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GROWING QUASI-MODES IN DYNAMICS OF SUPERSONIC COLLAPSE

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Growing Quasi-Modes in
Dynamics of Supersonic Collapse

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ABSTRACT

A possibility for destruction of the stable against infinitely small perturbations self-similar regime of supersonic collapse by growing quasi-modes is demonstrated via the numerical solution of Koshi problem for Zakharov equations. The quantitative criterion for the destruction of self-similar regimes is formulated.

INTRODUCTION

The hypothesis of globally stable self-similar regimes existence for supersonic Langmuir collapse [1] plays a significant role in the attempts to construct a theory of strong Langmuir turbulence (see, for example, Ref. 2). Natural wish of researchers to test the hypothesis was partly satisfied in recent years. First, in Ref. 3 the examples were numerically constructed for self-similar solutions of the so-called scalar model equations and one-dimensional equations assumingly describing the Langmuir collapse of a strongly flattened cavern on its short axis. Later, the authors of articles [4] succeeded in numerical finding of the self-similar regime with a nearly spherically symmetric cavern for Langmuir collapse (such a regime is realized at a nearly equal population of a slightly split triplet of the ground states, corresponding to the unit «orbital momentum» $l=1$). Further, the structure was revealed of a whole set of self-similar scalar collapse regimes and some of those were found out analytically [5]. Finally, in Ref. 6 it was produced the most interesting (from the viewpoint of strong turbulence theory) self-similar Langmuir collapse regime for a cavern with a single populated state. Simultaneously with the search of self-similar solutions, the attempts were made to study a stability problem for those. In Ref. 7 a continuous component of the spectrum was found out for linearized equations of a scalar model. This permitted the author to obtain a simple criterion for instability of self-similar solutions against the modes of continuous spectrum. For the scalar collapse the criterion mentioned coincides with the necessary condition for the self-similar solutions existence. It is worthwhile to note here, that self-similar solutions considered are related to the supersonic adiabatic limit. According to Ref. 7, an account of a small «sonic» correction turns the eigenmodes of continuous spectrum of stability problem for self-similar collapse into the so-called quasi-modes. The latter can grow only at a cavern compression by a finite, though a large on the Mach parameter, factor and, at sufficiently low initial amplitude, they have not enough time to develop.

The possibility of existence for true growing eigenmodes (corresponding to the point spectrum of a stability problem for self-similar solutions) has been studied for a wide class of self-similar scalar collapse regimes in Ref. 8. There, in particular, the stability was proved of the simplest self-similar solution from Ref. 3 against the infinitely small perturbations. The possibility of destruction for this and other self-similar solutions stable agianst the infinitely small perturbations by growing quasi-modes, at not too small initial amplitude of the latter, is of principle importance. This possibility, called in question by a number of specialists up to now, is worth to be studied in more detail.

2. BASIC EQUATIONS

In the scalar model of Langmuir wave collapse the evolution of an electric field time envelope ψ and perturbation of ion concentration n is described by the following dimensionless equations [1]:

$$\left(i\frac{\partial}{dt}+\Delta-n\right)\psi=0, \tag{2.1}$$

$$\left(\frac{\partial^2}{dt^2}-\Delta\right)n=\Delta|\psi|^2. \tag{2.2}$$

Let τ and a be the typical time and spatial scales for the function n variation, i. e. the deepening time and the size of cavern. At

$$\tau\ll a, \tag{2.3}$$

when in equation (2.2) the «sonic» term Δn can be neglected, the collapse is accepted to call the «supersonic». In the process of supersonic collapse the values τ, a, n and $W=|\psi|^2$ are bound by

the relation following from (2.2):

$$\frac{n}{\tau^2}\sim\frac{W}{a^2}. \tag{2.4}$$

According to the equation (2.1), the frequency ω of the function ψ oscillations and the values a, n satisfy the estimates

$$\omega\sim a^{-2}\sim n. \tag{2.5}$$

The second estimate inables one to represent (2.4) in the form

$$\tau^{-2}\sim W. \tag{2.6}$$

Under the condition

$$\tau^{-1}\ll\omega \tag{2.7}$$

the cavern deepening is slow (adiabatic) and an energy of trapped Langmuir waves does not change:

$$Wa^d=\text{const} \tag{2.8}$$

(d is a space dimension). Using estimates (2.5), (2.6), one can rewrite the condition (2.7) in the following form:

$$Wa^4\ll 1 \tag{2.9}$$

and make sure that its fulfilment is improved in the process of supersonic adiabatic collapse at $d\leqslant 3$. The applicability condition for a supersonic approximation (2.3) is equivalent to an inequality

$$Wa^2\gg 1, \tag{2.10}$$

which becomes stronger in the process of collapse at $d=3$, remains the same at $d=2$ and is weakened at $d=1$, which ultimately leads to the cease of one-dimensional collapse. Though, until (2.3) is satisfied one is free to consider the collapse at any dimension of a space $d\leqslant 3$. The separation of a rapidly oscillating factor from the functions ψ:

$$\psi(\vec{r},t)=\bar{\psi}(\vec{r},t)\exp\left\{i\int_{t_0}^{t}dt'\,\omega(t')\right\} \tag{2.11}$$

enables one to represent the equations, describing the supersonic adiabatic collapse, in the following form

$$(-\omega+\Delta-n)\,\bar{\psi}=0, \tag{2.12}$$

$$\frac{d}{dt}\int d^d\vec{r}\,|\psi|^2=0, \tag{2.13}$$

$$\frac{d^2}{dt^2}\,n=\Delta\,|\psi|^2. \tag{2.14}$$

With no loss in generality, the function $\bar{\psi}(\vec{r},t)$ can be choosed real. The equations (2.12) — (2.14) allow the self-similar substitution

$$n(\vec{r},t)=\omega(t)\,u(\vec{r}\sqrt{\omega(t)}\,), \qquad \bar{\psi}(\vec{r},t)=\tau^{-1}E(\vec{r}\sqrt{\omega(t)}\,),$$

$$\tau=t_s-t, \quad \omega(t)\propto\tau^{-4/d}. \tag{2.15}$$

New functions $u(\bar{\rho})$ and $E(\bar{\rho})$ satisfy the equations

$$(-1+\Delta-u)\,E=0, \tag{2.16}$$

$$\left(\frac{4}{d}+1+\frac{2}{d}\vec{\rho}\vec{\nabla}\right)\left(\frac{4}{d}+\frac{2}{d}\vec{\rho}\vec{\nabla}\right)u=\Delta E^2, \tag{2.17}$$

where $\vec{\nabla}$ already means the differentiation over the variable $\vec{\rho}$.

The equations (2.12) — (2.14), linearized on the background of self-similar solution, has the power-type over τ solutions:

$$\delta\omega(t)=\omega(t)\,\mathrm{Re}\,(\Omega_\alpha\tau^\alpha),$$

$$\delta n(\vec{r},t)=\omega(t)\,\mathrm{Re}\,[\,u_\alpha(\vec{\rho})\,\tau^\alpha]\,, \tag{2.18}$$

$$\delta\psi(\vec{r},t)=\tau^{-1}\mathrm{Re}\,[\,E_\alpha(\vec{\rho})\,\tau^\alpha].$$

The values of α, Ω_α and functions $u_\alpha(\vec{\rho})$, $E_\alpha(\vec{\rho})$ are defined by the set of equations

$$(-1+\Delta-u)\,E_\alpha=(\Omega_\alpha+u_\alpha)\,E\,, \tag{2.19}$$

$$\alpha\int d^d\vec{\rho}\,EE_\alpha=0, \tag{2.20}$$

$$\left(\frac{4}{d}+1+\frac{2}{d}\vec{\rho}\vec{\nabla}-\alpha\right)\left(\frac{4}{d}+\frac{2}{d}\vec{\rho}\vec{\nabla}-\alpha\right)u_\alpha=2\Delta(EE_\alpha) \tag{2.21}$$

on the class of functions regular at $\rho\to 0$ and decreasing at $\rho\to\infty$. The perturbations, growing faster than self-similar solution, correspond to the eigenvalues α located in the left semiplane:

$$\mathrm{Re}\,\alpha<0. \tag{2.22}$$

The eigenvalues $\alpha=-2/d$ and $\alpha=-1$ are generated by a symmetry of equations (2.12) — (2.14) against the space-time shifts

(see Ref. 7) and is not connected with the true instability. If one considers the mentioned above «regularity at $\rho\to 0$» of the functions u_α, E_α as their finiteness, the spectrum of eigenvalues α should have the continuous component filling the entire strip (see Ref. 7):

$$\frac{4}{d}>\mathrm{Re}\,\alpha>\frac{4}{d}+\frac{1}{2}-\left[\frac{1}{4}+2E^2(0)\right]^{1/2}=\alpha_0. \tag{2.23}$$

The value α_0 is negative at

$$E^2(0)>\frac{2}{d}\left(\frac{4}{d}+1\right). \tag{2.24}$$

That guarantees the existence of unstable quasi-modes. If one considers the «regularity at $\rho\to 0$» as a stronger condition of all corrections finiteness generated by the sonic term Δn (omitted during the passage from (2.2) to (2.14)), the spectrum of eigenvalues α turns out to be discrete and corresponding eigenmodes, according to Ref. 7, should be true. The results [8], obtained within the frame of the scalar collapse model at $d=3$, enable one to anticipate that the simplest self-similar solutions with

$$\frac{2}{d}\left(\frac{4}{d}+1\right)<E^2(0)<\frac{4}{d}\left(\frac{8}{d}+1\right) \tag{2.25}$$

are stable against infinitely small perturbations, i. e. that the true unstable eigenmodes for these solutions are absent. Nevertheless, the quasi-modes should grow on the background of solutions satisfying the condition (2.25).

3. QUALITATIVE EXPLANATION OF INSTABILITY

The evolution of a small perturbation $\delta\omega$, δn, $\delta\bar{\psi}$ of self-similar solution (2.15) is described by the linearized equations (2.12) — (2.14):

$$(-\omega+\Delta-n)\,\delta\bar{\psi}=(\delta\omega+\delta n)\,\bar{\psi}\,, \tag{3.1}$$

$$\int d^d\vec{r}\,\bar{\psi}\,\delta\bar{\psi}=0, \tag{3.2}$$

$$\frac{\partial^2}{\partial t^2}\,\delta n=2\Delta(\bar{\psi}\,\delta\bar{\psi}). \tag{3.3}$$

The equation (3.1), solvable for $\delta\bar{\psi}$ under the condition

$$\delta\omega = \frac{\int d^d\bar{r}\,\delta n \psi^2}{\int d^d\bar{r}\,\psi^2} \tag{3.4}$$

enables one to restore $\delta\psi$ by δn with an accuracy to a proportional to $\bar{\psi}$ item, which is unambiguously defined by the orthogonality condition (3.2). Thus, the set of equations (3.1) — (3.3) is reduced to the closed equation for δn. This equation has especially simple form in the case, when the perturbation δn is of a small, compared to a, spatial scale λ and is localized in the vicinity of the cavern center. Under the conditions mentioned, one can neglect in equation (3.1) (near the cavern center) the terms $(\omega+n)\delta\bar{\psi}$ and $\delta\omega\bar{\psi}$ and replace in equation (3.3) $\bar{\psi}(\bar{r}, t)$ by $\psi(0, t) = E(0)/\tau$ (see (2.15)). The substitution $\Delta\delta\bar{\psi} \approx \delta n \psi$ in (3.3) results in the equation

$$\frac{\partial^2}{\partial t^2}\delta n \approx \frac{2E^2(0)}{\tau^2}\delta n. \tag{3.5}$$

The sonic term Δn, omitted in (2.14), has no influence on the evolution of small-scale perturbation δn under the condition

$$\lambda \gg \tau. \tag{3.6}$$

The integration of (3.5) yields:

$$\frac{\delta n(\bar{r}, t)}{n(0, t)} \infty \tau^{\alpha_0}. \tag{3.7}$$

with the value α_0 given in (2.23). As seen from (3.7), at $\alpha_0 < 0$ the growth of small-scale perturbation of an arbitrary form proceeds faster than the cavern deepening, which is agreed with the conclusions given in Ref. 7 on the existence of unstable quasi-modes. The deeper relation between the considerations used here and in Ref. 7 can be traced by expanding the small-scale perturbation $\delta n(\bar{r}, t)$ over quasi-modes having the fixed values of $\mathrm{Re}\,\alpha = \alpha_0 + 0$ and all possible values of $\mathrm{Im}\,\alpha$. The instability described by the formula (3.7) is developing unless and until the size of cavern a noticeably exceeds the wave length of perturbation λ. For this time, the ratio $\delta n/n$ grows by a factor

$$\left(\frac{\tau_f}{\tau_0}\right)^{\alpha_0} \sim \left(\frac{\lambda}{a_0}\right)^{\alpha_0 d/2}$$

and reaches the value of

$$\left(\frac{\delta n}{n}\right)_f \sim \left(\frac{\delta n}{n}\right)_0 \left(\frac{a_0}{\lambda}\right)^{-\alpha_0 d/2}. \tag{3.8}$$

At $1 \gg \delta n_f \lambda^2 \sim \left(\frac{\delta n}{n}\right)_f$ the perturbation δn_f weakly distorts the «wave function» $\bar{\psi}$ and that justifies the using of the linear theory of instability. In the opposite case:

$$\left(\frac{\delta n}{n}\right)_0 \geqslant \left(\frac{\lambda}{a_0}\right)^{-\alpha_0 d/2} \tag{3.9}$$

the growing small-scale perturbation distorts noticeably the field $\bar{\psi}$ and the self-similar collapse regime is distroyed.

The amplification of a relative perturbation $\delta n/n$ during the given cavern compression is determined by the factor $\delta = -\alpha_0 d/2$. The values of δ for calculated in Ref. 3 self-similar solutions (2.16), (2.17) with $d = 1, 2, 3$ and $E^2(0)$ from the range (2.25) are given below:

	1	2	3
$E^2(0)$	17.88	4.277	1.913
$-\alpha_0$	1.50	0.467	0.185
δ	0.75	0.467	0.276

(3.10)

The maximum value of δ is achieved at $d=1$, which makes the one-dimensional model the most suitable for numerical studies of small-scale instability.

4. ONE-DIMENSIONAL MODEL

Prior to numerical calculations, one should be sure in the stability of one-dimensional self-similar solution with $E^2(0) = 17.88$ against infinitely small perturbations. At $d=1$ the true eigenmodes are analytical in the center of cavern $(x=0)$ and decreasing at $x \to \pm\infty$ solutions of equations (2.19) — (2.21). Since the considered self-similar solution is even, the eigenmodes are also of a definite parity. For the odd modes the condition (2.20) is satisfied automatically, the value of Ω_α in (2.19) is equal to zero and $E_\alpha(0) = 0$. The normalized by condition

$$\left.\frac{dE_\alpha}{dx}\right|_{x=0}=1 \qquad (4.1)$$

analytical at $x\to 0$ odd solution of equations (2.19), (2.21) depends on the sole complex parameter α. The limit

$$I_-(\alpha)=\lim_{x\to\infty} e^{-x}E_\alpha(x) \qquad (4.2)$$

is analytic function of α. The spectrum of true eigenvalues α, corresponding to odd eigenmodes, is determined by the following equation:

$$I_-(\alpha)=0\,. \qquad (4.3)$$

The number Z_- of growing odd modes, being equal to the number of zeros of the function $I_-(\alpha)$ in the left semiplane, can be found by the formulae

$$Z_-=P_-+J_-\,, \qquad (4.4)$$

$$J_-=\frac{1}{2\pi i}\int_\Gamma d\alpha\frac{d}{d\alpha}\ln I_-(\alpha)\,, \qquad (4.5)$$

where Γ is a contour passing the whole imaginary axis α in the up direction and closed by an infinite left semicircle, P_- is the number of the function's $I_-(\alpha)$ poles in the left semiplane. At $|\alpha|\to\infty$ ($\mathrm{Re}\,\alpha<0$) the function $I_-(\alpha)$ tends to the finite limit, therefore the integral along an infinite semicircle in the expression for J_- is equal to zero. Using the property that $I_-(\alpha^*)=I_-(\alpha)^*$, the integral along the imaginary axis α can be represented in the following form

$$J_-=\frac{1}{\pi}\int_0^\infty d\beta\frac{d}{d\beta}\arg I_-(i\beta)\,. \qquad (4.6)$$

The poles of function $I_-(\alpha)$ are those values of α at which the equation (2.19), (2.21) (with $d=1$) have a nontrivial odd solution, analytical at $x\to 0$ and satisfying the condition

$$\left.\frac{dE_\alpha(x)}{dx}\right|_{x=0}=0\,. \qquad (4.7)$$

A simple analysis shows that such a solution exists only at

$$(5+2p-\alpha)(4+2p-\alpha)=2E^2(0)\,, \qquad (4.18)$$

where $p=1, 3, 5, \ldots$ is an odd positive integer, i. e. at

$$\alpha=\alpha_p^\pm=2p+4+\frac{1}{2}\pm\left[\frac{1}{4}+2E^2(0)\right]^{1/2}. \qquad (4.8)$$

For the self-similar solution under consideration an inequality is satisfied $E^2(0)<21$, which provides that all numbers $\alpha_p^\pm$ with $p\geqslant 1$ are positive, i. e. the function $I_-(\alpha)$ has no poles in the left semiplane α and $P_-=0$. The numerical calculation yields $J_-=1$. Hence, $Z_-=1$. The only eigenvalue α, corresponding to an odd growing mode, is equal to -2. This mode is generated by a shift of the place, where the singularity is formed, and has no relation to the true instability.

The analytical in the centre of the cavern even solution of equations (2.19), (2.21) linearly depends on two parameters, at every value of α. The decrease condition for $E_\alpha(x)$ at $x\to\infty$ determines unambiguously the ratio of these parameters and, after introducing the normalization

$$\left.\frac{d^2E_\alpha}{dx^2}\right|_{x=0}=\left.\frac{d^2E}{dx^2}\right|_{x=0}, \qquad (4.9)$$

determines also the function $E_\alpha(x)$. As well as $E_\alpha(x)$ the integral

$$I(\alpha)=\int_{-\infty}^{\infty} dx\,E(x)\,E_\alpha(x) \qquad (4.10)$$

from (2.20) depends analytically on α. The nonzeroth eigenvalues of α, corresponding to even modes, are the roots of the equation

$$I(\alpha)=0\,. \qquad (4.11)$$

At $|\alpha|\to\infty$ ($\mathrm{Re}\,\alpha<0$) the function $I(\alpha)$ tends to the finite limit:

$$E_\alpha(x)\to E(x)\,,\qquad I(\alpha)\to\int_{-\infty}^{\infty} dx\,E^2(x)\,.$$

Therefore, the numbers of zeros Z_+ and poles P of the function $I(\alpha)$, located in the left semiplane, are bound by the following relation

$$Z_+ = P + \int_0^\infty d\beta \frac{d}{d\beta} \arg I(i\beta) . \tag{4.12}$$

For counting the number P it worth mentioning that at every pole meaning of α the equation (2.19), (2.21) have nontrivial even solution, analytical in the center of cavern, decreasing at $x \to \infty$ and satisfying the condition

$$\left.\frac{d^2 E_\alpha}{dx^2}\right|_{x=0} = 0 . \tag{4.13}$$

At any value of α the analytical in the center of cavern even solution of equations (2.19), (2.21), having the property (4.13), is determined with accuracy to multiplication by an arbitrary constant. Normalizing such a solution by the condition

$$E_\alpha(0) = 1 , \tag{4.14}$$

one can introduce an analytical function

$$I_+(\alpha) = \lim_{x \to \infty} e^{-x} E_\alpha(x) . \tag{4.15}$$

Its zeros are located in the poles of function $I(\alpha)$, which enables one to count the number of the latter in the left semiplane of α by the formula

$$P = P_+ + \frac{1}{\pi} \int_0^\infty d\beta \frac{d}{d\beta} \arg I_+(i\beta) . \tag{4.16}$$

In the poles of function $I_+(\alpha)$, the number of which in the left semiplane is denoted here by P_+, the equations (2.19), (2.21) have nontrivial analytical in the center of cavern even solution, satisfying the conditions

$$E_\alpha(0) = 0 , \quad \left.\frac{d^2}{dx^2} E_\alpha(x)\right|_{x=0} = 0 . \tag{4.17}$$

Such a solution exists only at $\alpha = \alpha_p^\pm$, where $\alpha_p^\pm$ is given by the formula (4.8) with even positive p: $p = 2, 4, 6, \ldots$ For the self-similar solution under consideration, all numbers $\alpha_p^\pm$ are located in the right semiplane α, hence, $P_+ = 0$. The calculation of Z_+ by the following from (4.12), (4.16) formula

$$Z_+ = P_+ + \frac{1}{\pi} \int_0^\infty d\beta \frac{d}{d\beta} \arg [I(i\beta) I_+(i\beta)] \tag{4.18}$$

yields $Z_+ = 1$. The sole growing even mode corresponding to the eigenvalue $\alpha = -1$ is related to the shift of the singularity generation moment, but not to the true instability.

Thus, the considered self-similar solution is stable against infinitely small perturbations and can be used for the study of quasi-modes' instability.

5. SMALL-SCALE INSTABILITY

In order to check the conclusion of an increase in small-scale perturbations and the condition (3.9) for the destruction of self-similar supersonic collapse by them, the Koshi problem for one-dimensional equations (2.12) — (2.14) was solved numerically. The initial conditions differed from the self-similar solution (2.15) by a small adding, which rapidly oscillates along x (see Fig. 1):

$$n(x, 0) = \omega_0 [\, u(x\omega_0^{1/2}) + \delta u(x, 0)] ,$$

$$\left.\frac{\partial n(x, t)}{\partial t}\right|_{t=0} = \frac{\omega_0}{\tau_0} \Big[4u(x\omega_0^{1/2}) + \tag{5.1}$$

$$+ 2x \frac{d}{dx} u(x\omega_0^{1/2}) + (4 - \alpha)\, \delta u(x, 0) \Big] ,$$

$$\delta u(x, 0) = b \frac{\cos(kx + \varphi_0)}{1 + x^4 \omega_0^2} .$$

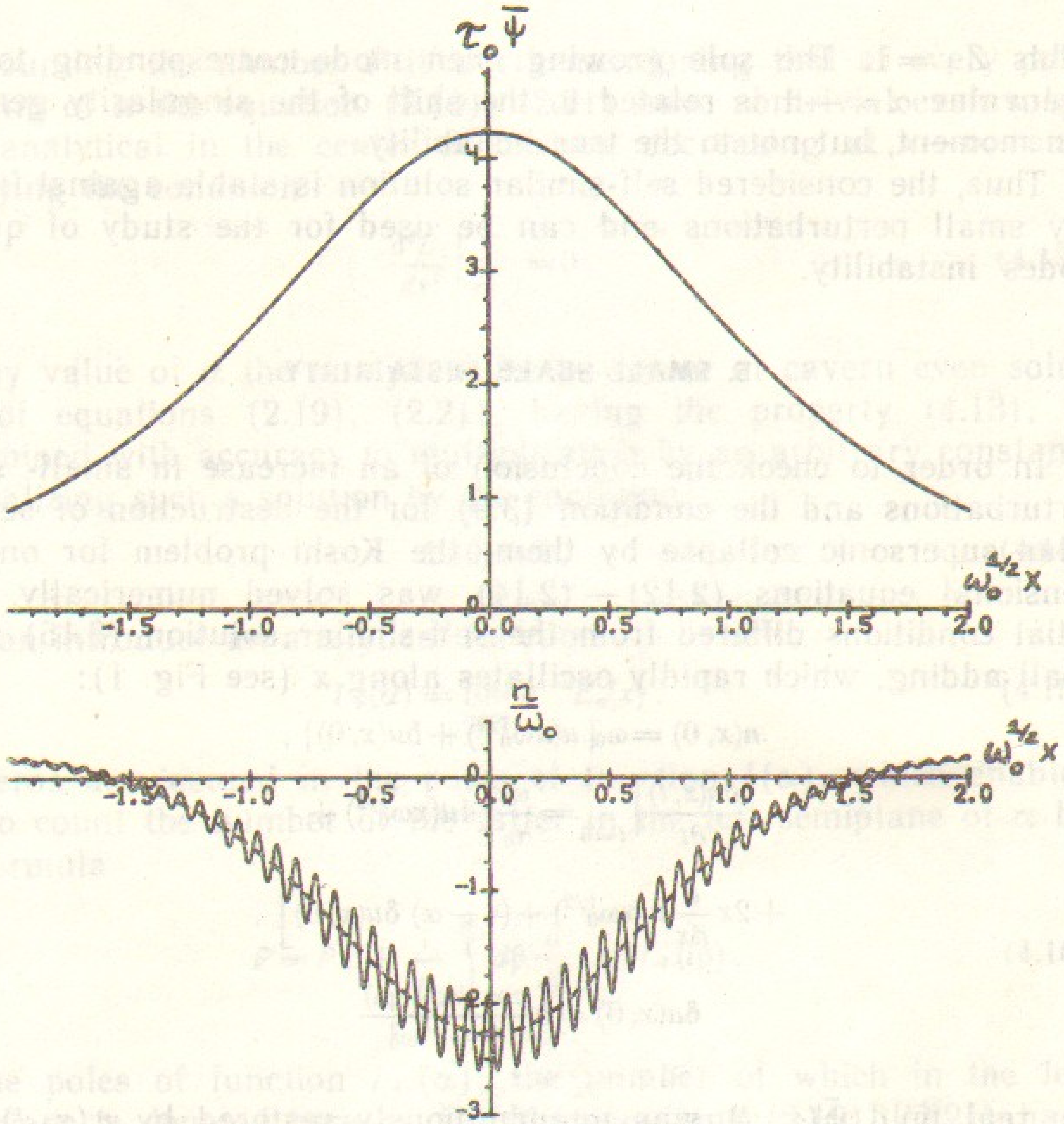
The real field $\bar\psi(x, t)$ was unambiguously restored by $n(x, t)$ via the equation (2.12) and the normalizing condition

$$\int_{-\infty}^{\infty} dx\, \bar\psi^2(x, t) = \frac{1}{\tau_0^2 \sqrt{\omega_0}} \int_{-\infty}^{\infty} d\xi\, E^2(\xi) . \tag{5.2}$$

The values of parameters $a_0 = z\omega_0^{-1/2}$, τ_0 satisfied the conditions (2.3), (2.7) for the supersonic adiabatic collapse:

$$a_0 = 2 \cdot 10^{-3}, \quad \tau_0 = 10^{-5}. \tag{5.3}$$

The wavelength $\lambda = 2\pi/k$ was chosen in the range $a_0 \gg \lambda \gg \tau_0$ in



order to make perturbation small-scaled, but practically uninfluenced by the sonic term Δn omitted in (2.14):

$$\lambda = 8 \cdot 10^{-5}. \tag{5.4}$$

The condition (3.9) for the destruction of self-similar collapse regime at the given values of parameters is equivalent to the inequality

$$b \geqslant 0.1\,. \tag{5.5}$$

The development of perturbation $\delta n(x, t) = \omega(t)\,\delta u(x, t)$ in the process of the cavern compression and deepening is presented in Figs 2 and 3. Fig. 2 corresponds to the small initial amplitude of perturbation: $b = 0.03$. In this case the collapse remains to be close to the self-similar one up to the cavern size $a \sim \lambda$ (at which the perturbation ceases to be small-scaled) and, consequently, further. In the case presented in Fig. 3 ($b = 0.3$) the small-scale perturbation has enough time to destruct completely the self-similar collapse regime.

As it is seen from the given figures, the small-scale perturbation grows without shifting in space and does not change its spatial scale. In the frame of coordinates compressed together with the cavern such a perturbation is stretched with time and moves out from the center of cavern. At a sufficiently small initial amplitude it goes outside the cavern without a noticeable influence on the collapse process. Thus, the small scale instability is of a convective character. In contrast to this, the instability of true eigenmodes (if those exist) is absolute. The development of an absolute instability of one-dimensional self-similar solution (2.15) with $E^2(0) = 51.45$ (also found in Ref. 3) is f iven in Fig. 4. As it is seen from the figure, the true eigenmode is compressed together with the cavern. This provides the possibility of an infinite growth of perturbation amplitude and, independently on the initial conditions, results in the destruction of the self-similar collapse regime.

6. ON THE TRANSITION OF SMALL-SCALE SONIC PERTURBATIONS INTO MODULATIONALLY UNSTABLE ONE

Only such a small-scale perturbations were considered above, for which the sonic term Δn, omitted in (2.14), is not essential. A shorter- wavelength perturbations, satisfying the condition opposite to (3.6):

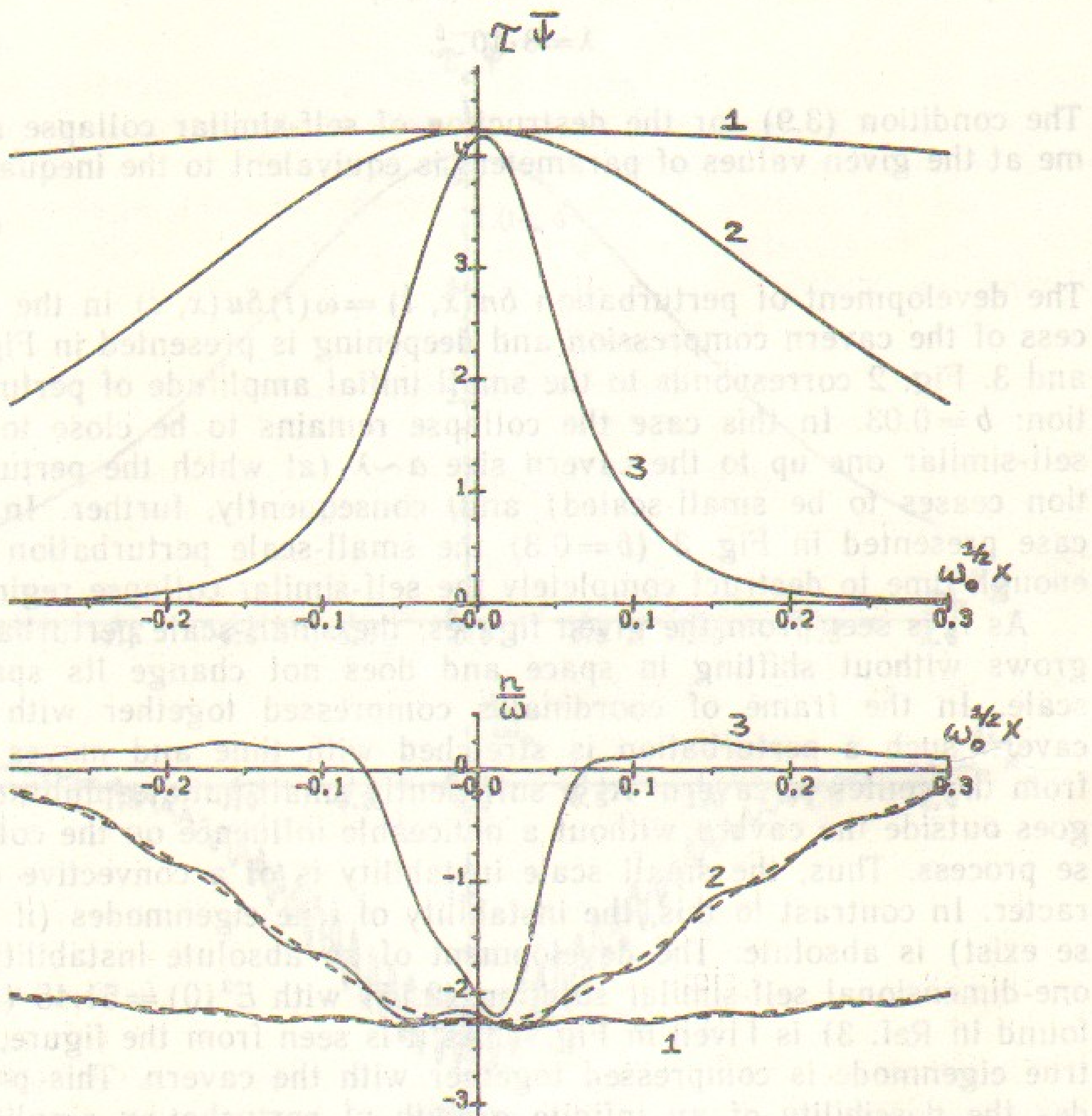


Fig. 2. Collapse dynamics at $b=0.03$: $\tau=\tau_0(\omega_0/\omega)^{1/4}$, line *1* represents initial state; line *2*—state with $\omega\approx 20\omega_0$; line *3* — $\omega\approx 400\omega_0$.

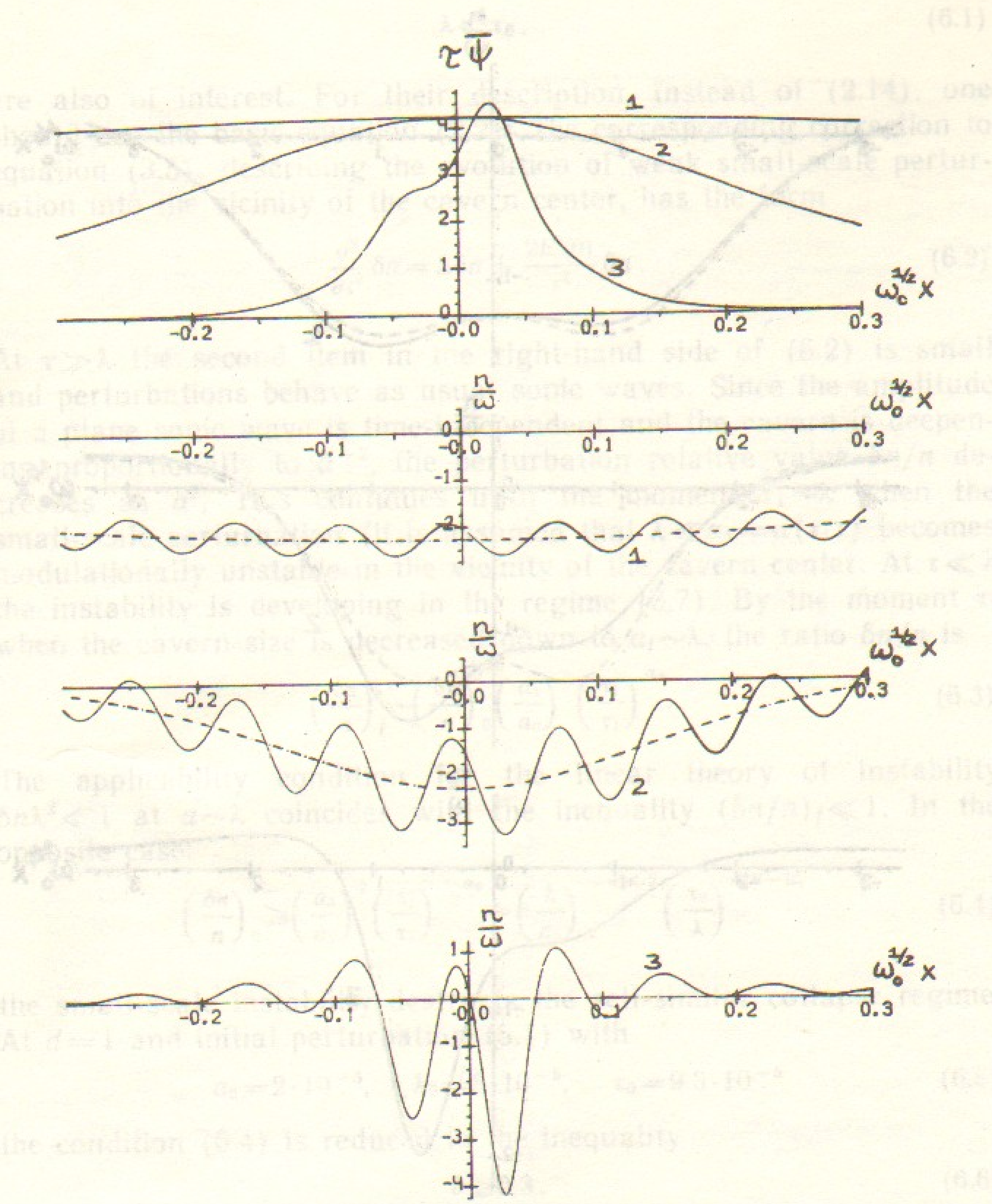


Fig. 3. Collapse dynamics at $b=0.3$: *1* — $\omega=\omega_0$; *2* — $\omega\approx 20\omega_0$; *3* — $\omega\approx 460\omega_0$.

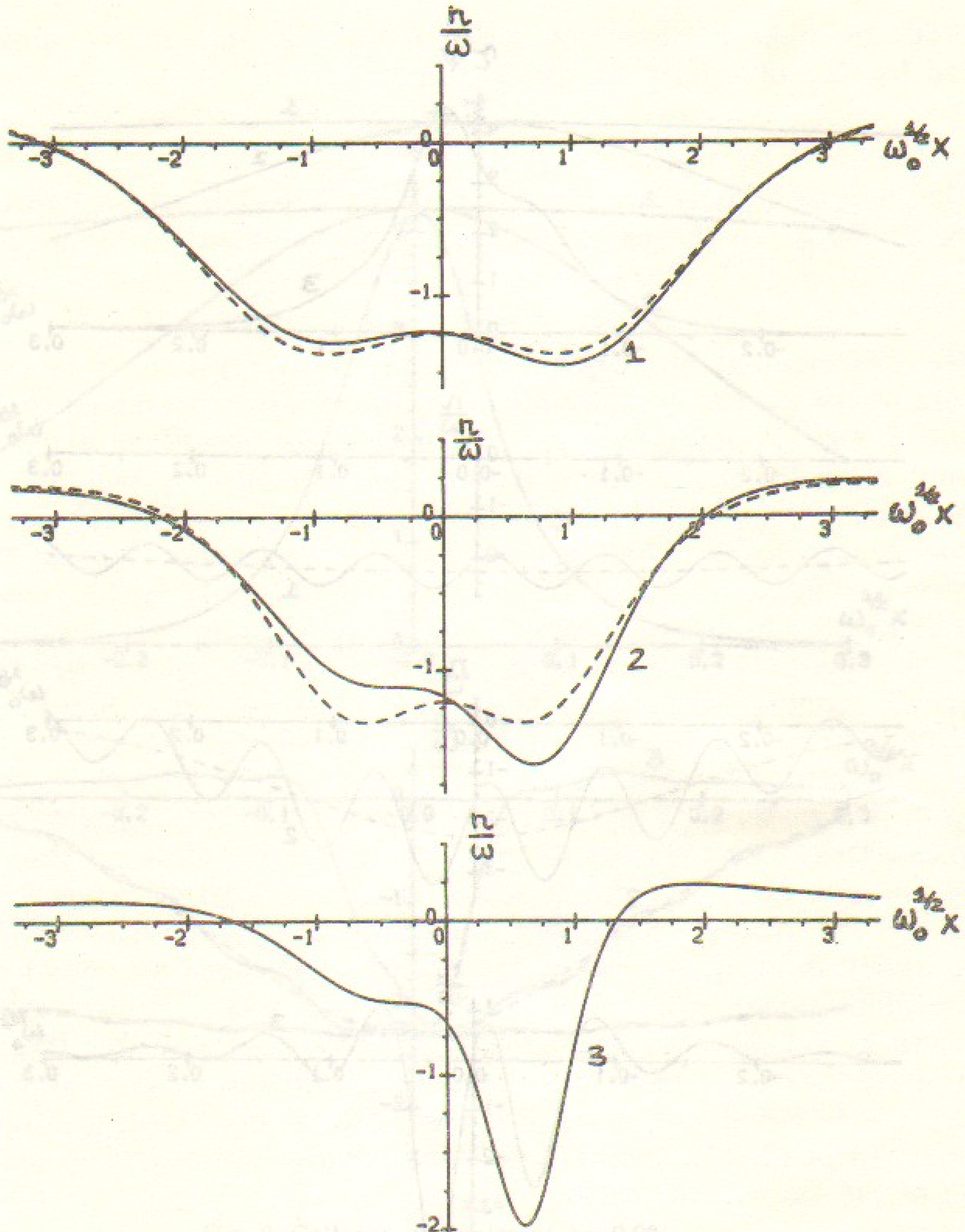


Fig. 4. Destruction of self-similar solution with $E^2(0)\approx 51.45$ by unstablest eigenmode and transition to self-similar regime with $E^2(0)\approx 17.88$:
1 — $\omega=\omega_0$; *2* — $\omega\approx 2.17\omega_0$; *3* — $\omega\approx 5.58\omega_0$.

$$\lambda \ll \tau_0, \tag{6.1}$$

are also of interest. For their description, instead of (2.14), one should use the basic equation (2.2). The corresponding correction to equation (3.5), describing the evolution of weak small-scale perturbation into the vicinity of the cavern center, has the form

$$\frac{\partial^2}{\partial t^2}\,\delta n = \Delta \delta n + \frac{2E^2(0)}{\tau^2}\,\delta n\,. \tag{6.2}$$

At $\tau \gg \lambda$ the second item in the right-hand side of (6.2) is small and perturbations behave as usual sonic waves. Since the amplitude of a plane sonic wave is time-independent and the cavern is deepening proportionally to a^{-2}, the perturbation relative value $\delta n/n$ decreases as a^2. This continues until the moment $\tau_1 \sim \lambda$ when the small-scale perturbation (it is assumed that $\lambda \ll a_1 = a(\tau_1)$) becomes modulationally unstable in the vicinity of the cavern center. At $\tau \ll \lambda$ the instability is developing in the regime (3.7). By the moment τ_f when the cavern size is decreased down to $a_f \sim \lambda$, the ratio $\delta n/n$ is

$$\left(\frac{\delta n}{n}\right)_f \sim \left(\frac{\delta n}{n}\right)_0 \left(\frac{a_1}{a_0}\right)^2 \left(\frac{\tau_f}{\tau_1}\right)^{\alpha_0}. \tag{6.3}$$

The applicability condition for the linear theory of instability $\delta n \lambda^2 \ll 1$ at $a \sim \lambda$ coincides with the inequality $(\delta n/n)_f \ll 1$. In the opposite case:

$$\left(\frac{\delta n}{n}\right)_0 \gtrsim \left(\frac{a_0}{a_1}\right)^2 \left(\frac{\tau_f}{\tau_1}\right)^{-\alpha_0} \sim \left(\frac{\lambda}{a_0}\right)^{-\alpha_0 d/2} \left(\frac{\tau_0}{\lambda}\right)^{4/d-\alpha_0} \tag{6.4}$$

the small-scale instability destructs the self-similar collapse regime. At $d=1$ and initial perturbation (5.1) with

$$a_0 = 2\cdot 10^{-3}, \quad \lambda_0 = 8\cdot 10^{-5}, \quad \tau_0 = 9.3\cdot 10^{-5} \tag{6.5}$$

the condition (6.4) is reduced to the inequality

$$b \gtrsim 0.3\,. \tag{6.6}$$

The evolution of such a perturbation at $b\approx 3$ is shown in Fig. 5. According to the estimate (6.6), the self-similar regime of collapse is destroyed.

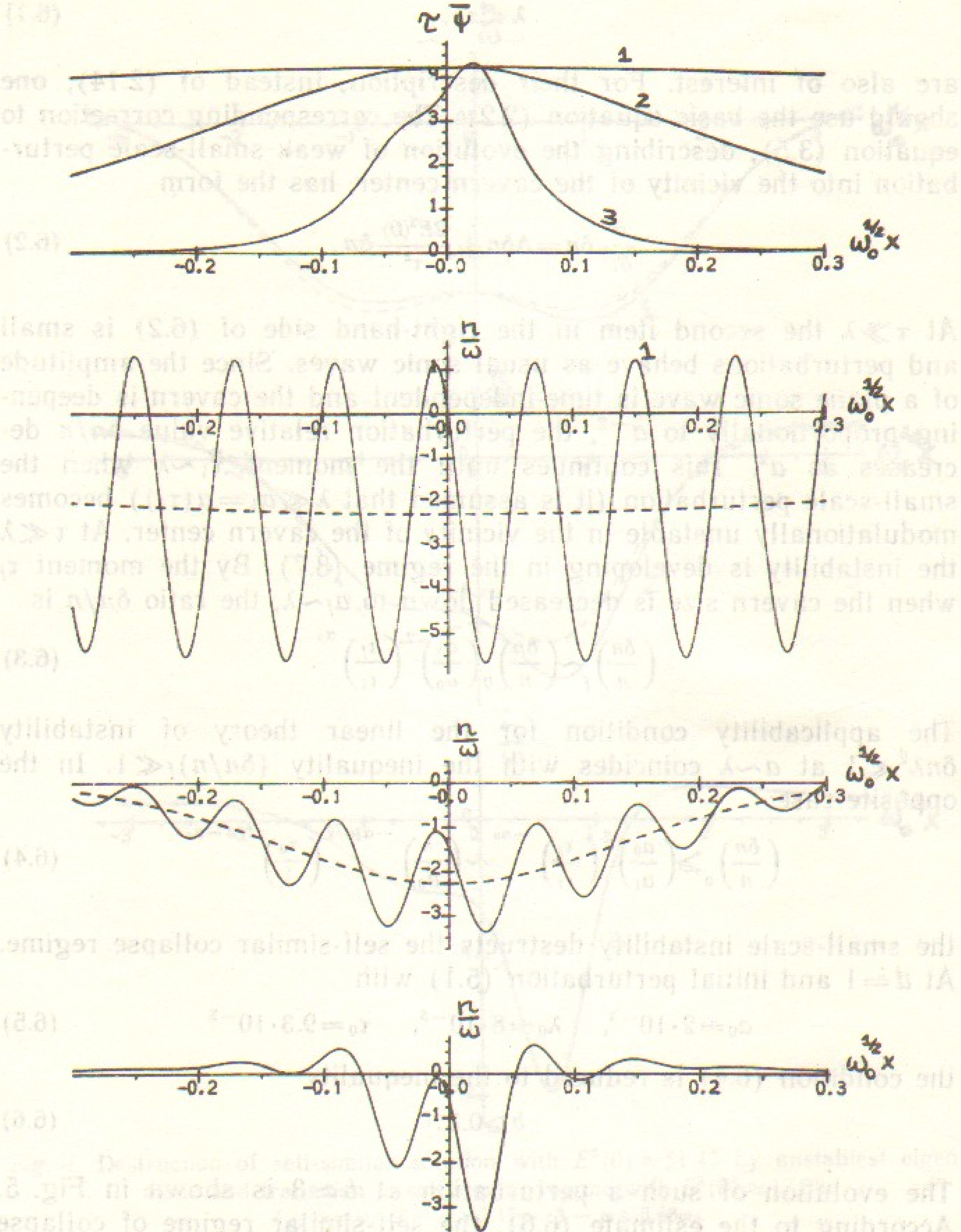


Fig. 5. Transition from sonic perturbation to modulationally unstable one and destruction of self-similar collapse regime:
$1 - \omega = \omega_0$; $2 - \omega \approx 20\omega_0$; $3 - \omega \approx 408\omega_0$.

7. CONCLUSION

The numerical calculations performed demonstrate the possibility to destruct the stable against infinitely small perturbations self-similar regimes of supersonic collapse by short-scale perturbations of finite amplitude. This possibility should be taken into account in the description of strong Langmuir turbulence of a nonisothermal plasma (with higher electron temperature), where the level of a short-wave sound is quite high. The collapse dynamics in the presence of sufficiently high-intensive sound is not universal. As the compression grew of captured Langmuir waves, the more and more short-scale perturbations becoming modulationally unstable in the cavern center, the new and new narrow gaps of ion concentration deepening and sucking the bound state populated by Langmuir waves. In this case, the characteristic size of collapsing bunch of Langmuir waves decreases faster than that according to the law $a \propto \tau^{2/d}$ available in the absence of small-scale perturbations. The collapse new law is of statistical character and it depends on the sonic turbulence spectrum (see Ref. 9).

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