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TETRAD AND SELF-DUAL FORMULATIONS OF REGGE CALCULUS

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НОВОСИБИРСК

Tetrad and Self-Dual Formulations of Regge Calculus

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ABSTRACT

The formulations of Regge-discretized general relativity are studied which are analogous to the tetrad and self-dual representations of the continuum theory.

Due to the general covariant nature of Einstein gravity a coordinateless formulation of this theory is of interest. Such the formulation is proposed by Regge calculus [1]. Regge calculus deals with the piecewise-flat Riemann manifolds which can be chosen to fit any smooth Riemann manifold with an arbitrarily high accuracy. A piecewise-flat manifold (or Regge lattice) can be considered as that composed of flat four-dimensional tetrahedra (or four-simplexes). The curvature takes the delta-function form with the support on two-simplexes or bones. It develops through the appearance of deficit angles: the sum of hyperdihedral angles meeting at the given two-simplex differs from 2π. Regge calculus possesses many analogies with the continuum theory. For example, the analog of the Bianci identities takes place [1]. At the same time the basic variables are link lengths which form a countable set. This fact is of importance in both computer and quantum applications.

This paper will consider Regge-analog of the description of general relativity with the help of four ($\mu=1, 2, 3, 4$) four-vectors e^a_μ (the tetrad) and four anti-symmetric 4×4-matrices $\omega_{\mu ab} = -\omega_{\mu ba}$ (the tetrad connection) [2]. The continuum action takes the form

$$S(e,\omega) = \int e^{a\mu} e^{b\nu} R_{ab\mu\nu}(\omega)\,(\det e)\,d^4x\,,$$

$$R_{ab\mu\nu}(\omega) = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_{\nu ac}\omega_{\mu cb} - \omega_{\mu ac}\omega_{\nu cb}\,. \qquad (1)$$

Here $e^{a\mu} e^b_\mu = \delta^{ab}$ (to simplify the notations we work with the Euclidean signature). Expressing ω in terms of e with the help of equations of motion $\delta S/\delta\omega = 0$ we reduce (1) to the usual Riemann tensor $R_{ab\mu\nu} = e^\lambda_a e^\rho_b R_{\lambda\rho\mu\nu}$ and Einstein—Hilbert action.

Regge lattice can be considered as an abstract simplicial complex equipped with a function which assignes to each 1-simplex σ^1 or link it's length $l(\sigma^1)$ [3]. The n-dimensional Einstein action on Regge lattice is proportional to

$$S_R = \sum_{\sigma^{n-2}} \mu(\sigma^{n-2}) \varphi(\sigma^{n-2}). \qquad (2)$$

The summation in (2) extends over $(n-2)$-simplexes σ^{n-2} of measure ($(n-2)$-dimensional volume) $\mu(\sigma^{n-2})$; $\varphi(\sigma^{n-2})$ are the deficit angles. Let us try to write down for (2) an analog of representation (1).

Such a representation would be of interest both by itself and also from the viewpoint of the problem of regularization of quantum general relativity in the so-called self-dual representation [4] by means of it's formulation on the lattice [5]. To introduce a lattice regularization parameter in a covariant way the metric is required. Meanwhile the metric in the self-dual representation is a complicated operator which involves second order functional derivatives and itself requires a regularization [6]. A possible way out of this difficulty is to proceed directly from discrete Regge formalism from the very beginning: the functional derivatives are substituted by the ordinary ones and do not require regularization. The first step in this way is to introduce for (2) representation of the type of (1) since it is (1) which can be considered as a starting point to pass to the self-dual representation [7].

Thus, we need the discrete analogs of the tetrad and connection. These were considered in Ref. [8]. Namely, to each simplex σ^n we assign an Euclidean coordinate system and to each one-subsimplex $\sigma^1 \subset \sigma^n$ we assign a vector in this system, $l^a(\sigma^1|\sigma^n)$. To each 2-subsimplex $\sigma^2 \subset \sigma^n$ we can assign the bivector $[l^a(\sigma^1_1|\sigma^n) l^b(\sigma^1_2|\sigma^n)]$ where σ^1_1, σ^1_2 are any two of it's three subsimplices $\sigma^1_i \subset \sigma^2$. Analogously to each k-subsimplex σ^k the k-vector corresponds:

$$V^{ab\ldots d}(\sigma^k|\sigma^n) = [\, l^a(\sigma^1_1|\sigma^n)\, l^b(\sigma^1_2|\sigma^n) \ldots l^d(\sigma^1_k|\sigma^n)] \qquad (3)$$

with the k-dimensional volume

$$\mu(\sigma^k) = ((V^{ab\ldots d})^2/k!)^{1/2}. \qquad (4)$$

The square brackets in (3) mean antisymmetrization over indexes $a, b, \ldots, d$ (the factor $1/k!$ being included). Each $(n-1)$-simplex σ^{n-1} is the common face of two n-simplexes σ^n_1, σ^n_2 and one can introduce an orthogonal rotation matrix as a function of (oriented) face $\Omega(\sigma^{n-1})$, which connects the systems of σ^n_1 and σ^n_2:

$$l^a(\sigma^1|\sigma^n_1) = \Omega^{ab}(\sigma^{n-1})\, l^b(\sigma^2|\sigma^n_2) \qquad (5)$$

for common links $\sigma^1 \subset \sigma^{n-1} = \sigma^n_1 \cap \sigma^n_2$.

The curvature is expressible in terms of matrices Ω [8]. A parallel transport of a vector around a $(n-2)$-vector through the sequence of faces σ^{n-1}_i results in the rotation

$$R(\sigma^{n-2}) = \prod_{i=1}^{N} \Omega(\sigma^{n-1}_i)^{\varepsilon(\sigma^{n-2}, \sigma^{n-1}_i)}, \qquad (6)$$

where the index $\varepsilon(\sigma^{n-2}, \sigma^{n-1}_i) = \pm 1$ takes into account orientation of faces. The Regge lattice corresponds to the particular case of our construction if the link vectors for the 1-subsimplexes of each 2-simplex sum to zero. On Regge lattice the curvature matrix is a rotation around $(n-2)$-vector $V(\sigma^{n-2})$ by the angle

$$\varphi(\sigma^{n-2}) = \arcsin \frac{1}{2} \frac{\varepsilon^{ab\ldots dfg}}{(n-2)!} \frac{V^{ab\ldots d}(\sigma^{n-2})}{\mu(\sigma^{n-2})} R^{fg}. \qquad (7)$$

Combining (2) — (4), (6), (7) we get a representation $S_R(l(\sigma^1|\sigma^n)\ \Omega(\sigma^{n-1}))$ which we are going to study. Namely, we are interested in the conditions (if any) that should be imposed for the theory with action $S_R(l, \Omega)$ to be equivalent to that with the Einstein—Hilbert action $S_R(l)$ (see (2)) and with the deficite angles $\varphi(\sigma^{n-2})$.

First, the set of variables l, Ω should be further specified. Namely, for each link σ^1 the n-simplex $\sigma^n \supset \sigma^1$ should be chosen in which the link vector $l^a(\sigma^1|\sigma^n)$ is considered as independent variable (in other simplexes $\sigma'^n \supset \sigma^1$ the vectors $l^a(\sigma^1|\sigma'^n)$ are expressible in terms of $l^a(\sigma^1|\sigma^n)$ and Ω). We shall say that the link σ^1 is referred to the n-simplex σ^n. Besides that, an ambiguity arises also when constructing k-vectors. Suppose the independent variables $l^a(\sigma^1_i|\sigma^n_i)$, $i = 1, 2$, $\sigma^n_1 \neq \sigma^n_2$ are assigned to the two 1-subsimplexes σ^1_i of a 2-simplex σ^2. To construct the bivector $V^{ab}(\sigma^2|\sigma^n)$ one should transform the components $l^a(\sigma^1_i|\sigma^n_i)$ to the simplex σ^n. This can be achieved by the action of matrices Ω in many ways according to the set of different paths connecting σ^n_i and σ^n. Besides that, we are also free to choose any two of three 1-subsimplexes of σ^2 whose link vectors can be used to construct $V^{ab}(\sigma^2)$.

Consider equations of motion for Ω. The formulas will be explicitly written out for the cases $n=3$ and $n=4$. The $(n-2)$-vectors V depend on Ω but it will be seen that correct equations of motion follow only if $(n-2)$-vectors do not vary with Ω in some simplexes σ^n, $\partial V(\sigma^{n-2}|\sigma^n)/\partial\Omega=0$. This means that either the links whose vectors are used to construct V are referred to the same n-simplex or, otherwise, those Ω that are used to transform link vectors to the same n-simplex should be substituted by their concrete expression $\Omega(l)$ corresponding to Regge lattice. When varying the action with Ω let us take into account the orthogonality conditions $\bar{\Omega}(\sigma^{n-1})\,\Omega(\sigma^{n-1})=1$ by introducing the Lagrange multipliers. The latter can be excluded by the action of operator $\varepsilon^{ab\ldots dgh}\Omega^{fg}\partial/\partial\Omega^{fh}$. The given matrix $\Omega=\Omega(\sigma^{n-1})$ appears in those $R(\sigma^{n-2})$ which correspond to the $(n-2)$-subsimplices of σ^{n-1}. Such R can be shortly written as $(\Gamma_1\Omega\Gamma_2)^{\varepsilon}$. Here $\varepsilon=\pm1$ and Γ_1, Γ_2 are some products of connection matrices different from the given one, Ω. As a result, the equations of motion read

$$\sum_{\sigma^1\subset\sigma^2}\varepsilon(\sigma^1,\sigma^2)\,\Gamma_2(\sigma^1,\sigma^2)\,\frac{\operatorname{tr}R(\sigma^1)-R(\sigma^1)^{\varepsilon(\sigma^1,\sigma^2)}}{\cos\varphi(\sigma^1)}\,\vec{l}(\sigma^1)=0 \qquad (8)$$

for each σ^2 at $n=3$;

$$\sum_{\sigma^2\subset\sigma^3}\varepsilon(\sigma^2,\sigma^3)\,\Gamma_2(\sigma^2,\sigma^3)\times$$

$$\times\frac{V(\sigma^2)\operatorname{tr}R(\sigma^2)-V(\sigma^2)\,R(\sigma^2)^{-\varepsilon(\sigma^2,\sigma^3)}-R(\sigma^2)^{\varepsilon(\sigma^2,\sigma^3)}\,V(\sigma^2)}{\cos\varphi(\sigma^2)}\;\vec{\Gamma}_2(\sigma^2,\sigma^3)=0 \qquad (9)$$

for each σ^3 at $n=4$. Here $\varphi(\sigma^{n-2})$ is given by (7).

Let us check that there is the solution to the above equations corresponding to some Regge lattice. Namely, let us specify Regge lattice by assigning the length $(l^a(\sigma^1)\,l^a(\sigma^1))^{1/2}$ to each 1-simplex σ^1. Further, to each n-simplex we assign an Euclidean system such that the link vectors of thus specified Regge lattice would coincide with $l^a(\sigma^1|\sigma^n)$ for those links which are referred to σ^n. Then Ω are chosen to connect these coordinate systems, $R(\sigma^{n-2})$ is rotation around $V(\sigma^{n-2})$ by the deficit angle $\varphi(\sigma^{n-2})$. In three dimensions, e. g., $(R(\sigma^1)-1)\vec{l}(\sigma^1)=0$, $\operatorname{tr}\mathrm{R}=1+2\cos\varphi$, and (8) reduces to

$$\sum_{\sigma^1\subset\sigma^2}\Gamma_2(\sigma^1,\sigma^2)\,\vec{l}(\sigma^1)=0\,. \qquad (8a)$$

This expresses the fact of closure of the link vectors of 2-simplex into the triangle. This property is fulfilled by construction. The matrices Γ_2 are needed to transform vectors to the same coordinate system. Analogously, in four dimensions we get the property of vanishing the sum of bivectors over the boundary of tetrahedron.

Thus, $S_R(l)$ realizes an extremum of $S_R(l,\Omega)$. In four dimensions we can use the group property $O(4)=SU(2)\times SU(2)$ to get once more representation analogous to the self-dual one in the continuum theory. In the latter the connection ω (i. e. infinitezimal rotation) can be decomposed into the (mutually commutative) (anti-) self-dual parts ${}^{\pm}\omega$:

$$\omega={}^{+}\omega+{}^{-}\omega\,,\qquad {}^{\pm}\omega=\pm({}^{\pm}\omega)^{*},$$

$$(\omega^{*})^{ab}:=\frac{1}{2}\varepsilon^{abcd}\,\omega^{cd}. \qquad (10)$$

The finite rotation $\Omega=\exp(\omega)$ (and R as well) decomposes multiplicatively:

$$\Omega=\Omega_{+}\,\Omega_{-}\,,\qquad \Omega_{\pm}=\exp({}^{\pm}\omega)\,. \qquad (11)$$

Let us call the set of $\Omega_{\pm}$ for all possible Ω the set of (anti-) self-dual rotations. Rotation around 2-simplex in some basis takes the block-diagonal form $R=\operatorname{diag}(L(\varphi),1)$ where $L(\varphi)$, 1 are 2×2-matrices, $L(\varphi)$ is the rotation by an angle φ. Then $R_{\pm}=\operatorname{diag}(L(\varphi/2),L(\pm\varphi/2))$ and it is seen that $S_R(l,\Omega_{\pm})=\frac{1}{2}S_R(l)$ for the Regge lattice solution $\Omega(l)$. Conversely, consider the action $S_R(l,\Omega)$ with Ω running over the set of self-dual rotations U (i. e. $U_{+}=U$). Each U is parameterized by three variables u^i as $U=\exp(iu^i\Sigma^i)$ where Σ^i $(i=1,2,3)$ is the basis of self-dual matrices chosen to satisfy the algebra of Pauli matrices. Write down the equations of motion for U. The self-duality conditions on $\ln U$ can be cast into the form $\bar{U}U=1$ and $(U-\bar{U})^{*}-(U-\bar{U})=0$ and taken into account by Lagrange multipliers. The latter can be excluded by the action of operator

$$\varepsilon^{abcd}\,U^{fc}\,\partial/\partial U^{fd}+U^{fa}\,\partial/\partial U^{fb}-U^{fb}\,\partial/\partial U^{fa}.$$

The resulting equations are simply self-dual part of eqs (9) with Ω substituted by U. It is easy to see that only self-dual parts of bivectors V enter these equations. According to ${}^{+}V=iv^i\Sigma^i$ these parts can be represented by the vectors $\vec{v}$ in the abstract 3-dimensional space on which the matrices U act as rotations:

$$\exp(iu^i \Sigma^i)\, v^k \Sigma^k \exp(-iu^i \Sigma^i) = \left[L\left(2u, \frac{\vec{u}}{u}\right) \vec{v} \right]^k \Sigma^k, \qquad (12)$$

where $L(\varphi, \vec{n})$ is a rotation by an angle φ around $\vec{n}$ axis. For the solution relevant to Regge lattice all the curvature matrices $R_+(\sigma^2) = \Pi\, U^e$ act as rotations around $\vec{v}(\sigma^2)$. In this case the equations of motions simply establish the closure of vectors $\vec{v}$ representing the faces of 3-simplex into a quadrangle.

Consider now the problem of uniqueness of solution of the equations of motion for Ω. The uniqueness is understood as that applied to the physical observables — the deficits $\varphi(\sigma^{n-2})$ which should coincide with the Regge lattice ones. Besides that there are the evident necessary conditions for the solution to be Regge lattice one:

$$(R(\sigma^1) - 1)\, \vec{l}(\sigma^1) = 0 \quad (n=3);$$

$$R(\sigma^2)\, V(\sigma^2)\, \overline{R(\sigma^2)} = V(\sigma^2) \quad (n=4), \qquad (13)$$

the $(n-2)$-vectors are not rotated by their curvature matrices. In three dimensions these conditions can be seen to be sufficient as well. Indeed, if $(R(\sigma^1) - 1)\, \vec{l}(\sigma^1) = 0$ for all the 1-simplexes then for each 3-simplex it's link vectors are unambiguously defined and by equations of motion they close into tetrahedron. These tetrahedra form the Regge lattice of interest.

Let us examine whether the conditions (13) are provided by equations of motion. Write down $\Omega = 1 + w$ and consider linear approximation in w (thus the uniqueness in some neighbourhood of $\Omega = 1$ will be analyzed; besides, it is linear approximation which proves to be essential while approaching the continuum limit). Consider n-simplex and write down $n+1$ equations of motion for $n+1$ it's faces. If $(n-2)$-vectors are not rotated by their curvature matrices we get simply vanishing the sum of $(n-2)$-vectors of subsimplexes of each $(n-1)$-face. Of these only n equations are independent ones. Conversely, requiring consistency of our $n+1$ equations we get condition on change of $(n-2)$-vectors under the action of curvature matrices. If turns that this condition involves only $(n-2)$-vectors which are referred to the considered n-simplex σ^n:

$$\sum_{\left\{\sigma^1 \,\middle|\, \frac{\partial l(\sigma^1|\sigma^3)}{\partial \Omega} = 0\right\}} (R(\sigma^1|\sigma^3) - 1)\, \vec{l}(\sigma^1|\sigma^3) = O(w^2) \quad (n=3),$$

$$\sum_{\left\{\sigma^2 \,\middle|\, \frac{\partial V(\sigma^2|\sigma^4)}{\partial \Omega} = 0\right\}} \left(R(\sigma^2|\sigma^4)\, V(\sigma^2|\sigma^4)\, \overline{R(\sigma^2|\sigma^4)} - V(\sigma^2|\sigma^4)\right) = O(w^2) \quad (n=4). \qquad (14)$$

In particular, it may be the case that only one $(n-2)$-vector is referred to the given n-simplex. Then the condition (13) follows for the $(n-2)$-vector (up to $O(w^2)$ terms). However, there are always the simplexes σ^n with $m = m(\sigma^n) > 1$ $(n-2)$-vectors referred to them. In this case the additional conditions (13) are required, but due to (14) these should be imposed on only $m-1$ $(n-2)$-vectors. If, on the other hand, less than $m-1$ polyvectors are subject to these conditions, all the remaining $(n-2)$-vectors are, generally speaking, changed by their curvature matrices in general solution, as it is demonstrated for three dimensions in the following example.

Namely, consider the periodic Regge lattice introduced by Roček and Williams [9]. The periodic cell is a cube divided into 6 tetrahedra whose edges are cube edges, face diagonals and common body diagonal. To each vertex 7 links can be assigned which can be labeled by unordered combinations of numbers i (three edges), ik (three face diagonals), 123 (body diagonal). Further, there are 12 2-simplexes (triangles) labeled by ordered pairs of numbers and of symbol d: $\{ik\}$, $\{id\}$, $\{di\}$. These pairs label 2-simplexes whose vertices are the original centre and it's successive translations in the two directions: along the edges i, k, or along the egde i and face diagonal d in the plane of two another edges, or in the inversed consequence. Correspondingly, there are 12 connection matrices $\Omega_{\sigma^2} \equiv \Omega(\sigma^2)$ and 7 curvature matrices $R_{\sigma^1} \equiv R(\sigma^1)$ per point expressible through the former, e. g., in the form

$$R_1 = \bar{\Omega}_{12} (\bar{T}_3 \bar{\Omega}_{31}) (\bar{T}_{23} \Omega_{d1}) (\bar{T}_2 \Omega_{21}) \Omega_{13} \Omega_{1d},$$

$$R_2 = \bar{\Omega}_{23} (\bar{T}_1 \bar{\Omega}_{12}) (\bar{T}_{31} \Omega_{d2}) (\bar{T}_3 \Omega_{32}) \Omega_{21} \Omega_{2d},$$

$$R_3 = \bar{\Omega}_{31} (\bar{T}_2 \bar{\Omega}_{23}) (\bar{T}_{12} \Omega_{d3}) (\bar{T}_1 \Omega_{13}) \Omega_{32} \Omega_{3d},$$

$$R_{23} = \bar{\Omega}_{32} (\bar{T}_1 \Omega_{1d}) \Omega_{23} \Omega_{d1}, \qquad (15)$$

$$R_{31} = \bar{\Omega}_{13} (\bar{T}_2 \Omega_{2d}) \Omega_{31} \Omega_{d2},$$

$$R_{12} = \bar{\Omega}_{21} (\bar{T}_3 \Omega_{3d}) \Omega_{12} \Omega_{d3},$$

$$R_{123} = \Omega_{d3} \Omega_{2d} \Omega_{d1} \Omega_{3d} \Omega_{d2} \Omega_{1d},$$

where T_{σ^1} is the translation along the link σ^1 to the neighbouring vertex. The entry (15) implies a definite orientation of 2-simplexes and, besides, a definite choice of tetrahedra to which link vectors are referred. In particular, vectors $\vec{l}_1$ and $\vec{l}_{123}$ are referred to the same tetrahedron while others are to different ones. The equations of motion for, e. g., the triangle {23} take the form

$$\frac{\operatorname{tr} R_2-\bar{R}_2}{\cos\varphi_2}\vec{l}_2+T_2\Omega_{31}\frac{\operatorname{tr} R_3-\bar{R}_3}{\cos\varphi_3}\vec{l}_3-\Omega_{d1}\frac{\operatorname{tr} R_{23}-R_{23}}{\cos\varphi_{23}}\vec{l}_{23}=0. \tag{16}$$

In the linear in $w=\Omega-1$ approximation and for uniform system (which admits the solution $w=0$) we get in terms of deficits the nonzero one-parametric general solution. Let $\vec{l}_i$, $\vec{l}_k=\delta_{ik}$, $\vec{l}_{ik}=\vec{l}_i+\vec{l}_k$, $\vec{l}_{123}=\vec{l}_1+\vec{l}_2+\vec{l}_3$. Then all the deficits vanish except for $\varphi_{23}=-\varphi_{123}\sqrt{3/2}=\lambda$, λ is an arbitrary parameter, the curvatures being $R_1^{ab}=R_{23}^{ab}=R_{123}^{ba}=\delta^{ab}+\lambda\varepsilon^{abc}n_{23}^c$. All φ's vanish if condition $(R_{123}-1)\vec{l}_{123}=0$ is imposed. Besides that, since l_1^a, l_{123}^a are referred to the same coordinate system, these are consistent with Regge lattice only if the following condition is fulfilled:

$$(\vec{l}_{123}-\vec{l}_1)^2=T_1\vec{l}_{23}^{\,2}. \tag{17}$$

Consider briefly the four-dimensional case. The 4-cube is devided into 24 4-tetrahedra [9]. To each vertex we can assign the 15 links, namely: 4 edges i, 6 face diagonals ik, 4 body diagonals ikl, hyperbody diagonal 1234 labeled by unordered combinations. These links form 50 2-simplexes and also 60 3-simplexes $\{ikl\}$, $\{dkl\}$, $\{idl\}$, $\{ikd\}$ per point (notations are analogous to those for three dimensions). Evidently, 15 links can be distributed among the 24 4-simplexes in such a way that no more than one link vector is referred to each 4-simplex. Then the conditions like (17) are not required and the vector variables $l^a(\sigma^1|\sigma^4)$ take on their values loosely.

Then, on the other hand, each bivector $V^{ab}(\sigma^2)$ contains the matrices Ω needed to transform the pair of generating vectors to the same 4-simplex. The paths connecting any two 4-simplexes in the given 4-cube can be chosen within the hypercube intersecting only internal 36 3-simplexes $\{...d...\}$. Correspondingly, it is sufficient to know only $\Omega_{...d...}(l)$ to define V. These matrices absorb an information on the curvatures R_A in the 14 internal 2-simplexes formed by link A and by hyperbody diagonal 1234, and are the solution to equations

$$\left.\begin{aligned} R_A(\Omega_{...d...})\,l_{1234}&=l_{1234}\\ R_A(\Omega_{...d...})\,l_A&=l_A\end{aligned}\right\} \tag{18a}$$

$$((\Pi\Omega_{...d...})_1 l_{AB}-(\Pi\Omega_{...d...})_2 l_A)^2=T_A l_B^2. \tag{18b}$$

The equations (18a) are simply the conditions for the link vectors to be unambiguously defined within the 4-simplexes. One can say that (18a) express the possibility of «glueing» together the 4-simplexes into the 4-cube with internal curvatures. Then eqs (18b) express the possibility of «glueing» together (with the help of «internal» connections Ω_{ikl}) the different 4-cubes: the original one and that obtained by translation T_A.

Thus, an information on internal curvatures is already partially absorbed into the definition of bivectors.

The Regge lattice solution of equations of motion should lead to vanishing of the 50 matrices $RV\bar{R}-V$. Taking into account 24 conditions of the type (14) following from the equations of motion it is sufficient to equate to zero only 26 of these matrices. The hypothesis is that these 26 conditions are both necessary and sufficient to get Regge lattice deficits in our theory, as it takes place in three dimensions. This assertion is presently under investigation. The immediate following step is to pass to the continuum time and construct the Hamiltonian formalism.

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