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A b s t r a c t

Charged particle pair production at beam-beam collision in electron-positron linear colliders has been discussed taking into account a finite size of the beams (both longitudinal and transverse) and end effects. Contributions of the main acting mechanisms are singled out which depend on the energy of initial particles and the masses of created particles. A spectral distribution of produced particles is presented.

1. In Refs. 1 to 7 the energy losses of particles in the linear colliders have been considered. The magnetic bremsstrahlung mechanism in a strong crossed field of the counter-moving beam dominates. The photon radiation probability (per unit time) owing to this mechanism is of the form [8]:

$$dw_\gamma/dt \equiv dW_\gamma(t) = \alpha\,\Phi\, d\omega/(\pi\sqrt{3}\,\gamma^2)\ ;$$

$$\Phi = \left(\frac{\varepsilon'}{\varepsilon}+\frac{\varepsilon}{\varepsilon'}\right)K_{2/3}(z) - \int\limits_z^\infty K_{1/3}(y)\,dy - \frac{\omega}{\varepsilon}\boldsymbol{\zeta}\mathbf{h}\,K_{1/3}(z), \qquad (1)$$

$$z = 2u/3\chi,\quad u=\omega/\varepsilon',\quad \varepsilon'=\varepsilon-\omega,\quad \chi=\gamma F/H_0,\quad \mathbf{h}=\mathbf{F}^*/F,$$

where $\alpha = 1/137$, ω is the photon energy, $\varepsilon(m)$ is the energy (mass) of colliding particles, $\gamma=\varepsilon/m$, $F=|\mathbf{F}|=|\mathbf{F}^*|$, $\mathbf{F}=\mathbf{E}_\perp+(\mathbf{v}\times\mathbf{H})$, $\mathbf{F}^*=\mathbf{H}_\perp+(\mathbf{E}\times\mathbf{v})$, $\mathbf{E}_\perp=\mathbf{E}-\mathbf{v}(\mathbf{v}\mathbf{E})$, $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic fields in the lab. system, $H_0=m^2/e = 4.41\cdot 10^{13}$ Oe, $\boldsymbol{\zeta}$ is the electron spin vector in the rest system, $\mathbf{v}$ is the velocity of the particle, and K_ν are the MacDonald functions. The probability of photon radiation during beam-beam collision in linear colliders turns out to be of the order of unity and the density of accompanying photons becomes comparable to the density of the charged particles in the colliding beams. At $\varkappa = \omega F/mH_0 \gtrsim 1$ these photons are converted with high probability to electron-positron pairs in the field of counter-moving beam. Recently the authors have considered the creation of electron-positron pairs by the photons with a definite energy in the beam field taking into account the field inhomogenety and end effects [9]. The present paper deals with the pair production by the photons, radiated at beam-beam collision. When $\chi \gg 1$ one has to make allowance for the direct electroproduction process ($e \to 3e$) owing to the contribution of virtual intermediate photons.

The charged particles and secondary photons may also produce pairs through incoherent processes the cross sections of which are modified essentially in the presence of the strong external field. The e^+e^- pair production by the photon on the particles of the counter-moving beam and by two virtual photons turns out to be the most important among these processes at $\chi \lesssim 1$. We discuss the production of heavy particle pairs as well.

2. Let us consider the radiation and subsequent pair creation in the field of counter-moving beam. Assuming that the photon formation length $\ell_c = \gamma \lambda\!\!\!^{-}_c / (\chi^2 u)^{1/3}$ ($u \lesssim \chi$, $\lambda\!\!\!^{-}_e = 1/m$ is the Compton wavelength) is considerably smaller than the longitudinal beam size σ_z , one can use formula (1) for radiation probability. At $\omega \ll \chi\varepsilon / (1+\chi)$ this probability may be presented as follows:

$$\frac{dW_\gamma}{d\omega} = \frac{\alpha \Gamma(2/3)}{\pi\sqrt{3}\,\gamma^2} \left(\frac{3\chi}{u}\right)^{2/3} \simeq \frac{\alpha}{2\gamma^2}\left(\frac{\chi\varepsilon}{\omega}\right)^{2/3} \qquad (2)$$

Since the main contribution to the total radiation probability W_γ is given by relatively soft photons $\langle\omega\rangle \sim \varepsilon\chi/2(1+2\chi)$ one can obtain an approximate expression for W_γ by integrating Eq.(2) over ω from 0 to $\varepsilon\chi/1+\chi$

$$W_\gamma \simeq 3\alpha\chi / 2\gamma \lambda\!\!\!^{-}_c (1+\chi)^{1/3}.$$

Using it we obtain for $\chi(t) = \chi_0 \exp(-2t^2/\sigma_z^2)$ an estimate for the total number of radiated photons

$$n_\gamma \simeq 9\alpha\sigma_z\chi_0 / (4\gamma \lambda\!\!\!^{-}_c\, (2+\chi_0)^{1/3}).$$

Here W_γ is consistent with the exact expression for W_γ within 18 percent.

The differential (over ε) probability of the pair production by the photon in the external field may be obtained from formula (1) by means of substitutions $\varepsilon \to -\varepsilon$, $\omega \to -\omega$, $\vec{\underline{\zeta}} \to -\vec{\underline{\zeta}}$, $\omega^2 d\omega \to -\varepsilon^2 d\varepsilon$.

Integrating over ε (in some terms, integration by parts was carried out) we obtain the following expression for the total probability (per unit time) of the pair creation by the photon in external field (here $\vec{\underline{\zeta}}$ is the spin vector of produced electron)

$$W_e = \frac{2\alpha m^2}{3\sqrt{3}\pi\omega} \int_0^1 dx \left[\left(\frac{1-3\xi_3}{2} + \frac{1}{x(1-x)}\right) K_{2/3}(y) - \frac{3\vec{\underline{\zeta}}\underline{h}}{2x} K_{1/3}(y)\right], \qquad (3)$$

here $y = 2/(3\varkappa x(1-x))$ and $x = \varepsilon/\omega$. A dependence on photon polarization is included into Eq.(3), where $\vec{\underline{\xi}}$ are the Stokes parameters for the following choice of axes: $\underline{e}_1 = (\underline{h} \times \underline{v})$, $\underline{e}_2 = \underline{h}$. For $\varkappa \ll 1$ the probability (3) is exponentially suppressed ($W_e \propto \exp(-8/3\varkappa)$). For $\varkappa \gg 1$ we have

$$W_e = 0.380\, \frac{\alpha m^2}{\omega} \varkappa^{2/3} (1 - \xi_3/5). \qquad (4)$$

The probability W_r of the process under consideration ($e \to e + \gamma \to e + e^+ + e^-$) is defined by the product of probabilities (1) and (3). We consider below unpolarized beams, so $\vec{\underline{\zeta}} = 0$. At $\chi \gg 1$ the contribution to W_r gives the following interval of ω : $\varepsilon/\chi\,(\varkappa = 1) < \omega < \varepsilon$ and the spectral distribution within this interval is $d\omega/\omega$. Then one can use, with logarithmic accuracy the asymptotic expressions (2) and (4) and take into account that for soft photons $\xi_3 = 1/2$ [8]. As a result of integration of this expression over time, we obtain the probability of pair production at beam-beam collision due to the real photons

$$w_r = (1/2)\int d\omega \int_{-\infty}^{\infty} dW_\gamma(t')/d\omega \, dt' \int_{-\infty}^{\infty} W_e(t)\,dt \simeq$$

$$\simeq C\,(\Delta\varepsilon/\varepsilon)^2 \ln \chi_0(\underline{\varrho}), \quad C = \left(\frac{3}{2}\right)^{12} \frac{\Gamma(2/3)}{\Gamma(1/6)} \frac{1}{7\pi^{3/2}\, 2^{1/3}} = 0.643\ldots \qquad (5)$$

The factor 1/2 in Eq.(5) appears in transformation of integrals over time. We make allowance for symmetric distribution of particles in beams along the axis z. In Eq.(5) the probability of the process is expressed in terms of the relative energy losses $\Delta\varepsilon/\varepsilon$ during one collision, we use that $\chi(t) = \chi_0 \exp(-2t^2/\sigma_z^2)$, (Eq.(15)[9]). To obtain the final result we have to average the expression obtained over the transverse coordinate $\underline{\varrho}$ with the corresponding particle density $n_\perp(\underline{\varrho})$. Below we assume that for the electron and radiated photons, $\underline{\varrho}$ does not change during the beam-beam collision.

The virtual intermediate photon contribution can be calculated using the equivalent photon approximation in the external field. Making allowance for the smallness of the transverse beam dimensions we have for the equivalent photon spectrum [10,11]

$$n(\omega) = \frac{2\alpha}{\pi\omega} \ln\left(\Delta/q_m(\omega)\right),$$

$$q_m(\omega) = m\frac{\omega}{\varepsilon}\left(1 + \frac{\varepsilon\chi}{\omega}\right)^{1/3} + 1/\sigma_y \equiv q_F(\omega) + q_\sigma \qquad (6)$$

here Δ is the upper boundary of a transverse momentum transfer which has no influence on photoproduction probability, and σ_y is the smallest of the transverse beam dimensions. Since the spectrum of the virtual photons is more singular (as a func-

tion of ω) than the spectrum of real photons, the main contribution to the probability to be found at $\chi \gg 1$ is given by $\omega \sim \varepsilon/\chi\,(\varkappa \sim 1)$. For this value of ω we have $\Delta \simeq m$, $q_m \simeq m\chi^{-1/3}$, so using Eq.(3) we obtain, with logarithmic accuracy [12,13], the contribution of the virtual photons to the pair production probability:

$$w_\sigma(\underline{\varrho}) = \frac{13\alpha^2 m^2 \ln\chi_0}{18\sqrt{3}\,\pi\varepsilon} \int_{-\infty}^{\infty} \chi(t)\,dt \,. \qquad (7)$$

Averaging the probabilities (5) and (7) over $\underline{\varrho}$ we find the final form of the contributions of real and virtual photons to electroproduction probabilities at $\chi_m \gg 1$ (χ_m is the maximum value of χ). In the case of Gaussian transverse distribution of density with $\sigma_x = \sigma_y$ we have for their ratio

$$\delta \equiv \frac{\overline{w_\sigma}}{\overline{w_r}} \simeq 0.84\,\gamma\lambda\!\!\!^{-}_c/(\sigma_z \chi_m^{1/3}) \,. \qquad (8)$$

When $\chi \lesssim 1$ the contribution to the process probability is given by the region $\omega \sim \varepsilon$ and the relative value of the virtual photon contribution is small ($\delta \sim \gamma\lambda\!\!\!^{-}_c/\sigma_z\chi \sim l_c/\sigma_z \ll 1$). In this case one can calculate the probability of pair production in the field using the asymptotic "exponentially small" expressions for W_γ Eq.(1) and W_e Eq.(3) and doing subsequent integrations by the Laplace method. Finally we obtain at fixed $\underline{\varrho}$ for $\chi_0(\underline{\varrho}) \lesssim 1$

$$w_r = \frac{\sqrt{3}\,\pi}{512}\left(\frac{\alpha\sigma_z}{\gamma\lambda\!\!\!^{-}_c}\right)^2 \chi_0^3 \exp\left(-16/3\chi_0\right). \qquad (9)$$

At such values of the parameter the energy of the initial particle is divided approximately into equal parts between three final particles. It is worth stressing that expression (9) is very sensitive to a change of the parameter χ_0, which in turn depends on a change of the shape of the beams in a course of the collision. One should make allowance for this fact when averaging over $\underline{\varsigma}$.

3. Let us discuss the contribution of incoherent processes (on separate particles) to total number of produced pairs. Here the most important process is a pair production by the photon (real or virtual) on charged particles. The cross section of photoproduction of the pair of spin 1/2 particles with a mass μ has with logarithmic, the form

$$\sigma_p = \frac{28\alpha^3}{9\mu^2} L\left(1+\frac{\varkappa^2}{3.03}\right)(\varkappa \ll 1); \quad \sigma_p \simeq \frac{2.18\alpha^3 L}{\mu^2 \varkappa^{2/3}}(\varkappa \gg 1);$$

$$\varkappa = (\omega + q_0)eF/\mu^3, \quad L = \ln\frac{\Delta}{q_m(q_0)},$$

$$\Delta^2 = \mu^2(1+\varkappa)^{2/3}, \quad \omega q_0 = \Delta^2 \tag{10}$$

where $q_m(q_0)$ is defined by Eq.(6). Using the photoproduction cross section (see the exact expression in Ref. 14) and distribution of real (see Eqs.(1) and (2)) and virtual (Eq.(6)) photons, one can calculate with logarithmic accuracy the corresponding probability. At $\varkappa \gtrsim 1$ incoherent contribution is essentially smaller than considered in Section 2, so one should consider incoherent processes only at $\varkappa \lesssim 1$. At $\chi_m \lesssim 1$ (which is the case for linear colliders projects with $\varepsilon \sim$ 0.5-1.0 TeV) an ef-

fective cross section of electron-positron pair photoproduction by real photons is of the form $\sigma_p^{inc} \simeq n_\gamma \sigma_p / 2$, where n_γ is total number of the photons radiated by the particle. An estimate of the corresponding contribution of virtual photons gives

$$\sigma(2e \to 4e) \simeq \sigma_{LL} \ln(\sigma_y / \lambda_c) / 2\ln\gamma, \tag{11}$$

where σ_{LL} is the Landau-Lifshitz cross section [15]. These two processes (with real and virtual photons) at $n_\gamma \sim 1$ contribute roughly in equal parts to the total number of produced pairs. Let us consider the probability of the pair creation (for spin 0 and 1/2 particles with mass μ) at the real photons collision. Averaging the corresponding two-photon cross sections with the spectral distribution (2) we obtain with logarithmic accuracy

$$\sigma_{ef}^{(s)} = \mathcal{D} C^{(s)} \alpha^4 \sigma_z^2 \left(\frac{\chi_0}{\gamma^2}\frac{m}{\mu}\right)^{4/3} \ln\frac{\omega_{max}}{\omega_{min}}, \quad \mathcal{D} = \frac{27}{40} 6^{1/3} \frac{\Gamma^4(2/3)}{\Gamma(1/3)} = 1.54..,$$

$$\frac{\omega_{min}}{\varepsilon} = \frac{4}{\chi_0^2}\left(\frac{2\lambda_c \gamma}{\sigma_z}\right)^3, \quad \omega_{max} = \frac{\mu^2}{\omega_{min}} \lesssim \frac{\varepsilon \chi_0}{(1+\chi_0)^2}, \tag{12}$$

where $C^{(1/2)} = 1$, $C^{(0)} = 1/8$ and ω_{min} follows from the condition $\ell_c < \sigma_z$. Formulae (12) are valid for $\omega_{min}/\mu \ll 1$. When an opposite inequality is fulfilled $\mu/\omega_{min} \ll 1$ (supercollider) the e^+e^- pair production by the mechanism discussed is suppressed as $(\mu/\omega_{min})^{4/3}$. Using Eqs.(12) we obtain an estimate for the effective cross section of the process under discussion

$$\bar{\sigma}_{ef}^{(s)} \simeq \frac{C^{(s)} n_\gamma^2 \alpha^2}{2(\bar{\omega}\mu^2)^{2/3}}\left(\frac{1+\chi_0}{1+2\chi_0}\right)\ln\frac{\mu}{\omega_{min}} \tag{13}$$

here $\bar{\omega} \simeq \varepsilon \chi_0/(3+4\chi_0)$. The relative contribution of this cross section as compared to Eq.(11) increases with μ as $(\mu/\bar{\omega})^{2/3}$ and at large μ the cross section (13) may dominate. At $\mu \gg \bar{\omega}$ it becomes exponentially small. It is worth noting that for two-photon processes we present the total cross sections whilst for coherent processes probabilities for one direction are presented.

Lastly, we would like to note that, when $\varepsilon' \gamma \lambda\!\!\!^{-}_c/\sigma_z \gg \omega \gg \omega_{min}$ (see Eq.(12)) side by side with "synchrotron" photons the end (collinear) photons are radiated with spectral distribution [7]

$$dW_i = \frac{\alpha}{2\pi}\frac{d\omega}{\omega}\left\{\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)\left(\ln\frac{2\varepsilon'\gamma\lambda\!\!\!^{-}_c\sqrt{\ln\frac{\omega}{\omega_{min}}}}{\omega\sigma_z} - 1.714\right) - \frac{2\varepsilon'}{\varepsilon}\right\} \quad (14)$$

At some conditions (compare Eqs.(6) and (14)) their contribution to pair creation becomes comparable with the contribution given by virtual photons.

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