}}{2p}-2}\,, \tag{28}$$

where integration occurs in the positive direction along the contour enclosing the points $\pm 1/2$,

$$C^{(T)}_{p1} = \left|\frac{4\pi p m\alpha^{(T)}}{1-\exp\left(-\frac{\pi m\alpha^{(T)}}{p}\right)}\right|^{1/2}\cdot\sqrt{1+\left(\frac{m\alpha^{(T)}}{2p}\right)^2}\,. \tag{29}$$

When substituting (28) and (26) into (25), the integral over τ is taken in a simple way, and after that the integral over τ' is reduced to the residue at the point $\tau = -\frac{P_0}{P}(1+2t)$. As a result, we obtain

$$A^{\lambda}_{T_f,T_i}(\vec{p},\vec{k}) = -\frac{g}{m\omega p}\, e^{i\delta_1^{(T_f)}}\, C_{p1}^{(T_f)*}\, \vec{e}^{\lambda}(\vec{k})\vec{p} \cdot I^{Q_f Q_i}_{(x)}, \qquad (30)$$

where

$$x = \frac{\omega}{E_0}, \quad Q_j = \frac{\alpha^{(T_j)}}{2}\sqrt{\frac{m}{E_0}}, \qquad (31)$$

$$I^{Q_f Q_i}_{(x)} = -x^2 \int_0^{\infty} dt \left(\frac{1+t}{t}\right)^{iQ_i} \frac{d}{dt}\left((1+2t+\sqrt{1-x})^{-\frac{iQ_f}{\sqrt{1-x}}-2} \cdot (1+2t-\sqrt{1-x})^{\frac{iQ_f}{\sqrt{1-x}}-2}\right) \qquad (32)$$

For arbitrary values of the parameters, the integral $I^{Q_f Q_i}_{(x)}$ cannot be expressed via elementary functions. Let us consider some limiting cases when the expression for $I^{Q_f Q_i}_{(x)}$ is simplified.

1) Low emission frequency limit, $x \to 0$:

$$I^{Q_f Q_i}_{(x)} \xrightarrow[x \to 0]{} \left(\frac{x}{4}\right)^{i(Q_f - Q_i)} \frac{\Gamma(1-iQ_i)\Gamma(2-i(Q_f-Q_i))}{\Gamma(2-iQ_f)}. \qquad (33)$$

2) $|Q_i| \ll 1$. In this case,

$$I^{Q_f Q_i}_{(x)} \simeq \left(\frac{1-\sqrt{1-x}}{1+\sqrt{1-x}}\right)^{\frac{iQ_f}{\sqrt{1-x}}} \qquad (34)$$

Regions 1) and 2) overlap each another. It is easy to see that (34) coincides with (33) for this case.

3) $|Q_i| \gg 1$. Here the asymptotic behaviour greatly depends on x. Since for small x there is the expression (33) true at any Q_i, Q_f, let us consider the case when $x \sim 1$. With this constraint, still two substantially different cases are possible. At $Q_f \neq Q_i$ we have

$$I^{Q_f Q_i}(x) = \frac{3x^2 Q_i}{2(Q_i - Q_f)^5} \qquad (35)$$

whereas at $Q_f = Q_i = Q$

$$I^{Q_f Q_i}(x) = i^{2/3}\,\Gamma\left(\frac{2}{3}\right)\left(\frac{2x}{3Q^2}\right)^{1/3} \qquad (36)$$

At small x, the applicability region of the equalities (35) and (36) is limited by the largeness condition for the r.h. sides of these equalities as compared with the r.h. side of eq.(33), i.e. at $Q_f \neq Q_i$

$$x^2 \gg \frac{2}{3}(Q_f - Q_i)^6 \cdot \sqrt{\frac{2\pi|Q_f - Q_i|}{|Q_i| \cdot |Q_f|^3}}\, e^{-\frac{\pi}{2}(|Q_i| - |Q_f| + |Q_i - Q_f|)}, \qquad (37)$$

while for $Q_f = Q_i = Q$

$$x \gg \frac{3}{2Q\Gamma^3\left(\frac{2}{3}\right)} \qquad (38)$$

When the opposite inequalities hold the expression (33) is applicable.

4. Photon emission

With the use of formula (30), the amplitude of photon emission during the heavy pair production is representable, if the Coulomb interaction between heavy particles is taken into account, in the following form

$$A^{\lambda}(\vec{p},\vec{k}) = \tau(x,Q) \cdot A^{\lambda}_B(\vec{p},\vec{k}), \qquad (39)$$

where A^{λ}_B is the emission amplitude in Born approximation,

$$A^{\lambda}_B(\vec{p},\vec{k}) = -2e\,\frac{\vec{e}^{\lambda}(\vec{k})\vec{p}}{m \cdot \omega}, \qquad (40)$$

and the coefficient

$$\tau(x,Q) = e^{i\delta_1} \frac{C_{p1}(Q)}{2p} I^{QQ}(x) \tag{41}$$

characterizes the influence of the interaction. Here $I^{QQ}(x)$ and $C_{p1}(Q)$ are defined in (32) and (29) respectively; note that

$$Q = \sqrt{\frac{m}{E_0}} \cdot \frac{\alpha}{2} \quad , \quad \alpha^{(T)} = \alpha \tag{42}$$

Formulae (40) and (42) are written for lepton production taking the QED interaction between them into account. It seems that even of greater interest is photon emission in the case of $Q\bar{Q}$ pair production. For this case the substitution $e \to e_Q$ (e_Q is the charge of a quark) should be made in (40). The α in (42) should be taken equal to $\frac{4}{3}\alpha_s$ in the singlet state and to $\left(-\frac{\alpha_s}{6}\right)$ in the octet state. (Mention should be made that the factor 4/3 is omitted in formulae (46) and (48) of Ref./10/.)

Now we would like to consider the properties of the coefficient $\tau(x,Q)$ in (41). At $x \to 0$, using eq.(33) and the relations

$$e^{i\delta_\ell} \frac{C_{p\ell}}{2p} = e^{\frac{\pi Q}{2\sqrt{1-x}}} \cdot \Gamma\left(\ell + 1 - \frac{iQ}{\sqrt{1-x}}\right) , \tag{43}$$

and

$$\left(\psi^{(-)}_{\vec{p}_0}(0)\right)^* = e^{\frac{\pi Q}{2}} \Gamma(1 - iQ) , \tag{44}$$

we obtain

$$\tau(x,Q) \xrightarrow[x \to 0]{} \left(\psi^{(-)}_{\vec{p}_0}(0)\right)^* . \tag{45}$$

Since $\left(\psi^{(-)}_{\vec{p}}(0)\right)^*$ is the factor incorporating the interaction between the pair components in the emissionless process (1), formula (45) proves the applicability of the general assertion on the infrared radiation factorization for our case /11/.

Another illustration can be seen at the opposite end of the spectrum. If the fraction of the energy carried away by a photon is rather large, the velocity of the charged final-state particles can become so small that $\frac{Q}{\sqrt{1-x}} \gtrsim 1$ at $Q \ll 1$. We here obtain from (41) and (34)

$$\tau(x,Q) = e^{i\delta_1} \frac{C_{p1}(Q)}{2p} . \tag{46}$$

This factor corresponds to making allowance for the interaction only after photon emission and is an analog of the factor $\left(\psi^{(-)}_{\vec{p}}(0)\right)^* = e^{i\delta_0} \frac{C_{p0}(Q)}{2p}$ in (1). The difference is due to the fact that after emission the angular momentum of a pair equals 1. As seen from (43),

$$\frac{C_{p1}(Q)}{C_{p0}(Q)} = \sqrt{1 + \frac{Q^2}{1-x}} , \tag{47}$$

i.e. taking the interaction into account results in a relative increase of the process probability for $L = 1$ as compared to $L = 0$.

Within $x \backsim 1$, simple expressions for $\tau(x,Q)$ are derived only in the asymptotic region of large $|Q|$. Using formulae (36), (29) and (41) we get for the case of attraction

$$|\tau(x,Q)| = \Gamma\left(\frac{2}{3}\right) \frac{\sqrt{2\pi Q}}{(1-x)^{3/4}} \left(\frac{2Qx}{3}\right)^{1/3} \tag{48}$$

at $Q \gg 1$, whereas for the case of repulsion the result is different from (48) by the factor $\exp\left(-\frac{\pi|Q|}{\sqrt{1-x}}\right)$.

As we should expect, attraction contributes to increasing the process probability as compared with the Born one, while repulsion reduces it exponentially.

Figures 1 and 2 show $|\tau(x,Q)|^2$ vs. x at Q = 1.0 and Q = 3.0, correspondingly. Solid lines stand for the result of the numerical integration, dotted lines mean the calculation according to formula (45) and the dash-dot lines imply the calculation according to formula (48). It is seen that the interval of the x at which formula (45) works well diminishes with increasing Q. Formula (48) at Q = 3.0 gives satisfactory agreement with numerical calculation.

We have omitted the plots for the case of repulsion ($Q<0$) since for $|\tau(x,Q)|^2$ the substitution $(Q\to -Q)$ has influence only on a readily-calculable function $C_{p_1}(Q)$ (29), while the quantity $|I^{QQ}(x)|$ (32), which requires for numerical integration, remains unchanged.

As seen from (15), the quantity

$$dW^\gamma(\vec k) = \sum_\lambda \frac{p}{p_0}\left|\frac{A^\lambda(\vec p,\vec k)}{\psi^{(-)}_{\vec p_0}(0)}\right|^2 \frac{d^3\vec k}{(2\pi)^3 2\omega} \qquad (49)$$

may be called the photon emission probability. After summation with respect to photon polarization coupled with integration over the angles of emission we obtain using (39) and (40)

$$x\frac{dW^\gamma}{dx} = \left|\frac{\tau(x,Q)}{\psi^{(-)}_{\vec p_0}(0)}\right|^2 x\frac{dW_B^\gamma}{dx}, \qquad (50)$$

where dW_B^γ is the corresponding probability in Born approximation:

$$x\frac{dW_B^\gamma}{dx} = \frac{2}{3}\left(\frac{e}{\pi}\right)^2\frac{E_0}{m}(1-x)^{3/2} \qquad (51)$$

An important integral characteristic is the energy fraction carried away by a photon:

$$\langle x_\gamma\rangle \equiv \int_0^1 x\frac{dW^\gamma}{dx}dx = \int_0^1\left|\frac{\tau(x,Q)}{\psi^{(-)}_{\vec p_0}(0)}\right|^2 x\frac{dW_B^\gamma}{dx}dx \qquad (52)$$

Let us introduce $\eta_\gamma \equiv \langle x_\gamma\rangle / \langle x_\gamma\rangle_B$, where

$$\langle x_\gamma\rangle_B = \frac{4}{15}\left(\frac{e}{\pi}\right)^2\frac{E_0}{m} \qquad (53)$$

is the known Born expression for $\langle x_\gamma\rangle$. To calculate $\langle x_\gamma\rangle$ within large Q, $Q\gg 1$, we make advantage of the asymptotics (48) since $x\sim 1$ gives the main contribution. In this case,

$$\eta_\gamma = \Gamma^2\left(\frac{2}{3}\right)\left(\frac{2Q}{3}\right)^{2/3}\cdot\frac{3}{2} \qquad (54)$$

For repulsion at $Q<0$, $|Q|\gg 1$, the $x\sim\frac{1}{\pi Q}$ region proves to be significant in the calculation of $\langle x_\gamma\rangle$ which is a transient one from the asymptotics (33) to the (36). A rough approximation can be obtained under assumption that $\pi\gg 1$. With the use of eqs.(33) and (29) we obtain

$$\eta_\gamma \simeq \frac{5}{2\pi|Q|} \qquad (55)$$

Figure 3 shows η_γ vs. Q. The $Q>0$ and $Q<0$ regions correspond to attraction and repulsion, respectively. The solid lines are the result of the numerical integration, and the dotted ones are the calculation according to formulae (54) and (55). It is seen that the asymptotical formulae begin yield satisfactory agreement with the numerical integration at $|Q|\simeq 1$.

5. Gluon emission

By analogy with (39), we introduce the quantity $\tau(x,Q_i,Q_f)$, characterizing the influence of the interaction if the $Q\bar{Q}$ pair is created in colour state T_i and transforms into T_f after emission:

$$A^{\lambda}_{T_f,T_i}(\vec{p},\vec{k}) = \tau(x,Q_i,Q_f)\,A^{\lambda}_{B}(\vec{p},\vec{k})\,. \qquad (56)$$

Here A^{λ}_{B} is given by formula (40) with a substitutiton $e\to g$ and the quantity $\tau(x,Q_i,Q_f)$ equals, as it follows from (30) and (40),

$$\tau(x,Q_i,Q_f) = e^{i\delta_1^{(T_f)}}\frac{C_{p1}^{(T_f)}}{2p}\cdot I^{Q_f Q_i}(x)\,, \qquad (57)$$

Note that $C_{p1}^{(T_f)}$ is given by formula (29), and $I^{Q_f Q_i}(x)$ is given by formula (32). When the $Q\bar{Q}$ pair is produced in the octet state and remains here after emission, gluon emission has no, in essence, distinction from photon emission at $Q<0$ treated in the foregoing section.

A distinctive feature of the processes with a change of the colour spin in the emission process is the absence of ifrared radiation factorization. At $x\to 0$ we derive from (29) and (33), instead of (45),

$$\tau(x,Q_i,Q_f)\underset{x\to 0}{\to}\left(\psi^{(-)}_{\vec{p}_0,T_i}(0)\right)^{*} e^{\frac{\pi}{2}(Q_f-Q_i)}\left(\frac{x}{4}\right)^{i(Q_f-Q_i)}\cdot\Gamma(2-i(Q_f-Q_i)), \qquad (58)$$

where $\left(\psi^{(-)}_{\vec{p}_0,T_i}(0)\right)^{*}$ is given by the r.h. side of formula (44) for $Q=Q_i$. The lack of factorization is evidently due to an infinitely large radius of the Coulomb interaction. In this respect, one can draw on a certain analogy with the absence of factorization at high energies /12,13/.

Let us dwell on the strong interaction region: $|Q_i|\gg 1$. The behaviour of $\tau(x,Q_i,Q_f)$ depends considerably on the signs of Q_i and Q_f. If a pair is produced in the octet state $(Q_i=-\frac{Q}{6},\ Q=\frac{m\alpha_s}{2p_0})$ and passes to the singlet one as a result of radiation $(Q_f=4/3\,Q)$, we obtain from (57) with the help of formulae (35) and (29)

$$\tau(x,-\tfrac{Q}{6},\tfrac{4}{3}Q) = e^{i\delta_1^{(0)}}\left(\frac{2}{3}\right)^{13/2}\frac{x^2}{(1-x)^{3/4}}\left(\frac{\pi}{Q}\right)^{1/2}\frac{1}{Q^2}\,. \qquad (59)$$

Emphasize that $|\tau|$ reduces with Q, although in the final state there exists the attraction. Repulsion in the intermediate state proves to be more significant. Formula (59) fails in the $x\to 0$ region, where, according to (58) we have at $Q\gg 1$

$$|\tau(x,-\tfrac{Q}{6},\tfrac{4}{3}Q)|\underset{x\to 0}{\to}\frac{3\pi Q^2}{2}e^{-\frac{\pi Q}{6}}\,. \qquad (60)$$

Increasing the lifetime in the intermediate state (at $\frac{\omega}{E_0}\to 0$) results in an exponential suppression of the probability as compared to the Born (60). The applicability of formula (59) for small x is bounded by the inequality (37):

$$x^2\gg\left(\frac{3}{2}\right)^{15/2}\sqrt{\pi}\,Q^{9/2}\cdot e^{-\frac{\pi Q}{6}}\,. \qquad (61)$$

For the range where the inequality, inverse to (61), holds formula (60) is applicable. For large Q this is an exponentially narrow range. Of most interest is the case when a pair is produced in colour-singlet state. Such a case is realized during the $Q\bar{Q}$ production in e^+e^- annihilation. After the gluon radiation the pair is in the octet state, i.e. $Q_i=\frac{4}{3}Q$, $Q_f=-\frac{Q}{6}$.

This case is analysed in what follows. At $Q\gg 1$ we get

$$\tau(x,\tfrac{4}{3}Q,-\tfrac{Q}{6}) = -e^{i\delta_1^{(8)}}\left(\frac{2}{3}\right)^5 \frac{2x^2}{Q^4}\left[\frac{\pi Q}{3\sqrt{1-x}}\left(1+\left(\frac{Q}{6\sqrt{1-x}}\right)^2\right)\right]^{\frac{1}{2}} \cdot e^{-\frac{\pi Q}{6\sqrt{1-x}}} \cdot \qquad (62)$$

Repulsion in the final state leads to an exponential smallness of $|\tau|$ for large Q. The radiation spectrum is substantially distorted and the region of high frequencies turns out to be suppressed. Pay our attention to the fact that in cases when the quarks in the final state are attracted, $|\tau|$ grows with increasing x (see formulae (48) and (59)).

At low frequencies, formula (62) is applicable for

$$x^2 \gg 8\left(\frac{3}{2}\right)^{15/2}\sqrt{\pi Q^9}\, e^{-4/3\,\pi Q} \qquad (63)$$

With the opposite inequality fulfilled, i.e. within a narrow, close-to-zero range we obtain from (58) for $Q \gg 1$:

$$|\tau(x,\tfrac{4}{3}Q,-\tfrac{Q}{6})| = \sqrt{2}\; 3\pi Q^2 e^{-\frac{3}{2}\pi Q} \qquad (64)$$

Figures 4-6 illustrate the dependence of $|\tau(x,\frac{4}{3}Q,-\frac{Q}{6})|^2$ on x for Q = 1.0, 3.0 and 4.0 respectively. Solid lines stand for the result of the numerical integration, the dotted ones denote the calculation according to formula (58), and the dot-dash lines denote the calculation according to formula (62). As in the case of photon radiation, it is seen that the interval within which the infrared approximation (58) works well, narrows with increasing Q. The strong-interaction approximation (62) works well with $Q \gtrsim 4.0$.

Similarly to (49), the quantity

$$dW_0^g(\vec{k}) = \frac{4}{3}\sum_\lambda \frac{P}{P_0}\left|\frac{A^\lambda_{3,0}(\vec{p},\vec{k})}{\psi^{(-)}_{\vec{p},0}(0)}\right|^2 \frac{d^3\vec{k}}{(2\pi)^3 2\omega} \qquad (65)$$

may be called the probability of the gluon emission in case when a pair is produced in a colour-singlet state. The coefficient 4/3 in (65) is of the group origin (see eq.(23)). After summation over gluon polarizations and after integration over the emission angles we have

$$x\frac{dW_0^g}{dx} = \left|\frac{\tau(x,\frac{4}{3}Q,-\frac{Q}{6})}{\psi^{(-)}_{\vec{p},0}(0)}\right|^2 x\,\frac{dW_{0\,B}^{g}}{dx}\,, \qquad (66)$$

where $x\frac{dW^g_{0B}}{dx}$ is given by the r.h. side of the equality (51) with the substitution $e^2 \to \frac{4}{3}g^2$.

In ref./14/ the average fraction of the energy for a heavy quark produced in e^+e^- collisions has been calculated in Born approximation: $\langle z_Q\rangle = \left\langle\frac{2E_Q}{\sqrt{s}}\right\rangle \simeq \frac{\langle E_Q\rangle}{m}$. If we define $\langle x_g\rangle$ similarly to (52), then we obtain, only taking the gluon emission into account,

$$1-\langle z_Q\rangle = \frac{E_0}{m}\frac{\langle x_g\rangle}{2} = \frac{E_0}{m}\eta_g\frac{\langle x_g\rangle_B}{2}\,, \qquad (67)$$

where the factor $\frac{E_0}{m}$ is accounted for by the fact we have determined $x=\frac{\omega}{E_0}$ rather than $\frac{\omega}{m}$. The coefficient η_g incorporates the effect of the $Q\bar{Q}$ interaction, and the $\langle x_g\rangle_B$ is a known Born expression for $\langle x_g\rangle$:

$$\langle x_g\rangle_B = \frac{16}{45}\left(\frac{g}{\pi}\right)^2\frac{E_0}{m}\,. \qquad (68)$$

It is worth emphasizing the lack of the factor 8/15 in formula (9) of ref./14/ and the factor ($1/\pi$) in formula (44) of ref. /10/.

To calculate $\langle x_g\rangle$ at large Q, one can take advantage of the asymptotics (62) since the main contribution is given by $x\sim\frac{6}{\pi Q}$ lying within the applicability range (63) of formula (62). As a result, we have

$$\eta_g = \frac{5}{6}\left(\frac{8}{3\pi}\right)^5 \frac{1}{Q^{11}}\, e^{-\frac{\pi Q}{3}} \quad . \qquad (69)$$

As one could expect, repulsion in a final state give rise to a sharp reduction of the energy losses of a nonrelativistic pair of quarks.

Numerical integration turns out to be necessary at moderate Q. We present the results of such integration on Fig. 7 by solid line, while the dotted curve corresponds to formula (69). As the diagrams show, the asymptotic formula (69) fails to work well at moderate Q. This is due to the approximations used to integrate the spectrum (62).

Much more efficient is formula

$$\eta_g = \frac{5}{9}\left(\frac{2}{3Q}\right)^8 \cdot \sqrt{\frac{8\pi}{\left(\frac{\pi Q}{(1-x_0)^{5/2}}+\left(\frac{4}{x_0}\right)\right)^2}} \cdot x_0^4 \cdot \left(1-x_0+\left(\frac{Q}{6}\right)^2\right) e^{-\frac{\pi Q}{3\sqrt{1-x_0}}} \qquad (70)$$

where x_0 is the solution of the equation

$$(1-x)^3 - \left(\frac{\pi Q}{24}\cdot x\right)^2 = 0 \qquad (71)$$

which lies within $0 \leqslant x_0 \leqslant 1$.

Formula (70) has been derived by the steep descent method when calculating $\langle x_g \rangle$ (see eq.(52)). To locate the saddle point, we have beared in mind that a strong dependence of x is of the form $x^4 \exp\left(-\frac{\pi Q}{3\sqrt{1-x}}\right)$ (see eqs.(52), (62) and (51)).

The function (70) is plotted on Fig. 7 as a dot-dash line. This formula works well with $Q \gtrsim 3.0$.

6. Conclusion

The results obtained show that a Coulomb-like interaction changes both its integral characteristics (such as the energy

losses) and the radiation spectrum during the production of a nonrelativistic pair of heavy particles, as compared with the relevant Born values. In the most interesting case of gluon emission during the $Q\bar{Q}$ pair production in e^+e^- collisions the final-state repulsion results in radiation suppression and to a reduction of energy losses. The radiation spectrum is distorted: the radiation is stronger suppressed the higher the frequency is. For photon radiation under these conditions the situation becomes different: the interaction enhances radiation, especially its hard part.

Mention should be made of the applicability of the above consideration, in particular for quark pairs production. On the one hand, the applicability region is restricted by the nonrelativism condition: $\left(\frac{P}{m}\right)^2 \ll 1$. On the other hand, the characteristic momenta should be sufficiently large to avoid unperturbative effects and to make use of perturbation theory with respect to α_s.

In addition to the $\ln\left(\frac{P}{\Lambda_{QCD}}\right) \gg 1$ condition (Λ_{QCD} is a mass parameter in QCD), this imposes the limitations of the type $E^4 \gg \alpha_s^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle$ (see refs./4,5,10/). A comprehensive analysis of the corrections is out of the scope of the present paper. With the indicated conditions fulfilled, one can use the above formulae where the coupling constant α_s must be taken at distances roughly equal to $\frac{1}{P}$.

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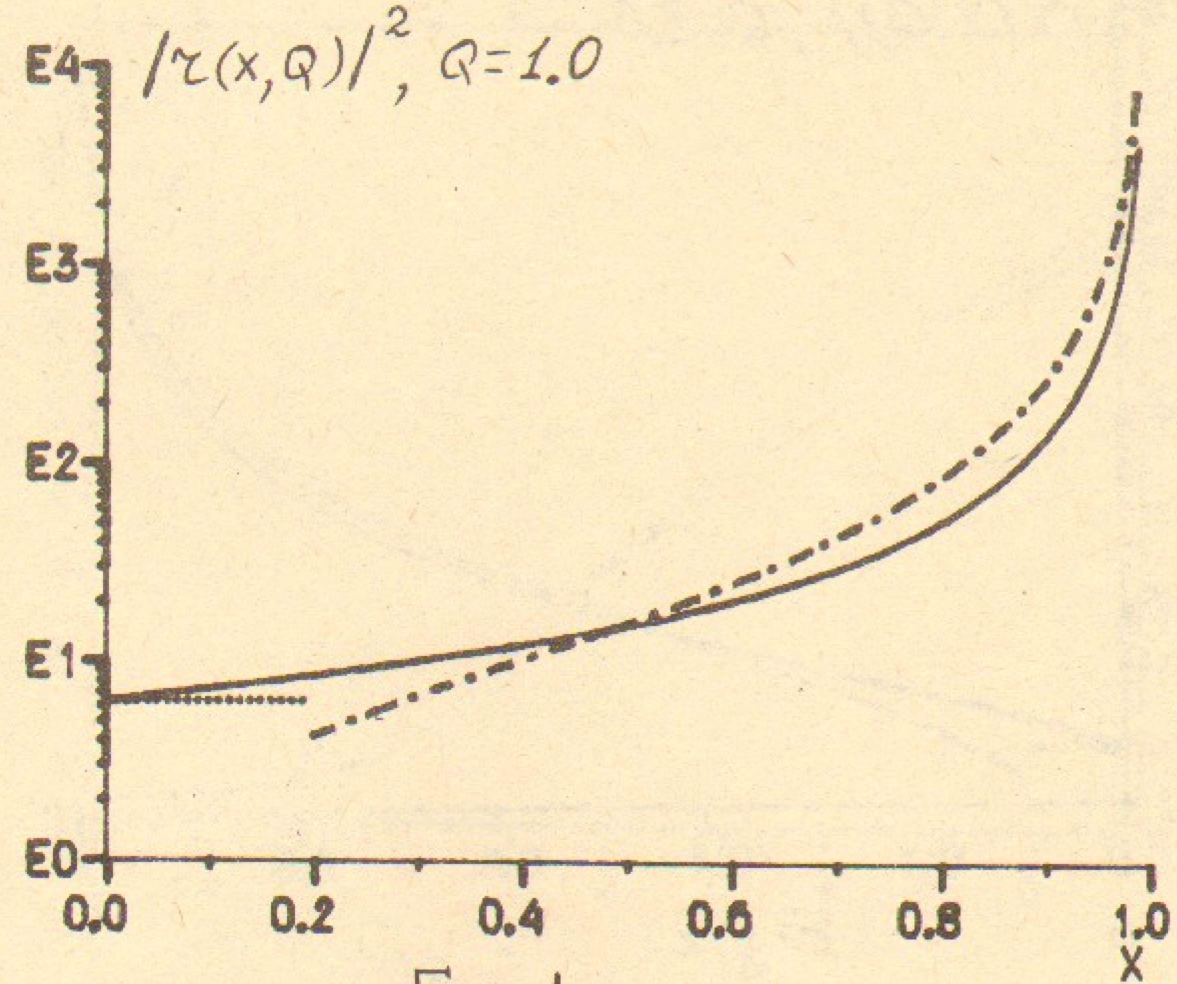


Fig. 1.

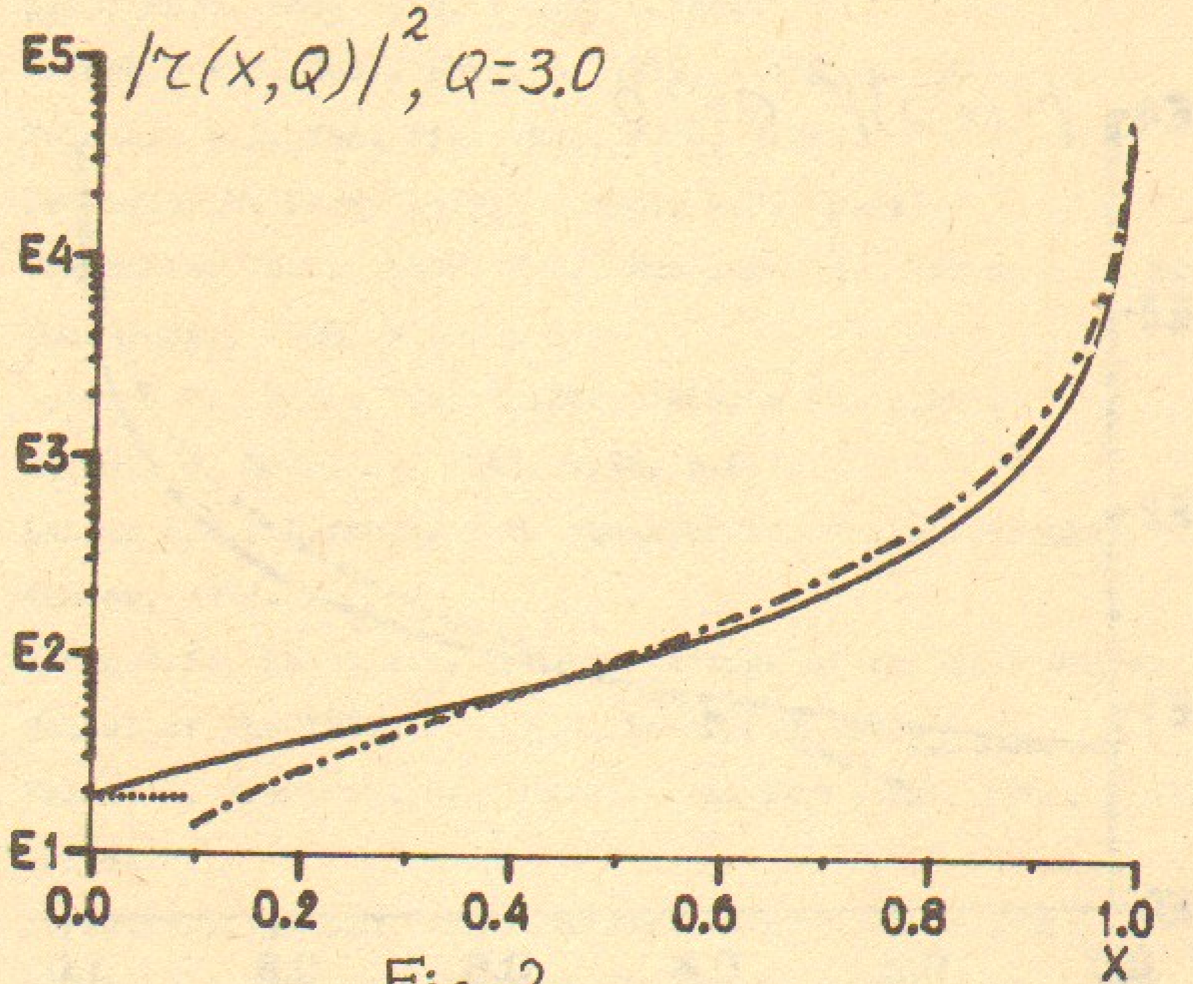

$|\tau(x,Q)|^2$, $Q=3.0$
E5
E4
E3
E2
E1
0.0
0.2
0.4
0.6
0.8
1.0
X

Fig. 2.

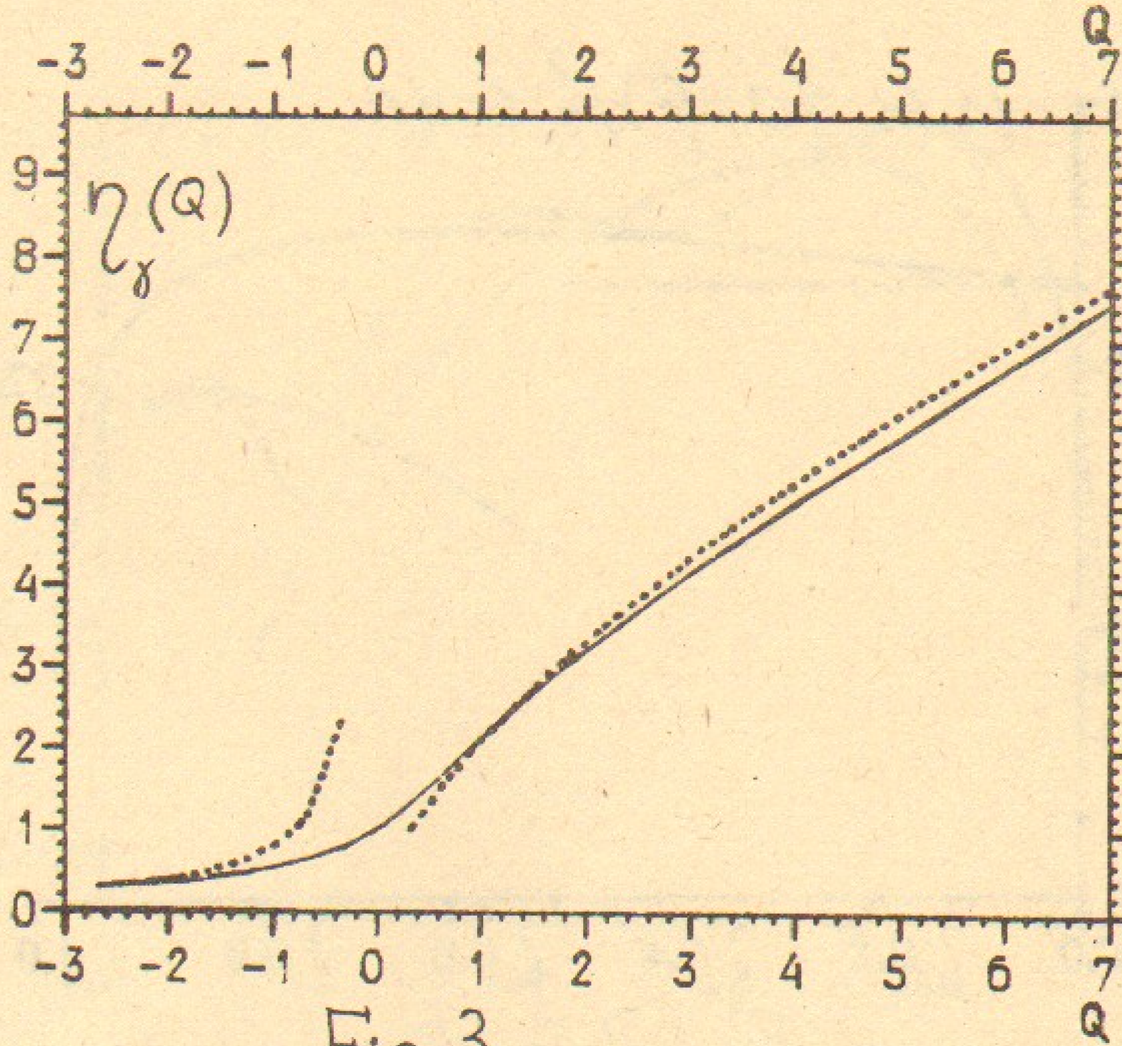

Q
-3 -2 -1 0 1 2 3 4 5 6 7
$\eta_\gamma(Q)$
0 1 2 3 4 5 6 7 8 9
-3 -2 -1 0 1 2 3 4 5 6 7
Q

Fig. 3.

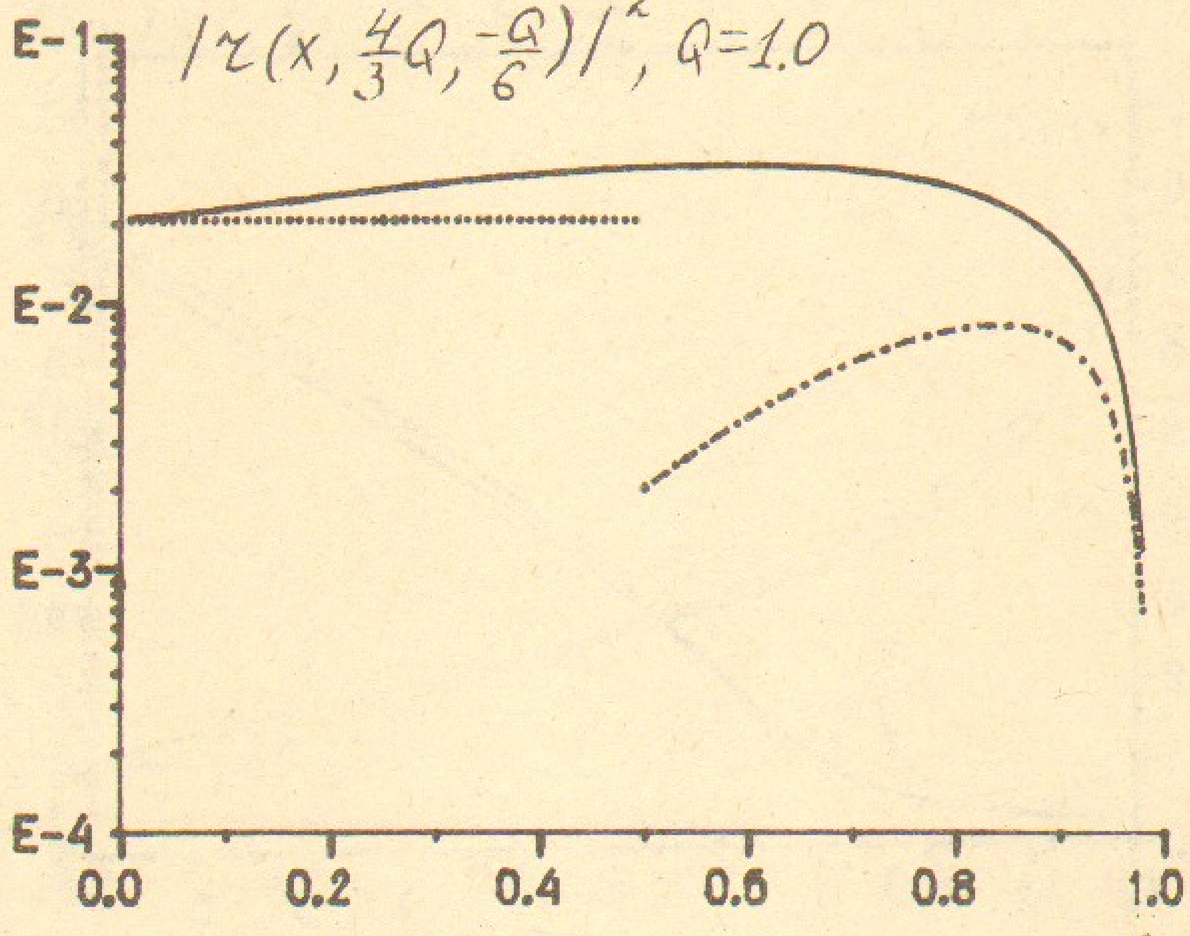

$|\tau(x, \frac{4}{3}Q, \frac{-Q}{6})|^2$, Q=1.0
E-1
E-2
E-3
E-4
0.0
0.2
0.4
0.6
0.8
1.0

Fig. 4.

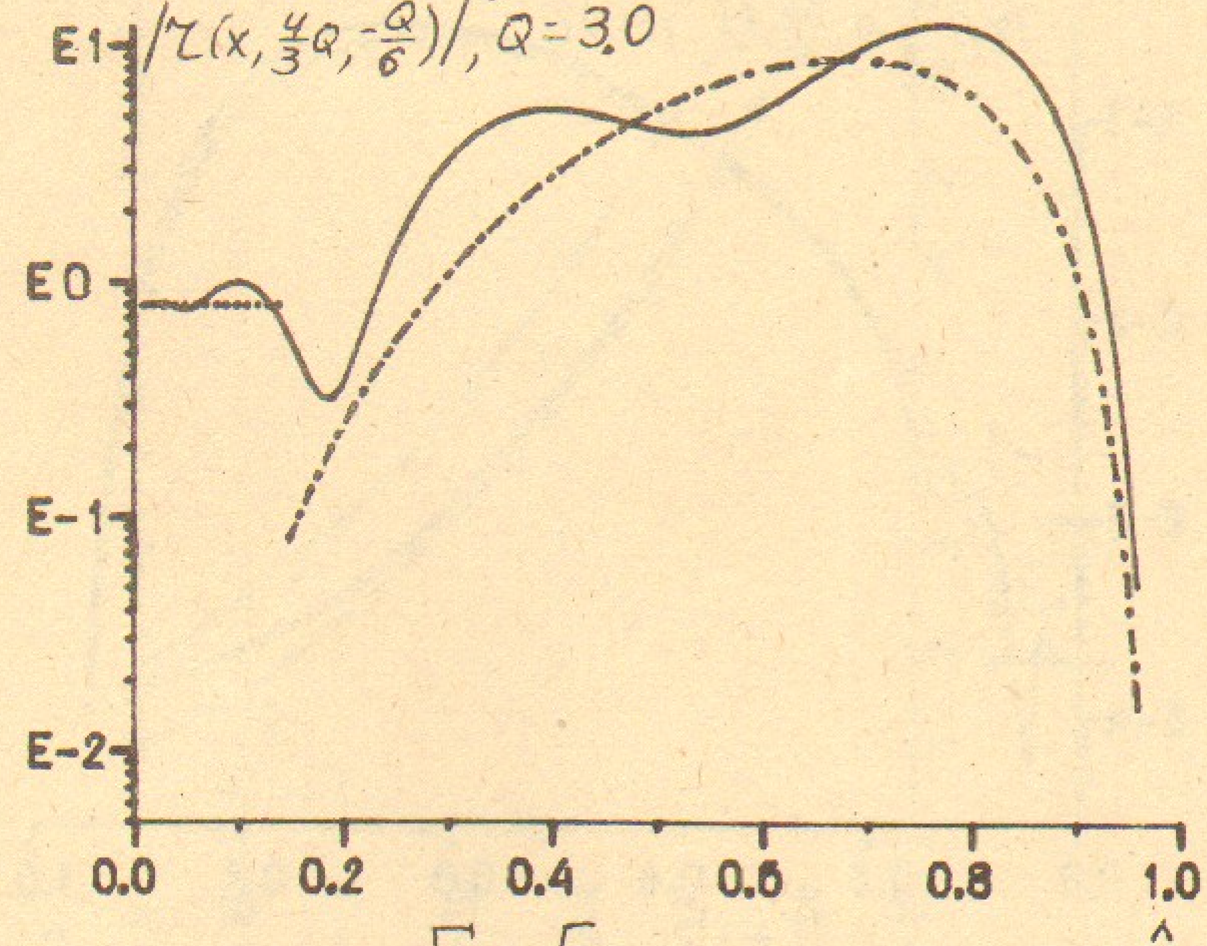
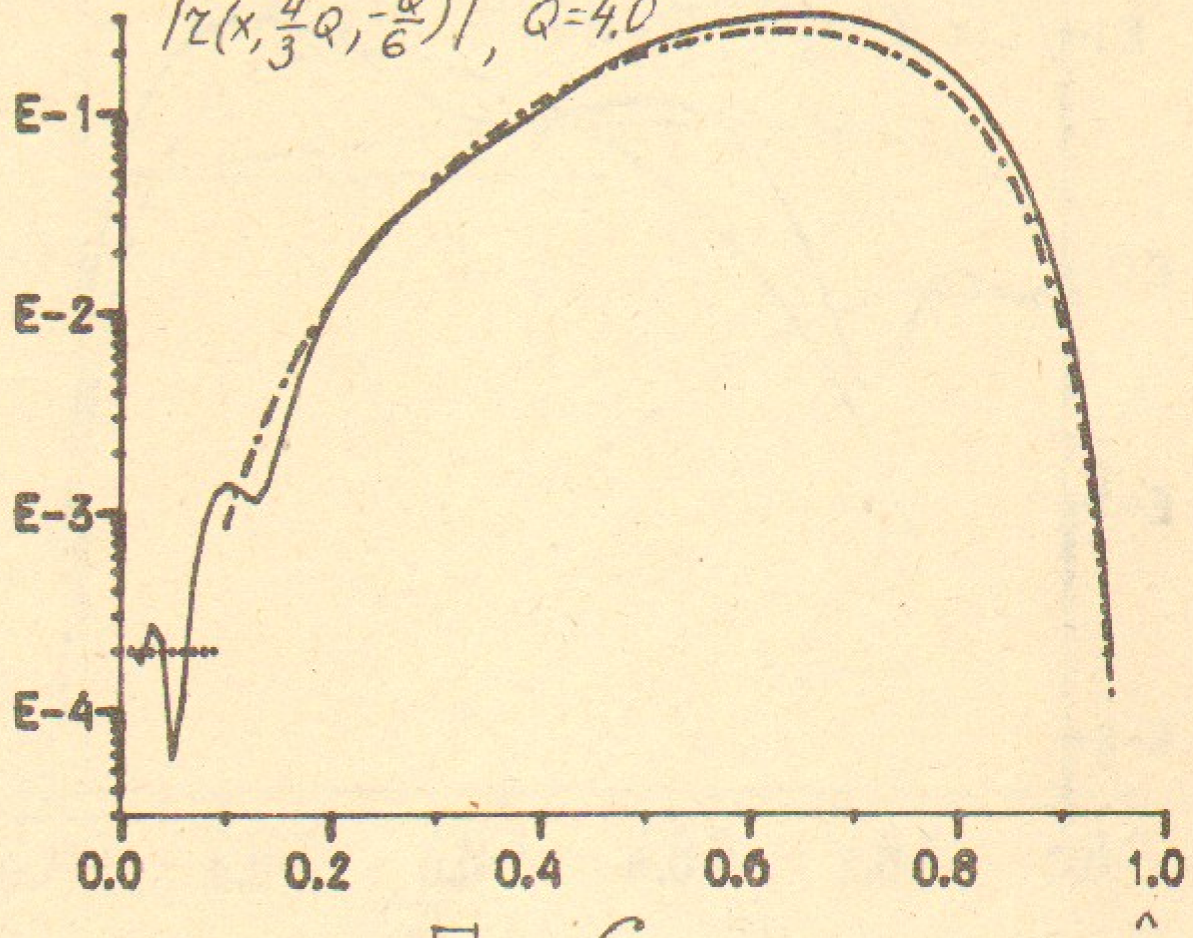
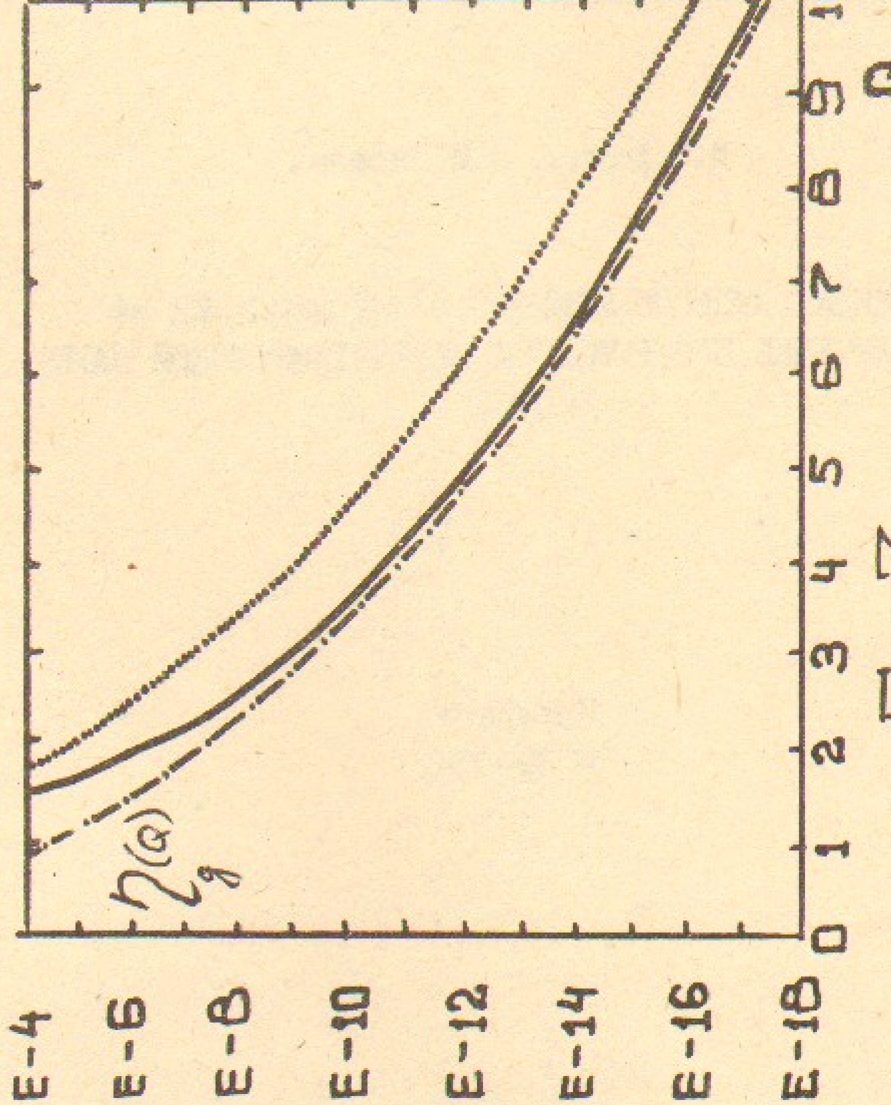

$|\tau(x, \frac{4}{3}Q, \frac{-Q}{6})|^2$, Q=3.0
E1
E0
E-1
E-2
0.0
0.2
0.4
0.6
0.8
1.0

Fig. 5.



$|r(x, \frac{4}{3}Q, -\frac{Q}{6})|^2$, $Q=4.0$
E-1
E-2
E-3
E-4
0.0
0.2
0.4
0.6
0.8
1.0
^

Fig. 6.



$\eta_g^{(Q)}$
E-4
E-6
E-8
E-10
E-12
E-14
E-16
E-18
0
1
2
3
4
5
6
7
8
9
10
Q

Fig. 7.

В.С.Фадин, О.И.Яковлев

ВЛИЯНИЕ КУЛОНОПОДОБНОГО ВЗАИМОДЕЙСТВИЯ НА ИЗЛУЧЕНИЕ ПРИ РОЖДЕНИИ НЕРЕЛЯТИВИСТСКОЙ ПАРЫ ЧАСТИЦ

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