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ON THE EFFECTIVE «CHIRAL DYNAMICS» IN THE PROBLEM OF A LARGE AMPLITUDE COLLECTIVE MOTION IN A FINITE FERMI-SYSTEM

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НОВОСИБИРСК

On the Effective «Chiral Dynamics»
in the Problem of
a Large Amplitude Collective Motion
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ABSTRACT

By means of a change of variables in the functional integral, the action of a Fermi-system with a two-body interaction is expressed through the group variables, corresponding to the fermion's evolution operator in external field. It offers a possibility to remove the strong time-nonlocality of the action making it available for introducing the collective variables.

The problem of a correct discription of collective excitations appears to be one of the fundamental problems in a theory of finite Fermi-system. The main difficulties are connected here with the problem of correct separation of the degrees of freedom of the system on the single-particle and the collective ones, the latter should correspond to a some kind of Bose variables, while the Fermi operators [1] are the original objects. The problem has been solved by the method of «boson expansion» of Fermi-operators, but only partially because of the fact that this technique is limited by the necessity to consider small amplitudes of the collective motion near a fixed «vacuum» only. Of the most interest in the physics of collective nuclear motion is the structure of the collective mode in the region of the phase transition to a finite deformation for large mean-field fluctuations. In the consideration of such nonperturbative motion, the question concerning the rigorousity of a change of variables used in the derivation of a collective Hamiltonian acquires to be of a greate importance. In a such context, the method of functional integration [2] seems to be the most adequate one.

Starting with the propagator $G(T)=\mathrm{Tr}[e^{-iHT}]$ in a form

$$G(T)=\int D\psi\, D\psi^+ \exp\left\{ i\int_0^T dt\left(\psi_i^+(i\partial_t-e_k)\,\delta_{ik}\psi_k-\frac{1}{2}\psi_i^+\psi_k V_{ik,i'k'}\psi_{i'}^+\psi_{k'}\right)\right\} \quad (1)$$

(repeated indices are assumed to be summed over) describing a system of nonrelativistic spinless fermions (ψ, ψ^+ are the Grassmanians), one may obtain the expression for $G(T)$ in terms of the

Bose field only [2], introducing a Bose field $\sigma(t)$ and then integrating over the ψ, ψ^+:

$$G(T)=\int D\sigma \exp\left\{ i \int_0^T dt \frac{1}{2} \sigma_{ik}(t) (V^{-1})_{ik,i'k'} \sigma_{i'k'}(t) + \ln \mathrm{Det}[(i\partial_t - e_k)\delta_{ik} + \sigma_{ik}]. \right. \quad (2)$$

Being equivalent to Eq. (1), the expression (2) contains the whole information on the system (in the functional integration, all the configurations of the field $\sigma_{ik}(t)$ should be included, not only the «equilibrium ones» i. e. the ones satisfying the relation $\sigma_{ik}(t) = V_{ik,i'k'} \psi_{i'}^{+} \psi_{k'}$ as it is widely assumed [2]). In the most general context, the problem of extraction of the collective variables reduces to finding the directions in the functional space of $\sigma(t)$ along which the action changes slowly while increasing sharply in all the orthogonal directions. However, it turns to be a difficult thing to do when manipulating directly with the (2) because of a strong temporary nonlocality of the second term in the action.

The main idea of the work* is to consider as an independent variable not the field $\sigma(t)$ itself, but the exact evolution operator for the fermion in the varying external field σ, which offers a possibility to deliver from the strong nonlocality of the action and to rewrite it in a form convenient for introducing the collective variables. In fact, it means the transition from the variables describing the fluctuating potential created the particles themselves to the ones describing the related scattering data. Let's introduce the following variable $U(t, 0)$ obeying the equations

$$(i\partial_t - h(t))\, U(t,0) = 0, \qquad h_{ik}(t) = e_i \delta_{ik} - \sigma_{ik}(t), \quad (3)$$

such as $\psi_i(t) = U_{ik}(t, 0)\psi_k(0)$. When having fermions of Ω kinds, the $U(t, 0)$ is a generator of the group $U(\Omega)$. Treating the components of the field $U(\Omega)$ as new variables, let us pass to the functional integration over the $U(t)$ in Eq. (2) using the Faddeev—Popov procedure:

$$G(T) = \int D[U]\, D[\sigma] \exp(iS[\sigma])\, \delta[h - (i\partial_t U) U^{-1}]\, J. \quad (4)$$

Here, the $\delta[h - (i\partial_t U) U^{-1}]$ means the $\prod_{ik} \delta(h_{ik} - ((i\partial_t U) U)_{ik})$ and J is the transition Jacobian. Note that Eq. (4) coincides with Eq. (2)

*) The same method has been used earlier by the authors of Refs [3, 4].

while J proves to be an irrelevant functional constant if $D[U]$ means the standard invariant measure on the $U(\Omega)$ [5]. Consider the functional determinant $|\partial\sigma_{ik}(t)/\partial\alpha_{i'k'}(t')|$ for the α_{ik} being the group parameters in an arbitrary realization. According to Eq. (3), it is equal to the determinant of the matrix

$$A_{i'k'}^{ik}(t, t') = \frac{\delta(t,t')\, \partial\sigma_{ik}(t)}{\partial\alpha_{i'k'}(t)} + \frac{\dot{\delta}(t,t')\, \partial\sigma_{ik}(t)}{\partial\dot{\alpha}_{i'k'}(t)}.$$

Up to a functional constant, we have

$$|A| = \left| \int_0^T dt\, \theta(t', \tau)\, A(t, t') \right| = \left| \frac{\theta(t,\tau)\, \partial\sigma_{ik}(t)}{\partial\alpha_{i'k'}(t)} - \frac{\delta(t,\tau)\, \partial\sigma_{ik}(t)}{\partial\dot{\alpha}_{i'k'}(t)} \right|,$$

where the $\theta(t, \tau)$ is an ordinary step function, and the first term does not contribute to the determinant being the triangular matrix with the zero diagonal elements (in time variables). Using Eq. (3), we obtain:

$$\frac{\partial\sigma_{ik}(t)}{\partial\dot{\alpha}_{i'k'}(t)} = \frac{\partial}{\partial\dot{\alpha}_{i'k'}(t)} ((i\partial_t U)\, U^{-1})_{ik} = \left(\left(\frac{i\partial}{\partial\alpha_{i'k'}} U \right) U^{-1} \right)_{ik}.$$

The determinant of the product of this matrix and the conjugated one, i. e. $((i\partial/\partial\alpha_{ij} U) U^{-1})_{ik}$, proves to be equal to $|M|$ for

$$M_{ij}^{ik} = \mathrm{Tr}\left[\left(\frac{\partial}{\partial\alpha_{ik}} U \right) U^{-1} \left(\frac{\partial}{\partial\alpha_{ij}} U \right) U^{-1} \right]$$

being the metric tensor on a group [5]. Then it follows $D[\sigma] = \prod_{ik} D[\alpha_{ik}]\, |A| = \prod_{ik} D[\alpha_{ik}]\, |M|^{1/2}$, which coincides with the definition of the invariant group measure $D[U]$ [5]. Integrating over the σ in (4), we have the $G(T)$ expressed over the U and its derivatives only:

$$G(T) = \int D[U] \exp(iS[(i\partial_t U)\, U^{-1}]).$$

The convenience of the new variables clears up below when a concrete realization of the group elements U is introduced. The temporarily nonlocal term in Eq. (2) may be expressed in the form

$$\int D[\sigma] \exp[\ln \mathrm{Det}(i\partial_t - h)] = \mathrm{Tr}\, U(T),$$

where the trace should be taken in the Fock space of the many-particle wave functions. According to the periodicity of $\sigma(t)$, the single-particle basis $\{|k\rangle\}$ of the Floquet functions there exists in which the monodromy matrix (i.e. evolution operator per period, $U(T)$) proves to be diagonal [2]. Thus, the problem of finding the fermion Green function in an arbitrarily varying field reduces to the much simpler problem of diagonalization of the time-independent operator, and we are able to use its formal solution. For this purpose, we represent an arbitrary element $U(t)$ in the form:

$$U(t)=O(t)\exp\left(-i\sum_{k}\omega_k(t)\,B_{kk}\right)O^{-1}(t)\,,\quad O(t)=\exp\left(i\sum_{i\neq k}\chi_{ik}(t)\,B_{ik}\right),\qquad(5)$$

where $B_{ik}=a_i^+a_k$ are the fermion pair operators in the reference basis, and $O(t)$ is some auxiliary unitary transformation for $\overset{*}{\chi}_{ki}=\chi_{ik}$. Then, the single-particle states $|\tilde{k}\rangle=O(t)\,|k\rangle$ proves to be the Floquet functions, and to use the Slater wave functions composed of them makes the problem of calculation the trace of $U(t)$ trivial. As a result, Eq. (4) takes the form:

$$G(T)=\int D[U]\exp\left\{i\int_0^T dt\,\mathrm{Tr}\,(\sigma[U(t)]\,V^{-1}\sigma[U(t)]\text{-}\ln Z[\omega_i(T)])\right\},\qquad(6)$$

where the $\sigma(t)$ is resolved over the $U(t)$ from (3), and the functional Z coincides in form with an ordinary statistical sum in a canonical ensemble:

$$Z[\omega_k(t)]=\mathrm{Tr}\exp\left[-i\sum_k a_k^+a_k\,\omega_k(t)\right]=\sum_{\{n_k\}}\exp\left[-i\sum_k n_k\,\omega_k(t)\right].\qquad(7)$$

Noting that, according to Eq. (3) it follows $\omega_k(0)=0$, we pass from (5) to the following expression for the $G(T)$:

$$G(T)=\int D[U]\exp\left(i\int_0^T dt\left[\mathrm{Tr}\,[(eI-i(\partial_tU)\,U^{-1})\,V^{-1}(eI-i(\partial_tU)\,U^{-1})]-\right.\right.$$

$$\left.\left.-i\dot{\omega}_k(t)\,Z^{-1}[\omega(t)]\frac{\partial}{\partial\omega_k}Z[\omega(t)]\right]\right).\qquad(8)$$

In this formula, the action turns out to be a «quasi-local» functional expressed through the parameters χ_{ik}, ω_k as well as their first derivatives, and Eqs (5, 8) may be considered as a final result. Let us point out two following facts concerning Eq. (8): i) in a considera-

tion of an adiabatic collective mode, (the typical frequency $\omega_C\ll\dot{\omega}_k$) the statistical sum in (6) is dominated by the one term for the «optimum» set $[\tilde{n}_k]$, and the second term in the action takes the form $-\sum_k\dot{\omega}_k(t)\,\tilde{n}_k=-\sum_k\tilde{n}_k\tilde{E}_k(t)$ where the $\tilde{E}_k(t)$ is the instantaneous eigenvalues of the time-dependent Hamiltonian corrected by the generalized Coriolis force [6], so the results of the adiabatic treatment are reproduced; ii) the formula (8) is an exact expression and it may be used for a numerical computer analysis. Moreover, in spite of its visible complexity, it turns out to be convenient for introducing the collective variables. Within the operator formalism, the collective degrees of freedom may be identified with new operator combinations, such as $C_\alpha=\sum_{ik}\varphi^{\alpha}_{ik}a_i^+a_k$, for the latter being closed (at least approximately) in terms of the commutator algebra. From the whole set of $U(\Omega)$ elements, there is a subset of «collective operators» forming a group of a small dimensionality, G_C. Realizing the least action principle for all the directions which are orthogonal to the «collective» one, let us take the $U(t)$ in the form

$$U(t)=U_C\exp\left[-i\sum_{ik}\chi'_{ik}(t)\,B'_{ik}\right]=\prod_{\alpha}e^{X_\alpha C_\alpha}\exp\left[-\sum_{ik}\chi'_{ik}(t)\,B'_{ik}\right],$$

where the last multiplier describes the small deviations from the «collective» path ($\chi'_{ik}\ll1$). Then, the measure in Eq. (9) may be taken as $D[U]=D[U_C]\prod_{ik}D[\chi'_{ik}]$, and the calculation of the time derivatives of the operator exponents in (8) is simplified because of the number of noncommuting (collective) operators being a few now in the correspondence of the small dimensionality of the collective group G_C (commutators of C_α with the transverse modes generators may be neglected). Also, the calculation of the «collective part» of Z depending on the several coordinates X_α, may be performed analytically in many cases. At the last step of the construction the collective dynamics, it should be verified that the minimization of the action in the variables χ'_{ik} reproduces a posteriori the initial assumption that $\chi'_{ik}=0$, after which one should expand the action up to a second order on these variables and then perform the corresponding Gaussian integration.

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Об эффективной «киральной динамике» в задаче о коллективном движении с большой амплитудой в конечной Ферми-системе

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