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НОВОСИБИРСК

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A b s t r a c t

The production of electron-positron pair by a photon in a field of a bunch of charged particles has been considered taking into account the field inhomogeneity and end effects. The results obtained are applied to the consideration of the process in electron-positron colliders.

1. The particle interaction during beam-beam collision in electron-positron linear colliders occurs in an electromagnetic field produced by the beams. This field turns out to be especially intense in the so-called supercolliders where the particle energy is $\varepsilon > 1$ TeV. As a result, first, the quantum processes in an external field turn out to be essential (radiation, pair production, etc) and, the second, the cross sections of the main quantum electrodynamics processes are drastically modified compared to the case of free particles. At present, widely discussed is an important question of the radiation energy losses under these conditions (see Refs./1/ to /5/ and literature cited). The present paper deals with an electron-positron pair production by a photon in inhomogeneous field of the beam.

The general theory of pair production by a photon in an external field is given in Ref.6. Using this theory the pair production probability for collider can be presented as a sum where the main contribution is given by a constant (along the particle formation length) field approximation and additional contributions are owing to field gradients and end effects, i.e. the very effects which have been actively discussed in the problem of radiation (see Refs./1/ to /5/).

The behaviour of the pair production probability in a constant field is determined by the parameter ($\hbar = c = 1$)

$$\varkappa = \frac{e}{m^3}\sqrt{|(F_{\mu\nu}\kappa^\nu)^2|} = \frac{\omega}{m}\frac{|\underline{F}|}{H_0} \qquad (1)$$

where $\kappa^\nu = (\omega, \omega\underline{n})$ is a photon four-momentum, $F_{\mu\nu}$ is an electromagnetic field tensor, $\underline{F} = \underline{E}_\perp + (\underline{n}\times\underline{H})$, $\underline{E}_\perp = \underline{E} - \underline{n}(\underline{n}\underline{E})$, $\underline{E}$ and $\underline{H}$ are electric and magnetic fields in the lab. system and $H_0 = m^2/e = 4.41\times10^{13}$ Oe.

Similarly to the case of radiation problem, the results of an analysis of the pair production by a high energy photon in aligned single crystals became important for understanding of the process in colliders, because in crystals under certain conditions the constant field approximation turns to be applicable and inhomogeneity effects are also essential /7,8/.

2. The general expression for the probability of electron-positron pair production by a photon in an arbitrary external field has the following form (/6/,see also /7/)

$$dw_e=\frac{\alpha\varepsilon d^3p}{(4\pi)^2\omega\varepsilon'}\int dt_1\int dt_2\left[\frac{4}{\gamma^2}-\beta(\mathbf{v}(t_1)-\mathbf{v}(t_2))^2\right]e^{iA}, \qquad (2)$$

here

$$A=-\frac{\varepsilon\omega}{2\varepsilon'}\int_{t_1}^{t_2}dt\left[\frac{1}{\gamma^2}+(\mathbf{n}-\mathbf{v}(t))^2\right],\quad \beta=\frac{\varepsilon}{\varepsilon'}+\frac{\varepsilon'}{\varepsilon},$$

where ε, $\varepsilon'=\omega-\varepsilon$ are the energies of the created particles, $\gamma=\frac{\varepsilon}{m}$, $\mathbf{v}(t)=\mathbf{p}(t)/\varepsilon$ is the velocity of one of the particles, ω and $\mathbf{n}$ are defined in Eq.(1).

The probabilities of a photon radiation by charged particle and pair production by a photon are interrelated by the substitution rules. In particular, the formula (2) is related to a photon radiation probability, and, in this case, one should take into account that $dw = dI/\omega$, dI is intensity of the radiation. In the case where only relativistic particles (with small outcoming angles) are involved in the process, the substitution rule is still valid for the probabilities integrated over angles, in particular, for the spectral distributions /6/. The corresponding formulae are mutually derived from each other with substitutions:

$$\varepsilon\to-\varepsilon,\quad \omega\to-\omega,\quad \omega^2d\omega\to-\varepsilon^2d\varepsilon. \qquad (3)$$

The spectral distribution of the probability over an energy of one of the particles of the produced pair may be obtained with the help of these substitutions from the formula (6) of Ref./5/

$$\frac{dw_e}{d\varepsilon}=\frac{i\alpha\varepsilon^2}{8\pi\omega^2}\iint\frac{dt\,d\tau}{\tau-i0}\left[\frac{4}{\gamma^2}-\beta(\mathbf{v}(t_1)-\mathbf{v}(t_2))^2\right]e^{iA_1}, \qquad (4)$$

where $t_{2,1}=t\pm\tau/2$,

$$A_1=-\frac{\omega\varepsilon\tau}{2\varepsilon'}\left[\frac{1}{\gamma^2}+\frac{1}{\tau}\int_{t_1}^{t_2}\boldsymbol{\Delta}^2(t')dt'\right],\quad \boldsymbol{\Delta}(t')=\mathbf{v}(t')-\frac{1}{\tau}\int_{t_1}^{t_2}\mathbf{v}(t')dt', \qquad (5)$$

where all the notations used are defined in the Eqs.(1), (2).The formula (4) represents the contribution of a certain trajectory of produced particle and its velocity $\mathbf{v}(t)$ depends (implicitly) on the coordinate of a production point $\mathbf{r}_0$ over which averaging should be made. For the crystals this procedure is performed in /7/. For the case, where photons are moving in the direction opposite to the beam of charged particles, it is reduced to the averaging*) over the distribution of photons in the transverse plane. In this sense the formulae given below (as well as formulae in Refs.1-5)describe the process for a certain impact parameter.

From the formula (7) of Ref.5 (see also formula (25) in Ref.7) for the field slightly varying along the particle formation length we get:($l_p=l_0(1+\varkappa x(1-x))^{1/3}/\varkappa$, $x=\varepsilon/\omega$, $l_0=\omega/m^2$)

$$\frac{dw_e}{d\varepsilon}=\frac{\alpha m^2}{\sqrt{3}\pi\omega^2}\int dt\Big\{\beta K_{2/3}(\lambda)+\int_\lambda^\infty K_{1/3}(y)dy-\frac{1}{3|\mathbf{b}|^4}\Big[\beta(\mathbf{b}(\mathbf{V}\nabla)^2\mathbf{b})\times$$
$$\times\left[K_{2/3}(\lambda)-\frac{2}{3\lambda}K_{1/3}(\lambda)\right]-\frac{1}{10}\left[((\mathbf{V}\nabla)\mathbf{b})^2+3(\mathbf{b}(\mathbf{V}\nabla)^2\mathbf{b})\right]\Big[\lambda K_{1/3}(\lambda)-$$
$$-\frac{4}{3}K_{2/3}(\lambda)-\beta\left(4K_{2/3}(\lambda)-\left(\lambda+\frac{16}{3\lambda}\right)K_{1/3}(\lambda)\right)\Big]\Big]\Big\}, \qquad (6)$$

where $\mathbf{V}=\mathbf{n}-\mathbf{v}$, $\mathbf{v}$ is the velcity of counter-travelling beam, $\lambda=\frac{2m^2\omega}{3\varepsilon\varepsilon'|\mathbf{b}|}$, $\mathbf{b}=e\mathbf{F}/m$, K_ν are the MacDonald functions. The first two terms give the constant field limit dw_F and all the remained terms is the correction taking into account the field inhomogeneity.

*) For the description of the beam radiation as a whole the formulae of Ref.5 should be also averaged over the distribution of irradiating particles in the transverse plane.

along the particle formation length. Similarly that as in Ref.5 one can evaluate the relative contribution of these corrections in Eq.(6)

$$(dw_e - dw_F)/dw_F \sim c_1 l_p^2/\sigma_z^2 + c_2 l_p^2 \sigma_\perp^2/\sigma_\perp^2 \tag{7}$$

where $\sigma_z(\sigma_\perp)$ is longitudinal (transversal) dimension of a beam. As in the case of radiation, the second term in Eq.(7) related to the field transversal inhomogeneity, turns to be of the order $D^2/(1 + D)$ (where D is a disruption parameter) with respect to the first term. At $D \ll 1$ it can be neglected. Let us now consider the first term in Eq.(7) related to the longitudinal inhomogeneity of the field. Let us remind that the expression (6) contains the integration over t and $\varkappa(t) < \varkappa_{max}$. If $\varkappa \ll 1$, the integrand in (6) is exponentially small ($K_\nu(\lambda) \propto e^{-\lambda}$ at $\lambda \gg 1$). In practice, this asymptotics can be extended up to $\varkappa \lesssim 1$, when formula (6) is valid at $l_o \ll \sigma_z$. At $\varkappa > 1$ we have $c_1 \sim 1$, so the condition $\varkappa(t) \gg (l_o/\sigma_z)^{3/2}$ should be fulfilled for Eq.(6) applicability. Since the main contribution to the integral is given by the region $\varkappa(t) \sim 1$, then at $l_o \ll \sigma_z$ one can use the formula (6), but at $\sigma_z \lesssim l_o$ one should return to the formula (4).

3. Let us discuss now the end (boundary) effects in pair production. If $l_p \ll \sigma_z$, $\sigma \ll l_o$ (σ_z is the beam longitudinal size, σ is the characteristic length of an inhomogeneity region on the beam boundary), then from formula (4) or from formulae (10)-(12) of Ref.5 with the substitutions (3) we can obtain the following formulae for the spectral distribution of the pair production probability

$$dw_e/d\varepsilon = dw_F/d\varepsilon + dw_b/d\varepsilon \tag{8}$$

where $dw_F/d\varepsilon$ is given by the first two terms of formula (6) and $dw_b/d\varepsilon$ gives the end effects ($dw_b = dw_{bi} + dw_{bf}$):

$$\frac{dw_{bi}}{dx} = \frac{\alpha}{\pi}\Big\{2x(1-x) + [x^2 + (1-x)^2]\Big[-C + \int d\tilde{\tau}(t) \sin\tilde{\tau}(t) \ln\Gamma(t) +$$

$$+ \int \frac{dt}{l_\omega} \int_0^\infty \frac{d\tau}{\tau}\big(R(t,\tau) - R_i(t,\tau) - R_o(t,\tau)\big)\Big]\Big\}, \tag{9}$$

where

$$\tilde{\tau}(t) = \frac{1}{l_\omega}\int_{-\infty}^{t} \psi^2(t')dt', \quad \underline{\psi}(t) = \frac{e}{m}\int_{-\infty}^{t} \underline{F}(t')dt', \quad \underline{\Phi}(t) = \int_{-\infty}^{t} \underline{\psi}(t')dt',$$

$$l_\omega = \frac{2\varepsilon\varepsilon'}{\omega m^2} = 2l_o x(1-x), \quad \Gamma(t) = l_\omega^2 \tilde{\tau}(t)/\underline{\Phi}^2(t), \quad C = 0.577216..,$$

$$R(t,\tau) = \left[\underline{\psi}(t+\tau/2) - \underline{\psi}(t-\tau/2)\right]^2 \sin\Big\{\tilde{\tau}(t+\tau/2) - \tilde{\tau}(t-\tau/2) - \frac{1}{l_\omega \tau}\Big[\underline{\Phi}(t+\tau/2) - \underline{\Phi}(t-\tau/2)\Big]^2\Big\}, \quad R_i(t,\tau) = \psi^2(t+\tau/2)\sin\Big[\tilde{\tau}(t+\tau/2) - \frac{1}{l_\omega \tau}\underline{\Phi}^2(t+\tau/2)\Big],$$

$$R_o(t,\tau) = \dot{\psi}^2(t)\tau^2 \sin\left[\dot{\psi}^2(t)\tau^3/12 l_\omega\right]. \tag{10}$$

For the details of calculation of the probability (9) describing the end effects at pair production with a power accuracy one can refer to Appendix A of Ref.5. It is essential that the characteristic size of a field inhomogeneity is included in Eq.(9) only in a function $\Gamma(t)$ whilst the remained functions depend only on the field asymptotical behaviour. This circumstance simplifies substantially the calculations. In Ref.5 the end effects are calculated for the fields of the type: $F(t) = F_o(t/\sigma)^n \vartheta(t)$, $F(t) = F_o(\sigma/t)^n$ ($n \geq 2$), $F(t) = F_o\exp(-t/\sigma)$, $F(t) = F_o\exp(-z^2/2\sigma_z^2) = F_o\exp(-2t^2/\sigma_z^2)$. The spectral distribution of pair production can be obtained for all these types of fields from the corresponding expressions of Ref.5. Given here are only the probabilities for some characteristic cases:

a) a step-like field

$$\frac{dw_b}{dx} = \frac{dw_{bi}}{dx} + \frac{dw_{bf}}{dx} = \frac{\alpha}{\pi}\Big\{2x(1-x) + [x^2 + (1-x)^2]\Big[\frac{2}{3}\ln(\varkappa_o x(1-x)) + C_1\Big]\Big\}, \tag{11}$$

where $x = \varepsilon/\omega$, $\varkappa_o = \omega e F_o/m^3$, $C_1 = (8/3)\ln 2 - \ln 3/3 - 2C/3 - 2 =$ $= -0.903..$. The regions where the field is switched on (i) and switched out (f) give the same contribution.

b) Gaussian field $F(t) = F_0\exp(-2t^2/\sigma_z^2)$.

Performing the substitutions (3) in formula (24) of Ref.5 we have

$$\frac{dw_b}{dx} = \frac{\alpha}{\pi}\left\{2x(1-x) + [x^2+(1-x)^2]\left[\ln\left(2l_0\sqrt{\ln a}\,x(1-x)/\sigma_z\right) + C_2\right]\right\}, \quad C_2 = \mathcal{E}_{exp} - C = -1.714.., \quad a = \frac{\varkappa^2}{4}(\sigma_z/l_0)^3. \tag{12}$$

The numerical integral $\mathcal{E}_{exp} = -1.137$ is introduced in Ref.5. The probability (12) is calculated with an accuracy of up to terms $\sim\frac{1}{\ln a}$.

For obtaining the total probability for pair production one has to carry out an elementary integration over the particle energy in Eqs.(11) and (12). As a result we obtain:

$$\text{a) } w_b = \frac{2\alpha}{3\pi}\left(\ln\varkappa_0^{2/3} + C_3\right); \quad \text{b) } w_b = \frac{2\alpha}{3\pi}\left[\ln\left(2l_0\sqrt{\ln a}/\sigma_z\right) + C_4\right], \tag{13}$$

where $C_3 = C_1 - 17/18 = -1.847$, $C_4 = C_2 - 5/3 = -3.381$.

4. Let us now consider the contribution into probability of the pair production mechanism in a constant (along the particle formation length) external field (the first term in formula (8)). We find for the Gaussian distribution $\varkappa(t) = \varkappa_0\exp(-2t^2/\sigma_z^2)$ at $\varkappa_0 \gg 1$:

$$w_F = \frac{\alpha\sigma_z}{l_0}\left\{\frac{\sqrt{3\pi}}{2}\left[\frac{5}{7}\frac{\Gamma^2(2/3)}{\Gamma^2(1/3)}(3\varkappa_0)^{2/3}\right] - \frac{2}{3}\sqrt{2\ln\varkappa_0}\left(1+\frac{C_e}{\ln\varkappa_0}+..\right)\right\} =$$

$$= \frac{\alpha\sigma_z}{l_0}\left\{0.583\varkappa_0^{2/3} - 0.943\sqrt{\ln\varkappa_0}\left(1+C_e/\ln\varkappa_0+..\right)\right\} \tag{14}$$

where $C_e = 7/8 - (\ln 3)/4 - C/2 = 0.312..$ The first term in a curly brackets in Eq.(14) corresponds to the integration over time of asymptotics for the function $dw_F(t)/dt$ at $\varkappa(t)\gg1$ and for obtaining the second term one already needs the full expression for $dw_F(t)/dt$ (the first two terms in Eq.(6)). For comparison, let us give the

relative energy losses owing to the radiation in the magnetic bremsstrahlung limit (see Ref.5) for $\chi(t) = \chi_0\exp(-2t^2/\sigma_z^2)$ at $\chi_0\gg1$:

$$(\Delta\varepsilon/\varepsilon)_F = \frac{\alpha\sigma_z}{l_0}\left\{\frac{\sqrt{3\pi}}{2}\left[\left(\frac{2}{3}\right)^5\Gamma(2/3)(3\chi_0)^{2/3}\right] - \frac{2}{3}\sqrt{2\ln\chi_0}\left(1-\frac{C_\gamma}{\ln\chi_0}+..\right)\right\} =$$

$$= \frac{\alpha\sigma_z}{l_0}\left\{0.569\chi_0^{2/3} - 0.943\sqrt{\ln\chi_0}\left(1-C_\gamma/\ln\chi_0+..\right)\right\}, \tag{15}$$

where in this formula $l_0 = \varepsilon/m^2$, $C_\gamma = (\ln 3)/4 + C/2 - 1/8 = 0.438..$

Let us give a numerical example. In a "standard" supercollider electron beam with an energy $\varepsilon = 5$ TeV having Gaussian distribution of density over all coordinates $\sigma_x = \sigma_y = 5\times10^{-8}$ cm, $\sigma_z = 4\times10^{-5}$ cm; $N = 1.2\times10^8$ collides with the photons with an energy $\omega = 5$ TeV. In this case $\varkappa_{max} = 4600$ and for $\varkappa_0 = \varkappa_{max}$ we have $w_F = 0.122$. Let us remind that the relative energy losses for the supercollider mentioned are $(\Delta\varepsilon/\varepsilon)_F = 0.120$. We want to attract attention to the fact that the difference between numerical factors in the main terms in Eqs.(14) and (15) does not exceed 3%. Under the same conditions the end effects (Eq.(13b)) are $w_b \approx 1\times10^{-3}$ and the second term contribution in (14) is $|\Delta w_F| \approx 2.2\times10^{-3}$.

Thus, similarly to the radiation losses, the main contribution at high energies is given by the pair production mechanism in a constant external field.

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