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НОВОСИБИРСК

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A B S T R A C T

Inelastic gluon-gluon scattering amplitudes in the Born approximation for the quasi-multi-Regge kinematics are calculated, starting with the Veneziano-type expression for the inelastic amplitude of the gluon-tachyon scattering with its subsequent simplification in the region of large energies and the Regge slope $\alpha' \to 0$. Results obtained allow one to determine the high order corrections to the gluon Regge trajectory, the reggeon-particle vertices and to the integral kernel of the Bethe—Salpeter equation for the vacuum t-channel partial waves.

1. INTRODUCTION

Due to the asymptotic freedom in QCD the glueball Regge trajectories $j(q^2)$ can be calculated at large momentum transfers $|\vec{q}| \gg \Lambda_{QCD} \simeq 100$ MeV [1]. In particular, in the leading logarithmic approximation (LLA) the pomeron having vacuum quantum numbers turns out to be a compound state of two reggeized gluons [2] and the odderon, which is a reggeon with the negative charge parity and signature, consists of three gluons [3]. These Regge poles are important for the high energy phenomenology [4]. In LLA the t-channel partial waves $f_j(q^2)$ for colourless particle scattering amplitudes are expressed in terms of the off-mass shell gluon-gluon scattering amplitudes satisfying an integral equation of the Bethe—Salpeter type [2, 5]. This equation is explicitly solved due to its conformal invariance in the two dimensional impact parameter space [1]. Taking into consideration the running of the QCD coupling constant one can compute the bare pomeron trajectories at large $\vec{q}$ and find the lower bounds for their intercepts [1]. To determine the region of applicability of these results one needs to find the QCD radiative corrections to LLA. In this paper we calculate inelastic gluon-gluon scattering amplitudes in the Born approximation for a quasi-multi-Regge kinematics (QMRK) of produced gluons. We call QMRK the momentum configuration of the final state particles in which all of them excepting one pair have large relative energies $\sqrt{s_{ij}}$ and fixed transverse momenta $k_{i\perp}$. The invariant mass of the above pair supposes to be of the same order of value as

$|k_{i\perp}|$. Below it is shown that these results allow one to determine the high order corrections to the gluon Regge trajectory, the reggeon-particle vertices and to the integral kernel of the Bethe—Salpeter equation for the vacuum t-channel partial waves.

In the next section we consider the simplest process in QMRK, namely—the production of an extra gluon in the fragmentation region of the initial gluon. It is convenient to start from the well known Veneziano-type expression [6] for the inelastic amplitude of the gluon-tachyon scattering with its subsequent simplification in the region of large energies $\sqrt{s}$ and the small Regge slope α' (cf. [7]). The third section is devoted to certain transformations of obtained expressions. In the fourth section we calculate the inelastic amplitude for the two gluon production in QMRK starting from the corresponding dual amplitude. The final formula is studied in the fifth section. In the conclusion we discuss obtained results and list out unsolved problems.

2. GLUON PRODUCTION IN THE FRAGMENTATION REGION

It is known that in the Regge asymptotics ($s\to\infty$, t fixed) the scattering amplitudes are factorized in the t-channel. In particular, the Born amplitude for production of a gluon in the fragmentation region of the incident gluon at high energies depends only on the colour spin of the target. Therefore in the case of the string theory it is sufficient to consider as a target the spinless tachyon with its

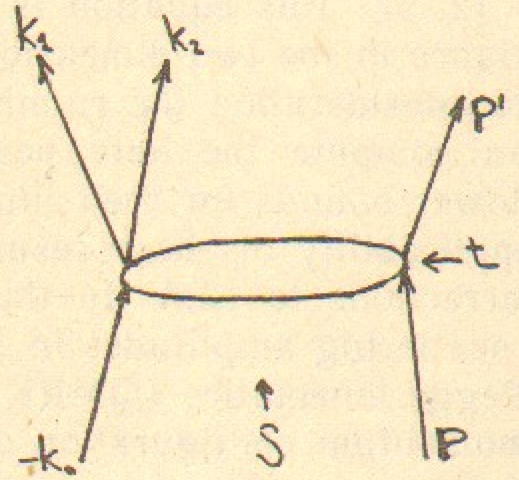

Fig. 1.

squared mass $m^2=-1/\alpha'$ where α' is the slope of the Regge trajectory (the gluon production in the tachyon-tachyon scattering was studied earlier in Ref. [8]).

Let the initial gluon and tachyon momenta are $p_A\equiv -k_0$, $p_B\equiv p$ and momenta of two gluons and one tachyon in the final state are k_1, k_2, p' correspondingly (see, Fig. 1). Then the kinematics in the fragmentation region is characterized by the relations:

$$s\equiv -2k_0p\gg(\alpha')^{-1}, \quad \beta_1=\frac{2k_1p}{-2k_0p}\sim 1, \quad \beta_2=\frac{2k_2p}{-2k_0p}\sim 1,$$
$$t_1\equiv 2k_0k_1\sim t_2\equiv 2k_0k_2\sim t\equiv -2pp'+2m^2\lesssim m^2, \qquad (1)$$
$$s_{12}\equiv 2k_1k_2\lesssim 1/\alpha'; \quad m^2=-1/\alpha'.$$

Here β_1 and β_2 are Feynman's parameters of the final state gluons, t_1, t_2, s_{12} are the invariants of the gluonic subprocess. We have also the following relations among the various invariants:

$$\beta_1+\beta_2=1+\frac{t}{s}\simeq 1, \quad s_{12}=t-t_1-t_2,$$
$$2k_0p'=s+t_2+t_1, \quad 2k_1p'=s\beta_1+t_2-t, \quad 2k_2p'=s\beta_2+t_1-t. \qquad (2)$$

In the tree approximation the scattering amplitude for this reaction is given by the functional integral [6]

$$A^{\mu_0\mu_1\mu_2}\sim\int Dx\exp\left[\frac{1}{8\pi\alpha'}\int d^2z\sum_{i=1}^{2}\left(\frac{\partial x^\mu}{\partial z^i}\right)^2\right]\times$$
$$\times\prod_{r=0}^{2}V^{\mu_r}(k_r)\,V(p)\,V(p'), \qquad (3)$$

where μ_0, μ_1, μ_2 are the Lorentz indices of external gluons and $V^\mu(k)$ and $V(p)$ are the string vertices for the gluon and tachyon emission:

$$V^\mu(k_r)=\int d\sigma_r\frac{\partial x^\mu(\sigma_r)}{\partial\sigma_r}\exp\left(ik_rx(\sigma_r)\right),$$
$$V(p)=\int d\sigma\, e^{-ipx(\sigma)}, \quad V(p')=\int d\sigma'\, e^{ip'x(\sigma')}. \qquad (4)$$

After integrating (3) over x we obtain

$$A^{\mu_0\mu_1\mu_2} \sim \int \prod_{r=0}^{2} d\sigma_r \frac{d\sigma\, d\sigma'}{(\sigma-\sigma')^2} \left| \frac{(\sigma_0-\sigma')(\sigma_1-\sigma)}{(\sigma_0-\sigma)(\sigma_1-\sigma')} \right|^{\alpha' s\beta_1} \times$$
$$\times \left| \frac{(\sigma_0-\sigma')(\sigma_2-\sigma)}{(\sigma_0-\sigma)(\sigma_2-\sigma')} \right|^{\alpha' s\beta_2} \left| \frac{(\sigma_0-\sigma')(\sigma_1-\sigma_2)}{(\sigma_2-\sigma')(\sigma_0-\sigma_1)} \right|^{\alpha' t_1} \times$$
$$\times \left| \frac{(\sigma_0-\sigma')(\sigma_1-\sigma_2)}{(\sigma_1-\sigma')(\sigma_0-\sigma_2)} \right|^{\alpha' t_2} \left| \frac{(\sigma_2-\sigma')(\sigma_1-\sigma')(\sigma_0-\sigma)}{(\sigma_1-\sigma_2)(\sigma_0-\sigma')(\sigma-\sigma')} \right|^{\alpha' t} C^{\mu_0\mu_1\mu_2} \qquad (5)$$

Here the tensor $C^{\mu_0\mu_1\mu_2}$ equals

$$C^{\mu_0\mu_1\mu_2} = (2\alpha')^3 C^{\mu_0} C^{\mu_1} C^{\mu_2} - (2\alpha')^2 \left(\frac{g^{\mu_1\mu_2} C^{\mu_0}}{(\sigma_1-\sigma_2)^2} + \right.$$
$$\left. + \frac{g^{\mu_0\mu_1} C^{\mu_2}}{(\sigma_0-\sigma_1)^2} + \frac{g^{\mu_0\mu_2} C^{\mu_1}}{(\sigma_0-\sigma_2)^2} \right), \qquad (6)$$

$$C^{\mu_r} = \left(\frac{p}{\sigma-\sigma_r} - \frac{p'}{\sigma'-\sigma_r} - \sum_{\substack{l=0 \\ l\neq r}}^{2} \frac{k_l}{(\sigma_l-\sigma_r)} \right)^{\mu_r}. \qquad (7)$$

Expression (5) respects the Boze symmetry $r \leftrightarrow r'$ and the on-mass shell **gauge** invariance $e_i \to e_i + ck_i$ for the gluon polarization vectors e_i due to the identities of the type

$$k_0^{\mu_0} C_{\mu_0\mu_1\mu_2} \exp G = \frac{\partial}{\partial\sigma_0} \left[-(2\alpha')^2 C_{\mu_1} C_{\mu_2} + \frac{2\alpha' g_{\mu_1\mu_2}}{(\sigma_1-\sigma_2)^2} \right] \exp G, \qquad (8)$$

where $\exp G$ is the integrand in eq. (5) without the factor $C^{\mu_0\mu_1\mu_2}$.

Furthermore, for physical polarizations ($k_i e_i = 0$) eq. (5) is invariant under the Möbious transformations of the integration variables:

$$\sigma_r \to \frac{a\sigma_r + b}{c\sigma_r + d}, \qquad (9)$$

where a, b, c, d are real parameters. This symmetry allows us to fix values of three variables:

$$\sigma = 1, \quad \sigma' = \infty, \quad \sigma_0 = 0 \qquad (10)$$

ommiting in eq. (5) an infinite volume of the space of the Möbious parameters. Instead of σ_1, σ_2 we introduce the other variables y, z:

$$y = \sigma_2 - \sigma_1, \quad z = \frac{\sigma_1}{\sigma_2 - \sigma_1}, \quad d\sigma_1 d\sigma_2 = |y|\, dz\, dy. \qquad (11)$$

Then in the asymptotic region (1) one has

$$|y| \sim \frac{1}{\alpha' s} \ll 1, \quad |z| \sim 1 \qquad (12)$$

and eq. (5) can be rewritten as follows

$$A^{\mu_0\mu_1\mu_2} \sim \int \frac{dz\, dy}{z^2 y |y|} \exp[-\alpha' szy - \alpha' t_2 \ln(1+z) - \alpha'\beta_2 sy] \times$$
$$\times |z|^{-\alpha' t_1} |y|^{-\alpha' t} \Big\{ (2\alpha')^3 \tilde{C}_0^{\mu_0} \tilde{C}_1^{\mu_1} \tilde{C}_2^{\mu_2} - (2\alpha')^2 \times$$
$$\times \left(g^{\mu_1\mu_2} z \tilde{C}_0^{\mu_0} + g^{\mu_0\mu_1} \tilde{C}_2^{\mu_2} + \frac{z}{(1+z)^2} g^{\mu_0\mu_2} \tilde{C}_1^{\mu_1} \right) \Big\}, \qquad (13)$$

where

$$\tilde{C}_0 = zyp - k_1 - \frac{z}{1+z} k_2, \quad \tilde{C}_1 = zyp + k_0 - zk_2,$$
$$\tilde{C}_2 = yp + \frac{k_0}{1+z} + k_1. \qquad (14)$$

To introduce in eq. (13) the colour indices i, i', i_0, i_1, i_2 for external tachyons and gluons we use the following combination of the Chan—Paton factors [6]:

$$C^{-}_{i_{r_1} i_{r_2} i_{r_3} i_{r_4} i_{r_5}} = C_{i_{r_1} i_{r_2} i_{r_3} i_{r_4} i_{r_5}} - C_{i_{r_5} i_{r_4} i_{r_3} i_{r_2} i_{r_1}},$$
$$C_{i_1 i_2 i_3 i_4 i_5} = 2Sp[t_{i_1} t_{i_2} t_{i_3} t_{i_4} t_{i_5}],$$
$$Sp[t_i t_j] = \frac{1}{2} \delta_{ij}, \qquad (15)$$

for each region of integration in eq. (5) restricted by the inequalities $\sigma_{r_1} < \sigma_{r_2} < \sigma_{r_3} < \sigma_{r_4} < \sigma_{r_5}$ where i_r can take five values: $i_r = i$, i', i_0, i_1, i_2. In the kinematics (1) the main contribution arises from those integration regions where the relations (12) hold. There are six such regions. In each of them the values of s, β_2, $(1-\beta_2)$ have to be chosen positive or negative in such a way that the integrals converge. The integration result has to be analytically continued into the kinematics region considered. Thus, eq. (13) can be written as follows (see (10) — (12)):

$$A^{\mu_0\mu_1\mu_2} \sim \Big\{ [-C^{-}_{i_2 i_1 i_0 i i'} (-\alpha' s)^{1+\alpha' t} + C^{-}_{i_0 i_1 i_2 i i'} (\alpha' s)^{1+\alpha' t}] \times$$

$$\times \int_0^\infty \frac{dx}{x^2} \int_0^\infty dz\, \chi^{\mu_0\mu_1\mu_2}(x, z, k_0, k_1, k_2, p, p') +$$

$$+ C_{\bar{i}_1 i_0 i_2 i i'} (\alpha' s)^{1+\alpha' t} \int_0^\infty \frac{dx}{x^2} \int_{-1}^0 dz\, \chi^{\mu_0\mu_1\mu_2}(x, z, k_0, k_1, k_2, p, p') \Big\} +$$

$$+ \left\{ \begin{matrix} \mu_1 \leftrightarrow \mu_2 \\ k_1 \leftrightarrow k_2 \\ i_1 \leftrightarrow i_2 \end{matrix} \right\} , \quad x = \alpha' s y , \tag{16}$$

where

$$\chi^{\mu_0\mu_1\mu_2}(x, z, k_0, k_1, k_2\, p, p') = e^{-x(z+\beta_2)} |x|^{-\alpha' t} |z|^{-\alpha' t_1} |1+z|^{-\alpha' t_2} \times$$

$$\times \Big\{ (2\alpha')^3 C_0^{\mu_0} C_1^{\mu_1} C_2^{\mu_2} - (2\alpha')^2 \Big[g^{\mu_1\mu_2} C_0^{\mu_0} + \frac{g^{\mu_0\mu_1}}{z^2} C_2^{\mu_2} + \frac{g^{\mu_0\mu_2}}{(1+z)^2} C_1^{\mu_1} \Big] \Big\} \tag{17}$$

and

$$C_0 = \frac{xp}{\alpha' s} - \frac{k_1}{z} - \frac{k_2}{1+z}, \quad C_1 = \frac{xp}{\alpha' s} + \frac{k_0}{z} - k_2, \quad C_2 = \frac{px}{\alpha' s} + \frac{k_0}{1+z} + k_1 . \tag{18}$$

To obtain eq. (16) the following easily verified identity was used:

$$\chi^{\mu_0\mu_1\mu_2}(x, z, k_0, k_1, k_2, p, p') \equiv -\chi^{\mu_0\mu_2\mu_1}(-x, -(1+z), k_0, k_2, k_1, p, p') . \tag{19}$$

This identity helps one to express one term in eq. (16) as a linear combination of others in the local limit $\alpha' \to 0$:

$$\int_0^\infty \frac{dx}{x^2} \int_{-1}^0 dz\, \chi^{\mu_0\mu_1\mu_2} \Big|_{\alpha' \to 0} = \int_0^\infty \frac{dx}{x^2} \int_0^\infty dz (-\chi^{\mu_0\mu_1\mu_2}) \Big|_{\alpha' \to 0} + \begin{pmatrix} \mu_1 \leftrightarrow \mu_2 \\ k_1 \leftrightarrow k_2 \end{pmatrix} . \tag{20}$$

In the $\alpha' \to 0$ limit one can put $(-1)^{\alpha' t_i} = 1\ (t_i = t, t_1, t_2)$, i. e. consider $\chi^{\mu_0\mu_1\mu_2}$ as analytic function of x, z. After using of the identity (19) and integration over x the last term in (20) is represented by the integral over z from $-\infty$ to -1, therefore, eq. (20) is a consequence of the analyticity of $\chi^{\mu_0\mu_1\mu_2}$ and the integral convergence. The following equality between the Chan—Paton factors and the colour generators $T^c_{i_1 i_2}$ in the gluon representation

$$C_{\bar{i}_2 i_1 i_0 i i'} + C_{\bar{i}_0 i_1 i_2 i i'} - C_{\bar{i}_1 i_0 i_2 i i'} - C_{\bar{i}_2 i_0 i_1 i i'} = T^{c_2}_{i' i}\, T^{i_2}_{c_1 c_2}\, T^{c_1}_{i_1 i_0} . \tag{21}$$

is also helpful.

Integration over x and z in eq. (16) can be performed in the

Yang—Mills limit $\alpha' \to 0$ with the use of eqs (20) and formulas of Appendix 1. Finally we obtain for the gluon production amplitude in QCD the following expression

$$A^{\mu_0\mu_1\mu_2}|_{\alpha' \to 0} = 8 g^3 s\, T^{c_2}_{i' i}\, T^{i_2}_{c_2 c_1}\, T^{c_1}_{i_1 i_0}\, a^{\mu_0\mu_1\mu_2}(k_1, k_2, k_0, p, p') +$$

$$+ \begin{pmatrix} \mu_1 \leftrightarrow \mu_2 \\ i_1 \leftrightarrow i_2 \\ k_1 \leftrightarrow k_2 \end{pmatrix} , \tag{22}$$

where we used eq. (21) and introduced $a^{\mu_0\mu_1\mu_2}$ by definition

$$a^{\mu_0\mu_1\mu_2} = \frac{p^{\mu_0}}{s} \Big[\frac{p^{\mu_1} p^{\mu_2}}{\beta_2 s^2} - \frac{p^{\mu_1} k_1^{\mu_2} - k_2^{\mu_1} p^{\mu_2}}{s_{12} s} - \frac{k_0^{\mu_1} p^{\mu_2}}{\beta_2 s t_1} +$$

$$+ \frac{k_0^{\mu_1} k_1^{\mu_2}}{t} \Big(\frac{1}{t_1} + \frac{1}{s_{12}} \Big) - \frac{k_2^{\mu_1} k_0^{\mu_2}}{s_{12} t} + \frac{k_0^{\mu_1} k_0^{\mu_2}}{t t_1} \Big] + k_1^{\mu_0} \Big[\frac{p^{\mu_1} p^{\mu_2}}{\beta_2 s^2 t_1} +$$

$$+ \frac{(k_2^{\mu_1} p^{\mu_2} - p^{\mu_1} k_1^{\mu_2})}{st} \Big(\frac{1}{t_1} + \frac{1}{s_{12}} \Big) - \frac{p^{\mu_1} k_0^{\mu_2}}{s t_1 t} \Big] - k_2^{\mu_0} \Big[\frac{p^{\mu_1} k_1^{\mu_2} - k_2^{\mu_1} p^{\mu_2}}{s_{12} s t} +$$

$$+ \frac{k_0^{\mu_1} p^{\mu_2}}{s t_1 t} \Big] - \frac{g^{\mu_1\mu_2}}{2} \Big[\frac{(t\beta_2 - t_2)\, p}{s_{12} s t} + \frac{\beta_2}{t} \Big(\frac{1}{t_1} + \frac{1}{s_{12}} \Big) k_1 - \frac{\beta_1 k_2}{s_{12} t} \Big]^{\mu_0} -$$

$$- \frac{g^{\mu_0\mu_1}}{2} \Big[\Big(\frac{t}{t_1 \beta_2} - \frac{t_2}{t_1} \Big) \frac{p}{st} - \frac{\beta_1}{t_1 t} k_0 - \Big(\frac{1}{t_1} + \frac{1}{s_{12}} \Big) \frac{k_1}{t} \Big]^{\mu_2} -$$

$$- \frac{g^{\mu_0\mu_2}}{2} \Big[- \frac{p}{st} - \frac{\beta_2 k_0}{t_1 t} + \frac{k_2}{s_{12} t} \Big]^{\mu_1} . \tag{23}$$

The common numerical factor in eq. (22) was fixed from the requirement of coincidence with the QCD perturbation theory result. For example, the Feynman diagrams with all external gluon lines attached to the tachyon line give in the asymptotic region (1) the following contribution to $A^{\mu_0\mu_1\mu_2}|_{\alpha'=0}$

$$\Delta A^{\mu_0\mu_1\mu_2} = -g^3 \frac{(2p)^{\mu_0} (2p)^{\mu_1} (2p)^{\mu_2}}{s^2} \Big[\frac{T^{i_2} T^{i_1} T^{i_0}}{\beta_2} + \frac{T^{i_0} T^{i_1} T^{i_2}}{\beta_2} +$$

$$+ \frac{T^{i_1} T^{i_2} T^{i_0}}{\beta_1} + \frac{T^{i_0} T^{i_2} T^{i_1}}{\beta_1} - \frac{T^{i_2} T^{i_0} T^{i_1}}{\beta_1 \beta_2} - \frac{T^{i_1} T^{i_0} T^{i_2}}{\beta_1 \beta_2} \Big]_{i' i} \simeq$$

$$\simeq 8 g^3 s\, T^{c_2}_{i' i}\, T^{i_2}_{c_2 c_1}\, T^{c_1}_{i_1 i_0} \frac{p^{\mu_0} p^{\mu_1} p^{\mu_2}}{s \beta_2 s^2} + (1 \leftrightarrow 2) , \tag{24}$$

which agrees with the corresponding terms in eqs (22), (23). In the next section the factorization properties of expression (22) are verified and it is rewritten in a more symmetrical form.

3. t-CHANNEL FACTORIZATION OF THE GLUON BREMSSTRAHLUNG AMPLITUDE

Formula (22) can be reproduced by using the t-channel dispersion relations. To begin with let us write down the Born amplitude for the elastic gluon-gluon scattering in the helicity basis:

$$A_{\lambda_0\lambda_1\lambda_2\lambda_3}=-2g^2\Big[\frac{\delta_{\lambda_0,-\lambda_1}\delta_{\lambda_2,-\lambda_3}}{(k_0k_1)}\,((k_0k_2)\,T^d_{i_0i_3}T^d_{i_1i_2}+(k_0k_3)\times$$

$$\times T^d_{i_0i_2}T^d_{i_1i_3})+\frac{\delta_{\lambda_0,-\lambda_2}\delta_{\lambda_1,-\lambda_3}}{(k_0k_2)}\,((k_0k_1)\,T^d_{i_0i_3}T^d_{i_2i_1}+(k_0k_3)\,T^d_{i_0i_1}T^d_{i_2i_3})+$$

$$+\frac{\delta_{\lambda_0,-\lambda_3}\delta_{\lambda_2,-\lambda_1}}{(k_0k_3)}\,((k_0k_2)\,T^d_{i_0i_1}T^d_{i_3i_2}+(k_0k_1)\,T^d_{i_0i_2}T^d_{i_3i_1})\Big], \qquad (25)$$

where $\lambda_i=\pm1$ are helicities of the outgoing gluons (see Fig. 2),

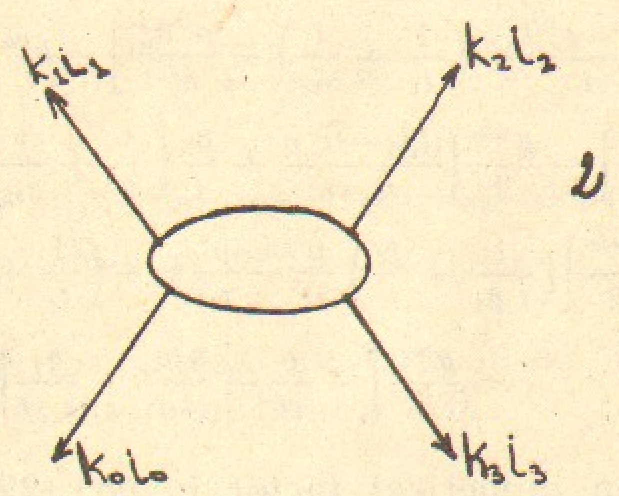

Fig. 2.

$$A_{\lambda_0\lambda_1\lambda_2\lambda_3}=A^{\mu_0\mu_1\mu_2\mu_3}\,\overset{*}{e}{}^{\lambda_0}_{\mu_0}(k_0)\,\overset{*}{e}{}^{\lambda_1}_{\mu_1}(k_1)\,\overset{*}{e}{}^{\lambda_2}_{\mu_2}(k_2)\,\overset{*}{e}{}^{\lambda_3}_{\mu_3}(k_3)\,,$$

$$e^{\lambda}_{\mu}(k_n)=\frac{-i\lambda\mathscr{P}(nmjl)}{\sqrt{2(k_0k_1)(k_0k_2)(k_1k_2)}}\Big[\frac{k^{\nu}_n}{3}(k_{m\mu}(k_j-k_l)_{\nu}+$$

$$+k_{l\mu}(k_m-k_j)_{\nu}+k_{j\mu}(k_l-k_m)_{\nu})+i\lambda\varepsilon_{\mu\nu\rho\delta}\,k^{\nu}_m k^{\rho}_j k^{\delta}_l\Big], \qquad (26)$$

$$n\neq m\neq l\neq j,\qquad e^{\lambda}_{\mu}(k)=\overset{*}{e}{}^{\lambda}_{\mu}(-k)=\overset{*}{e}{}^{-\lambda}_{\mu}(k)=e^{-\lambda}_{\mu}(-k)\,.$$

Here m, j, l are indices of gluons complementary to n, $\mathscr{P}(nmjl)$ is the parity of the permutation $\begin{pmatrix}0\,1\,2\,3\\ n\,m\,j\,l\end{pmatrix}$. Polarization vectors $e^{\lambda}_{\mu}(k_n)$ can differ from those presented in formula (26) by gauge-dependent terms containing $k_{n\mu}$.

The colour structures used above are linear dependent due to the equality

$$T^d_{i_0i_3}T^d_{i_1i_2}-T^d_{i_0i_2}T^d_{i_1i_3}=T^d_{i_0i_1}T^d_{i_3i_2}\,. \qquad (27)$$

Expression (25) can be transformed into a Lorentz covariant form $A^{\mu_0\mu_1\mu_2\mu_3}$ using the following substitutions (cf. [2]):

$$\delta_{\lambda_m,-\lambda_n}\rightarrow\sum_{\lambda_m,\lambda_n}\delta_{\lambda_m,-\lambda_n}\,e^{\lambda_m}_{\mu_m}e^{\lambda_n}_{\mu_n}=$$

$$=-g_{\mu_m\mu_n}-\frac{(k_j)_{\mu_m}(k_l)_{\mu_n}}{(k_jk_m)}-\frac{(k_l)_{\mu_m}(k_j)_{\mu_n}}{(k_lk_m)}, \qquad (28)$$

where the equality sign stands for the gauge independent part, i. e. for the terms without $(k_m)_{\mu_m}$, $(k_n)_{\mu_n}$.

To calculate the inelastic amplitude of the gluon production in the high energy gluon collisions we use the following asymptotic relation

$$\delta_{\mu_3\mu_{3'}}\simeq\frac{p_{\mu_3}(k_0)_{\mu_{3'}}}{(pk_0)},$$

for the tensor structure of the virtual gluon propagator in the t-channel (see Fig. 1) in combination with the substitution (28) for $n=3$ in eq. (25). It allows us to find the pole singularity of $A^{2\rightarrow3}$ at $t\rightarrow0$ (cf. (25)):

$$A^{2\rightarrow3}_{\lambda_0\lambda_1\lambda_2}\big|_{t\rightarrow0}=-2gT^c_{i'i}\frac{2g^2}{t}\Big[\delta_{\lambda_0,-\lambda_1}\overset{*}{e}{}^{\lambda_2}_{\mu}(k_2)\,B^{\mu}_2\times$$

$$\times\Big(T^d_{i_0c}T^d_{i_1i_2}\frac{t_2}{t_1}+T^d_{i_0i_2}T^d_{i_1c}\frac{s_{12}}{t_1}\Big)+\delta_{\lambda_0,-\lambda_2}\overset{*}{e}{}^{\lambda_1}_{\mu}(k_1)\,B^{\mu}_1\times$$

$$\times\Big(T^d_{i_0c}T^d_{i_2i_1}\frac{t_1}{t_2}+T^d_{i_0i_1}T^d_{i_2c}\frac{s_{12}}{t_2}\Big)+\delta_{\lambda_1,-\lambda_2}\overset{*}{e}{}^{\lambda_0}_{\mu}(k_0)\times$$

$$\times B^{\mu}_0\Big(T^d_{i_0i_2}T^d_{ci_1}\frac{t_1}{s_{12}}+T^d_{i_0i_1}T^d_{ci_2}\frac{t_2}{s_{12}}\Big)\Big], \qquad (29)$$

where

$$B_0=p+\frac{\beta_1 s}{t_2}k_2+\frac{\beta_2 s}{t_1}k_1,\qquad B_1=p+\frac{\beta_2 s}{t_1}k_0-\frac{s}{s_{12}}k_2,$$

$$B_2=p+\frac{\beta_1 s}{t_2}k_0-\frac{s}{s_{12}}k_1\,. \qquad (30)$$

To reconstruct the total amplitude $A^{2\to 3}$ from the calculated t-channel pole singularity we add to eq. (29) some terms regular at $t \to 0$ which are necessary for analyticity requirement in other invariants:

$$A^{2\to 3}_{\lambda_0\lambda_1\lambda_2} = -T^c_{i'i}\frac{4g^3}{t}\Big\{\big[\,\delta_{\lambda_0,-\lambda_1}\overset{*}{e}{}^{\lambda_2}_{\mu}(k_2)\,(T^d_{i_0 i_2}T^d_{i_1 c}\times$$

$$\times D^{\mu}(k_0,k_1,k_2) + T^d_{i_0 c}T^d_{i_1 i_2}D^{\mu}(k_1,k_0,k_2))\big] +$$

$$+\begin{bmatrix} k_1\leftrightarrow k_2 \\ \lambda_1\leftrightarrow\lambda_2 \\ i_1\leftrightarrow i_2\end{bmatrix} + \begin{bmatrix} k_0\leftrightarrow k_2 \\ \lambda_0\leftrightarrow\lambda_2 \\ i_0\leftrightarrow i_2\end{bmatrix}\Big\}, \qquad (31)$$

where the gauge invariant vertices D^{μ} are built from B^{μ}_t (30) by adding the terms proportional to t:

$$D(k_0,k_1,k_2) = B_2\frac{s_{12}}{t_1} + \frac{t}{t_1}\left(\frac{\beta_1}{\beta_2}p - \frac{s\beta_1}{t_2}k_0\right) =$$

$$= \frac{1}{t_1}\left[\left(s_{12} + t\frac{\beta_1}{\beta_2}\right)p + \frac{s\beta_1}{t_2}(s_{12}-t)\,k_0 - sk_1\right];$$

$$k_2 D(k_0, k_1 k_2) = 0. \qquad (32)$$

Note that D (32) does not have simultaneous poles in t_1 and t_2 in accordance with analytic properties of the Born amplitude because of the equality

$$s_{12} - t = -t_1 - t_2. \qquad (33)$$

Furthermore, for fixed $k_{1\perp}$ and large $k_{2\perp} \simeq -p'_{\perp}$ one obtains

$$A^{2\to 3}\big|_{|t|\to\infty} \sim O\left(\frac{1}{\sqrt{|t|}}\right) \qquad (34a)$$

due to the relations

$$\left(s_{12} + \frac{t\beta_1}{\beta_2}\right)\Big|_{|k_{2\perp}|\approx\sqrt{|t|}\gg|k_{1\perp}|} \approx k_{1\perp}k_{2\perp}, \qquad (34b)$$

$$s_{12} \sim |t_2| \sim |t|.$$

It guarantees the logarithmic-like behaviour of $\sigma_{tot} = \int \frac{d\sigma}{dt}\,dt$ with energy in accordance with the renormalizability property of the Yang—Mills theory (cf. [2]).

To make the Bose symmetry of expression (31) especially obvious we pass to the tensor representation

$$A^{2\to 3}_{\lambda_0\lambda_1\lambda_2} = \overset{*}{e}{}^{\lambda_1}_{\mu_1}(k_1)\,\overset{*}{e}{}^{\lambda_2}_{\mu_2}(k_2)\,\overset{*}{e}{}^{\lambda_0}_{\mu_0}(k_0)\,A^{\mu_0\mu_1\mu_2}, \qquad (35)$$

$$A^{\mu_0\mu_1\mu_2} = T^c_{i'i}\frac{4g^3}{t}\Big\{\big[\,\Delta^{\mu_0\mu_1}(k_0,k_1,p)\,(T^d_{i_0 i_2}\times$$

$$\times T^d_{i_1 c}D^{\mu_2}(k_0,k_1,k_2) + T^d_{i_0 c}T^d_{i_1 i_2}D^{\mu_2}(k_1,k_0,k_2))\big] +$$

$$+\begin{bmatrix} k_1\leftrightarrow k_2 \\ \mu_1\leftrightarrow\mu_2 \\ i_1\leftrightarrow i_2\end{bmatrix} + \begin{bmatrix} k_0\leftrightarrow k_2 \\ \mu_0\leftrightarrow\mu_2 \\ i_0\leftrightarrow i_2\end{bmatrix}\Big\}, \qquad (36)$$

where instead of eq. (28) for e^{λ}_{μ} in the light-cone gauge $(ep) = 0$ we have (see [2]):

$$\Delta_{\mu_0\mu_1}(k_0,k_1,p) = -\sum_{\lambda} e^{-\lambda}_{\mu_0}(k_0)\,e^{\lambda}_{\mu_1}(k_1) =$$

$$= g_{\mu_0\mu_1} - \frac{(k_1)_{\mu_0}p_{\mu_1}}{(k_1 p)} - \frac{p_{\mu_0}(k_0)_{\mu_1}}{(k_0 p)} + (k_0 k_1)\frac{p_{\mu_0}p_{\mu_1}}{(k_0 p)(k_1 p)}. \qquad (37)$$

It can be verified that formulas (36), (37), (27) give the amplitude which coincides with that obtained above from the string theory (see (22), (23)).

In the infrared limit of small momentum transfer we obtain from eq. (31)

$$A^{2\to 3}_{\lambda_0\lambda_1\lambda_2}\big|_{p'^2_{\perp}=t\to 0} \sim \frac{1}{|p'_{\perp}|} \qquad (38)$$

due to the formulas:

$$D^{\mu}(k_0,k_1,k_2)\big|_{p'_{\perp}\to 0} = k^{\mu}_2\frac{s}{t_1}, \qquad k_2 e^{\lambda}(k_2) = 0. \qquad (39)$$

Further, in the multi-Regge limit (see Fig. 1)

$$s_1 \equiv -2k_0k_2 \gg \vec{k}^2_{2\perp}, \qquad s_2 \equiv 2pk_2 \gg \vec{k}^2_{2\perp}, \qquad s_1 s_2 = \vec{k}^2_{2\perp}\,s, \qquad (40)$$

we have from eq. (31) the following factorized result (see [2]):

$$A^{2\to 3}_{\lambda_A\lambda_1\lambda_2} = 2g^3 s\,\delta_{\lambda_A\lambda_1}\,T^c_{i_1 i_0}\frac{1}{t_1}\overset{*}{e}{}^{\lambda_2}_{\mu}(k_2)\,\mathscr{P}^{\mu}(q_1,q)\,T^{i_2}_{dc}\frac{1}{t}T^d_{i'i},$$

$$\lambda_A = -\lambda_0, \qquad q = p - p', \qquad q_1 = k_0 + k_1, \qquad (41)$$

where the effective emission vertex $\mathscr{P}^{\mu}$ equals

$$\mathscr{P}(q_1,q) = \frac{2t_1}{s}D(k_0,k_1,k_2)\big|_{s_{1,2}\to\infty} - k_2 = -(q_1+q)_{\perp} +$$

$$+k_0\left(\frac{s_2}{s}+\frac{2t_1}{s_1}\right)+p\left(\frac{s_1}{s}+\frac{2t}{s_2}\right). \tag{42}$$

and the identity (27) was used.

4. GLUON PAIR PRODUCTION IN THE TACHYON-TACHYON COLLISIONS

Let us consider now a more complicated process of two gluon production in QMRK when the pair of produced gluons has a fixed invariant mass $\sqrt{\varkappa}$ at high energies (see Fig. 3)

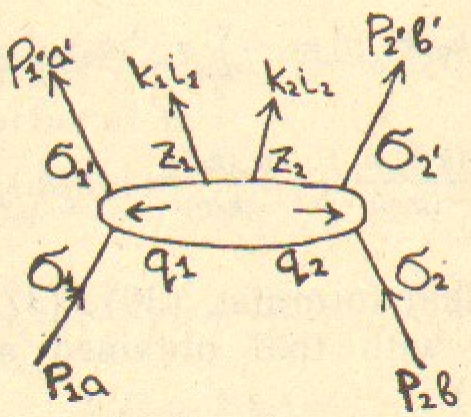

Fig. 3.

$$s\equiv 2p_1p_2\sim s'\equiv 2p_{1'}p_{2'}\sim u_1\equiv 2p_1p_{2'}\sim u_2\equiv 2p_{1'}p_2\gg 1/\alpha',$$

$$s_1\equiv 2p_1k_1\gg\frac{1}{\alpha'},\quad s_2\equiv 2p_2k_2\gg\frac{1}{\alpha'},\quad \varkappa\equiv 2k_1k_2\sim\frac{1}{\alpha'},$$

$$t_1\equiv -2p_1p_{1'}+2m^2\sim t\equiv t_1+2p_{1'}k_1-2p_1k_1\sim t_2\equiv -2p_2p_{2'}+2m^2\sim\frac{1}{\alpha'},$$

$$u_1+u_2=s+s'-\varkappa+t_1+t_2,\quad k_{i\perp}^2\sim\frac{1}{\alpha'}. \tag{43}$$

The scalar products of all external particle momenta can be expressed in terms of the above invariants. In particular, we have

$$s_{12}\equiv 2p_1k_2=s+t_1-u_1-s_1,\quad 2p_2k_1=s+t_2-u_2-s_2\equiv s_{21},$$

$$2p_{1'}k_1=s_1-t_1+t,\quad 2p_{1'}k_2=s-s_1-\varkappa-u_1+t_1+t_2-t,$$

$$2p_{2'}k_2=s_2-t_2+t,\quad 2p_{2'}k_1=s-s_2-\varkappa-u_2+t_2+t_1-t. \tag{44}$$

Using these relations one can obtain for the dual amplitudes of the gluon pair production in the tachyon-tachyon scattering the following expression in the Koba — Nielsen variables (cf. (5) — (7))

$$A^{2\to 4}\sim\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\int\frac{\prod_{i=1}^{2}(dz_i\,d\sigma_i\,d\sigma_{i'})}{\sigma_{11'}^2\sigma_{22'}^2}\times$$

$$\times\left|\frac{\sigma_{21}\rho_{21'}}{\sigma_{21'}\rho_{21}}\right|^{-\alpha's}\left|\frac{\sigma_{1'2'}\rho_{12}}{\sigma_{1'2}\rho_{12'}}\right|^{-\alpha's'}\left|\frac{\sigma_{21'}\rho_{12'}\rho_{21}}{\sigma_{12'}\rho_{12}\rho_{21'}}\right|^{-\alpha'u_1}\times$$

$$\times\left|\frac{\rho_{11'}\rho_{21}}{\rho_{11}\rho_{21'}}\right|^{-\alpha's_1}\left|\frac{\rho_{12}\rho_{22'}}{\rho_{12'}\rho_{22}}\right|^{-\alpha's_2}\left|\frac{z_{12}\sigma_{21'}}{\rho_{12}\rho_{21'}}\right|^{-\alpha'\varkappa}\times$$

$$\times\left|\frac{\sigma_{11'}\rho_{12}\rho_{21'}}{\sigma_{21'}\rho_{11'}\rho_{21}}\right|^{-\alpha't_1}\left|\frac{\sigma_{22'}\rho_{21'}}{\sigma_{21'}\rho_{22'}}\right|^{-\alpha't_2}\left|\frac{\rho_{11'}\rho_{22'}}{\rho_{12'}\rho_{21'}}\right|^{-\alpha't}\times$$

$$\times\left[(-2\alpha')^2\left\{\sum_{i=1}^{2}\left(\frac{p_i}{\rho_{1i}}-\frac{p_{i'}}{\rho_{1i'}}\right)-\frac{k_2}{z_{12}}\right\}^{\mu_1}\left\{\sum_{i=1}^{2}\left(\frac{p_i}{\rho_{2i}}-\frac{p_{i'}}{\rho_{2i'}}\right)-\frac{k_1}{z_{21}}\right\}^{\mu_2}-\right.$$

$$\left.-2\alpha'\frac{g^{\mu_1\mu_2}}{z_{12}^2}\right]; \tag{45}$$

$$\sigma_{ij}\equiv\sigma_i-\sigma_j,\quad \rho_{ij}\equiv z_i-\sigma_j,\quad z_{12}=-z_{21}\equiv z_1-z_2.$$

This expression is invariant under the gauge transformation $e(k_i)\to e(k_i)+ck_i$ and under the Möbious group, which allows one to fix the values of three variables

$$\sigma_{1'}=0,\quad \sigma_2=1,\quad \sigma_{2'}=\infty. \tag{46}$$

Instead of the other variables we use new ones

$$y_1=\frac{\sigma_1}{z_1},\quad y_2=z_2,\quad z=\frac{z_1}{z_2}. \tag{47}$$

Then in the asymptotic regime (43) we have

$$y_1\sim\frac{1}{\alpha's_1}\ll 1,\quad y_2\sim\frac{1}{\alpha's_2}\ll 1,\quad z\sim 1 \tag{48}$$

and therefore expression (45) can be simplified in such a way:

$$A^{2\to 4}\sim\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\int\frac{dz\,dy_1\,dy_2}{y_1^2y_2^2z|z|}|y_1|^{-\alpha't_1}|y_2|^{-\alpha't_2}|z|^{-\alpha't}\times$$

$$\times|1-z|^{-\alpha'\varkappa}\exp\{\alpha'[y_1y_2zs-y_1zs_{112}-y_2zs_{221}-y_1(1-z)s_1-y_2(1-z)s_2]\}\times$$

$$\times\left[D_1^{\mu_1} D_2^{\mu_2}+\frac{g^{\mu_1\mu_2}}{2\alpha'}\frac{z}{(1-z)^2}\right]\Phi(y_1, y_2, z)\,, \tag{49}$$

where

$$D_1=-q_1+p_1y_1-p_2y_2z+k_2\frac{z}{1-z}, \qquad q_1=p_{1'}-p_1\,;$$

$$D_2=-q_2+p_2y_2-p_1y_1z+k_1\frac{z}{1-z}, \qquad q_2=p_{2'}-p_2\,;$$

$$s_{112}\equiv 2p_1(k_1+k_2)=s+t_1-u_1\sim s_1\,,$$
$$s_{221}\equiv 2p_2(k_1+k_2)=s+t_2-u_2\sim s_2\,. \tag{50}$$

In the integrand of eq. (49) we introduced the additional multiplier $\Phi(y_1, y_2, z)$ which takes into account that for each ordering of values of initial variables σ_i, $\sigma_{i'}$, z_i there should be corresponding Chan—Paton factors (cf. (16))

$$\begin{aligned}\Phi(y_1, y_2, z)=\{&\theta(z)\,\theta(1-z)\,[\,\theta(y_2)\,\theta(1-y_2)\times\\ &\times(\theta(y_1)\,\theta(1-y_1)\,C^{+}_{a'ai_1i_2bb'}+\theta(-y_1)\,C^{+}_{aa'i_1i_2bb'})+\\ &+\theta(-y_2)\,(\theta(y_1)\,\theta(1-y_1)\,C^{+}_{i_2i_1aa'bb'}+\\ &+\theta(-y_1)\,\theta(1-y_1y_2z)\,C^{+}_{i_2i_1a'abb'})]+\\ &+\theta(-z)\,\theta(y_1)\,\theta(1-y_1)\,[\,\theta(y_2)\,\theta(1-y_2)\,C^{+}_{i_1aa'i_2bb'}+\\ &+\theta(-y_2)\,\theta(1-zy_2)\,C^{+}_{i_2a'ai_1bb'}]\}+\\ &+\left\{\begin{array}{c} z\to 1/z\\ y_i\to zy_i\\ i_1\leftrightarrow i_2\end{array}\right\},\end{aligned} \tag{51}$$

where a, a', b, b', i_1, i_2 are colour indices of scattered tachyons and produced gluons with their momenta equal to p_1, $p_{1'}$ p_2, $p_{2'}$, k_1, k_2 correspondingly and

$$C^{+}_{j_1j_2j_3j_4j_5j_6}=C_{j_1j_2j_3j_4j_5j_6}+C_{j_6j_5j_4j_3j_2j_1}\,. \tag{52}$$

In the sum (51) we took into account only those kinematical regions in which y_1 and y_2 are near zero in accordance with (48). The other integration limits in these variables can be put to $\pm\infty$. It is convenient to introduce the new variables (see (48)):

$$x_1=\alpha' s_{112}\,y_1\sim 1\,; \qquad x_2=\alpha' s_{221}\,y_2\sim 1\,. \tag{53}$$

As it is seen from eqs (49) — (51) one needs to consider the integrals

$$\int_0^1 dz\,T^{\mu_1\mu_2}(z)\,, \quad \int_1^\infty dz\,T^{\mu_1\mu_2}(z)\,, \quad \int_{-\infty}^0 dz T^{\mu_1\mu_2}(z)\,, \tag{54}$$

where

$$T^{\mu_1\mu_2}(z)=\frac{1}{z\,|z|}\int_0^\infty\frac{dx_1}{x_1^2}\int_0^\infty\frac{dx_2}{x_2^2}\,x_1^{-\alpha' t_1}\,x_2^{-\alpha' t_2}|z|^{-\alpha' t}\,|1-z|^{-\alpha'\varkappa}\times$$

$$\times\exp\left\{-x_1(r_1+z(1-r_1))-x_2(r_2+z(1-r_2))+x_1x_2z\frac{s}{\alpha' s_{112}\,s_{221}}\right\}\times$$

$$\times\left[D_1^{\mu_1} D_2^{\mu_2}+\frac{g^{\mu_1\mu_2}}{2\alpha'}\,\frac{z}{(1-z)^2}\right]; \qquad r_1\equiv\frac{s_1}{s_{112}}, \qquad r_2\equiv\frac{s_2}{s_{221}}. \tag{55}$$

They must be calculated for the values of invariants lying in the region of their convergency with the subsequent analytic continuation of the result to other kinematical regions. It can be verified by using the new integration variables $\tilde{x}_{1,2}$, $\tilde{z}$:

$$z=\frac{1}{\tilde{z}}, \qquad x_i=\tilde{x}_i\,\tilde{z}\,, \tag{56}$$

that

$$\int_1^\infty dz\,T^{\mu_1\mu_2}(z)=\int_0^1 dz\,T^{\mu_1\mu_2}(z)\begin{bmatrix}k_1\leftrightarrow k_2\\ \mu_1\leftrightarrow\mu_2\end{bmatrix}. \tag{57}$$

Further, as in the case of $A^{2\to 3}$ we have the relation (cf. (20))

$$\int_{-\infty}^0 dz\,T^{\mu_1\mu_2}(z)\,|_{\alpha'\to 0}=\int_0^1 dz\,T^{\mu_1\mu_2}(z)\,|_{\alpha'\to 0}+\begin{bmatrix}k_1\leftrightarrow k_2\\ \mu_1\leftrightarrow\mu_2\end{bmatrix}, \tag{58}$$

which allows us to restrict ourself in the field limit $\alpha'\to 0$ to the calculation of the integrals of the type

$$J^{n_1n_2n_3n_4}=\int_0^1\frac{dz}{z^{n_3}(1-z)^{n_4}}\int_0^\infty\frac{dx_1}{x_1^{n_1}}\times$$

$$\times\int_0^\infty\frac{dx_2}{x_2^{n_2}}\,x_1^{-\alpha' t_1}\,x_2^{-\alpha' t_2}\,z^{-\alpha' t}(1-z)^{-\alpha'\varkappa}\times$$

$$\times\exp\left\{-x_1(r_1+z(1-r_1))-x_2(r_2+z(1-r_2))+x_1x_2z\frac{s}{\alpha' s_{112}s_{221}}\right\}. \tag{59}$$

An example of the recurrence formulae for the functions J and their values at several n_j are given in Appendix 2. Using the table of Appendix 2 we can find the inelastic amplitude $A^{2\to 4}$ (see (49) — (51)) in the form

$$A^{2\to 4}=2sg^2 T^c_{a'a}\frac{1}{t_1}\gamma^{i_1 i_2}_{cd}(q_1,q_2)\frac{1}{t_2}T^d_{b'b}, \tag{60}$$

where

$$\gamma^{i_1 i_2}_{cd}(q_1,q_2)=g^2 T^{i_1}_{cj}T^{i_2}_{jd}\overset{*}{e}^{\mu_1}(k_1)\,\overset{*}{e}^{\mu_2}(k_2)\,A_{\mu_1\mu_2}+\binom{k_1\leftrightarrow k_2}{i_1\leftrightarrow i_2}. \tag{61}$$

The tensor $A^{\mu_1\mu_2}$ equals

$$\begin{aligned}A^{\mu_1\mu_2}=&-\frac{a_1^{\mu_1}a_2^{\mu_2}}{t}+\frac{b_1^{\mu_1}b_2^{\mu_2}}{s}\left(1+\frac{s_1s_2}{st}\right)+\\&+\frac{b_1^{\mu_1}c_2^{\mu_2}}{s}\left(\frac{t_2s}{s_2s_{221}}-\frac{s_1s_{12}}{st}\right)+\frac{c_1^{\mu_1}b_2^{\mu_2}}{s}\left(\frac{t_1s}{s_1s_{112}}-\frac{s_2s_{21}}{st}\right)-\\&-\frac{c_1^{\mu_1}c_2^{\mu_2}}{s}\left(1+\frac{\varkappa}{t}-\frac{s_{12}s_{21}}{st}\right)-2\left(g^{\mu_1\mu_2}-\frac{2k_2^{\mu_1}k_1^{\mu_2}}{\varkappa}\right)\times\\&\times\left(1+\frac{t}{\varkappa}+\frac{s_1s_2}{st}+\frac{s_1s_2-s_{12}s_{21}}{\varkappa s}-\frac{t_1s_{12}}{\varkappa s_{112}}-\frac{t_2s_{21}}{\varkappa s_{221}}\right).\end{aligned} \tag{62}$$

where

$$a_i=-2\left[q_i+\left(\frac{s_{ji}}{s}+\frac{t_i}{s_i}\right)p_i-\frac{s_i}{s}p_j-\frac{t}{\varkappa}k_j\right],$$

$$b_i=2\left(p_j-\frac{s_{ji}}{\varkappa}k_j\right),\quad c_i=2\left(p_i-\frac{s_i}{\varkappa}k_j\right), \tag{63}$$

$$i,j=1,2,,\quad i\neq j.$$

The common numerical factor in eq. (60) was fixed as in eq. (22) from the requirement of its coincidence with the QCD perturbation theory result. For example, using eq. (24) one easily obtains, that the Feynman diagrams with external gluon lines attached to the first tachyon line give in the asymptotic region (43) the following contribution to $A^{2\to 4}$

$$\Delta A^{2\to 4}=-\frac{g}{t_2}T^{i_0}_{b'b}(2p_2)_{\mu_0}\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\,\Delta A^{\mu_0\mu_1\mu_2}\left[\begin{matrix}p\to p_1\\ s\to s_{112}\\ \beta_2 s\to s_{12}\\ i\to a\end{matrix}\right]+$$

$$+\binom{k_1\leftrightarrow k_2}{i_1\leftrightarrow i_2}=2sg^2T^c_{a'a}\frac{1}{t_1}g^2T^{i_1}_{cj}T^{i_2}_{jd}\frac{1}{t_2}T^d_{b'b}\times$$

$$\times\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\frac{2p_1^{\mu_1}2p_1^{\mu_2}}{s_1s_{112}}t_1+\binom{k_1\leftrightarrow k_2}{i_1\leftrightarrow i_2}, \tag{64}$$

which agrees with the corresponding term in eqs (60) — (63), containing s_1s_{112} in denominator.

The simple colour structure of eqs (60), (61) appears due to the relations (58) and (27).

5. SOME PROPERTIES OF $A^{2\to 4}$

Here we investigate the above obtained formulas for $A^{2\to 4}$. Let us begin with the factorization property of the residue of $A^{2\to 4}$ in $t_1=0$ or $t_3=0$ (cf. (29)). This property follows from the verified identities (see (22), (23)):

$$\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\,A^{\mu_1\mu_2}|_{t_1=0}=\frac{8t_2s_{221}}{s}(p_1)_{\mu_0}\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\times$$
$$\times a^{\mu_0\mu_1\mu_2}(k_1,k_2,q_1,p_2,p_{2'}),$$
$$\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\,A^{\mu_1\mu_2}|_{t_2=0}=\frac{8t_1s_{112}}{s}(p_2)_{\mu_0}\overset{*}{e}_{\mu_1}(k_1)\,\overset{*}{e}_{\mu_2}(k_2)\times$$
$$\times a^{\mu_0\mu_2\mu_1}(k_2,k_1,q_2,p_1,p_{1'}). \tag{65}$$

Furthermore, $A^{\mu_1\mu_2}$ for $q_{i\perp}\to 0$ proportional to the tensor which vanishes when it is multiplied by the physical polarization vectors $\overset{*}{e}_{\mu_i}(k_j)$:

$$A^{\mu_1\mu_2}|_{q_{i\perp}\to 0}=-\frac{4}{t}\left(\frac{s_2}{s}p_1+\frac{s_1}{s}p_2+\frac{t}{\varkappa}k_j\right)^{\mu_i}k_j^{\mu_j},$$

$$i,j=1,2,\quad i\neq j \tag{66}$$

and therefore we have

$$A^{2\to 4}|_{q_{i\perp}\to 0}\leqslant\frac{1}{|q_{i\perp}|}, \tag{67}$$

which can lead only to a moderate logarithmic divergency of the total cross-section $\sigma=\int dt_i\frac{d\sigma}{dt_i}$ in the infrared region $t_i\to 0$.

In the region of large $|t_i|$ the total cross-section can have only logarithmic contributions $\sigma\sim\ln^k s$ in accordance with the renormalizability of QCD. This property follows in our case from the relation (cf. (34a))

$$A^{2\to 4}|_{|t_i|\to\infty} \lesssim O\left(\frac{1}{\sqrt{|t_i|}}\right). \tag{68}$$

Let us consider, for example, the kinematical situation in which $|q_{2\perp}|$, $|k_{2\perp}|$ and $\frac{s_1 s_2}{s}$ are fixed and $|q_{1\perp}|$ grows:

$$|k_{1\perp}| \simeq |q_{1\perp}| \gg |q_{2\perp}| \sim |k_{2\perp}|, \quad \frac{s_1 s_2}{s} \sim |q_{2\perp}|^2 \sim |k_{2\perp}|^2. \tag{69}$$

In this kinematical region the Lorentz scalars t_1, s_{21}, s_{221}, $\varkappa$ grow:

$$t_1 \approx -|q_{1\perp}|^2, \quad s_{21} \approx s_{221} \approx -\frac{t_1 s}{s_1}, \quad \varkappa \approx \frac{s_{12} s_{21}}{s} \tag{70}$$

as well as the Lorentz vectors q_1, k_1 components

$$k_1 \approx -q_1 \approx p_1\left(\frac{-t_1}{s_1}\right) - q_{1\perp}. \tag{71}$$

The terms in eqs (61), (62) growing as $\bar{q}^2_{1\perp}$ either cancel due to eqs (70), (71), or vanish due to $\overset{*}{e}_\mu(k_1)k_1^\mu = 0$, and, hence, the relation (68) is fulfilled.

It is evident from eqs (60)—(63) that the amplitude $A^{2\to 4}$ is invariant under the gauge transformation of one of polarization vectors $e(k_i) \to e(k_i) + ck_i$ independently from the another vector value. Let us stressed that at the same time the amplitude $A^{2\to 4}$ does not have simultaneous poles in overlapping channels.

Further, in the multi-Regge limit

$$s_{12} \gg s_1 \gg |k_{i\perp}|^2, \quad s_{21} \gg s_2 \gg |k_{i\perp}|^2, \quad \varkappa \approx \frac{s_{12} s_{21}}{s} \gg |k_{i\perp}|^2,$$

$$s_{21} s_1 \approx |k_{1\perp}|^2 s, \quad s_{12} s_2 \approx |k_{2\perp}|^2 s, \tag{72}$$

we have from eqs (61)—(63) the following factorized result (see [2]):

$$\gamma^{i_1 i_2}_{cd}(q_1, q_2) = \gamma^{i_1}_{cj}(q_1, q)\, \frac{1}{t}\, \gamma^{i_2}_{jd}(q, -q_2), \tag{73}$$

where

$$\gamma^{i_1}_{cj}(q_1, q) = -g T^{i_1}_{cj} \overset{*}{e}_\mu(k_1)\, \mathscr{P}^\mu(q_1, q),$$

$$\mathscr{P}(q_1, q) \approx a_1 - k_1 \approx -q_1 - q - 2p_1\left(\frac{s_{21}}{s} + \frac{t_1}{s_1}\right) + p_2\left(\frac{s_1}{s} + \frac{t}{s_{21}}\right),$$

$$q = k + q_1. \tag{74}$$

6. CONCLUSION

In this paper we obtained the inelastic amplitudes for gluon production in QMRK for high energy collisions. The formulas derived from string amplitudes for processes with tachyons can be used for interactions of gluons or quarks due to the conservation of s-channel helicities for each scattered particle in the Born approximation.

It should be noted that the gluon amplitudes up to six external particles have recently been calculated in helicity basis [9, 10]. In principle, it is possible to obtain our formulas starting with these amplitudes. But this task is rather complicated because of the transformation from helicity basis to tensor form as well as because the full expressions for gluon amplitudes are rather involved.

The knowledge of the amplitudes obtained is necessary for calculation of QCD radiative corrections to the scattering amplitudes in LLA (see [2]). Indeed, to find the three-particle contribution to the imaginary part of the gluon Regge trajectory we need to know $A^{2\to 3}$ in QMRK (see Fig. 4) The corresponding two-particle contribution can be found by using the t-channel unitarity conditions by iterating the Born amplitude for the gluon-gluon scattering (see Fig. 5,*a*,*b*). Further, the radiative corrections to the effective vertex for the gluon emission from the corresponding reggeon can be determined from t_1- and t_2-channel unitarity conditions by using again the inelastic amplitude $A^{2\to 3}$ (see Fig. 6,*a*,*b*). At last, the radiative corrections to the Bethe—Salpeter equation for the scattering amplitude in LLA contain besides the above contributions also the product of the tensors $A^{\mu_1\mu_2}$ in terms of which the amplitude $A^{2\to 4}$ is expressed (see Fig. 7). We hope to publish the results of calculations of these radiative corrections in the nearest future.

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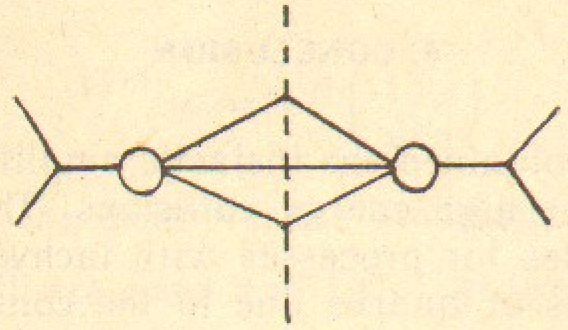
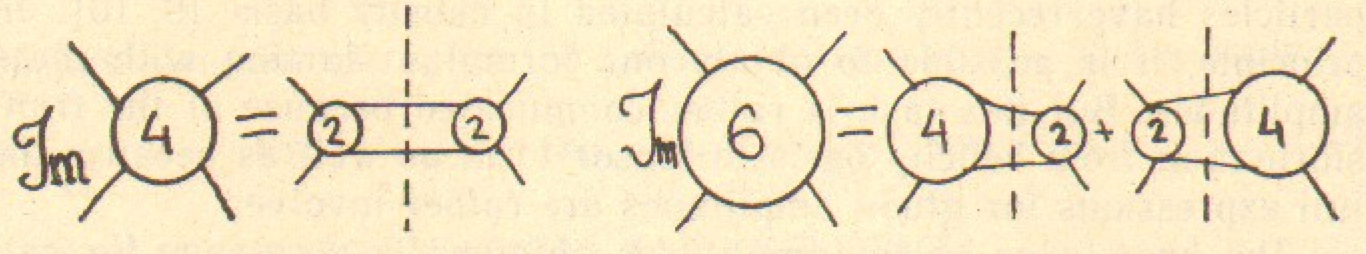
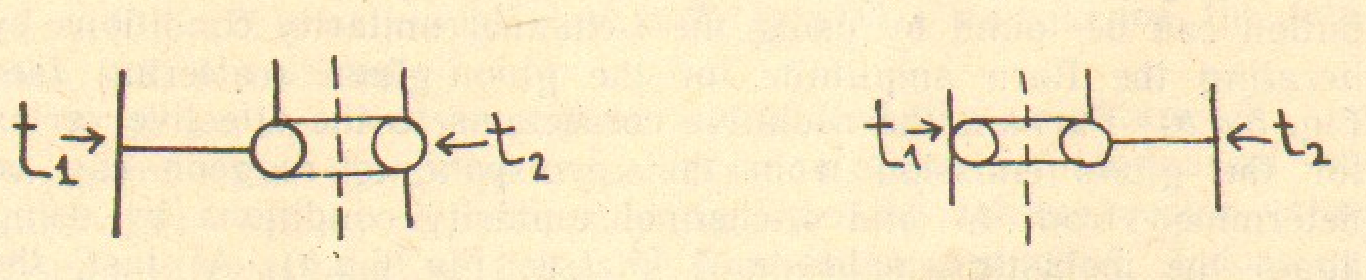


Fig. 4.



a *b*

Fig. 5.



a *b*

Fig. 6.

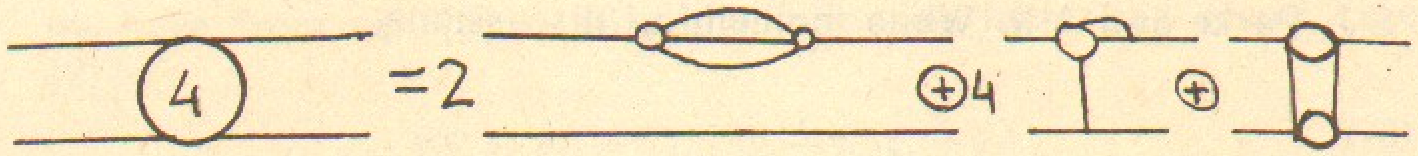


Fig. 7.

Appendix 1

Here we list out some integrals of the form

$$J^{n_1 n_2 n_3} = \int_0^\infty \frac{dx}{x^{n_1}} \int_0^\infty \frac{dz\, e^{-x(z+\beta_2)}}{z^{n_2}(1+z)^{n_3}} x^{-\alpha' t} z^{-\alpha' t_1} (1+z)^{-\alpha' t_2} \tag{A1.1}$$

in the limit $\alpha' \to 0$:

$$J^{-100} \approx \frac{1}{\beta_2}, \quad J^{0-10} \approx \frac{\beta_2}{\alpha' s_{12}}, \quad J^{000} \approx -\frac{1}{\alpha' s_{12}},$$

$$J^{010} \approx -\frac{1}{\alpha' \beta_2 t_1}, \quad J^{001} = O(1), \quad J^{100} \approx \frac{-t_2 + \beta_2 t}{\alpha' t\, s_{12}},$$

$$J^{020} \approx \frac{1}{\alpha' \beta_2^2 t_1}, \quad J^{110} \approx \frac{1}{\alpha'^2 t}\left(\frac{1}{t_1} + \frac{1}{s_{12}}\right),$$

$$J^{101} \approx \frac{1}{\alpha'^2 t\, s_{12}}, \quad J^{111} \approx \frac{1}{\alpha'^2 t_1 t}, \quad J^{210} \approx -\frac{\beta_2}{\alpha'^2 t}\left(\frac{1}{t_1} + \frac{1}{s_{12}}\right),$$

$$J^{201} \approx \frac{1-\beta_2}{\alpha'^2 t\, s_{12}}, \quad J^{102} \approx -\frac{1}{\alpha' t}, \quad J^{120} \approx \frac{1}{\alpha' t_1}\left(\frac{1}{\beta_2} - \frac{t_2}{t}\right),$$

$$J^{220} \approx -\frac{1}{\alpha'^2 t}\left(\frac{1}{t_1} + \frac{1}{s_{12}}\right), \quad J^{202} \approx -\frac{1}{\alpha'^2 t\, s_{12}},$$

$$J^{211} = J^{210} - J^{201} \approx -\frac{1}{\alpha'^2 t}\left(\frac{\beta_2}{t_1} + \frac{1}{s_{12}}\right),$$

$$J^{221} = J^{220} - J^{211} \approx -\frac{(1-\beta_2)}{\alpha'^2 t_1 t},$$

$$J^{212} = J^{211} - J^{202} \approx -\frac{\beta_2}{\alpha'^2 t_1 t};$$

$$s_{12} \equiv -t_1 - t_2 + t. \tag{A1.2}$$

Appendix 2

For calculations $J^{n_1 n_2 n_3 n_4}$ (59) it is helpfull to use the recurrence relations of the type:

$$J^{n_1 n_2 n_3 n_4} = \frac{1}{(n_3 - 1 + \alpha' t)}\Big[(n_4 + \alpha' \varkappa)\, J^{n_1 n_2 (n_3-1)(n_4+1)} -$$

$$-(1-r_1)\, J^{(n_1-1)\, n_2 (n_3-1)\, n_4} - (1-r_2)\, J^{n_1 (n_2-1)(n_3-1)\, n_4} +$$

$$+\frac{1}{\alpha'\Lambda} J^{(n_1-1)(n_2-1)(n_3-1)\,n_4}\Big]\,; \qquad \Lambda \equiv \frac{s_{112}\,s_{221}}{s}. \tag{A2.1}$$

We obtain for $\alpha' \to 0$:

$$J \equiv J^{1110} \approx -\frac{1}{\alpha'^3 t_1 t_2 t}; \qquad J^{1100} \approx -\alpha' t J; \qquad J^{2110} \approx -r_1 J;$$

$$J^{1210} \approx -r_2 J; \qquad J^{1120} \approx \alpha' J\left[\varkappa + t + t_1\left(\frac{1}{r_1}-1\right) + t_2\left(\frac{1}{r_2}-1\right) + \frac{t_1 t_2}{r_1 r_2 \Lambda}\right];$$

$$J^{2120} \approx -J\left(1 - r_1 + \frac{t_2}{r_2\Lambda}\right); \qquad J^{1220} \approx -J\left(1 - r_2 + \frac{t_1}{r_1\Lambda}\right);$$

$$J^{2010} \approx \frac{\alpha' t_2}{r_2} J\left(r_1 + \frac{t}{\Lambda}\right); \qquad J^{0210} \approx \frac{\alpha' t_1}{r_1} J\left(r_2 + \frac{t}{\Lambda}\right);$$

$$J^{2220} \approx \frac{J}{\alpha'\Lambda}; \qquad J^{2211} \approx -J^{2202} \approx \frac{J}{\alpha'\Lambda}\,\frac{t}{\varkappa};$$

$$J^{1201} \approx -\frac{Jt}{\varkappa}\left(1 + \frac{t_1}{\Lambda}\right); \qquad J^{2101} \approx -\frac{Jt}{\varkappa}\left(1 + \frac{t_2}{\Lambda}\right);$$

$$J^{2111} \approx -J\left(r_1 + \frac{t}{\varkappa}\left(1 + \frac{t_2}{\Lambda}\right)\right); \qquad J^{1211} \approx -J\left(r_2 + \frac{t}{\varkappa}\left(1 + \frac{t_1}{\Lambda}\right)\right);$$

$$J^{2212} \approx J\left[r_1 r_2 - \frac{t}{\varkappa}(1 - r_1 - r_2) + \frac{t}{\varkappa\Lambda}\left(\varkappa + t - t_1(1 - r_1) - t_2(1 - r_2)\right)\right] \tag{A2.2}$$

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