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A FRESH LOOK AT Φ—ω MIXING

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A Fresh Look at $\Phi-\omega$ Mixing

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ABSTRACT

An unprejudiced analysis is made of the possibility of different theoretical descriptions of the decays $\Phi\to\rho\pi\to\pi^+\pi^-\pi^0$, $\Phi\to\pi^0\gamma$ taking into account both the $\Phi-\omega$ mixing and the direct decays $\Phi\to\rho\pi$, $\Phi\to\pi^0\gamma$. For this purpose the data from the Neutral Detector on the reactions $e^+e^-\to\pi^+\pi^-\pi^0$, $\pi^0\gamma$, K_SK_L in the interval $\sqrt{s}=700\div1050$ MeV are used. Experimental data are shown not to permit one to answer the question what is the origin of the decay $\Phi\to\rho\pi$ ($\pi^0\gamma$). This decay can be completely caused both by the $\Phi-\omega$ mixing and by the direct decay $\Phi\to\rho\pi$ ($\Phi\to\pi^0\gamma$). Intermediate cases are certainly possible. It is shown that to elucidate the situation at least an order of magnitude improvement of accuracy is required.

1. INTRODUCTION

The problem of Φ-ω mixing, being a classical one in the spectroscopy of light quarks, during last 25 years was studied both theoretically and in experiment. There exists a well established view which always give meaningful answers when applied to the analysis of the experimental data. Briefly, the matters are as follows: Φ-meson is almost pure quarkonium $s\bar{s}$ while ω-meson is almost pure quarkonium $(u\bar{u}+d\bar{d})/\sqrt{2}$. A small magnitude of the $\Phi-\omega$ mixing parameter is well determined in a simple quark model, a so-called mass mixing model. Qualitatively, the smallness of this parameter is explaned in QCD by the absence of large nonperturbative effects in the vector channel. In this scheme the decays $\Phi\to\rho\pi$ and $\Phi\to\pi^0\gamma$, i. e. those which demand the OZI suppressed intermediate $s\bar{s}$ quark pair annihilation to proceed, are explained by the admixture of ω-meson in the Φ-meson wave function. The direct decays $\Phi\to\rho\pi$ and $\Phi\to\pi^0\gamma$ were usually neglected since they are believed to correspond to more complex nonplanar quark diagrams and in the framework of $1/N$ expansion are expected to be suppressed. However, this argument is only qualitative one and demands a special verification.

In the present paper we present the study of the possibility of different theoretical descriptions of the decays $\Phi\to\rho\pi\to\pi^+\pi^-\pi^0$, $\Phi\to\pi^0\gamma$ taking into account both the $\Phi-\omega$ mixing and the direct decays $\Phi\to\rho\pi$ and $\Phi\to\pi^0\gamma$. For this purpose the data are used on the reactions $e^+e^-\to\pi^+\pi^-\pi^0$ [1–4], $e^+e^-\to\pi^0\gamma$ [4, 5] and $e^+e^-\to K_SK_L$ (4, 6, 7] in the energy interval $\sqrt{s}=700\div1050$ MeV.

In Section 2 we give the expressions for the amplitude of the reactions $e^+e^-\to\pi^+\pi^-\pi^0$, $\pi^0\gamma$, K_SK_L in all orders in $\Phi-\omega$ mixing allowing for the $\pi^+\pi^-\pi^0$, $\pi^0\gamma$, $K\bar{K}$ and $\eta\gamma$ intermediate states.

In Section 3 the results are given of the analysis of experimental data on $e^+e^-\to\pi^+\pi^-\pi^0$, $e^+e^-\to\pi^0\gamma$ and $e^+e^-\to K_SK_L$ and the discussion from point of view of existing pictures of $\Phi-\omega$ mixing is presented. It is shown that the data [1—7] do not permit one to answer the question what is the mechanism of the decays $\Phi\to\rho\pi$ and $\Phi\to\pi^0\gamma$. These decays can almost completely be caused by both $\Phi-\omega$ mixing and by the direct decays $\Phi\to\rho\pi$ and $\Phi\to\pi^0\gamma$ alone.

In Section 4, as a conclusion, we discuss the merits of different approaches to the experimental study of $\Phi-\omega$ mixing.

2. FUNDAMENTAL PHENOMENOLOGY OF $\Phi-\omega$ MIXING

In principle, there exist different ways to take the mixing of resonances into account. We shall adopt here the field theory inspired approach [8] which is based on summation in all orders of the loop corrections to bare, i. e. not distorted by the mixing, propagators of ρ-, ω- and Φ-meson. The convenience of this approach in comparison with equivalent N/D method [9] consists in its transparency from point of view of physics, an easy generalization to any number of resonances and any number of their common decay channels. The formulae obtained are unitary [8]. The adopted approach is most suitable when applied to weakly mixed resonances which we deal with choosing

$$\rho^{(0)}=(u\bar{u}-d\bar{d})/\sqrt{2}\,,\qquad \omega^{(0)}=(u\bar{u}+d\bar{d})/\sqrt{2}\,,\qquad \Phi^{(0)}=s\bar{s}\,. \tag{1}$$

Hereafter all the quantities with subscript (0) refer to the states (1). Let us write the general production amplitude in the e^+e^- annihilation allowing for the mixing of resonances

$$M(e^+e^-\to\rho^{(0)},\omega^{(0)},\Phi^{(0)}\to f)=\sum_{VV'}M(e^+e^-\to V)\,(G^{-1})_{VV'}\,M(V'\to f)\,, \tag{2}$$

where $f=\pi^+\pi^-\pi^0$, $\pi^0\gamma$, K_SK_L etc., the summation in (2) goes over

$V, V'=\rho^{(0)}$, $\omega^{(0)}$, $\Phi^{(0)}$ from (1); $M(e^+e^-\to V)$ $(M(V\to f))$ is the amplitude of the transition $e^+e^-\to V$ $(V\to f)$. $G_{VV'}$ is the matrix of inverse propagators

$$G_{VV'}=\begin{pmatrix} D_\rho^{(0)} & -\Pi_{\rho\omega} & -\Pi_{\rho\Phi} \\ -\Pi_{\rho\omega} & D_\omega^{(0)} & -\Pi_{\omega\Phi} \\ -\Pi_{\rho\Phi} & -\Pi_{\omega\Phi} & D_\Phi^{(0)} \end{pmatrix}, \tag{3}$$

$$D_V^{(0)}=m_V^{(0)^2}-s-i\sqrt{s}\;\Gamma_V^{(0)}(s)\,,$$

where $\Gamma_V^{(0)}$ denotes the full width of V-meson which is not distorted by mixing. All essential decay modes having the branching ratios greater then 1% are taken into account in $\Gamma_V^{(0)}$, see below for more detail.

The polarization operator of the $\Phi-\omega$ transition looks as follows:

$$\Pi_{\omega\Phi}=\mathrm{Re}\,\Pi_{\omega\Phi}+i\,\mathrm{Im}\,\Pi_{\omega\Phi}\,, \tag{4}$$

where

$$\mathrm{Im}\,\Pi_{\omega\Phi}=s^{1/2}\Big\{g^{(0)}_{\omega\rho\pi}g^{(0)^*}_{\Phi\rho\pi}W(s)/4\pi+ +(g^{(0)}_{\omega K^+K^-}g^{(0)^*}_{\Phi K^+K^-}q^3_{K^+K^-}+g^{(0)}_{\omega K^0\bar{K}^0}g^{(0)^*}_{\Phi K^0\bar{K}^0}q^3_{K^0\bar{K}^0})/6\pi s+ +\frac{1}{3}g^{(0)}_{\omega\pi\gamma}g^{(0)^*}_{\Phi\pi\gamma}q^3_{\pi^0\gamma}+\frac{1}{3}g^{(0)}_{\omega\eta\gamma}g^{(0)^*}_{\Phi\eta\gamma}q^3_{\eta\gamma}\Big\}. \tag{5}$$

$\mathrm{Im}\,\Pi_{\omega\Phi}$ is determined by the real intermediate states $\omega^{(0)}\to\rho\pi\to\Phi^{(0)}$. (if any), $\omega^{(0)}\to K\bar{K}\to\Phi^{(0)}$, $\omega^{(0)}\to\pi^0\gamma\to\Phi^{(0)}$ (if any), $\omega^{(0)}\to\eta\gamma\to\Phi^{(0)}$. In order to avoid the confusion let us note that though in the case of complex coupling constants each term in (5) has an imaginary part, in sum these are cancelled. This can be varified by cutting the diagrams for $\Pi_{\omega\Phi}$ over all possible two-particle intermediate states and summing the corresponding contributions. Thus, when doing the phenomenological analysis one cannot arbitrary introduce the phases into the vector meson couplings to various channels.

In (3) — (5) s is the total e^+e^- energy squared in the c.m.s. of e^+e^-, q_f $(f=\pi\pi$, $K\bar{K}$, $\pi^0\gamma$ and $\eta\gamma)$ is the momentum of the final particle in the same system, $W(s)$ is the phase space factor for decay $\omega,\Phi\to\rho^+\pi^-+\rho^-\pi^++\rho^0\pi^0$ allowing for the finite ρ meson width, see (17) — (19) for $W(s)$.

The real part $\mathrm{Re}\,\Pi_{\omega\Phi}$ in the resonance region is considered to be the function only slightly depending on s which fact to some degree is supported by experiment, see Sect. 3.

The coupling constants $g^{(0)}_{\Phi\rho\pi}$ and $g^{(0)}_{\Phi\pi\gamma}$ which are set usually to zero correspond to direct decays $\Phi^{(0)}\to\rho\pi\to 3\pi$, $\Phi^{(0)}\to\pi^0\gamma$ *)

The $\rho-\omega$ and $\rho-\Phi$ mixing are unessential in our case and hence will not be taken into account when analyzing the data [1—7], i. e. $\Pi_{\rho\omega}=0$ $\Pi_{\rho\Phi}=0$ in (3), see, however the end of the paper.

To reduce the number of free parameters we use the simple quark model predictions for the decay channels $\rho^{(0)}\to K\bar K$ and $\omega^{(0)}\to K\bar K$ since they lie under the $K\bar K$-threshold as well as for the decays $\rho^{(0)}\to\eta\gamma$, $\omega^{(0)}\to\eta\gamma$. The necessary relations are

$$\frac{g^{(0)}_{\rho K^+K^-}}{g^{(0)}_{\Phi K^+K^-}}=\frac{g^{(0)}_{\omega K^+K^-}}{g^{(0)}_{\Phi K^+K^-}}=\frac{g^{(0)}_{\omega K^0\bar K^0}}{g^{(0)}_{\Phi K^+K^-}}=-\frac{g^{(0)}_{\rho K^0\bar K^0}}{g^{(0)}_{\Phi K^+K^-}}=-\frac{1}{\sqrt2}\,, \tag{6}$$

$$\frac{g^{(0)}_{\omega\eta\gamma}}{g^{(0)}_{\Phi\eta\gamma}}=\frac{1}{\sqrt2},\qquad \frac{g^{(0)}_{\rho\eta\gamma}}{g^{(0)}_{\Phi\eta\gamma}}=\frac{3}{\sqrt2}. \tag{7}$$

When obtaining (7) we take the pseudoscalar octet mixing angle $\theta_p=-18°$.

Taking into account the above consideration let us write the reaction amplitudes of our interest for $e^+e^-\to\pi^+\pi^-\pi^0$, $\pi^0\gamma$, K_SK_L

$$\sigma_{3\pi}(s)=\frac{4\pi\alpha}{s^{3/2}}\frac{W(s)}{|\Delta|^2}\,\Big|g^{(0)}_{\gamma\omega}g^{(0)}_{\omega\rho\pi}D^{(0)}_{\Phi}+g^{(0)}_{\gamma\Phi}g^{(0)}_{\Phi\rho\pi}D^{(0)}_{\omega}+\Pi_{\omega\Phi}\big(g^{(0)}_{\gamma\omega}g^{(0)}_{\Phi\rho\pi}+g^{(0)}_{\gamma\Phi}g^{(0)}_{\omega\rho\pi}\big)\Big|^2, \tag{8}$$

$$\sigma_{\pi^0\gamma}(s)=\frac{(4\pi)^2\alpha}{3s^{3/2}}q^3_{\pi^0\gamma}(s)\,\Big|\frac{g^{(0)}_{\gamma\rho}g^{(0)}_{\rho\pi\gamma}}{D^{(0)}_{\rho}}+g^{(0)}_{\gamma\omega}g^{(0)}_{\omega\pi\gamma}\frac{D^{(0)}_{\Phi}}{\Delta}+g^{(0)}_{\gamma\Phi}g^{(0)}_{\Phi\pi\gamma}\frac{D^{(0)}_{\omega}}{\Delta}+\big(g^{(0)}_{\gamma\omega}g^{(0)}_{\Phi\pi\gamma}+g^{(0)}_{\gamma\Phi}g^{(0)}_{\omega\pi\gamma}\big)\frac{\Pi_{\omega\Phi}}{\Delta}\Big|^2, \tag{9}$$

$$\sigma_{K_SK_L}(s)=\frac{8\pi\alpha}{3s^{5/2}}q^3_{K_SK_L}\,\Big|\frac{g^{(0)}_{\gamma\rho}g^{(0)}_{\rho K^0\bar K^0}}{D^{(0)}_{\rho}}+g^{(0)}_{\gamma\omega}g^{(0)}_{\omega K^0\bar K^0}\frac{D^{(0)}_{\Phi}}{\Delta}+g^{(0)}_{\gamma\Phi}g^{(0)}_{\Phi K^0\bar K^0}\frac{D^{(0)}_{\omega}}{\Delta}+\frac{\Pi_{\omega\Phi}}{\Delta}\big(g^{(0)}_{\gamma\omega}g^{(0)}_{\Phi K^0\bar K^0}+g^{(0)}_{\gamma\Phi}g^{(0)}_{\omega K^0\bar K^0}\big)\Big|^2. \tag{10}$$

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*) Let us note for experts that the situation here is analogous to that in the phenomenology of CP violation. In the present case $g^{(0)}_{\Phi\rho\pi}/g^{(0)}_{\omega\rho\pi}$ is analogous to the parameter ε'. which appears in models of CP violation and which is caused by the direct decay $K_L\to\pi^+\pi^-$. The $\Phi-\omega$ mixing parameter ε, see below, is analogous to the parameter ε which describes the transition $K_L\to K_S\to\pi\pi$.

In (8) — (10) $\alpha=1/137$, $g^{(0)}_{\gamma V}=em^{(0)^2}_V/f^{(0)}_V$,

$$\Delta=D^{(0)}_{\omega}D^{(0)}_{\Phi}-\Pi^2_{\omega\Phi}. \tag{11}$$

Inverse propagators for the states (1) look like

$$D^{(0)}_{\rho}(s)=m^{(0)^2}_{\rho}-s-i\sqrt{s}\,\big(\Gamma^{(0)}(\rho\to\pi^+\pi^-,s)+\Gamma^{(0)}(\rho\to K^0\bar K^0,s)+\Gamma^{(0)}(\rho\to K^+K^-,s)+\Gamma^{(0)}(\rho\to\pi^0\gamma,s)+\Gamma^{(0)}(\rho\to\eta\gamma,s)\big)\,, \tag{12}$$

$$D^{(0)}_{V}(s)=m^{(0)^2}_{V}-s-i\sqrt{s}\,\big(\Gamma^{(0)}(V\to3\pi,s)+\Gamma^{(0)}(V\to K^+K^-,s)+\Gamma^{(0)}(V\to K^0\bar K^0,s)+\Gamma^{(0)}(V\to\pi^0\gamma,s)+\Gamma^{(0)}(V\to\eta\gamma,s)\big)\,, \tag{13}$$

$$V=\omega^{(0)},\Phi^{(0)}.$$

Let us write the expressions for the partial widths. The width of V-meson decay into two pseudoscalar particles is

$$\Gamma^{(0)}(V\to P_1P_2,s)=\frac{|g^{(0)}_{VP_1P_2}|^2}{6\pi s}q^3_{P_1P_2}(s)\,C_{VP_1P_2}(s)\times\theta\big(\sqrt{s}-m_{P_1}-m_{P_2}\big)\,, \tag{14}$$

P_1, $P_2=\pi^+\pi^-$, K^+K^-, K_SK_L, θ is the step function.

The width of the radiative decay

$$\Gamma^{(0)}(V\to P\gamma,s)=\frac{1}{3}\,|g^{(0)}_{VP\gamma}|^2q^3_{P\gamma}C_{VP\gamma}(s)\,, \tag{15}$$

P is the pseudoscalar particle.

The width of the $\pi^+\pi^-\pi^0$ decay is

$$\Gamma^{(0)}(V\to3\pi,s)=\frac{|g^{(0)}_{V\rho\pi}|^2}{4\pi}W(s)\,C_{V\rho\pi}(s)\,, \tag{16}$$

where $W(s)$ is

$$W(s)=\frac{2}{\pi}\int\limits_{2m_\pi}^{s^{1/2}-m_\pi}dm\,F(m)\,\Gamma^{(0)}(\rho\to\pi^+\pi^-,m^2)\,\frac{q^3(\sqrt{s},m,m_\pi)}{|D^{(0)}_{\rho}(m^2)|^2}, \tag{17}$$

$$F(m)=1+\frac{3}{2}\,\mathrm{Re}\left\{\frac{D^{(0)}_{\rho}(m^2)}{C^{1/2}_{\rho\pi\pi}(m^2)}\int\limits_{-1}^{1}dx\,(1-x^2)\,\frac{C^{1/2}_{\rho\pi\pi}(m^2_+)}{D^{(0)}_{\rho}(m^2_+)}\right\}. \tag{18}$$

Here

$$m^2_+=\frac{1}{2}\,(s+3m^2_\pi-m^2)+xq\,(\sqrt{s},m,m_\pi)\,\,(s(1-4m^2_\pi/m^2))^{1/2},$$

$$q(\sqrt{s}, m, m_\pi) = \left[\frac{1}{4s}\left(s-(m-m_\pi)^2\right)\left(s-(m+m_\pi)^2\right)\right]^{1/2}. \qquad (19)$$

In (14) — (16), (18) we introduce, as usual, the form factors which restrict a too fast growth with energy of the partial widths, from the demand that $\sqrt{s}\,\Gamma(s)\to$const in the limit of large s. These look like

$$C_{VP_1P_2}(s) = \frac{1+(R_{VP_1P_2}q_{P_1P_2}(m_V^2))^2}{1+(R_{VP_1P_2}q_{P_1P_2}(s))^2}, \qquad (20)$$

$$C_{V\rho\pi}(s) = \left(\frac{1+(R_{V\rho\pi}m_V)^2}{1+R^2_{V\rho\pi}s}\right)^2. \qquad (21)$$

$R_{VP_1P_2}$ and $R_{V\rho\pi}$ are so-called range parameters. Allowing for the vector dominance model one can set in (15)

$$C_{VP\gamma}(s) = C_{V\rho\pi}(s)\,. \qquad (22)$$

The expressions (8) — (10) for appropriate cross sections look somewhat unconventional. However, they can be rewritten in almost convenient way if one diagonalizes the states. Let us do this for the channel $e^+e^-\to\pi^+\pi^-\pi^0$ since only ω- and Φ-mesons contribute to the cross section. Allowing for in (8) the terms of the first order in $\Phi-\omega$ mixing in the coupling constants and the terms of the second order in $\Phi-\omega$ mixing in the mass shifts of the resonances we obtain from (8)

$$\sigma_{3\pi}(s) \simeq \frac{4\pi\alpha W(s)}{s^{3/2}}\left|\frac{(g^{(0)}_{\gamma\omega}-\varepsilon g^{(0)}_{\gamma\Phi})(g^{(0)}_{\omega\rho\pi}-\varepsilon g^{(0)}_{\Phi\rho\pi})}{M^{(0)^2}_\omega - s - \varepsilon^2(M^{(0)^2}_\Phi - M^{(0)^2}_\omega)} + \right.$$

$$\left. + \frac{(g^{(0)}_{\gamma\Phi}+\varepsilon g^{(0)}_{\gamma\omega})(g^{(0)}_{\Phi\rho\pi}+\varepsilon g^{(0)}_{\omega\rho\pi})}{M^{(0)^2}_\Phi - s + \varepsilon^2(M^{(0)^2}_\Phi - M^{(0)^2}_\omega)}\right|^2. \qquad (23)$$

Hereafter the parameter of $\Phi-\omega$ mixing ε is

$$\varepsilon = -\Pi_{\omega\Phi}/(M^{(0)^2}_\Phi - M^{(0)^2}_\omega)\,, \qquad M^{(0)^2}_V = m^{(0)^2}_V - i\sqrt{s}\;\Gamma^{(0)}_V(s)\,, \qquad (24)$$

From (23) we extract the coupling constants of the physical states, i. e. the states the wave function of which includes the $\Phi-\omega$ mixing:

$$g_{\gamma\omega} \simeq g^{(0)}_{\gamma\omega} - \varepsilon g^{(0)}_{\gamma\Phi},$$

$$g_{\omega\rho\pi} \simeq g^{(0)}_{\omega\rho\pi} - \varepsilon g^{(0)}_{\Phi\rho\pi} \simeq g^{(0)}_{\omega\rho\pi}, \qquad (25)$$

$$g_{\gamma\Phi} \simeq g^{(0)}_{\gamma\Phi} + \varepsilon g^{(0)}_{\gamma\omega},$$

$$g_{\Phi\rho\pi} \simeq g^{(0)}_{\Phi\rho\pi} + \varepsilon g^{(0)}_{\omega\rho\pi}. \qquad (26)$$

Analogous considerations of the $K\bar{K}$ and $P\gamma$ ($P=\pi^0$, η) final states show that

$$g_{\omega K\bar{K}} \simeq g^{(0)}_{\omega K\bar{K}} - \varepsilon g^{(0)}_{\Phi K\bar{K}},$$

$$g_{\omega P\gamma} \simeq g^{(0)}_{\omega P\gamma} - \varepsilon g^{(0)}_{\Phi P\gamma},$$

$$g_{\Phi K\bar{K}} \simeq g^{(0)}_{\Phi K\bar{K}} + \varepsilon g^{(0)}_{\omega K\bar{K}},$$

$$g_{\Phi P\gamma} \simeq g^{(0)}_{\Phi P\gamma} + \varepsilon g^{(0)}_{\omega P\gamma}. \qquad (27)$$

Picking out the real and imaginary parts of ε, the inverse propagators in (23) can be written as

$$M^{(0)^2}_{\Phi,\omega} - s \pm \varepsilon^2(M^{(0)^2}_\Phi - M^{(0)^2}_\omega) \simeq m^{(0)^2}_{\Phi,\omega} \pm \mathrm{Re}\,[\,\varepsilon^2(M^{(0)^2}_\Phi - M^{(0)^2}_\omega)\,] -$$

$$- s - i\,s^{1/2}\{\Gamma(\Phi,\omega\to 3\pi, s) + \Gamma(\Phi,\omega\to K^+K^-, s) + \Gamma(\Phi,\omega\to K_SK_L, s) +$$

$$+ \Gamma(\Phi,\omega\to\pi^0\gamma, s) + \Gamma(\Phi,\omega\to\eta\gamma, s)\}\,, \qquad (28)$$

The upper (lower) sign in (28) refers to $\Phi(\omega)$-meson. Up to small corrections which quadratically depend on $\Phi-\omega$ mixing, the partial widths in (28) must be calculated by the formulae (14) — (16) but with the substitution $g^{(0)}_{Vab}\to g_{Vab}$ according to (25) — (27). It is seen from (28) that there appears the correction to the resonance mass which is of the second order in $\Phi-\omega$ mixing

$$\delta m^2_{\Phi,\omega} \equiv m^2_{\Phi,\omega} - m^{(0)^2}_{\Phi,\omega} = \pm\,\mathrm{Re}\,[\,\varepsilon^2(M^{(0)^2}_\Phi - M^{(0)^2}_\omega)]\,, \qquad (29)$$

where $m_{\Phi,\omega}$ are the masses of the physical states.

3. RESULTS AND DISCUSSION

The analysis of the present Section is based on the data on the reactions $e^+e^-\to\pi^+\pi^-\pi^0$ [1—4], $e^+e^-\to\pi^0\gamma$ [4, 5] and $e^+e^-\to K_SK_L$ [4, 6, 7] in the energy range $\sqrt{s}=700\div1050$ MeV. Though the statistics of experiment is relatively rich it is still poor to determine all the parameters $f^{(0)}_\omega$, $f^{(0)}_\Phi$, $f^{(0)}_\rho$, $g^{(0)}_{\omega\rho\pi}$, $g^{(0)}_{\Phi\rho\pi}$, $g^{(0)}_{\omega K\bar{K}}$, $g^{(0)}_{\Phi K\bar{K}}$, $g^{(0)}_{\omega\pi\gamma}$, $g^{(0)}_{\Phi\pi\gamma}$, Re $\Pi_{\omega\Phi}$, $m^{(0)}_\omega$, and $m^{(0)}_\Phi$ with sufficient accuracy. Our main goal here is to determine the parameters of $\Phi-\omega$ mixing $g^{(0)}_{\Phi\rho\pi}$, $g^{(0)}_{\Phi\pi\gamma}$ and Re $\Pi_{\omega\Phi}$.

To reduce a number of free parameters let us fix the coupling constants of $\rho^{(0)}$, $\omega^{(0)}$, $\Phi^{(0)}$ to $K\bar{K}$ channel via (6), through the width of the decay $\Phi\to K_SK_L$ which is determined by the resonance excitation curve $e^+e^-\to K_SK_L$ in the vicinity of Φ peak:

$$|g^{(0)}_{\Phi K^0\bar{K}^0}|=\left(\frac{6\pi m^2_\Phi\Gamma(\Phi\to K_SK_L,m^2_\Phi)}{q^3_{K_SK_L}(m^2_\Phi)}\right)^{1/2}\frac{1}{|1-\varepsilon/\sqrt{2}|}. \qquad (30)$$

In the same manner we use (7) for the $\eta\gamma$ channel in order to express $g^{(0)}_{\rho\eta\gamma}$, $g^{(0)}_{\Phi\eta\gamma}$ and $g^{(0)}_{\omega\eta\gamma}$ through the partial $\eta\gamma$ width of the physical Φ-meson

$$|g^{(0)}_{\Phi\eta\gamma}|=\left(\frac{3\Gamma(\Phi\to\eta\gamma,m^2_\Phi)}{q^3_{\eta\gamma}(m^2_\Phi)}\right)^{1/2}\frac{1}{|1+\varepsilon/\sqrt{2}|}. \qquad (31)$$

$\Gamma(\Phi\to K_SK_L,m^2_\Phi)$ and $\Gamma(\Phi\to\eta\gamma,m^2_\Phi)$ are taken from [10].

The real part of the polarization operator $\Pi_{\omega\Phi}$ is considered to be the constant independent of energy, $\mathrm{Re}\,\Pi_{\omega\Phi}=A_{\omega\Phi}$. This assumption was tested using the parametrization $\mathrm{Re}\,\Pi_{\omega\Phi}=A_{\omega\Phi}+B_{\omega\Phi}s$. As a result the values $A_{\omega\Phi}=-0.02$ GeV^2 and $B_{\omega\Phi}=(0\pm11)\cdot10^{-3}$ was obtained, which does not contradict $B_{\omega\Phi}=0$.

The results of analysis of the different theoretical possibilities to describe the data [1—7] are presented in Table 1. The coupling constants cited there are obtained when the range parameters R, see (14)—(16), (20), (21), were set to zero. This is definitely justified in the resonance region of ρ-, ω-, Φ-mesons; nonzero values of the range parameters will only imply a slight numerical modification of the coupling constants without change of any qualitative conclusions.

Our natural intention is to follow the predictions of a simple quark model for the lepton couplings of the vector mesons (1)

$$\frac{1}{f^{(0)}_\rho}:\frac{1}{f^{(0)}_\omega}:\frac{1}{f^{(0)}_\Phi}=\frac{1}{\sqrt{2}}:\frac{1}{3\sqrt{2}}:-\frac{1}{3}, \qquad (32)$$

The variants 1, 2 and 4 of the Table 1 refer to this case. The parameter $\varepsilon_0=-A_{\omega\Phi}/(m^{(0)2}_\Phi-m^{(0)2}_\omega)$ for the variants 1—3 turns out to be $\varepsilon_0=0.05\div0.06$ in accord with the value calculated from the simple quark model or, equivalently, the mass mixing model.

An accuracy of the present data does not exclude a sizeable coupling constant of the direct decay $g^{(0)}_{\Phi\rho\pi}$ at the level $g^{(0)}_{\Phi\rho\pi}/g^{(0)}_{\omega\rho\pi}\simeq0.014$ which magnitude is about 30% of ε_0, see the variant 1 from the Table 1. At least an order of magnitude increase of experimental accuracy is required to answer the question if the

Table 1

The errors in this Table are only statistical ones. The systematical errors of determination of the resonance masses are estimated at the level 0.1 MeV. The systematical error in the coupling constant $g^{(0)}_{\omega\rho\pi}$, $g^{(0)}_{\Phi\rho\pi}$ is estimated at level 4%. For the variants 1 and 2 we adduce the values of χ^2/n_D separately for all variety of the data on $e^+e^-\to3\pi$ (upper part of the entry) and for the data in the vicinity of Φ peak (lower part of the entry)

$e^+e^-\to\pi^+\pi^-\pi^0$

N	$f^{(0)}_\omega$	$f^{(0)}_\Phi$ $f^{(0)}_\omega/f^{(0)}_\Phi$	$g^{(0)}_{\omega\rho\pi}$, GeV^{-1}	$g^{(0)}_{\Phi\rho\pi}$, GeV^{-1}	$A_{\omega\Phi}$, GeV^2	$m^{(0)}_\omega$, MeV	$m^{(0)}_\Phi$, MeV	χ^2/n_D $(\chi^2/n_D)_\Phi$
1	19.1 ± 2.2	$-f^{(0)}_\omega/\sqrt{2}$ $-\sqrt{2}$	14.3 ± 0.1	0.2 ± 0.3	-0.022 ±0.022	782.7 ±0.8	1019.0 ±0.6	109/145 48/47
2	19.7 ± 0.2	$-f^{(0)}_\omega/\sqrt{2}$ $-\sqrt{2}$	14.3 ± 0.1	0	-0.029 ±0.002	783.3 ±0.1	1018.5 ±0.1	121/145 58/50
3	19.6 ± 0.2	-13.3 ± 1.7 -1.5 ± 0.2	14.3 ± 0.1	0	-0.027 ±0.005	783.2 ±0.4	1018.6 ±0.3	114/145
4	16.9 ± 0.1	$-f^{(0)}_\omega/\sqrt{2}$ $-\sqrt{2}$	14.3 ± 0.1	0.78 ± 0.02	0	782.1 ±0.1	1019.6 ±0.1	126/145
5	16.9 ± 0.1	-15.3 ± 0.6 -1.1 ± 0.1	14.3 ± 0.1	1.1 ± 0.2	0	782.0 ±0.1	1019.6 ±0.1	106/145

$e^+e^-\to K_SK_L$

2	19.1 ± 1.0	$-f^{(0)}_\omega/\sqrt{2}$ $-\sqrt{2}$	from 3π	0	-0.023 ±0.002	—	1018.9 ±0.1	53/50
4	19.5 ± 0.8	$-f^{(0)}_\omega/\sqrt{2}$ $-\sqrt{2}$	from 3π	0.84 ±0.03	0	—	1019.5 ±0.1	53/50

direct decay coupling constant $g^{(0)}_{\Phi\rho\pi}$ at the level cited above does exist in this variant of the fit, i. e. when (32) is used.

In the present paper we consider $g^{(0)}_{\Phi\rho\pi}$ to be purely real. Note, that there exists the contribution to $\mathrm{Im}\,g^{(0)}_{\Phi\rho\pi}$ which arises due to the chain of real intermediate processes $\Phi\to K\bar{K}$, $K\bar{K}\to\rho\pi$ via K^*

exchange. It makes up about 10% of $g^{(0)}_{\Phi\rho\pi}+\varepsilon_0 g^{(0)}_{\omega\rho\pi}$. Since $\operatorname{Im} g^{(0)}_{\Phi\rho\pi}$ enters quadratically to the decay width the statistics of the experiments [1—7] does not permit one to determine this quantity by means of introduction of the phase, i. e. $g^{(0)}_{\Phi\rho\pi}=|g^{(0)}_{\Phi\rho\pi}|\exp(i\Psi)$. This remark is true as well for the coupling constants $g^{(0)}_{\omega\rho\pi}$, $g^{(0)}_{\Phi K\bar{K}}$ etc. the imaginary parts of which arise due to corresponding intermediate processes. Note that the variant 2 discribes the modern data rather well.

The variant 3 is close to variant 2 but at this time $f^{(0)}_{\omega}$ and $f^{(0)}_{\Phi}$ are fitted independently. With a good accuracy the $f^{(0)}_{\omega}/f^{(0)}_{\Phi}$ turns out to be very near the quark model one, see variant 3.

It is seen from the Table 1 that $m^{(0)}_{\omega}$ and $m^{(0)}_{\Phi}$ in the variant 1—3 do not coincide with values m_ω and m_Φ from the Particle Data Group (PDG) [10]. Corresponding shifts δm^2_ω and δm^2_Φ are explained with a good accuracy by the formula (29).

In the variant 4 and 5 the $\Phi-\omega$ mixing is absent in old classic sence, i. e. $\operatorname{Re}\Pi_{\omega\Phi}=0$. The quark model relation $f^{(0)}_{\omega}/f^{(0)}_{\Phi}=-\sqrt{2}$ is fixed in the variant 4. When doing so a good description is obtained, see the Table 1. If the lepton coupling constants $f^{(0)}_{\omega}$ and $f^{(0)}_{\Phi}$ are fitted independently then their ratios turns out to be -1.1 ± 0.1 which differs by 3 standard deviations from the quark model value $-\sqrt{2}$.

Only extreme variants corresponding to those 2 and 4 for the channel $e^+e^-\to\pi^+\pi^-\pi^0$ were considered for the channel $e^+e^-\to K_SK_L$. The results are given in two lower lines of the Table 1. The values of $\Phi-\omega$ mixing parameters obtained in these cases are in accord with those obtained for the channel $e^+e^-\to\pi^+\pi^-\pi^0$.

For the channel $e^+e^-\to\pi^0\gamma$ an additional parameter $f^{(0)}_{\rho}$ is fixed according to (32) while $g^{(0)}_{\omega\pi\gamma}$ and $g^{(0)}_{\Phi\pi\gamma}$ are fixed by the VDM relations

$$g^{(0)}_{\omega\pi\gamma}=\sqrt{\alpha}\, g^{(0)}_{\omega\rho\pi}/f^{(0)}_{\rho} \qquad g^{(0)}_{\Phi\pi\gamma}=\sqrt{\alpha}\, g^{(0)}_{\Phi\rho\pi}/f^{(0)}_{\rho}.$$

The results of fitting this channel confirm the results obtained for $e^+e^-\to\pi^+\pi^-\pi^0$, K_SK_L. However, the accuracy of determination of the $\Phi-\omega$ mixing parameters is considerably lower than in the main cases so we do not give here the results.

The partial widths of the decays Φ, $\omega\to e^+e^-$ are given by

$$\Gamma(\Phi\to e^+e^-,s)=\frac{4\pi\alpha}{3s^{3/2}}\,|g^{(0)}_{\gamma\Phi}+\varepsilon\, g^{(0)}_{\gamma\omega}|^2,$$

$$\Gamma(\omega\to e^+e^-,s)=\frac{4\pi\alpha}{3s^{3/2}}\,|g^{(0)}_{\gamma\omega}-\varepsilon\, g^{(0)}_{\gamma\Phi}|^2. \qquad (33)$$

Corresponding values for the decay widths ω, $\Phi\to e^+e^-$ side by side with the Φ, $\omega\to\pi^+\pi^-\pi^0$ ones calculated with the data from the Table 1 are given in the Table 2. Up to the experimental uncertainties they coincide with the values from PDG [10]. It is interesting to note that the leptonic ratio $R_{e^+e^-}$ where

$$R_{e^+e^-}=\Gamma(\Phi\to e^+e^-,m^2_\Phi)/\Gamma(\omega\to e^+e^-,m^2_\omega) \qquad (34)$$

is in principle rather sensitive to the $\Phi-\omega$ mixing, since ε appears with opposite sign in $\Gamma(\Phi\to e^+e^-)$ and $\Gamma(\omega\to e^+e^-)$. The quark model without $\Phi-\omega$ mixing, see (32), gives $R_{e^+e^-}=2.6$. If one calculates the magnitude of the $\Phi-\omega$ mixing parameter ε_0 (and, hence, ε) which is equal to tangens of the angle of deviation in the vector nonet from ideal value 35.3° one obtains $\varepsilon_0\simeq0.05$. Incerting this to (33), (34) one obtains $R_{e^+e^-}=1.8$ (the error is negligible if one postulates the quark model relations (32)).

Table 2

The errors quoted include both the statistical and systematical ones from the Table 1

	1	2	3	4	5
$\Gamma(\omega\to e^+e^-)$, keV	0.60 ± 0.06	0.61 ± 0.04	0.62 ± 0.04	0.61 ± 0.04	0.61 ± 0.04
$\Gamma(\Phi\to e^+e^-)$, keV	1.20 ± 0.13	1.11 ± 0.10	1.23 ± 0.32	1.59 ± 0.13	0.98 ± 0.22
$\dfrac{\Gamma(\Phi\to e^+e^-)}{\Gamma(\omega\to e^+e^-)}$	2.00 ± 0.29	1.82 ± 0.15	1.98 ± 0.52	2.6 ± 0.2	1.61 ± 0.34
$B(\omega\to3\pi)$, %	87.4 ± 7.0	87.1 ± 7.0	87.5 ± 7.0	87.5 ± 7.0	87.5 ± 7.0
$B(\Phi\to3\pi)$, %	16 ± 28	17.8 ± 2.4	16.0 ± 5.5	11.4 ± 0.6	21.0 ± 5.9

Up to recent time the experimental value of $R_{e^+e^-}$ was 1.9 ± 0.4 (PDG'82), 2.0 ± 0.4 (PDG'84—86) and psychologically witnessed for the sizeable $\Phi-\omega$ mixing. However, the situation is changed, and the value $R_{e^+e^-}$ calculated with the help of [10] is $R_{e^+e^-}=2.3\pm0.1$ mainly due to 15% reduction in the total ω width. The values of $R_{e^+e^-}$ from the present work, see the Table 2, show that more then order of magnitude increase of accuracy is demanded to select at least extreme variants 2 and 4.

It seems a somewhat unexpected that the data [1—7] do not contradict $\mathrm{Re}\,\Pi_{\omega\Phi}=0$, see variants 4, 5. Of course, it should be remembered that there remains the mixing between ω and Φ due to common decay channels, i.e. due to $\mathrm{Im}\,\Pi_{\omega\Phi}\neq 0$, see (5). Does the absence of $\Phi-\omega$ mixing in old classic sense ($\mathrm{Re}\,\Pi_{\omega\Phi}=0$) contradict to existing views? Apparently, no, since the two historically first and most popular models of $\Phi-\omega$ mixing—mass mixing and current mixing models give approximately equal in magnitude and opposite in sign values of $\mathrm{Re}\,\Pi_{\omega\Phi}$ *) [11]. Naturally, any intermediate models are conceivable (and do exist) which give negligible values of $\mathrm{Re}\,\Pi_{\omega\Phi}$.

The theoretical curves for the cross sections of $e^+e^-\to 3\pi$, $\pi^0\gamma$, K_SK_L corresponding to the variant 2 are shown in Figs 1—3.

4. CONCLUSION

When summarizing the situation with the $\Phi-\omega$ mixing it should be stressed that the channel $e^+e^-\to\pi^+\pi^-\pi^0$ is most suitable for obtaining the information about the $\Phi-\omega$ mixing. The drawback of the channel $e^+e^-\to\pi^0\gamma$ is a small value of its cross section.

The study of the leptonic ratio $R_{e^+e^-}$ should be used for drawing conclusion on $\Phi-\omega$ mixing only together with the information from the $\pi^+\pi^-\pi^0$ and $\pi^0\gamma$ channels.

The fact is that the relations (32) are valid in the case when the wave function of the bound $q\bar{q}$ state at the origin behaves like $|\Psi(0, m_V)|^2 \propto m_V^3$ depending on the vector meson mass m_V which implies $f_\omega^{(0)}/f_\Phi^{(0)}=-\sqrt{2}$. However, the data on the decays $J/\Psi\to e^+e^-$, $\Upsilon(1S)\to e^+e^-$ [10] correspond to the behavior $|\Psi(0, m_V)|^2 \propto m_V^2$ which implies the leptonic ratio is equal to the ratio of charges squared of corresponding quarks [14, 15]) **). If one extrapolates such a dependence from J/Ψ and $\Upsilon(1S)$ to ω and Φ one obtains without $\Phi-\omega$ mixing $R_{e^+e^-}\simeq 2$ i. e. approximately the same result as in the case of classic $\Phi-\omega$ mixing and of the

*) The $\Phi-\omega$ mixing experiment [13] was considered to be in favour [12] of mass mixing model. However, this conclusion is based on the assumption $g^{(0)}_{\Phi\rho\pi}=0$ and hence is not necessary.

**) From the point of view of the potential models such a behavior is implied by the $q\bar{q}$ potential $V(r)\propto r^{-\nu}$ with $\nu=0\div0.5$ [15] at small distances.

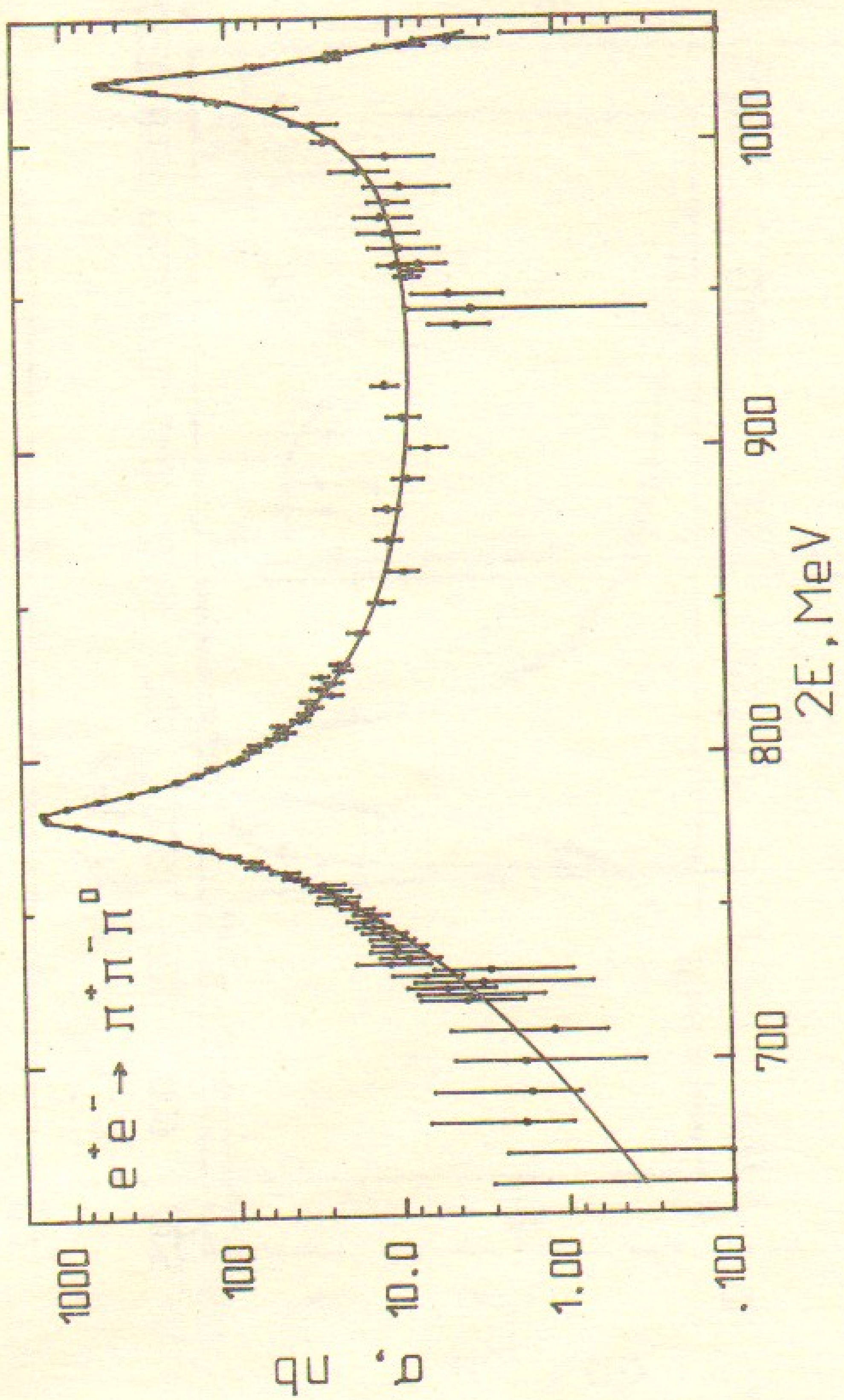

Fig. 1. The total cross-section of the reaction $e^+e^-\to\pi^+\pi^-\pi^0$.

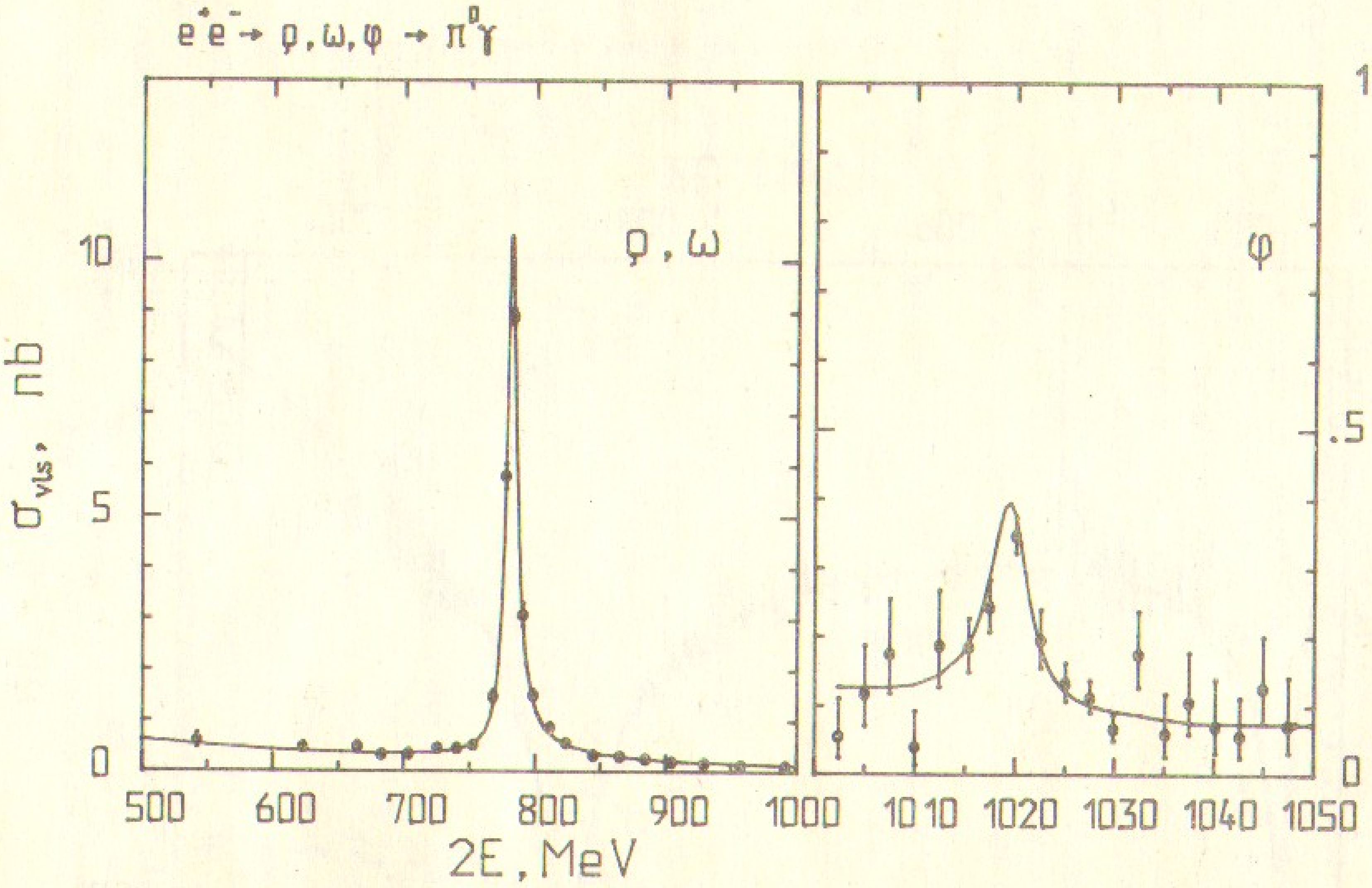

Fig. 2. The visible cross-section of the reaction $e^+e^- \to \pi^0\gamma$.

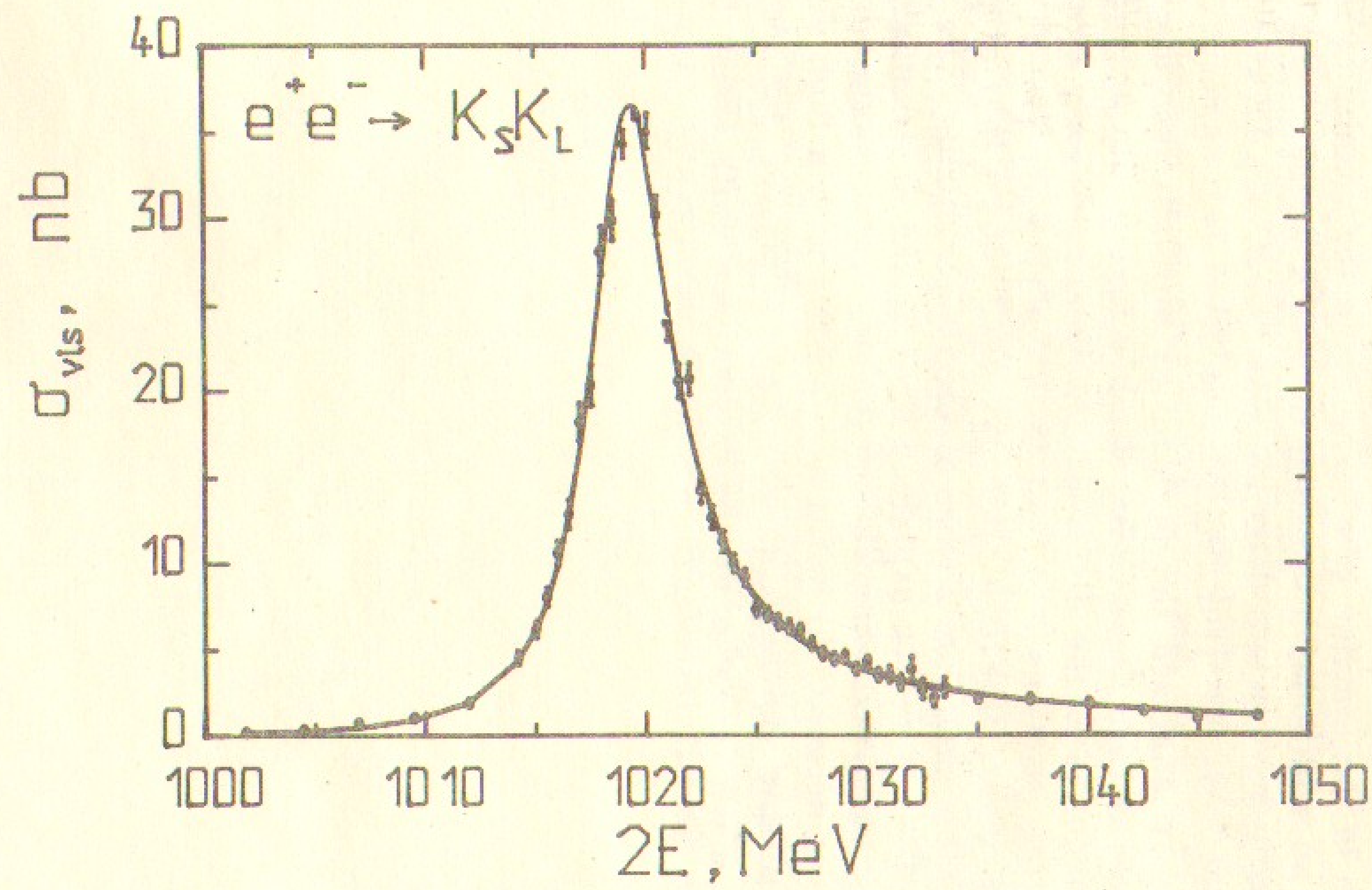

Fig. 3. The visible cross-section of the reaction $e^+e^- \to K_SK_L$.

standard behaviour $|\Psi(0, m_V)|^2 \infty m_V^3$. Note that the behaviour $|\Psi(0, m_V)|^2 \infty m_V^2$ implies $f_\omega^{(0)}/f_\Phi^{(0)} = -1.08$ which coincides with variant 5, see Table 1.

Let us note that the study of the decay $\Phi \to \pi^+\pi^-$ by interference pattern in the reaction $e^+e^- \to \pi^+\pi^-$ in the vicinity of Φ resonance gives an additional interesting approach to the $\Phi - \omega$ mixing since the mixing parameters determine the $\Phi \to \pi^+\pi^-$ decay amplitude in essential way. See [16] for more detail.

So, we have analyzed the problem of different theoretical descriptions of the decays $\Phi \to \rho\pi \to 3\pi$, $\Phi \to \pi^0\gamma$ taking into account both the $\Phi - \omega$ mixing and the direct decays $\Phi \to \rho\pi$, $\Phi \to \pi^0\gamma$ employing the data on the reactions $e^+e^- \to \pi^+\pi^-\pi^0$, $\pi^0\gamma$, $K_S K_L$ [1—7] in the energy range $\sqrt{s} = 700 - 1050$ MeV. We have shown that at present we have a rather poor knowledge of the $\Phi - \omega$ mixing. The decays $\Phi \to \rho\pi \to 3\pi$, $\Phi \to \pi^0\gamma$ can be explained both by the $\Phi - \omega$ mixing and by the direct decays $\Phi \to \rho\pi$, $\Phi \to \pi^0\gamma$ alone. In our opinion, there exist an urgent necessity to fulfill new studies with a high statistics and on a unique machine of all the variety of the reactions $e^+e^- \to \pi^+\pi^-\pi^0$, $\pi^0\gamma$, $K_S K_L$, $\eta\gamma$, $\pi^+\pi^-$ in the energy range mentioned above.

REFERENCES

1. *V.M. Aulchenko et al.* Phys. Lett., 186B (1987) 432.
2. *I.B. Vasserman et al.* Yad. Phys., 48 (1988) 753; Sov. Journ. Nucl. Phys., 48 (1988) 480.
3. *A.D. Bukin et al.* Accepted to Yad. Phys., 11—12 (1989).
4. *S.I. Dolinsky et al.* Submitted to Phys., Rep.
5. *S.I. Dolinsky et al.* Z.Phys. C-Particles and Fields, 42 (1989) 511.
6. *V.P. Druzhinin et al.* Phys. Lett., 144B (1984) 136.
7. *V.P. Druzhinin et al.* Preprint INP 89-87, Inst. of Nucl. Phys., Novosibirsk (1985).
8. *N.N. Achasov, S.A. Devyanin and G.N. Shestakov.* Usp. Fiz. Nauk, 142 (1984) 361; Sov. Phys.-Uspekhi, 142 (1984) 161.
9. *F.M. Renard.* Nucl. Phys., B82 (1974) 1.
10. Review of Particle Properties. Particle Data Group, Phys. Lett., 204B (1988) 1.
11. *N.M. Kroll, T.D. Lee and B. Zumino.* Phys. Rev., 157 (1967) 1376.
12. *N.N. Achasov, A.A. Kozhevnikov and G.N. Shestakov.* Pisma v ZhETF, 21 (1975) 497.
13. *A.D. Bukin et al.* Yad. Phys., 27 (1978) 976; Sov. Jour. Nucl. Phys., 27 (1978) 521.
14. *F.E. Close.* An introduction to Quark and Partons, Academic Press, 1979, ch.16.
15. *J.J. Kokkedee.* The Quark Model, W.A. Benjamin, Inc., 1969, ch.12.
16. *N.N. Achasov and A.A. Kozhevnikov.* $\Phi - \omega$, $\Phi - \rho$ Mixing and the Decay $\Phi \to \pi^+\pi^-$.—Preprint Inst. Mathematics TPh-26(175), Novosibirsk (1989).

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Новое о Φ — ω смешивании

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